Shape Reconstruction

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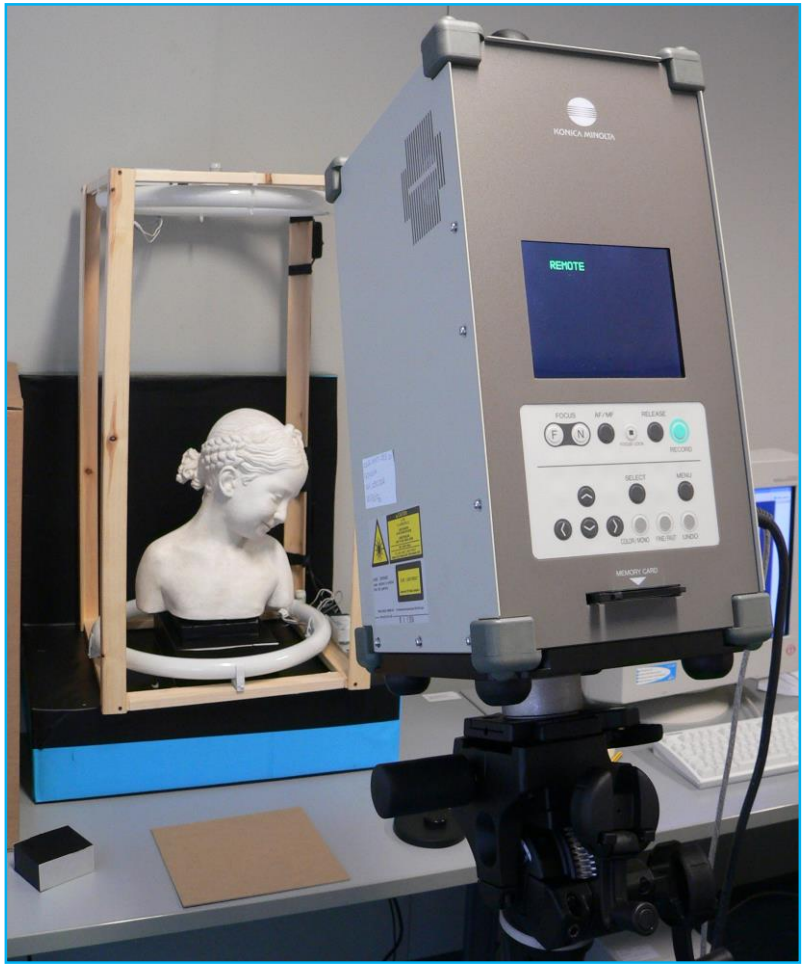
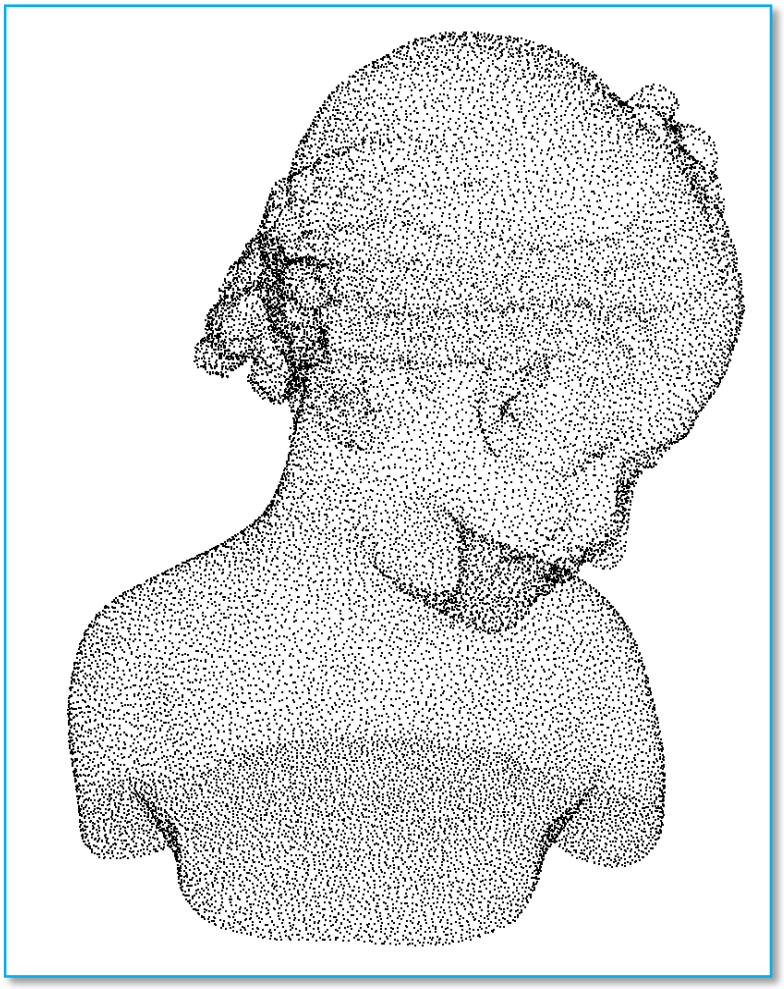
Outline

- Sensors
- Problem statement
- Computational Geometry
  - Convex hull, Voronoi/Delaunay, alpha-shapes
- Variational formulations
  - Poisson / spectral
- Quest for robustness
- Optimal transportation

