Optimal Control of Urban Human Mobility for Epidemic Mitigation

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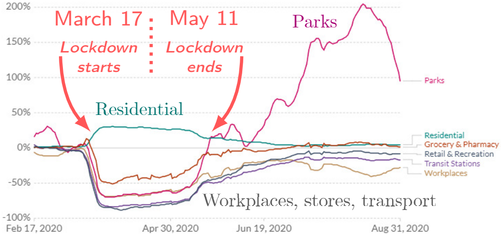
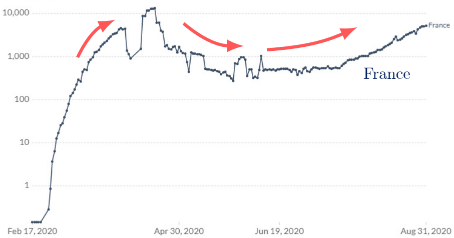
Influence of mobility restrictions on Covid-19

Occupancy in different locations



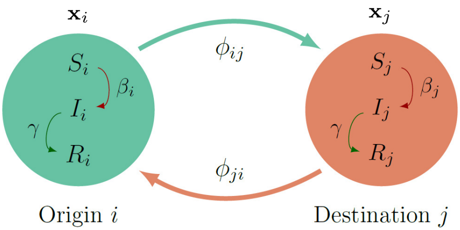
Daily cases in France



Literature review

- ***Linear mobility models*** (Sattenspiel '95; Balcan '10; Poletto '13)
  - \+ capture inter-city human mobility in slower time-scales
  - − cannot capture the human mobility in urban environments
- ***Agent-based models*** (Frias-Martinez '11; Pappalardo '15; Nadini '20)
  - \+ good for insights
  - − uncertain, reproducibility not guaranteed
- ***Machine learning-based approaches*** (Song '20, Wan '21)
  - \+ easy when data is available
  - − no guarantees, error-susceptibility, requires rich data

Our contribution



- Developed a model of human mobility in an **urban environment**
- Incorporated **local epidemic spread** process
- Devised **optimal control policies** for human mobility

Model

Capturing mobility patterns through gating functions

- **Supply gating function**: $\sigma_j(t) \in \{0, 1\}$

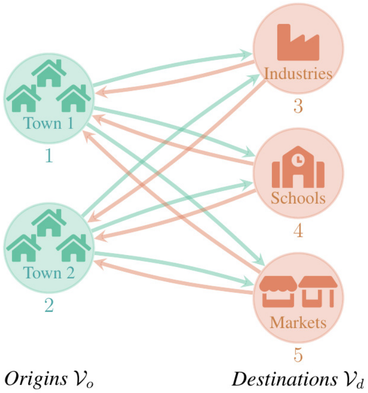
$$\sigma_j(t) = \begin{cases} 1, & \text{if } j \text{ is open at } t \\ 0, & \text{otherwise} \end{cases}$$

captures opening hours of $\mathcal{V}_d$

- **Demand gating function**: $\delta_{ij}(t) \in [0, 1]$

$$\int_{t_d}^{t_d+1} \delta_{ij}(t)dt = 1$$

captures daily mobility pattern b/w i, j

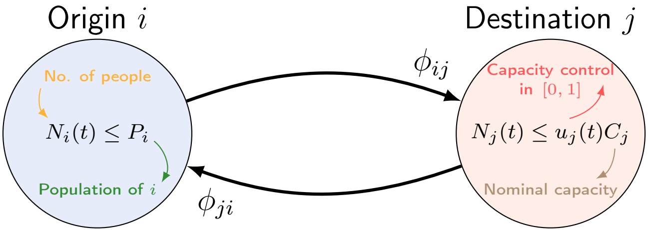


Model assumptions

- constant population
- bipartite mobility network
- visitors return the same day
- mobility pattern repeats weekly