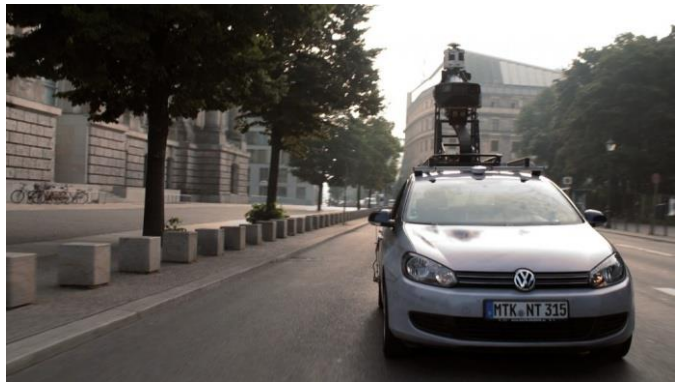
SENSORS

Laser scanning

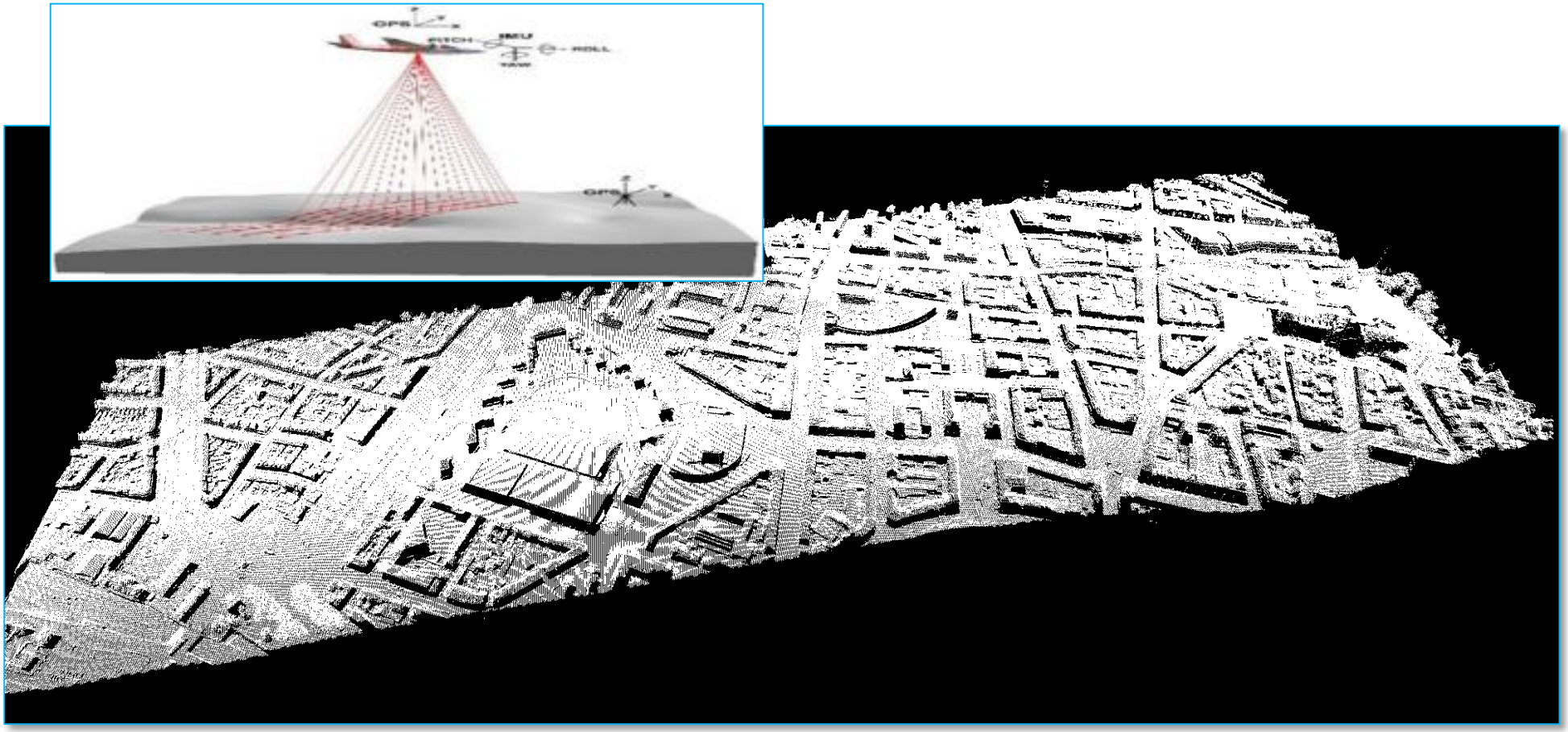




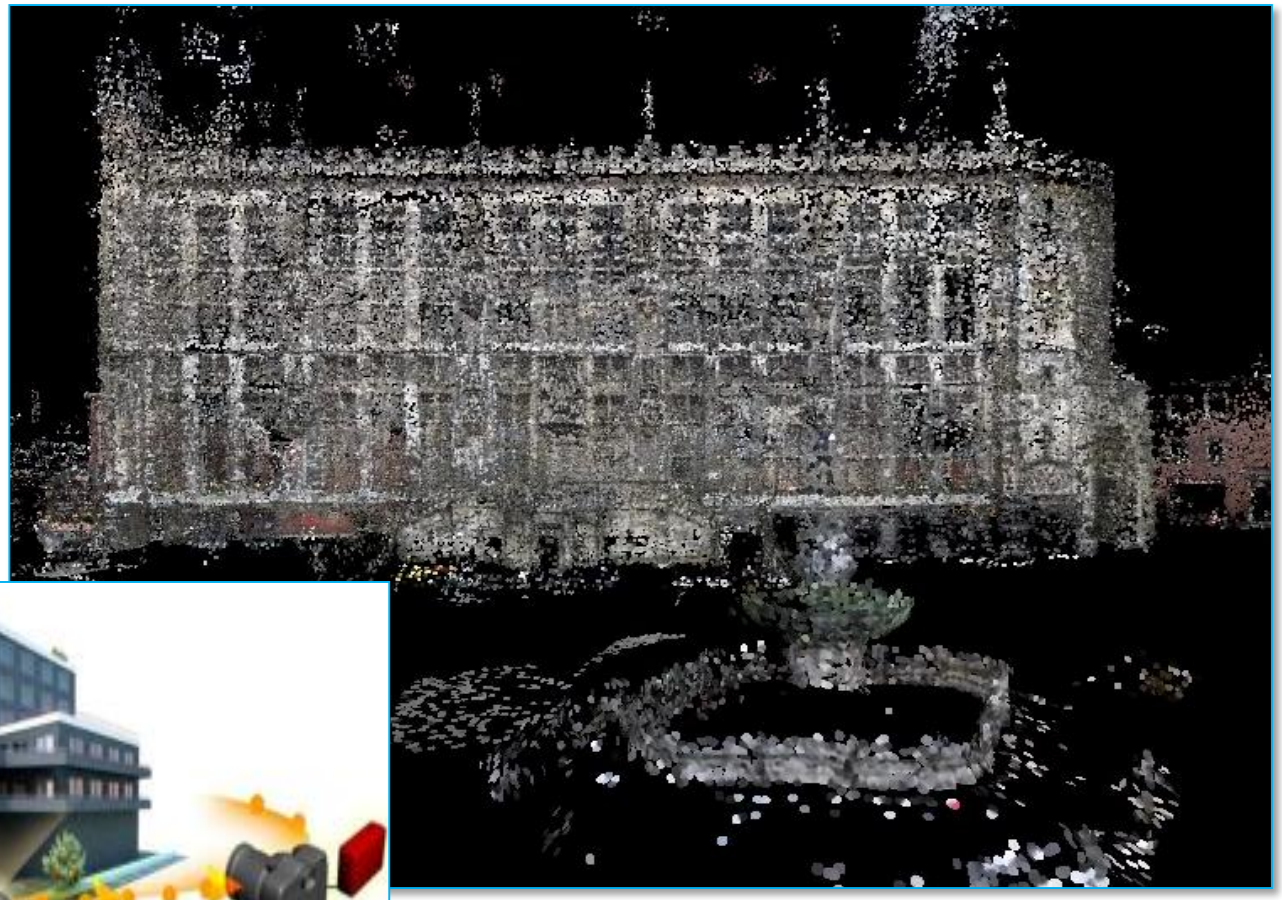
Car-based Laser



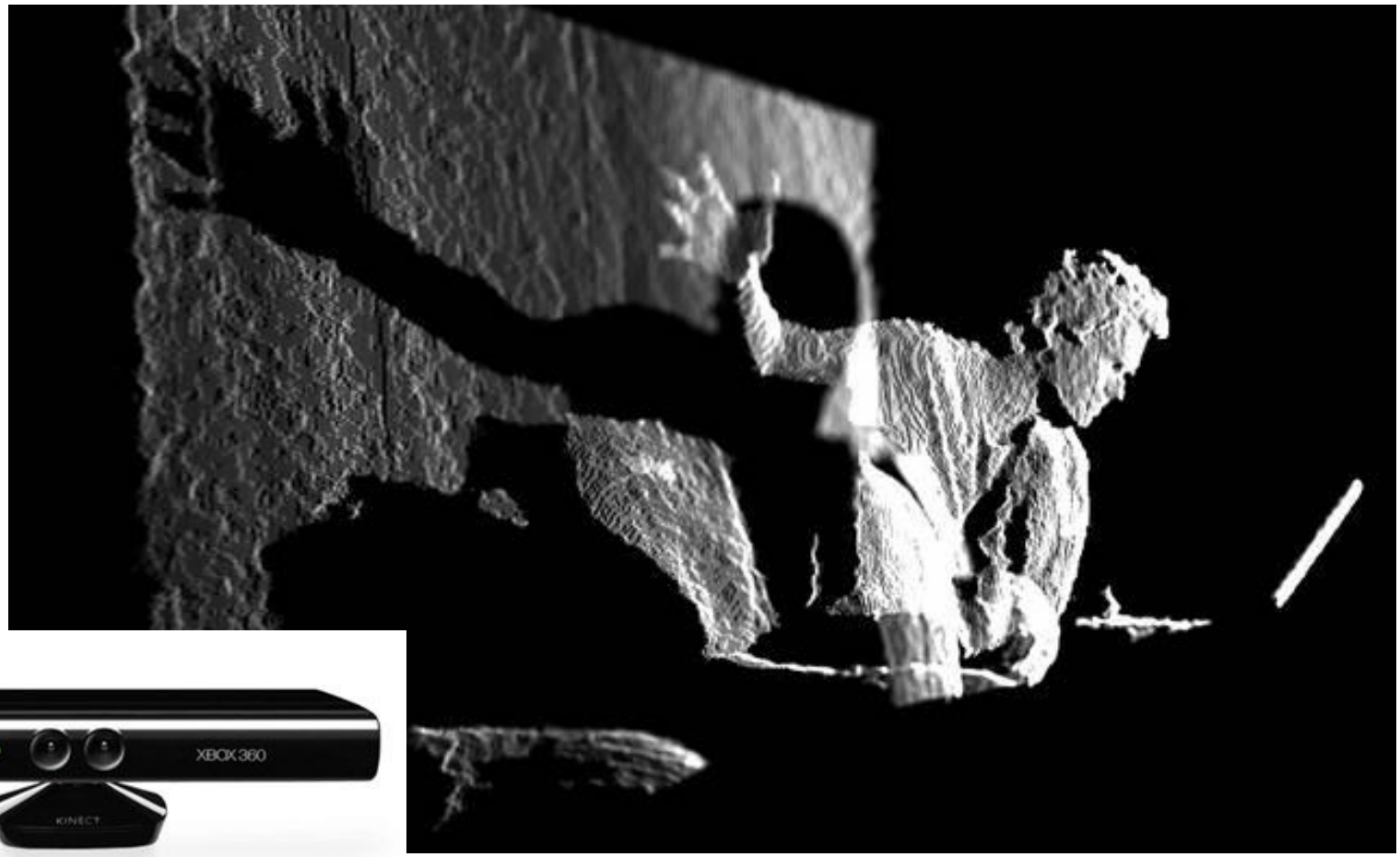
Airborne Lidar



Multi-View Stereo (MVS)



Depth Sensors

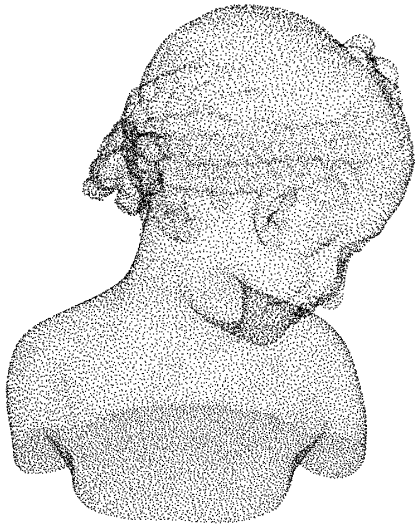


PROBLEM STATEMENT

Reconstruction Problem

Input: point set *P* sampled over a surface *S*:

- Non-uniform sampling
- With holes
- With uncertainty (noise)



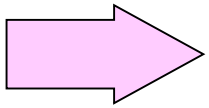
point set

Output: surface

Approximation of *S* in terms of topology and geometry

Desired:

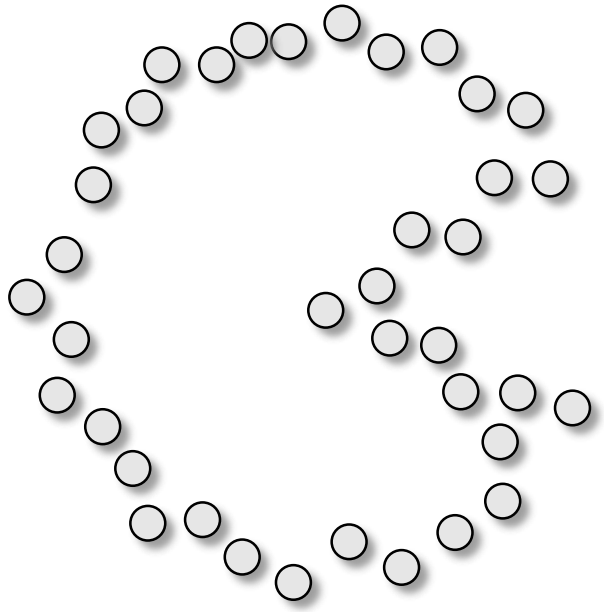
- Watertight
- Intersection free



reconstruction

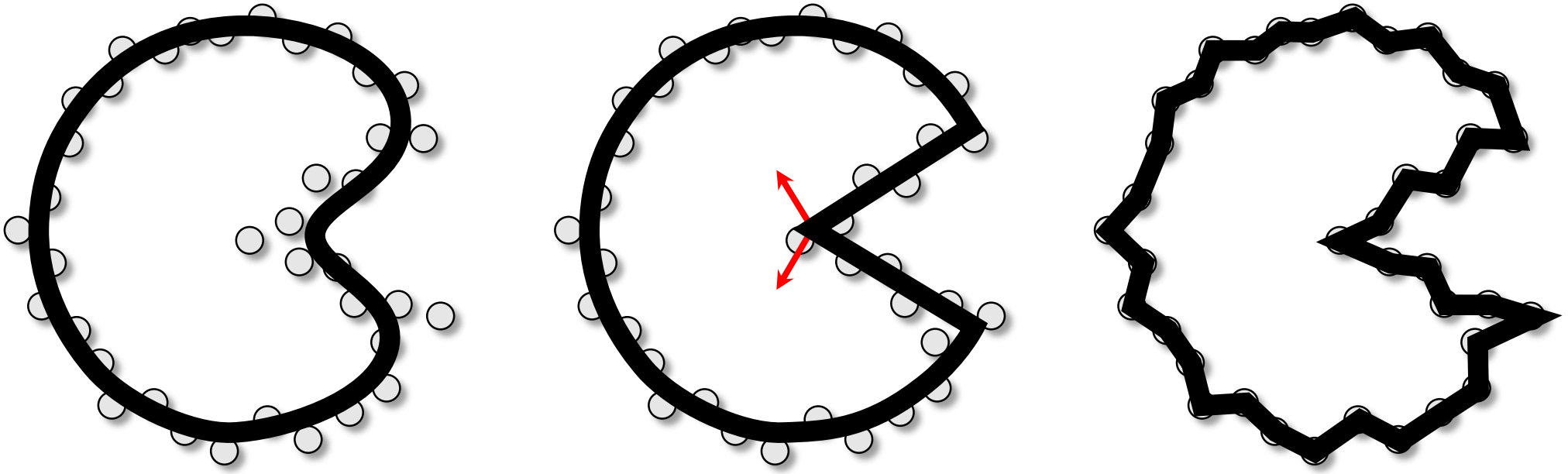
surface

Ill-posed Problem



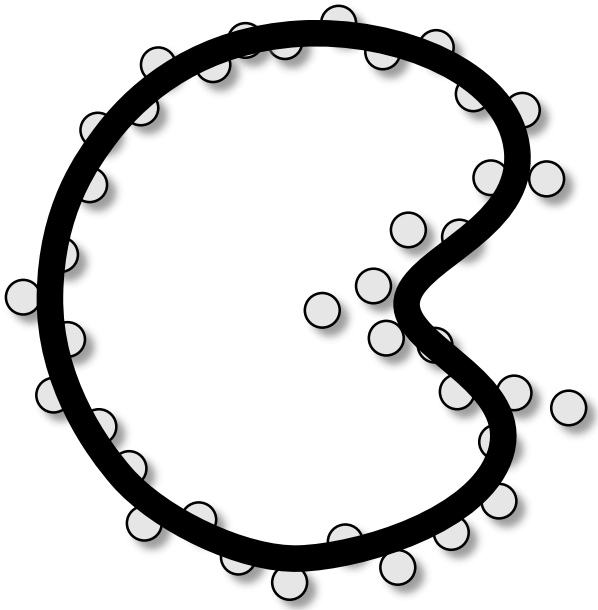
Many candidate surfaces for the reconstruction problem!

Ill-posed Problem

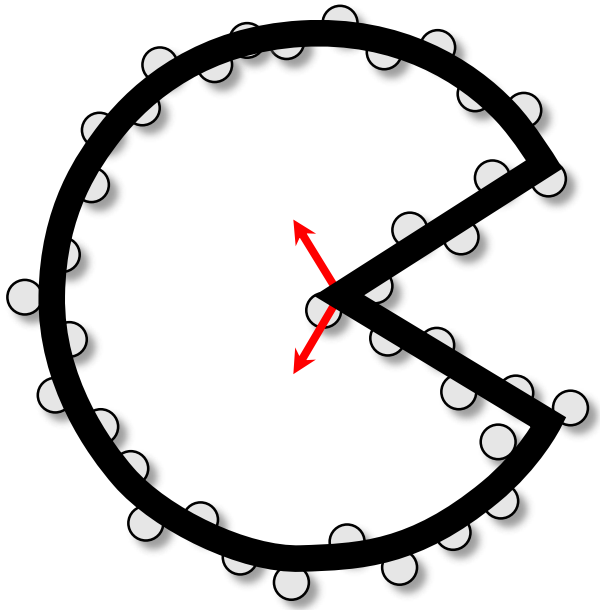


Many candidate surfaces for the reconstruction problem! How to pick?

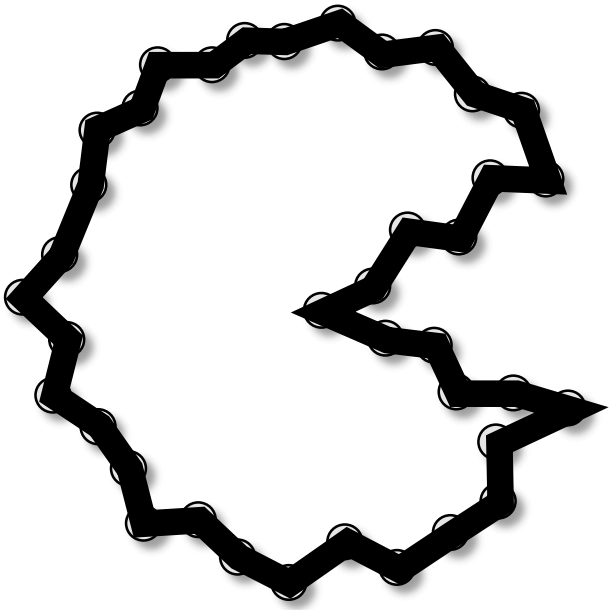
Priors



Smooth



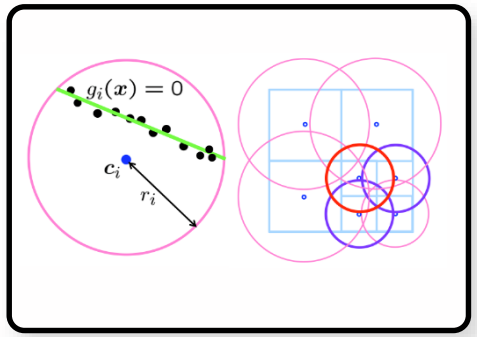
Piecewise Smooth



“Simple”

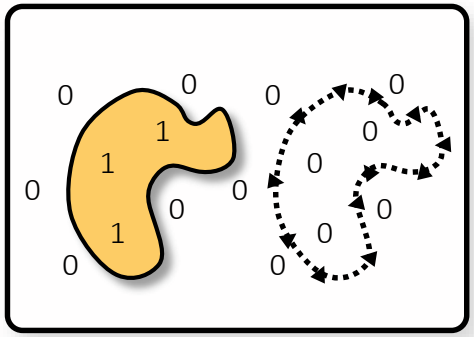
Surface Smoothness Priors

Local Smoothness



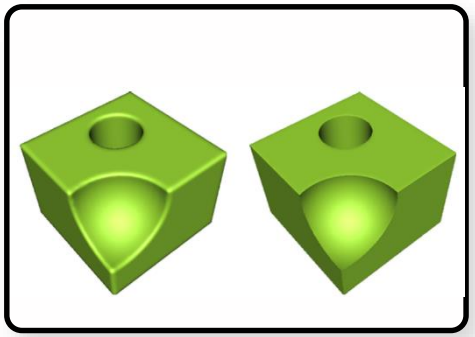
Local fitting
No control away from data
Solution by interpolation

Global Smoothness



Global: linear, eigen, graph cut, …
Robustness to missing data

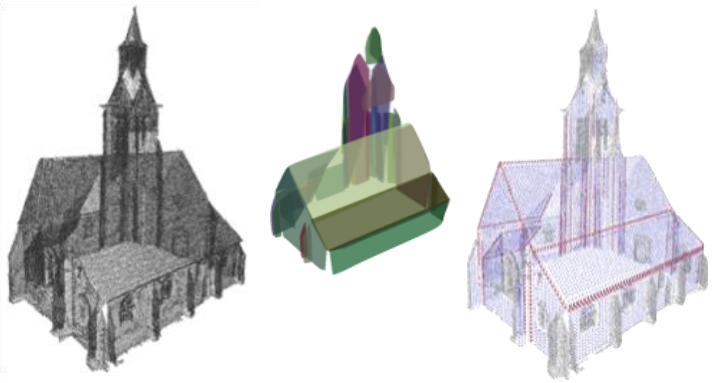
Piecewise Smoothness



Sharp near features
Smooth away from features

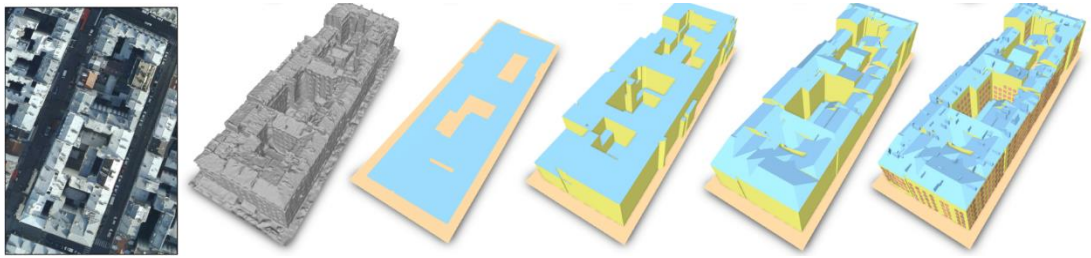
Domain-Specific Priors

Surface Reconstruction by Point Set Structuring



[Lafarge - A. EUROGRAPHICS 2013]

LOD Reconstruction for Urban Scenes

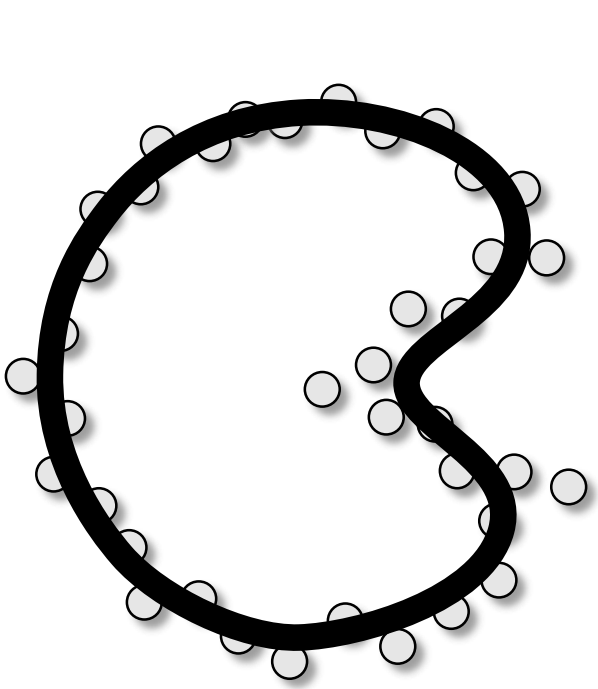


[Verdie, Lafarge - A. ACM Transactions on Graphics 2015]

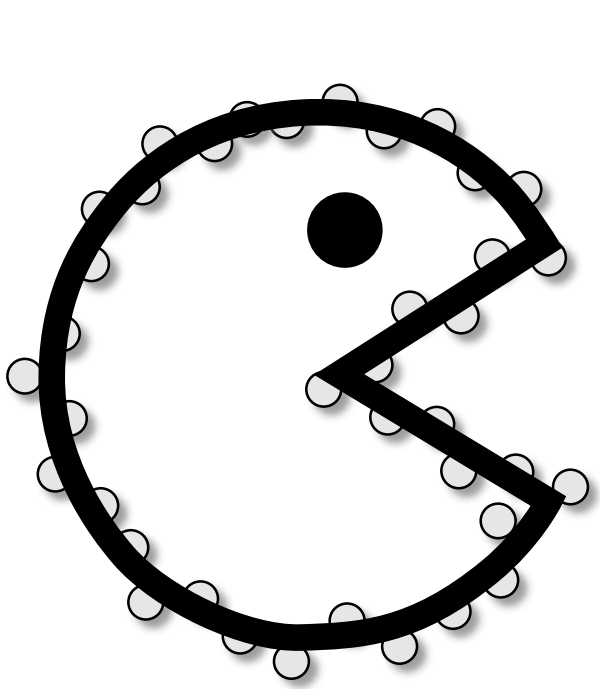
Previous Work

[1] GVU Center Georgia Tech, Graphics Research Group, Variational Implicit Surfaces Web site: http://www.cc.gatech.edu/gvu/geometry/implicit/. [6] T. Gentils R. Smith A. Hilton, D. Beresford and W. Sun. Virtual people: Capturing human models to populate virtual worlds. In Proc. Computer Animation, page 174185, Geneva, Switzerland, 1999. IEEE Press. [7] Anders Adamson and Marc Alexa. Approximating and intersecting surfaces from points. In Proceedings of the Eurographics/ACM SIG- GRAPH Symposium on Geometry Processing 2003, pages 230{239. ACM Press, Jun 2003. [8] Anders Adamson and Marc Alexa. Approximating bounded, nonorientable surfaces from points. In SMI '04: Proceedings of Shape Modeling Applications 2004, pages 243{252, 2004. 153 [9] U. Adamy, J. Giesen, and M. John. Surface reconstruction using umbrella, Computational Geometry, 21(1-2):63{86, 2002. [10] G. J. Agin and T. O.Binford. Computer description of curved objects. In Proceedings of the International Joint Conference on Artìlial In- telligence, pages 629{640, 1973. [11] Irene Albrecht, J¨org Haber, and Hans-Peter Seidel. Construction and Animation of Anatomically Based Human Hand Models. In Proceedings ACM SIGGRAPH/Eurographics Symposium on Computer Animation (SCA '03), pages 98{108, July 2003. [12] M. Alexa, J. Behr, D. Cohen-Or, S. Fleishman, D. Levin, and C. T. Silva. Point set surfaces. In IEEE Visualization 2001, pages 21{28, October 2001. [13] Marc Alexa and Anders Adamson. On normals and projection operators for surfaces de¿ed by point sets. In Marc Alexa, Markus Gross, Hanspeter P¿ter, and Szymon Rusinkiewicz, editors, Proceedings of Eurographics Symposium on Point-based Graphics, pages 149{156. Eurographics, 2004. [14] Marc Alexa, Johannes Behr, Daniel Cohen-Or, Shachar Fleishman, David Levin, and Claudio T. Silva. Computing and rendering point set surfaces. IEEE Transactions on Computer Graphics and Visualization, 9:3{15, 2003. [15] International 2003, pages 49{58, Seoul, Korea, May 2003. [17] H. Alt, B. Behrends, and J. Blomer. Approximate matching of polygonal shapes. Annals of Mathematics and Artìlial Intelligence, pages 251{265, 1995. 154 [18] N. Amenta, M. Bern, and M. Kamvysselis. A new Voronoi-based Surface Reconstruction Algorithm. In Proceedings of ACM SIGGRAPH' 98, pp.415-421, 1998. [19] N. Amenta, M. Bern, and M. Kamvysselis. A new Voronoi-based surface reconstruction algorithm. In Proceedings of ACM SIGGRAPH 1998, pages 415{421, 1998. [20] N. Amenta, S. Choi, T. K. Dey, and N. Leekha. A simple algorithm for homeomorphic surface reconstruction. In Proc. 16th Annu. ACM Sympos. Comput. Geom., pages 213{222, 2000. [21] N. Amenta, S. Choi, and R. Kolluri. The power crust. In Proceedings of 6th ACM Symposium on Solid Modeling, pages 249{260, 2001. [22] N. Amenta, S. Choi, and R. Kolluri. The power crust. In ACM Solid Modeling, 2001. [23] N. Amenta, S. Choi, and R. K. Kolluri. The power crust, unions of balls, and the medial axis transform. Comput. Geom. Theory Appl., 19:127{153, 2001. [24] N. Amenta and Y. Kil. De¿ing point-set surfaces. Acm Transactions on Graphics, 23, Aug 2004. [25] N. Amenta and Y. Kil. The domain of a point set surface. In Symposium on Point-Based Graphics 2004, 2004. [26] N. Amenta, S.Choi, t. K. Dey, and N. Leekha. A Simple Algorithm for Homeomorphic Surface Reconstruction, International Journal of Computational Geometry and Applications, vol.12 n.1-2, pp.125-141, 2002. [27] Nina Amenta and Marshall Bern. Surface reconstruction by Voronoi ¿tering. Discrete Comput. Geom., 22(4):481{504, 1999. [28] P. Anandan. A computational framework and an algorithm for the measurement of visual motion. Int. Journal of Computer Vision, 2:283{ 310, 1989. [29] Anonymous. The Anthropometry Source Book, volume I & II. NASA Reference Publication 1024. 155 [30] Anonymous. Nasa man-systems integration manual. Technical Report NASA-STD-3000. [31] H.J. Antonisse. Image segmentation in pyramids. Computer Vision, Graphics and Image Processing, 19(4):367{383, 1982. [32] A. Asundi. Computer aided moire methods. Optical Laser Engineering, 17:107{116, 1993. [33] D. Attali. r-regular Shape Reconstruction from Unorganized Points. In Proceedings of the ACM Symposium on Computational Geometry, pp.248-253, 1997. [34] D. Attali and J. O. Lachaud. Delaunay conforming iso-surface; skeleton extraction and noise removal, Computational Geometry: Theory and Applications. 19(2-3): 175-189, 2001. [35] D. Attali and J.-O. Lachaud. Constructing Iso-Surfaces Satisfying the Delaunay Constraint; Application to the Skeleton Computation. In Proc. 10th International Conference on Image Analysis and Processing (ICIAP'99), Venice, Italy, September 27-29, pages 382-387, 1999. [36] Dominique Attali and Jean-Daniel Boissonnat. Complexity of the Delaunay triangulation of points on polyhedral surfaces. In Proc. 7th ACM Symposium on Solid Modeling and Applications, 2002. [37] M. Attene, B. Falcidino, M. Spagnuolo, and J. Rossignac. Sharpen & bend: Recovering curved edges in triangle meshes produced by featureinsensitive sampling. Technical report, Georgia Institute of Technology, 2003. GVU-GATECH 34/2003. [38] M. Attene and M. Spagnuolo. Automatic Surface Reconstruction from point sets in space. In EUROGRAPHICS 2000 Proceedings, pp. 457- 465, Vol.19 n.3, 2000. [39] M. Attene and M. Spagnuolo. Automatic surface reconstruction from point sets in space. Computer Graphics Forum, 19(3):457{465, 2000. Proceedings of EUROGRAPHICS 2000. [40] Marco Attene, Bianca Falcidieno, Jarek Rossignac, and Michela Spagnuolo. Edge-sharpener: Recovering sharp features in triangulations of non-adaptively re-meshed surfaces. 156 [41] Marco Attene and Michela Spagnuolo. Automatic surface reconstruction from point sets in space. Computer Graphics Forum, 19(3):457{ 465, 2000. [42] C.K. Au and M.M.F. Yuen. Feature-based reverse engineering of mannequin for garment design. Computer-Aided Design 31:751-759, 1999. [43] S. Ayer and H. Sawhney. Layered representation of motion video using robust maximum-likelihood estimation of mixture models and mdl encoding. International Conference on Computer Vision, pages 777{784, 1995. [44] Z. Popovic B. Allen, B. Curless. Articulated body deformation from range scan data. In Proceedings SIGGRAPH 02, page 612619, San Antonio, TX, USA, 2002. Addison-Wesley. [45] Z. Popovic B. Allen, B. Curless. The space of all body shapes: reconstruction and parameterization from range scans. In Proceedings SIGGRAPH 03, page 587594, San Diego, CA, USA, 2003. Addison- Wesley. [46] ed B. M. ter haar Romeny. Geometry-Driven Di¨sion in Computer Vision. Kluwer Academic Pubs., 1994. [47] A. Bab-Hadiashar and D. Suter. Robust optic ow estimation using least median of squares. Proceedings ICIP-96, September 1996. Switzerland. [48] C. Bajaj, Fausto Bernardini, and Guoliang Xu. Automatic reconstruction of surfaces and scalar Îlds from 3d scans. In International Confer- ence on Computer Graphics and Interactive Techniques, pages 109{118, 1995. [49] C .L. Bajaj, F. Bernardini, J. Chen, and D. Schikore. utomatic Reconstruction of 3D Cad Models. In Proceedings of Theory and Practice of Geometric Modelling, 1996. [50] C.L. Bajaj, E.J. Coyle, and K.N. Lin. Arbitrary topology shape reconstruction from planar cross sections. Graphical.Models.and Image Processing., 58:524{543, 1996. 157 [51] G. Barequet, M.T. Goodrich, A. Levi-Steiner, and D. Steiner. Contour interpolation by straight skeletons. Graphical.models., 66:245{260, 2004. [52] G. Barequet, D. Shapiro, and A. Tal. Multilevel sensitive reconstruction of polyhedral surfaces from parallel slices. The Visual Computer, 16(2):116{133, 2000. [53] G. Barequet and M. Sharir. Piecewise-linear interpolation between polygonal slices. Computer Vision and Image Understanding, 63(2):251{272, 1996. [54] G. Barequet and M. Sharir. Partial surface and Beraldin. Practical considerations for a design of a high precision 3d laser scanner system. Proceedings of SPIE, 959:225{246, 1988. [74] B. Blanz and T. Vetter. A morphable model for the synthesis of 3d faces. In Proceedings SIGGRAPH 99, page 187194, Los Angeles, CA, USA, 1999. Addison-Wesley. [75] Volker Blanz, Curzio Basso, Tomaso Poggio, and Thomas Vetter. Reanimating Faces in Images and Video. In Pere Brunet and Dieter Fellner, editors, Computer Graphics Forum (Proceedings of Eurographics 2003), volume 22, pages 641{650, September 2003. [76] Volker Blanz and Thomas Vetter. A Morphable Model for the Synthesis of 3D Faces. In Alyn Rockwood, editor, Computer Graphics (SIGGRAPH '99 Conference Proceedings), pages 187{194. ACM SIGGRAPH, August 1999. [77] J. F. Blinn. A generalization of algebraic surface drawing. ACM Trans- actions on Graphics, 1(3):235{256, July 1982. [78] J. Bloomenthal, editor. Introduction to Implicit Surfaces. Morgan Kaufmann, 1997. [79] J. D. Boissonnat. Geometric Structures for Three-dimensional Shape Representation. ACM Transaction on Graphics, pp.266-286, Vol.3, 1984. [80] J.-D. Boissonnat. Geometric structures for three-dimensional shape representation. ACM Transactions on Graphics, 3(4):266{286, October 1984. [81] J.D. Boissonnat. Shape reconstruction from planar cross sections...[403] M.J. Zyda, A.R. Jones, and P.G. Hogan. Surface construction from planar contours. Computers.and Graphics, 11:393{408}, 1987,