Urban human mobility model

$$\dot{N}_i = \text{inflow} - \text{outflow} = \sum_j [\phi_{ji} - \phi_{ij}]$$



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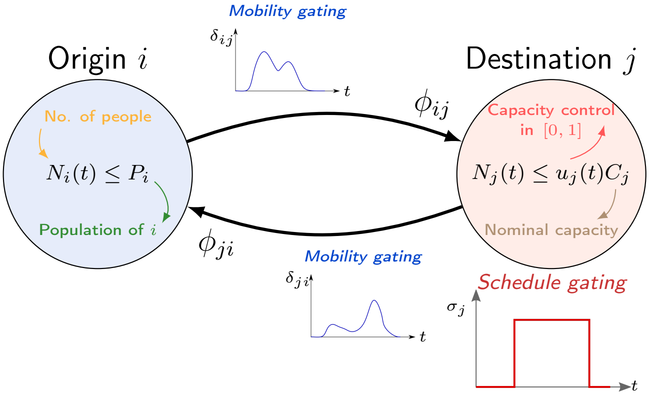


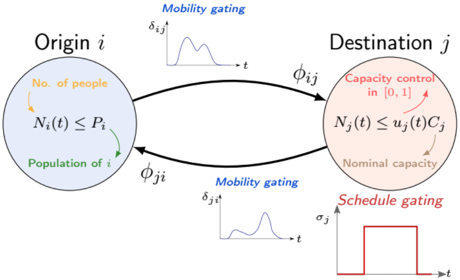
Urban human mobility model

$$\dot{N}_i = \text{inflow} - \text{outflow} = \sum_j [\phi_{ji} - \phi_{ij}]$$

Flow $\phi_{ij} = \min(\Delta_{ij}, \Sigma_j)$

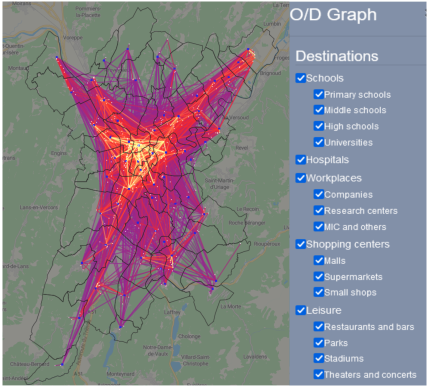
Demand $\Delta_{ij} = \delta_{ij}(t) \cdot f_{ij} \mathbb{1}_{N_i(t)>0}$

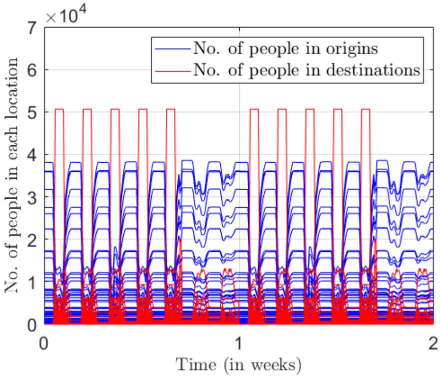
Supply $\Sigma_j = \sigma_j(t) \cdot (\sum_i f_{ij}) \mathbb{1}_{N_j(t)<u_j(t)C_j}$



$\mathbb{1}_X = 1$ if X is true and $\mathbb{1}_X = 0$ if X is false.

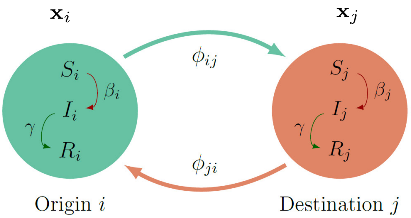
http://gtlville.inrialpes.fr/covid-19





* *Thanks to Ujjwal Pratap, Leo Senique, & Vadim Bertrand*

Incorporating epidemic spread process



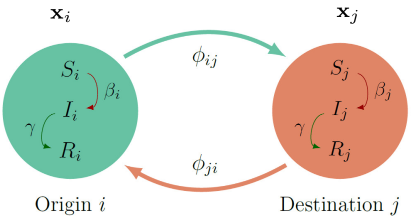
Assumption

all S_i, I_i, R_i are mobile

Local Epidemic Spread

$$\boldsymbol{\xi}_i(\mathbf{x}_i) = \left[\ -\beta_i S_i \frac{I_i}{N_i} \ , \quad \beta_i S_i \frac{I_i}{N_i} - \gamma I_i \ , \quad \gamma I_i \ \right]^\intercal$$