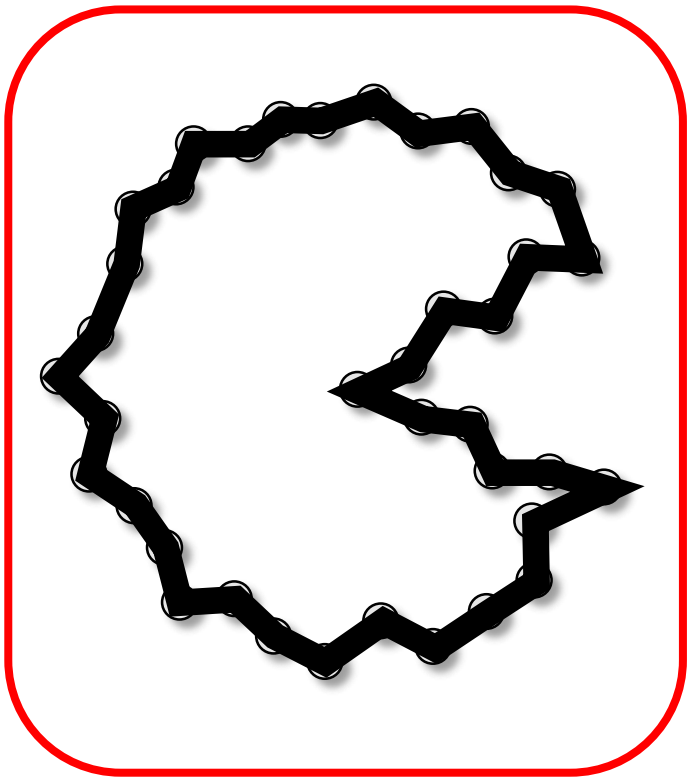
Warm-up



Smooth



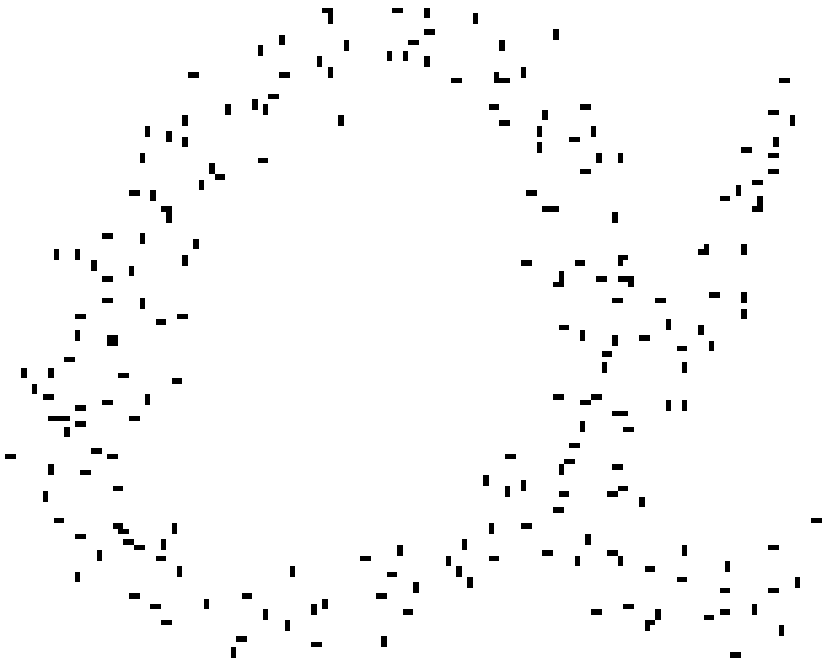
Piecewise Smooth

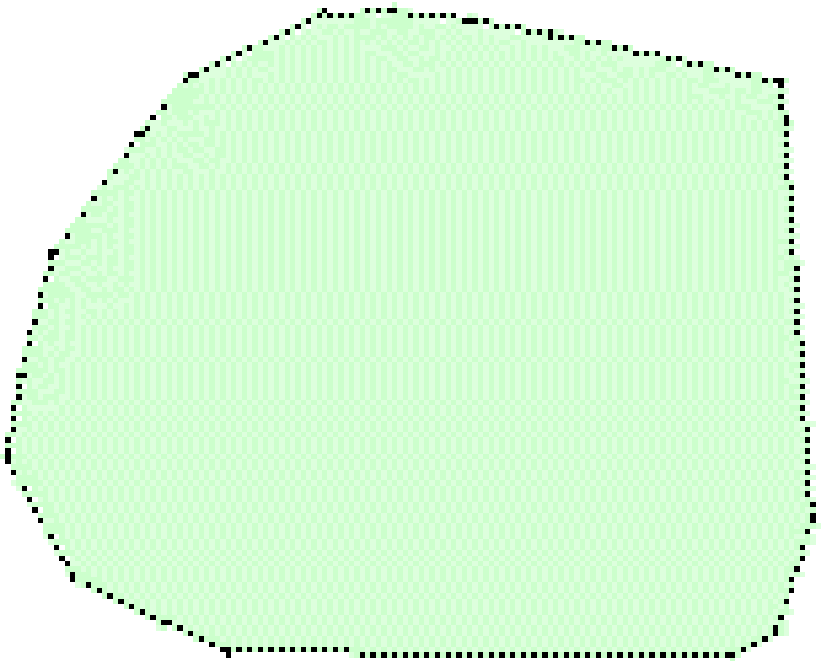


“Simple”

CONVEX HULL

Convex Hull



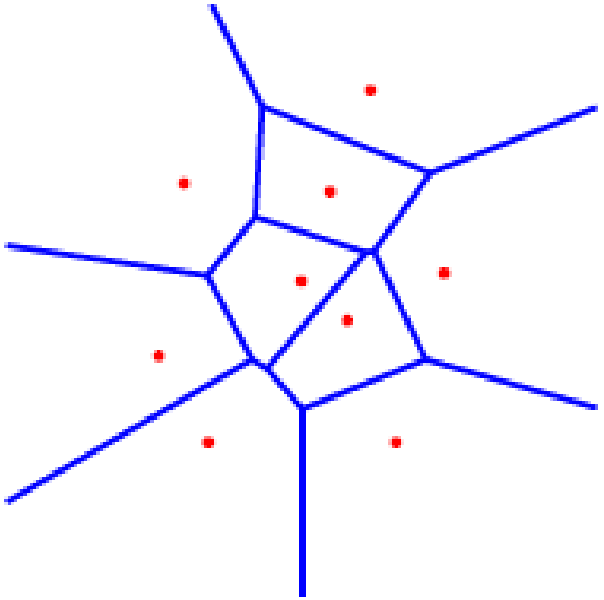


VORONOI / DELAUNAY

Voronoi Diagram

Let $\mathcal{E} = \{\mathbf{p_1}, \ldots, \mathbf{p_n}\}$ be a set of points (so-called sites) in $\mathbb{R}^d$. We associate to each site $\mathbf{p_i}$ its Voronoi region $V(\mathbf{p_i})$ such that:

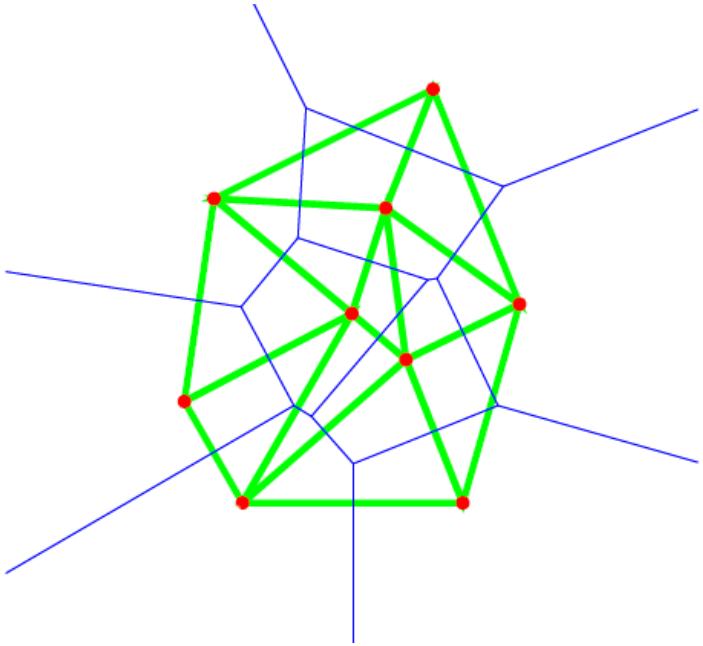
$$V(\mathbf{p_i}) = \{\mathbf{x} \in \mathbb{R}^d : \|\mathbf{x} - \mathbf{p_i}\| \leq \|\mathbf{x} - \mathbf{p_j}\|, \forall j \leq n\}.$$



Delaunay Triangulation

Dual structure of the Voronoi diagram.

The Delaunay triangulation of a set of sites E is a simplicial complex such that k+1 points in E form a Delaunay simplex if their Voronoi cells have nonempty intersection

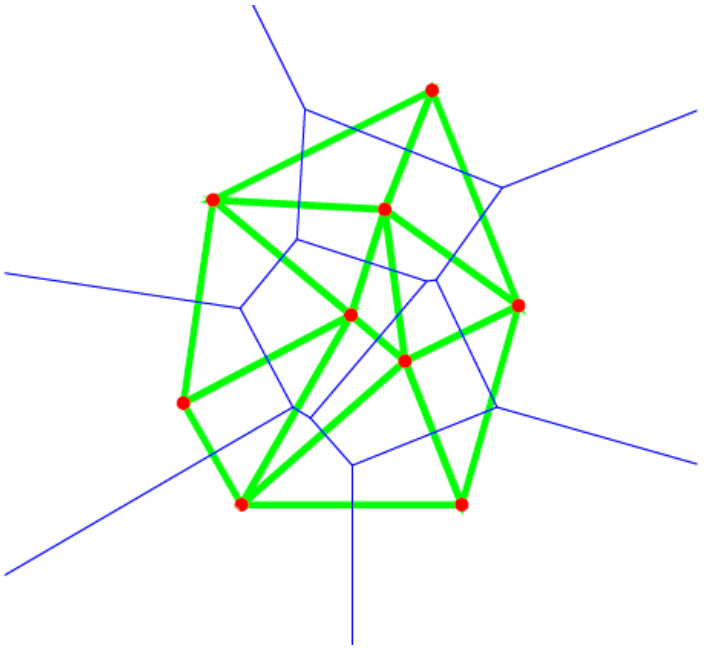


Delaunay-based

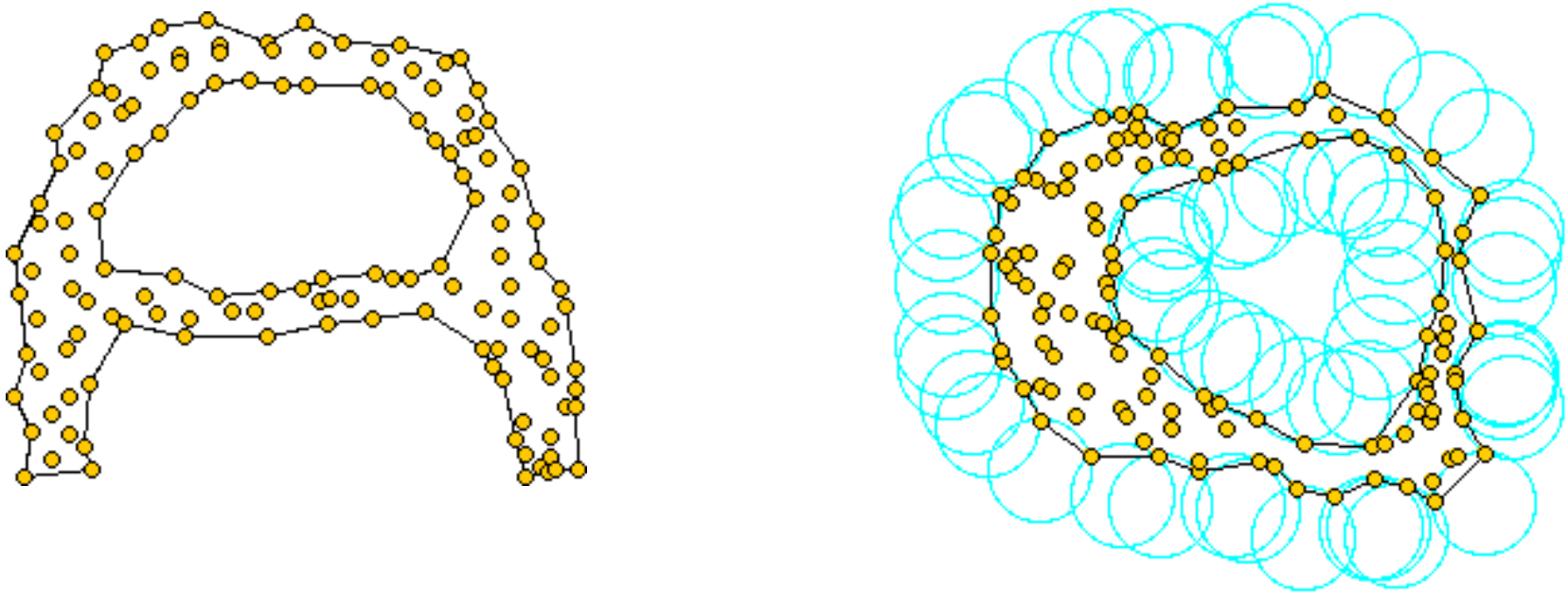
Key idea: assuming dense enough sampling, reconstructed triangles are Delaunay triangles.

First define

- Medial axis
- Local feature size
- Epsilon-sampling



Alpha-Shapes [Edelsbrunner, Kirkpatrick, Seidel]



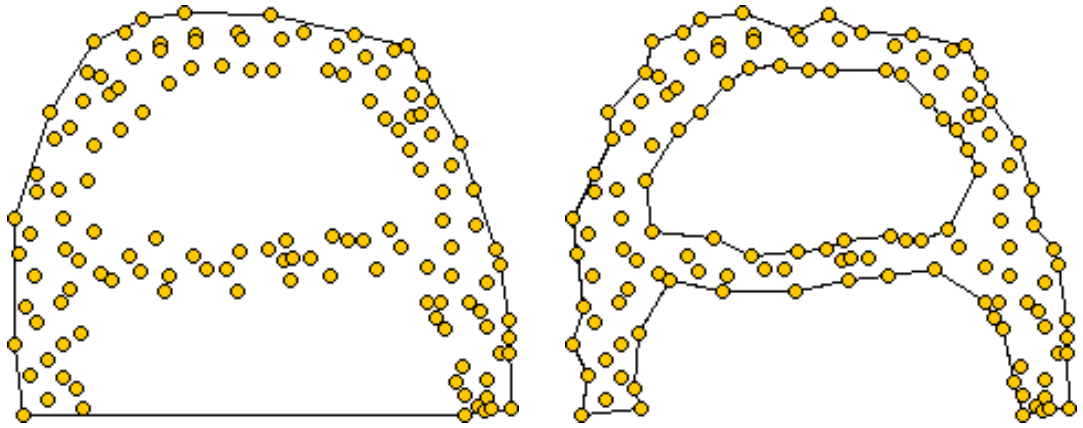
Segments: point pairs that can be touched
by an empty disc of radius alpha.

Alpha-Shapes

In 2D: family of piecewise linear simple curves constructed from a point set P.

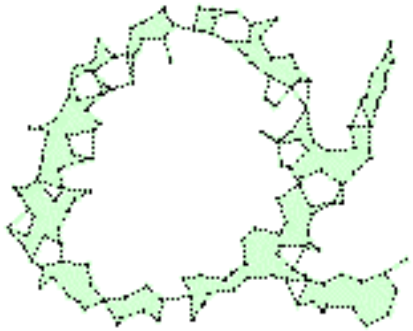
Subcomplex of the Delaunay triangulation of P.

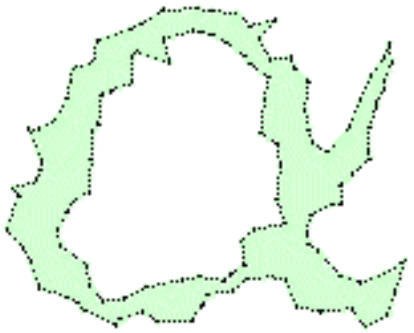
Generalization of the concept of the convex hull.



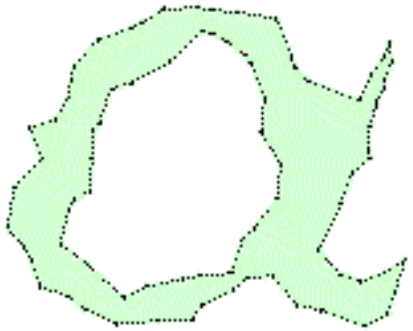
Alpha-Shapes

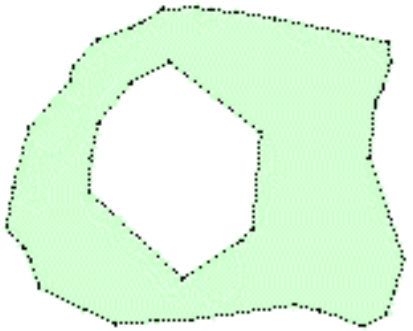
$\alpha = 0$

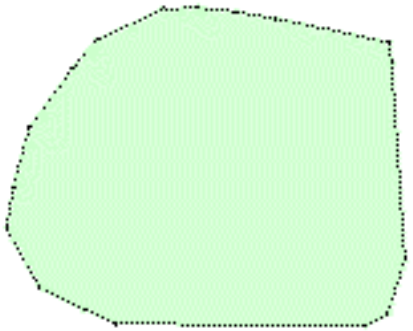




Alpha controls the desired level of detail.

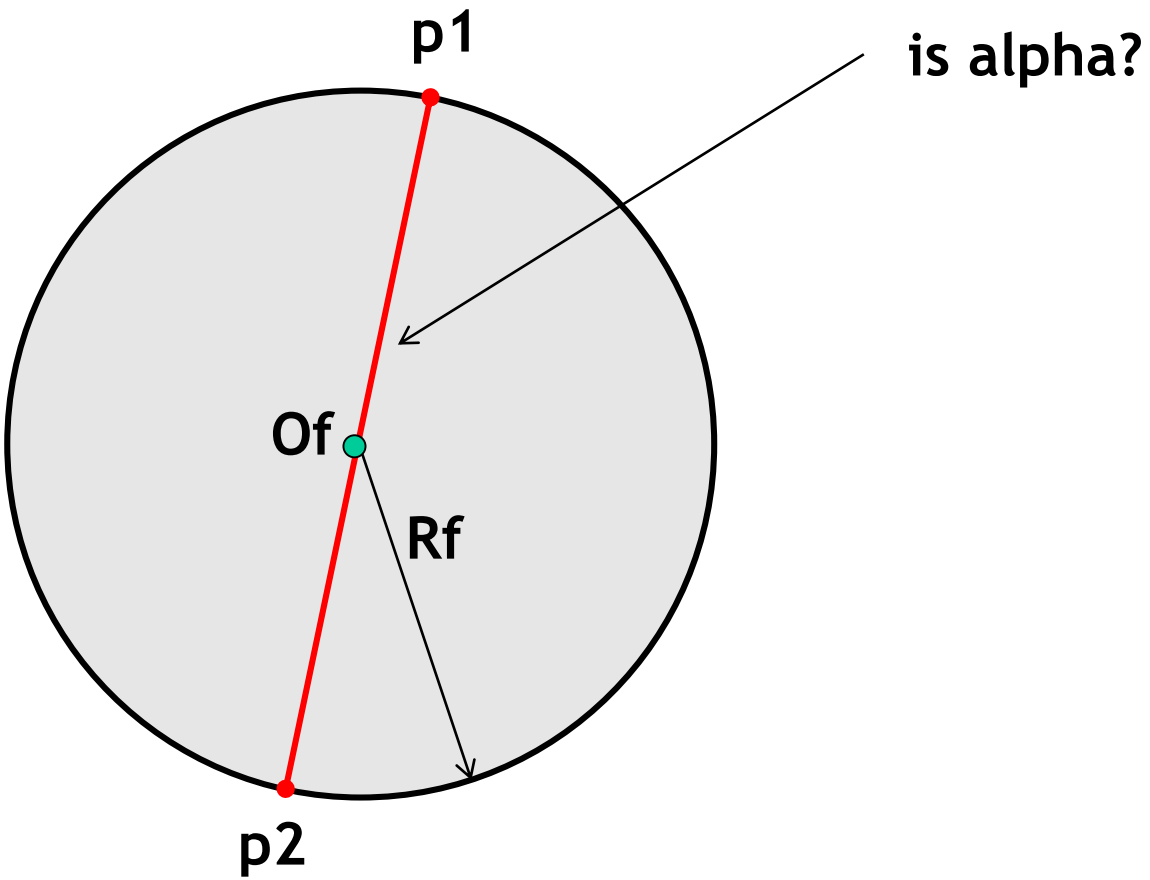




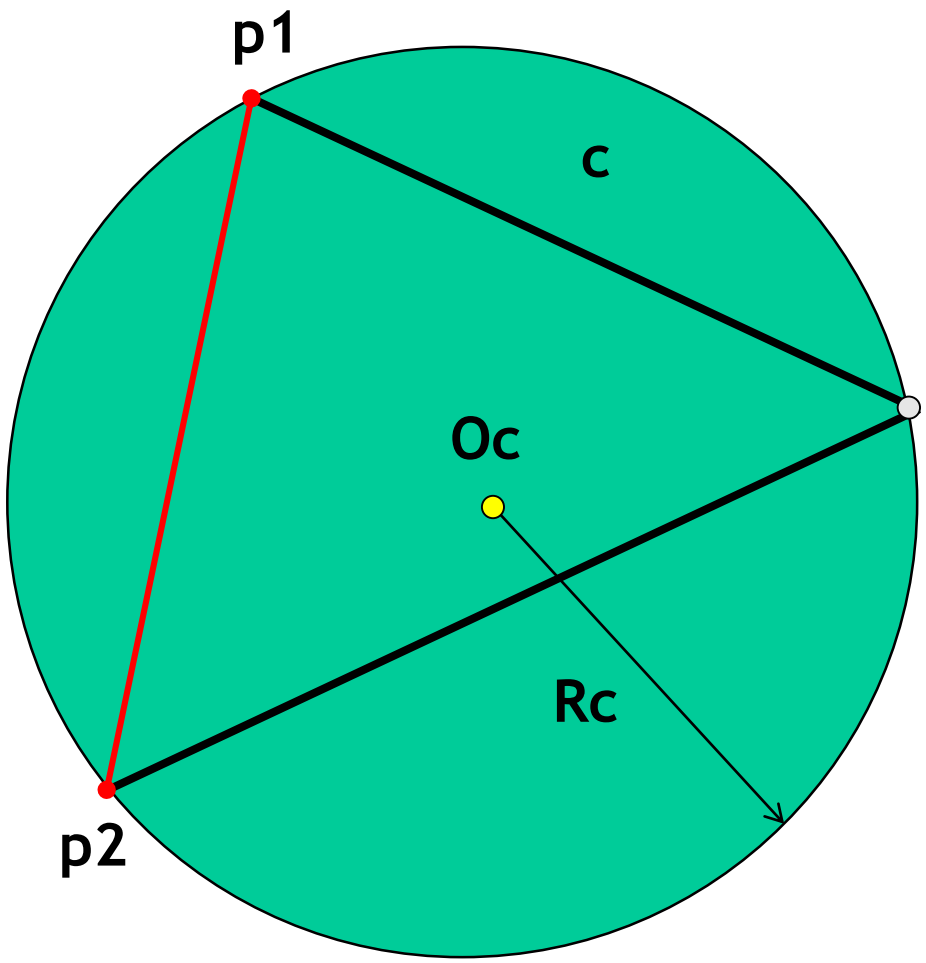


$\alpha = \infty$

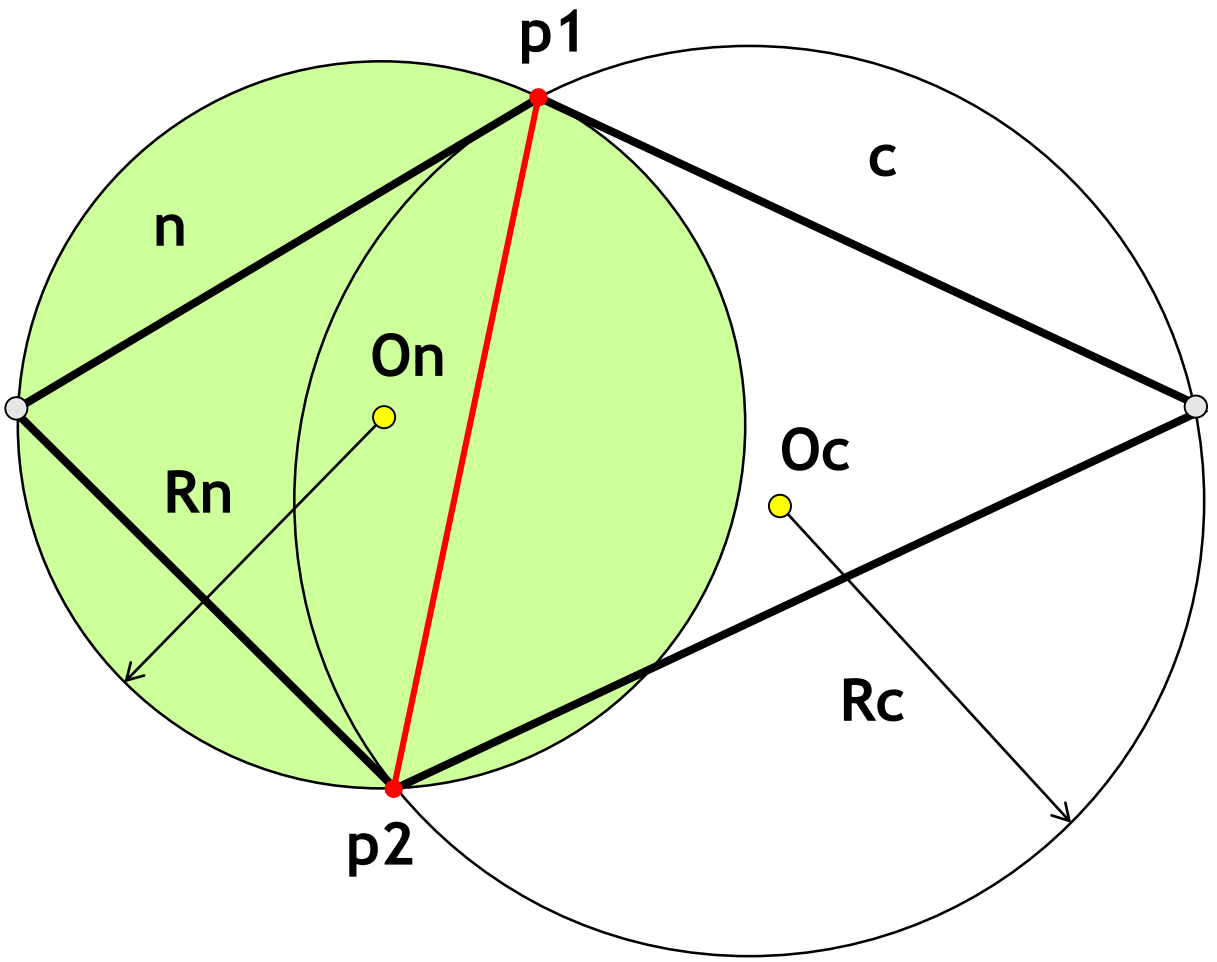
Computing Alpha-Shapes



Computing Alpha-Shapes

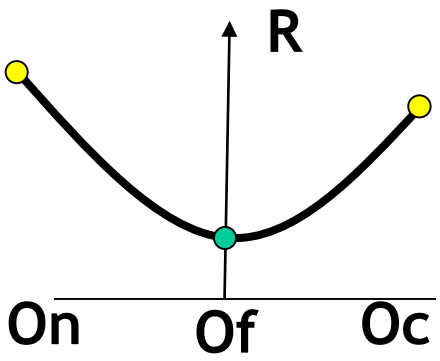


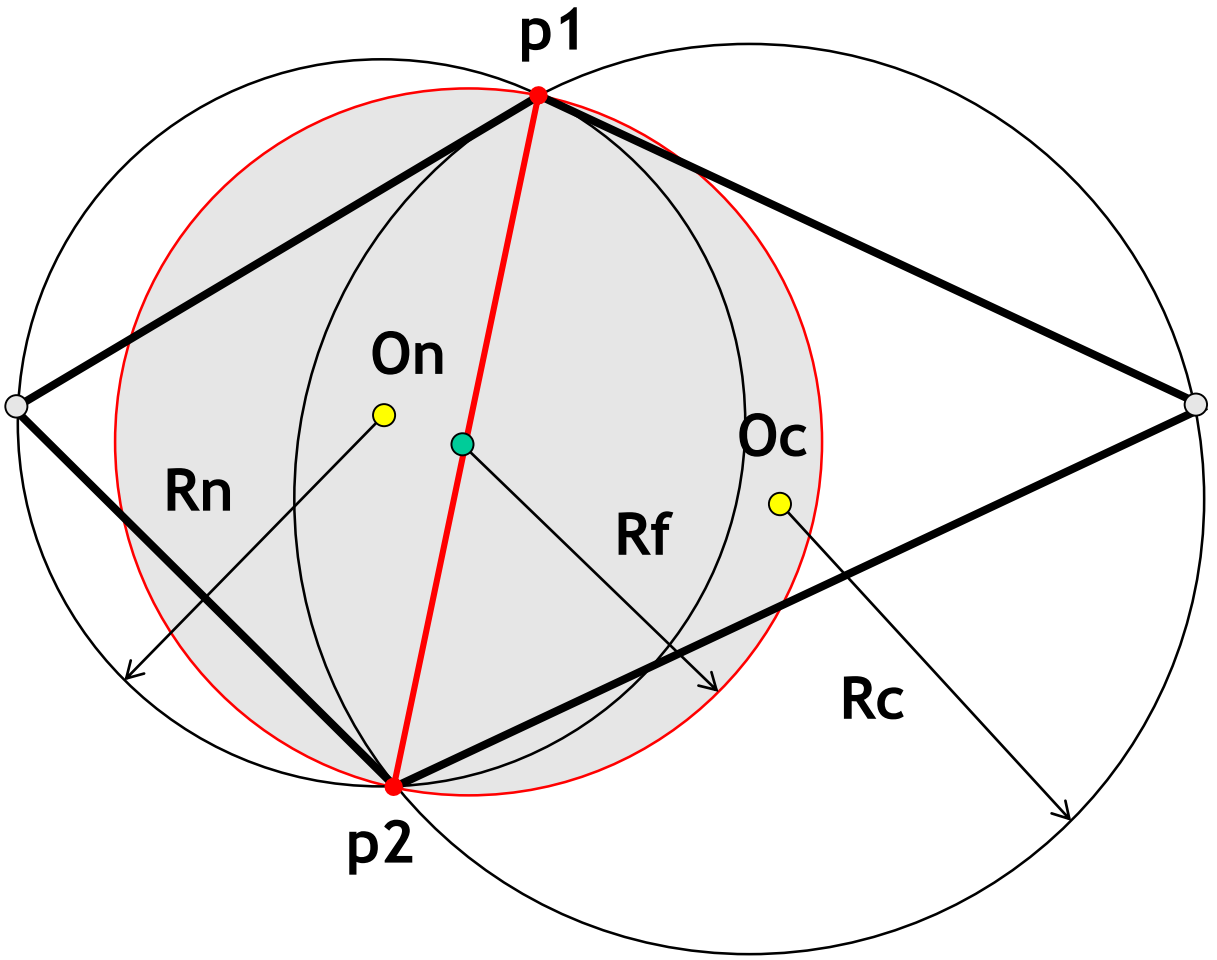
Computing Alpha-Shapes



Computing Alpha-Shapes

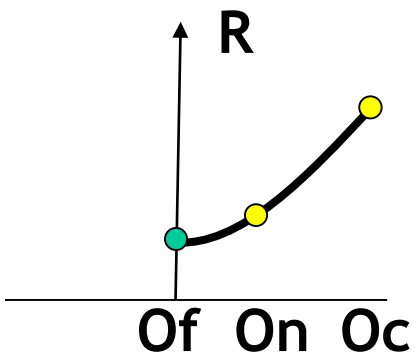
is_alpha if
alpha > Rf
AND
alpha <
max(Rc,Rn)