Incorporating epidemic spread process



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Local Epidemic Spread

$$\boldsymbol{\xi}_i(\mathbf{x}_i) = \left[\ -\beta_i S_i \tfrac{I_i}{N_i}\ ,\quad \beta_i S_i \tfrac{I_i}{N_i} - \gamma I_i\ ,\quad \gamma I_i\ \right]^\intercal$$

Model of urban human mobility with epidemic spread

$$\dot{\mathbf{x}}_i = \boldsymbol{\xi}_i(\mathbf{x}_i) + \sum_{j \in \mathcal{N}_i} \left[\phi_{ji}(u_j) \frac{\mathbf{x}_j}{N_j} - \phi_{ij}(u_j) \frac{\mathbf{x}_i}{N_i} \right]; \quad N_i = S_i + I_i + R_i$$

$$\Rightarrow \quad \boxed{\dot{\mathbf{x}} = \boldsymbol{\xi}(\mathbf{x}) + \Phi(\mathbf{x}, \mathbf{u})\mathbf{x}}$$

- **Economic activity**

$$E(t) = \sum_{j \in \mathcal{V}_d} \chi_j \frac{N_j(t)}{C_j} = \mathbf{e}^\mathsf{T} \mathbf{x}(t)$$

χ_j, economic weight of j ,

$$\boxed{\mathbf{e} = \begin{bmatrix} \mathbf{0}_m^\mathsf{T} & \frac{\chi_{d_1}}{C_{d_1}} & \cdots & \frac{\chi_{d_n}}{C_{d_n}} \end{bmatrix}^\mathsf{T} \otimes \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^\mathsf{T}}$$

- **Active infected cases**

$$I(t) = \sum_{i \in \mathcal{V}_o \cup \mathcal{V}_d} I_i(t) = \mathbf{g}^\mathsf{T} \mathbf{x}(t)$$

$$\boxed{\mathbf{g} = \begin{bmatrix} \mathbf{1}_m^\mathsf{T} & \mathbf{1}_n^\mathsf{T} \end{bmatrix}^\mathsf{T} \otimes \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^\mathsf{T}}$$

- **Infection peak**

$$I_{\text{peak}} = \sup_{t \in [0,T]} I(t)$$

Optimal Control for Epidemic Mitigation

Optimal capacity control

Capacity control input $u_j(t) \in [0,1]$ p.w. constant; $\mathbf{u} = \begin{bmatrix} u_{d_1} & \dots & u_{d_n} \end{bmatrix}^\mathsf{T}$

Operating capacities $u_j C_j$

Optimal capacity control

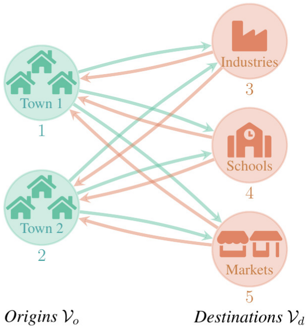
Capacity control input $u_j(t) \in [0,1]$ p.w. constant; $\mathbf{u} = [\ u_{d_1} \quad \dots \quad u_{d_n}\ ]^\intercal$

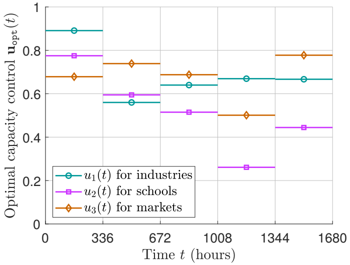
Operating capacities $u_j C_j$

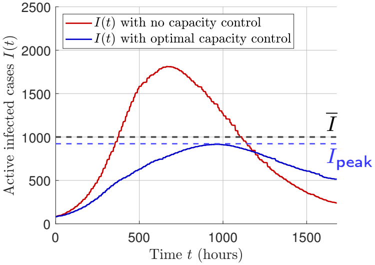
OCC Problem

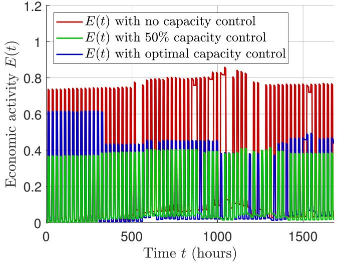
$$\begin{aligned} &\underset{\mathbf{u}\in\mathcal{U}}{\text{maximize}} \quad L(\mathbf{u}) := \frac{1}{T}\int_0^T \mathbf{e}^\intercal \mathbf{x}(t;\mathbf{x}_0,\mathbf{u})dt \\ &\text{subject to} \quad \begin{cases} \dot{\mathbf{x}} = \boldsymbol{\xi}(\mathbf{x}) + \Phi(\mathbf{x},\mathbf{u})\mathbf{x}; \quad \mathbf{x}(0) = \mathbf{x}_0 \\ I_{\text{peak}}(\mathbf{u}) := \sup\limits_{t\in[0,T]} \mathbf{g}^\intercal \mathbf{x}(t;\mathbf{x}_0,\mathbf{u}) \leq \overline{I} \end{cases} \end{aligned}$$

Can be solved numerically using CASADI or `fmincon` solver in MATLAB


Industries
3
Town 1
1
Schools
4
Town 2
2
Markets
5
Origins $\mathcal{V}_o$
Destinations $\mathcal{V}_d$

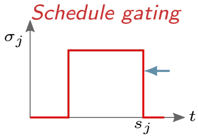

Optimal capacity control $\mathbf{u}_{\mathrm{opt}}(t)$
$u_1(t)$ for industries
$u_2(t)$ for schools
$u_3(t)$ for markets
Time t (hours)
0 336 672 1008 1344 1680


Active infected cases $I(t)$
$I(t)$ with no capacity control
$I(t)$ with optimal capacity control
$\overline{I}$
I_{peak}
Time t (hours)


Economic activity $E(t)$
$E(t)$ with no capacity control
$E(t)$ with 50% capacity control
$E(t)$ with optimal capacity control
Time t (hours)

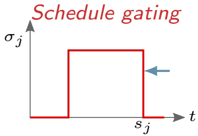
Optimal schedule control

- Fixed operating capacities, i.e., $\mathbf{u}(t) = \boldsymbol{\mu}$
- Closing hours $\mathbf{s} = [\, s_1 \; \dots \; s_n \,]^\intercal$ of destinations $\mathcal{V}_d$



Optimal schedule control

- Fixed operating capacities, i.e., $\mathbf{u}(t) = \boldsymbol{\mu}$
- Closing hours $\mathbf{s} = [\, s_1 \; \dots \; s_n \,]^\intercal$ of destinations $\mathcal{V}_d$

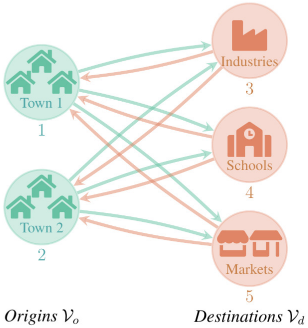


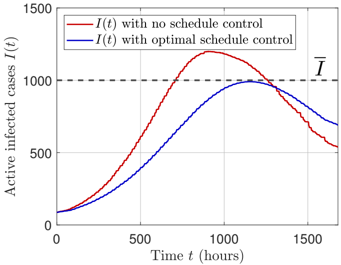
OSC Problem

$$\underset{\mathbf{s}\in[\,\underline{s},24)^p}{\text{maximize}} \quad L(\mathbf{s}) := \frac{1}{T}\int_0^T \mathbf{e}^\intercal \mathbf{x}(t;\mathbf{x}_0,\mathbf{s})dt$$

$$\text{subject to} \quad \begin{cases} \dot{\mathbf{x}} = \boldsymbol{\xi}(\mathbf{x}) + \Phi(\mathbf{x},\mathbf{s})\mathbf{x}; \quad \mathbf{x}(0) = \mathbf{x}_0 \\ I_{\text{peak}}(\mathbf{s}) \le \bar{I} \end{cases}$$

Let $\mathbf{u}(t) = 0.75\,\mathbf{1}_3$, i.e., 25% reduction in the capacities





	Industries	Schools	Markets
Closing hours	02:32 pm	01:45 pm	04:06 pm

Conclusions

Conclusions & Prospects

- Model of urban human mobility with local epidemic spread process
- Optimal mobility control for epidemic mitigation
  - Optimal capacity control
  - Optimal schedule control

- Prospects
  - *Online demonstrator for the network of Grenoble*
  - Model predictive control framework
  - Theoretic guarantees for optimal control problems

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