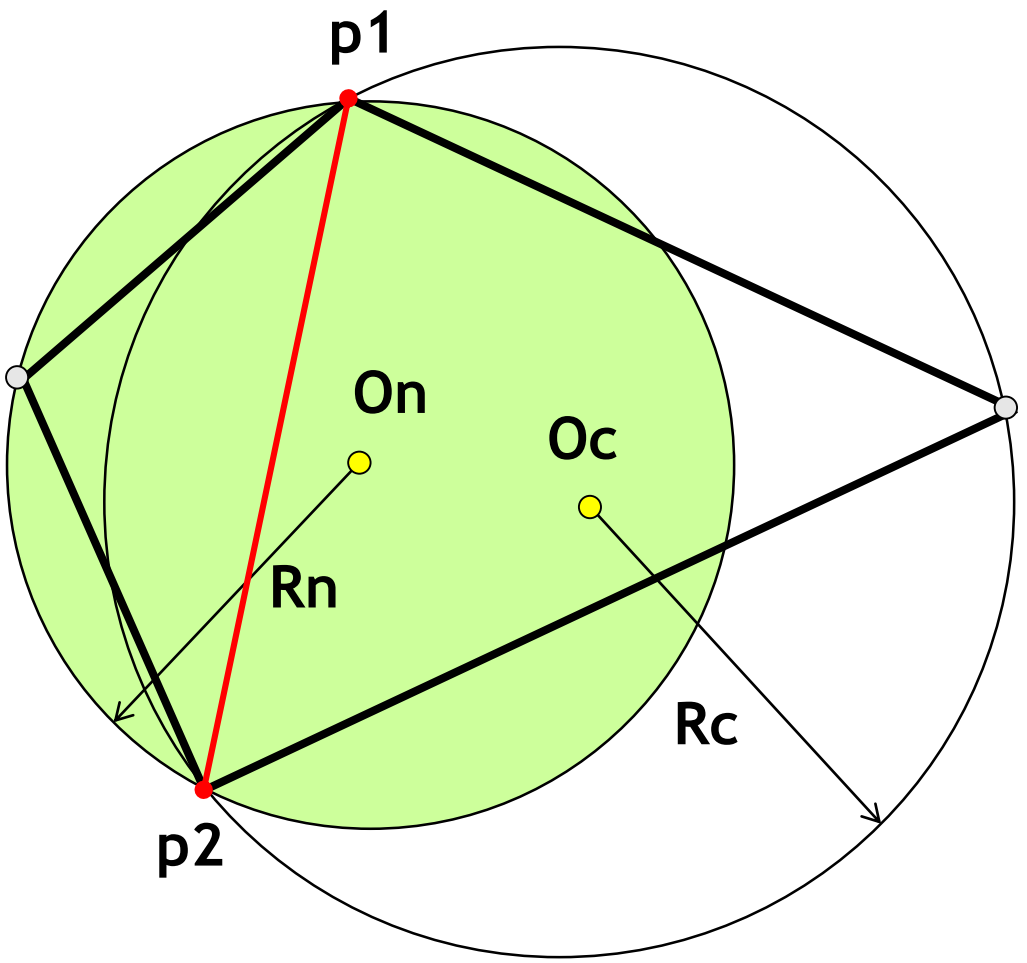




Computing Alpha-Shapes

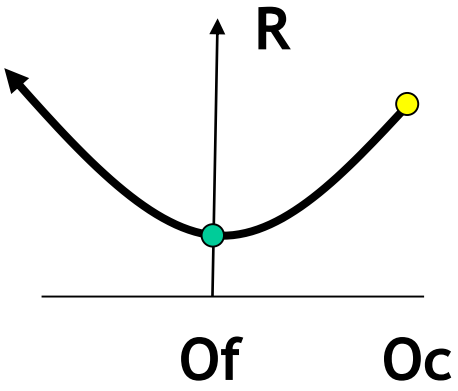
is_alpha if
alpha >
min(Rc,Rn)
AND
alpha <
max(Rc,Rn)

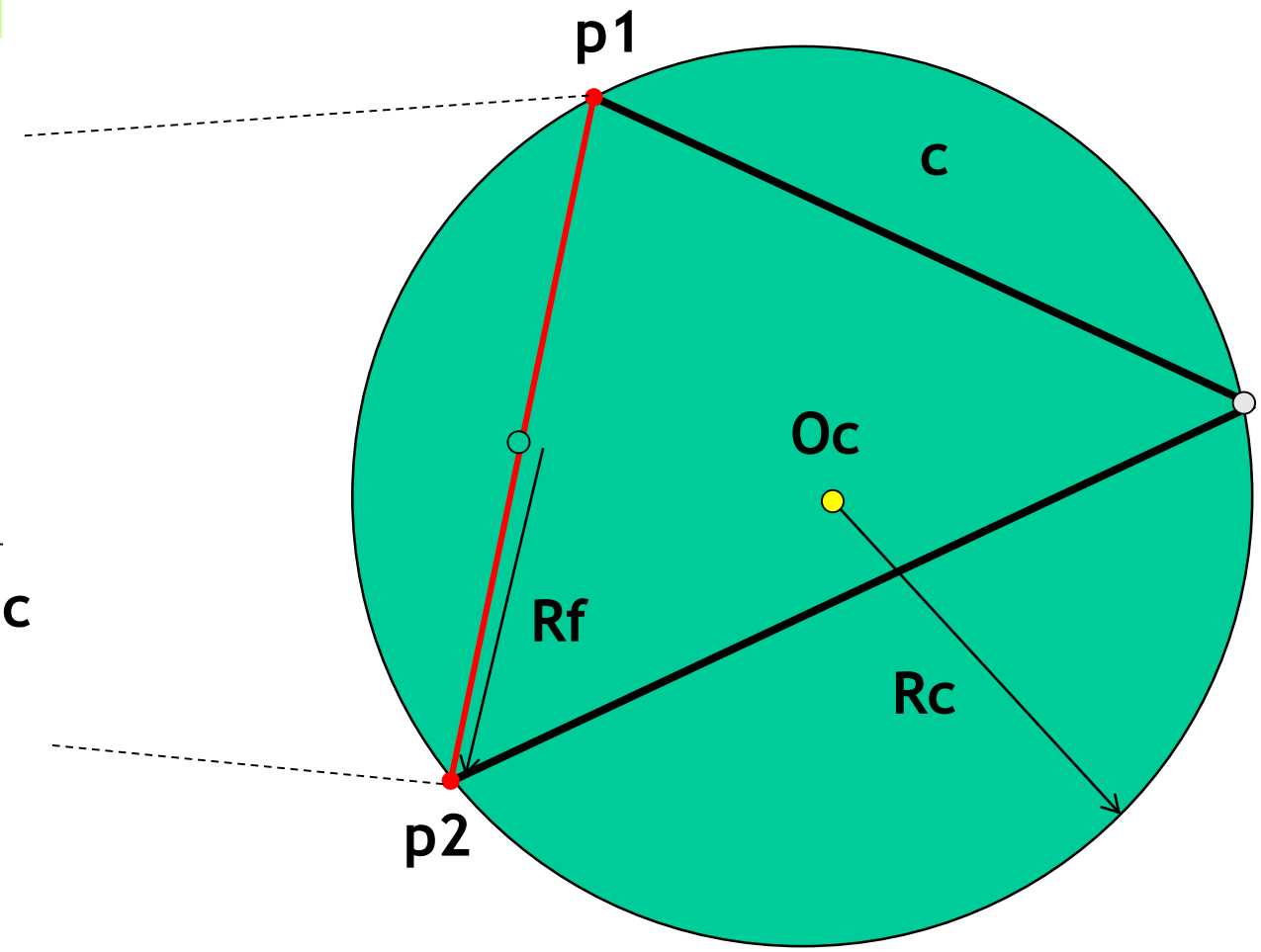




Computing Alpha-Shapes

is_alpha if
alpha > Rf

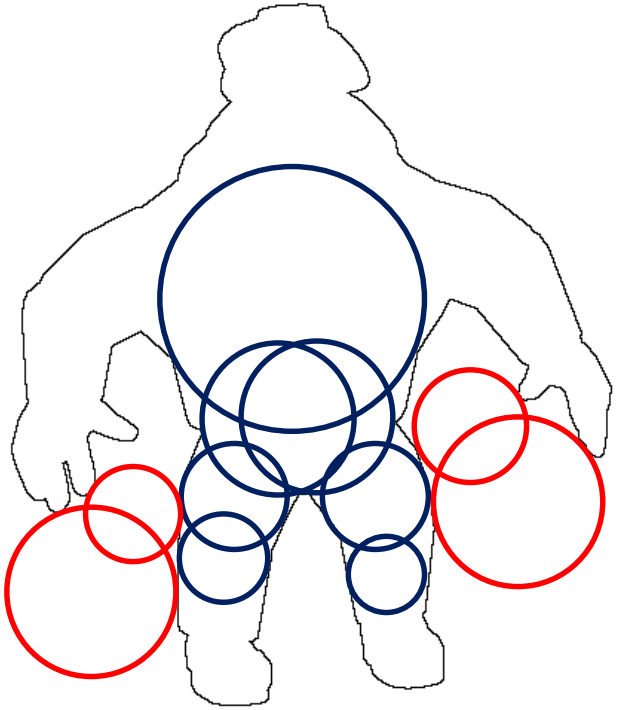




MEDIAL AXIS

Medial Axis

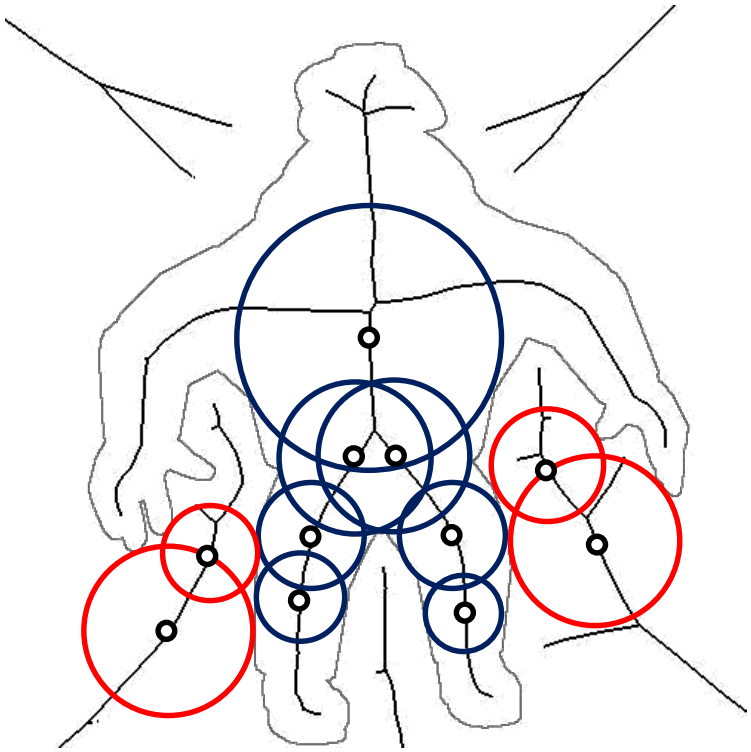
For a shape (curve/surface) a *Medial Ball* is a circle/sphere that only meets the shape tangentially, in at least two points.



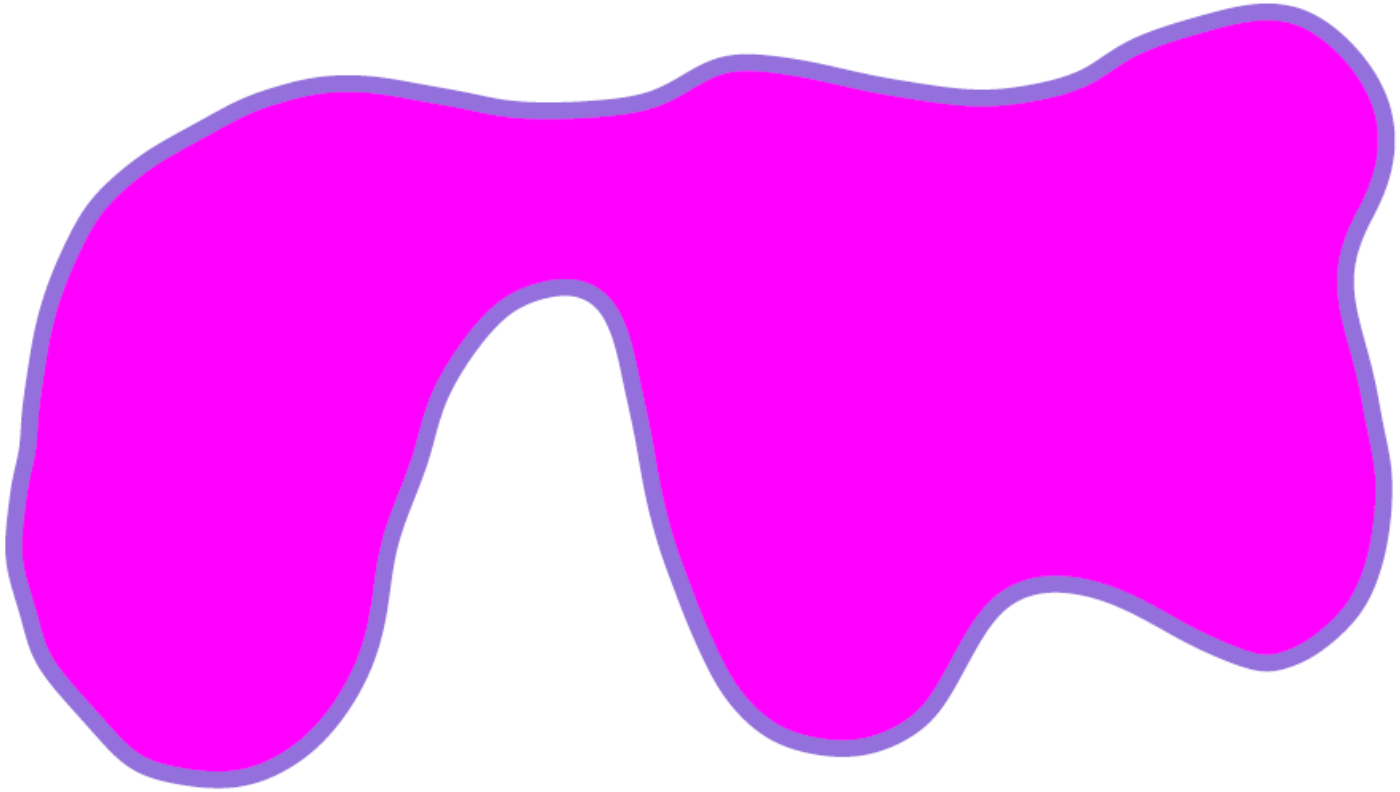
Medial Axis

For a shape (curve/surface) a *Medial Ball* is a circle/sphere that only meets the shape tangentially, in at least two points.

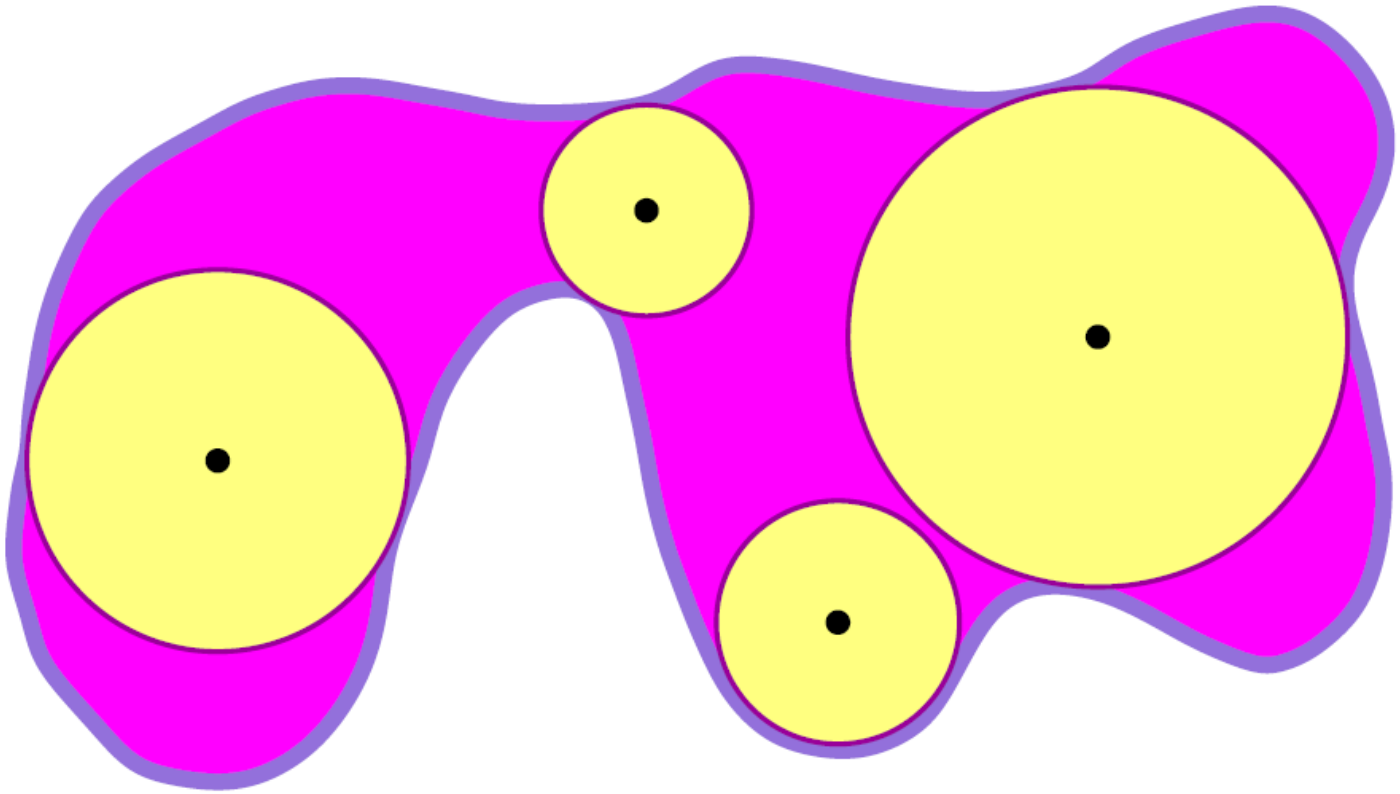
The centers of all such balls make up the *medial axis/skeleton*.



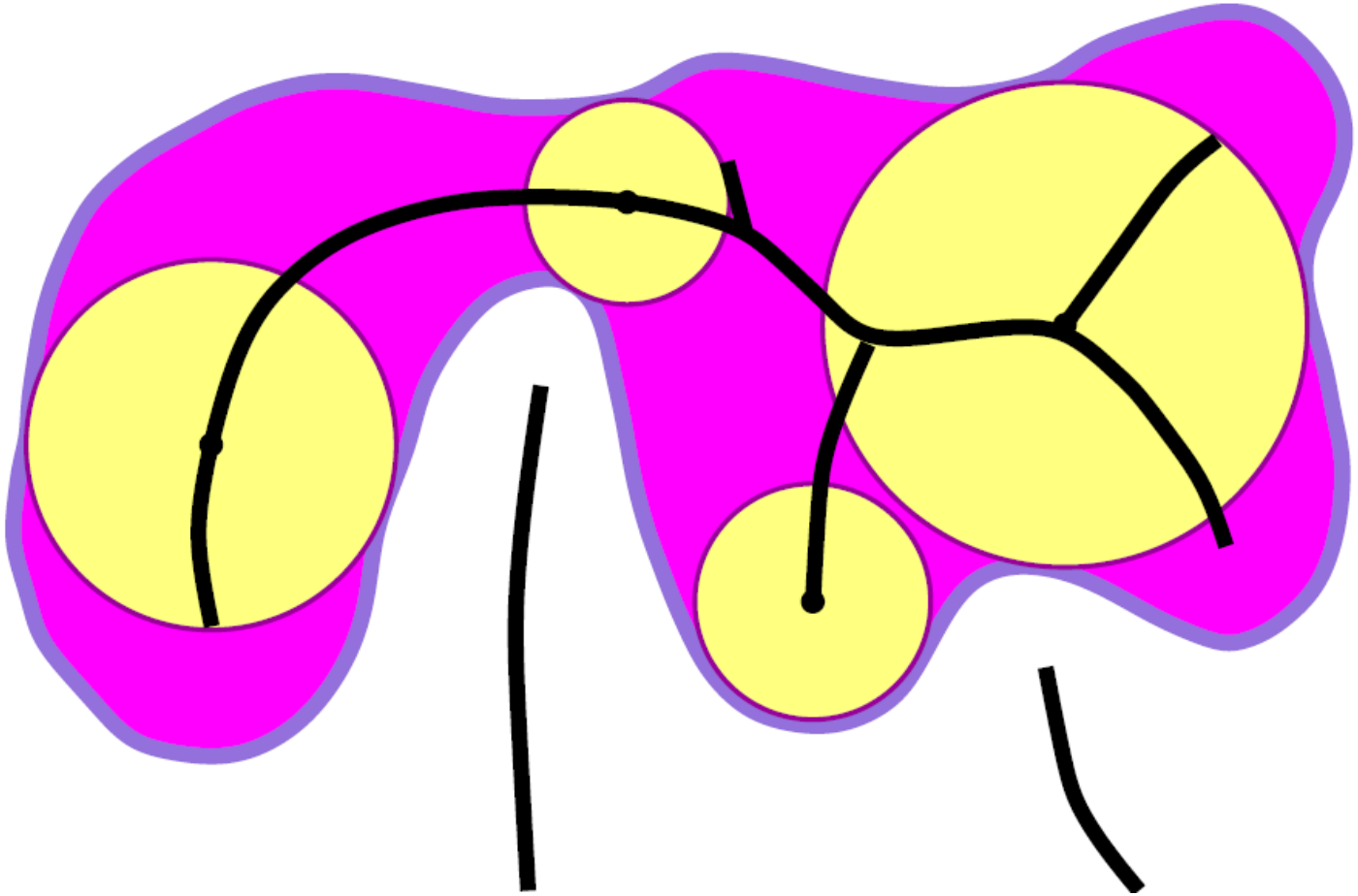
Medial Axis



Medial Axis



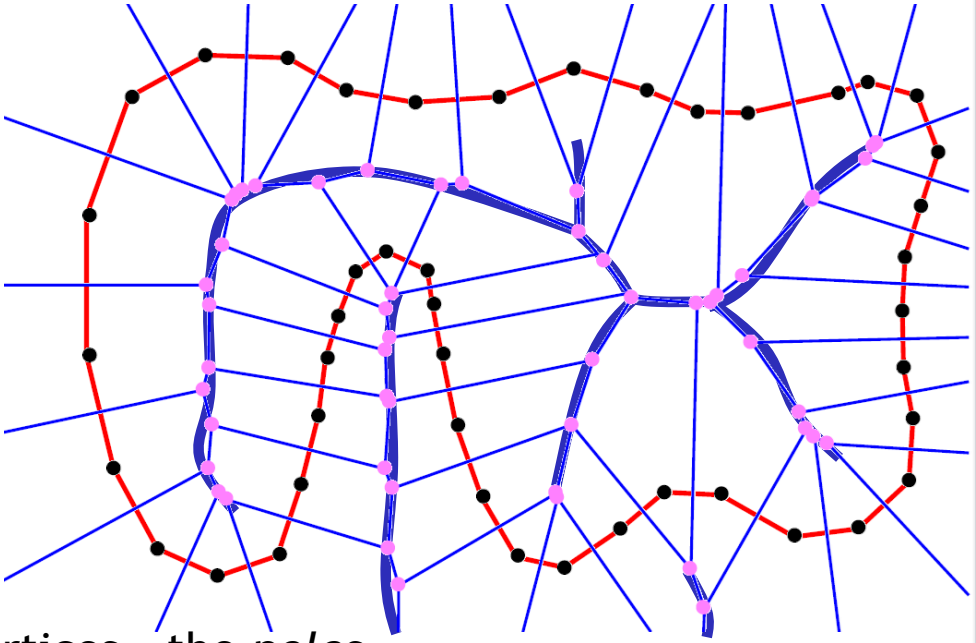
Medial Axis



Medial Axis

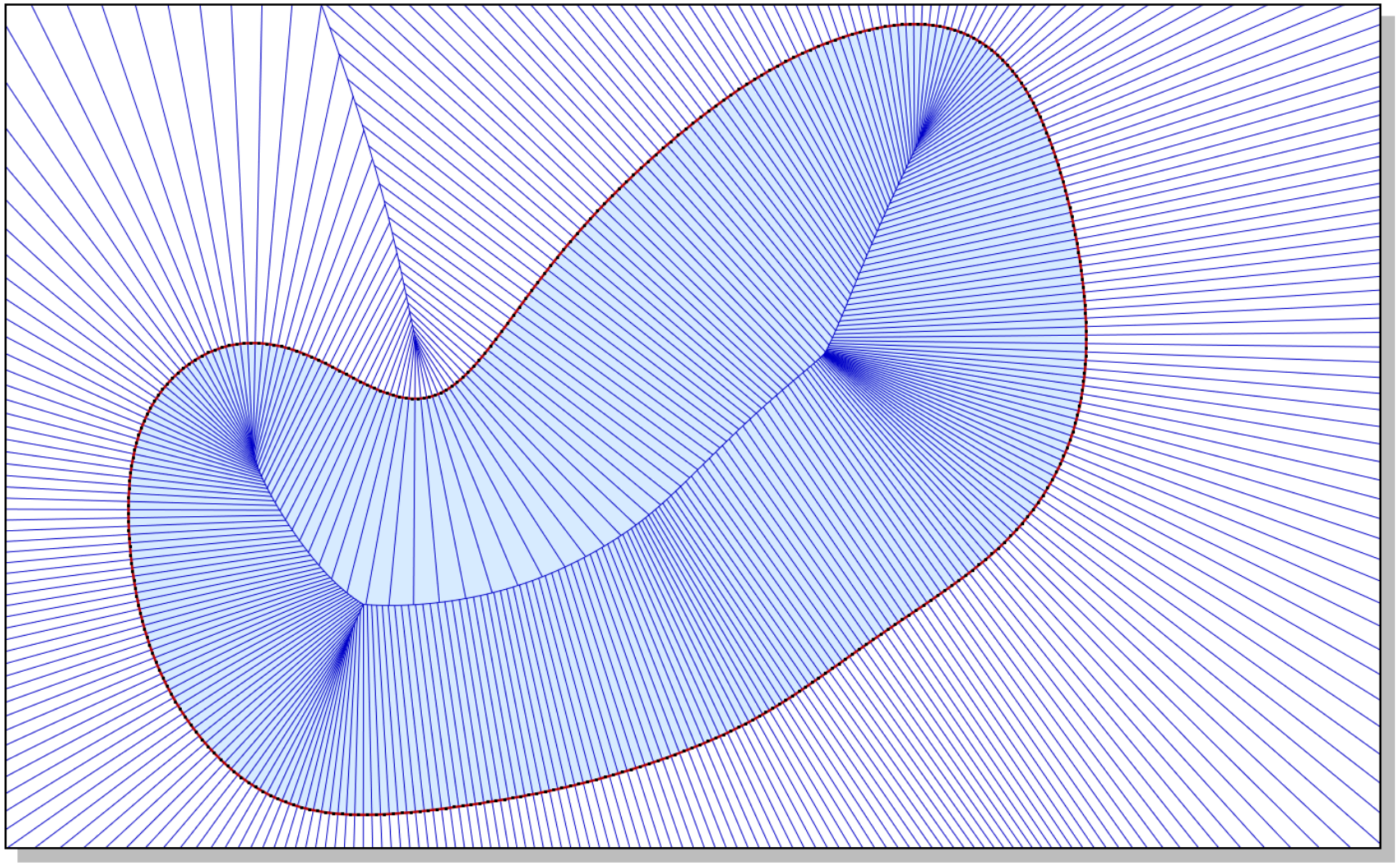
Observation*:

For a reasonable point sample, the medial axis is well-sampled by the Voronoi vertices.

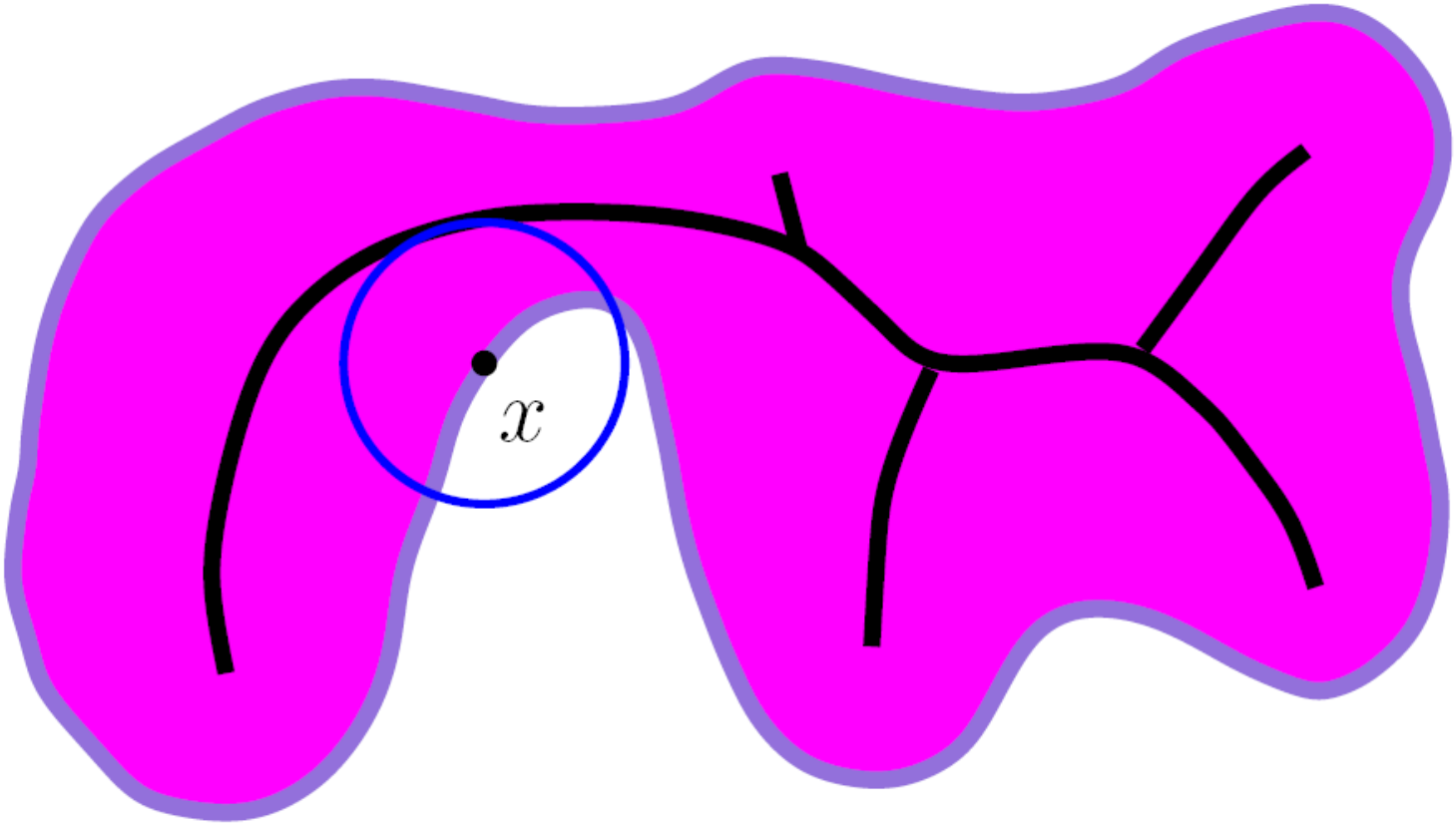


*In 3D, this is only true for a subset of the Voronoi vertices – the *poles*.

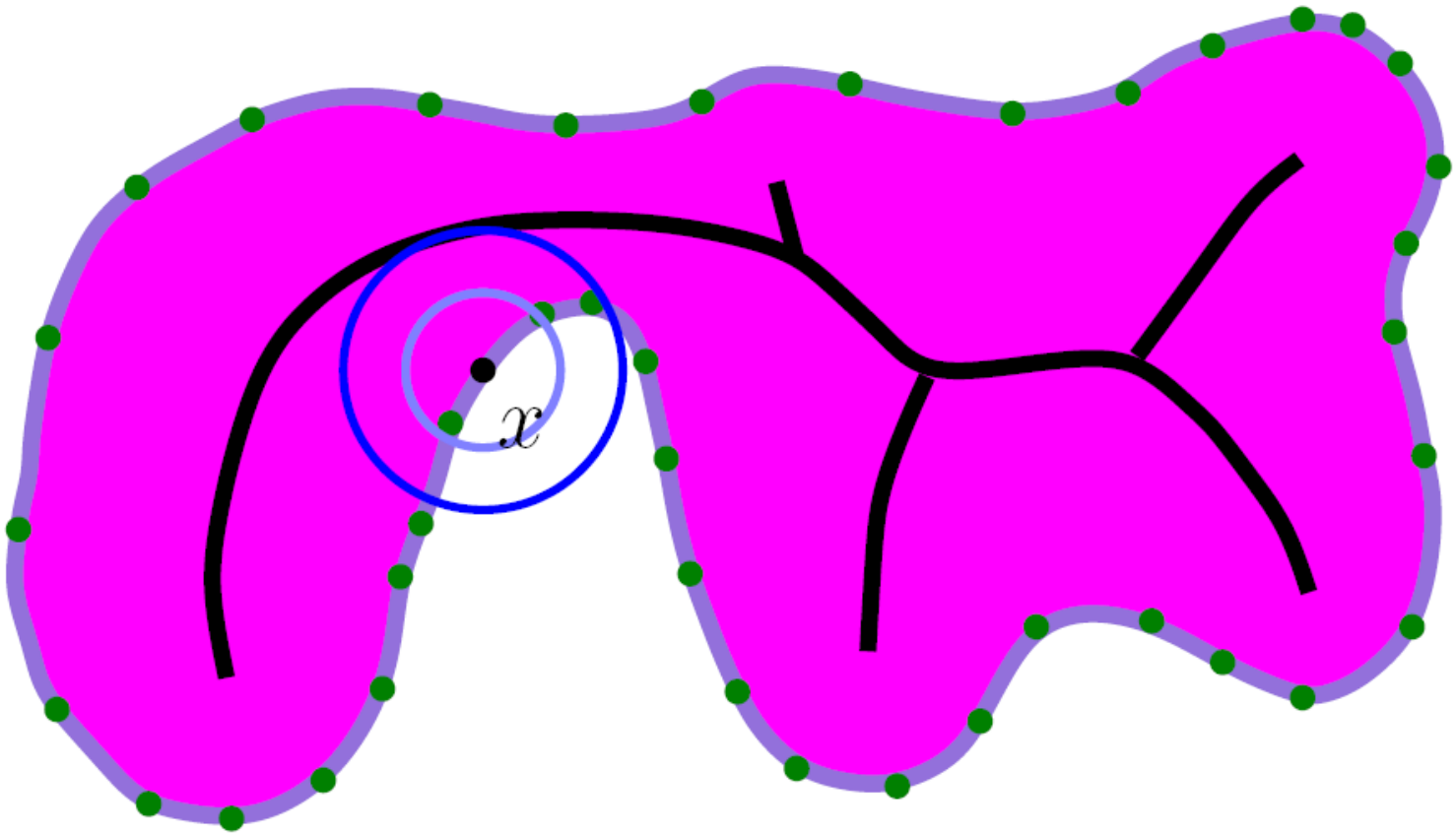
Voronoi & Medial Axis



Local Feature Size



Epsilon-Sampling



Crust [Amenta et al. 1998]

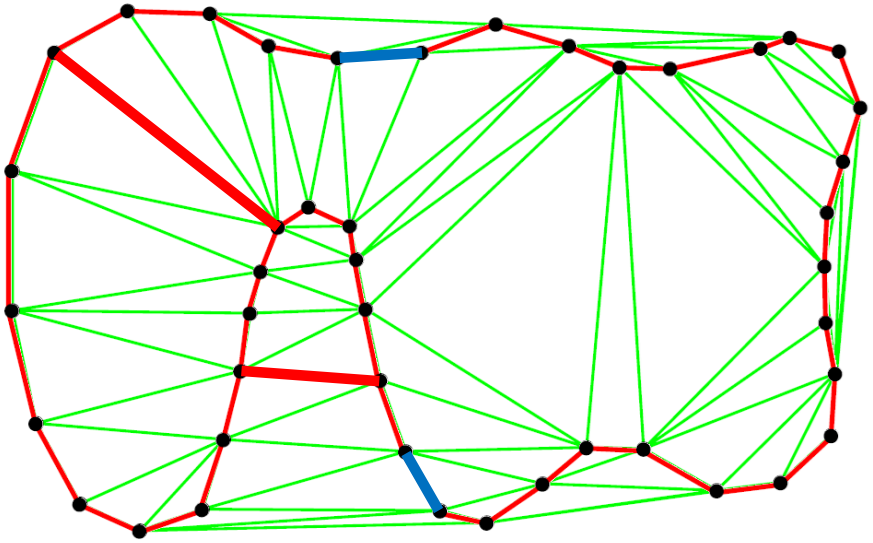
If we consider the Delaunay Triangulation of a point set, the shape boundary can be described as a subset of the Delaunay edges.

Q: How do we determine which edges to keep?

A: Two types of edges:

1. Those connecting adjacent points on the boundary
2. Those traversing the shape.

Discard those that traverse.



Crust [Amenta et al. 1998]

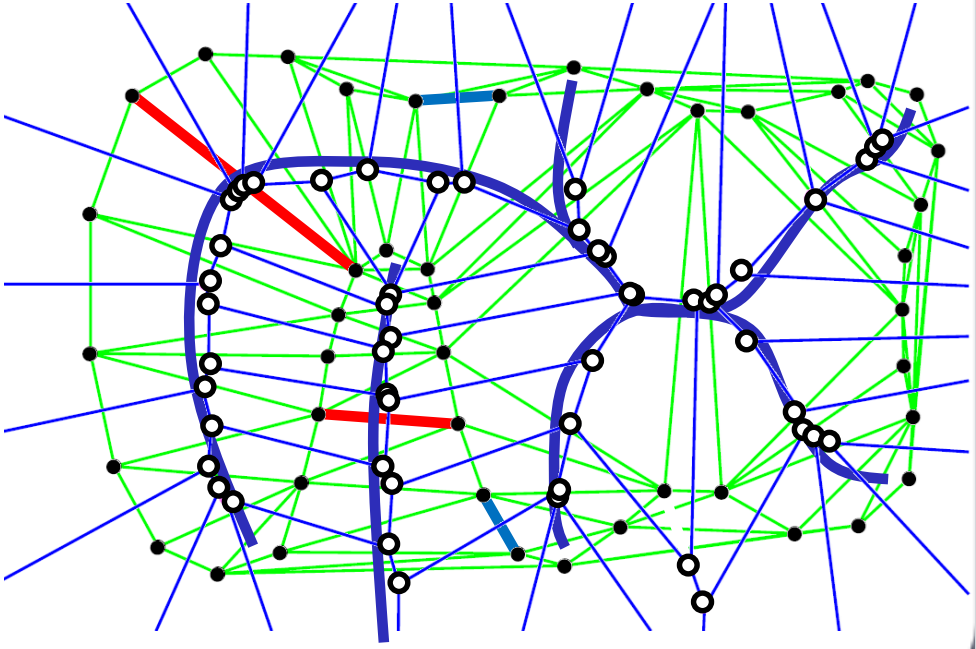
Observation:

Edges that traverse cross the medial axis.

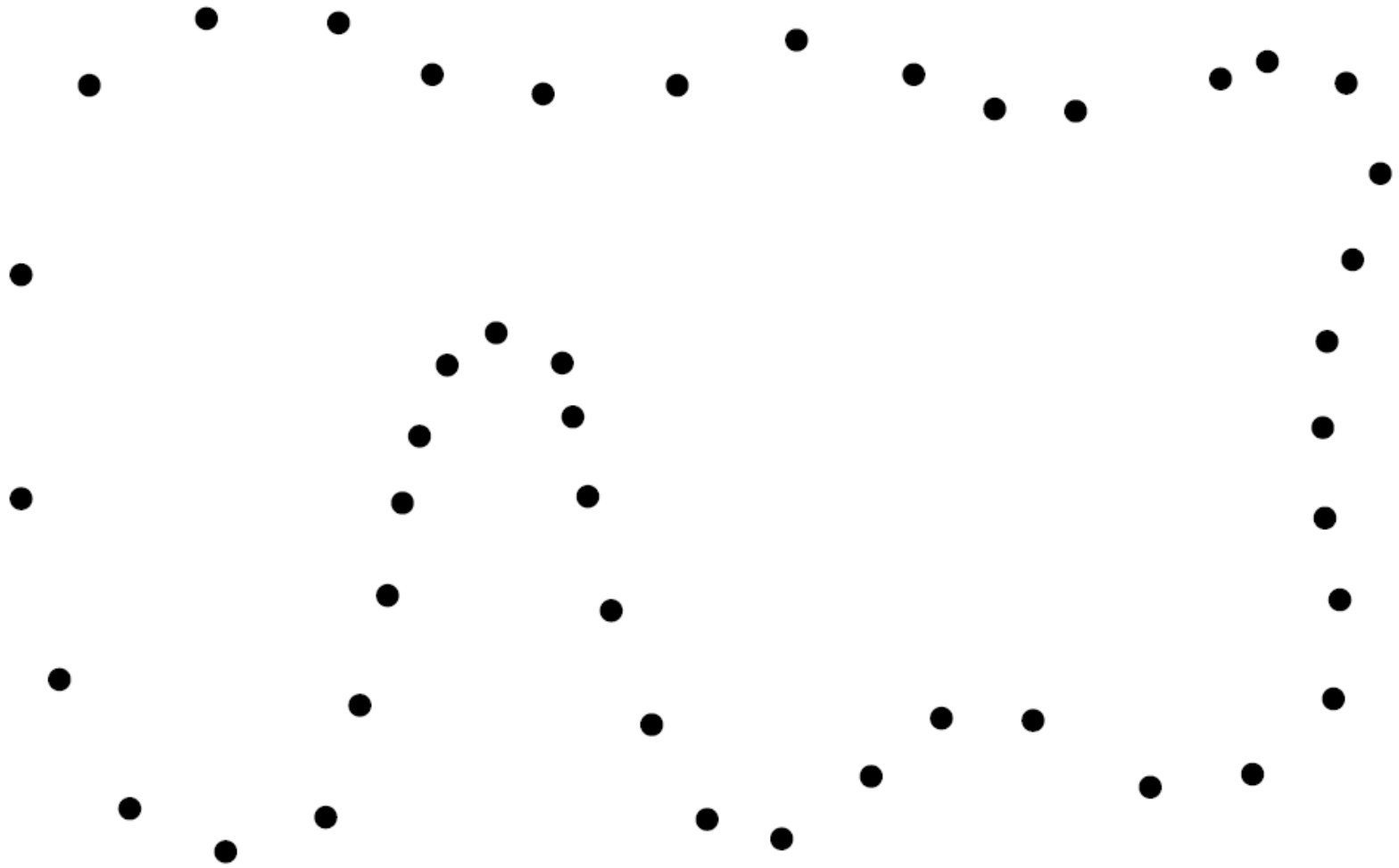
Although we don't know the axis, we can sample it with the Voronoi vertices.

Edges that traverse must

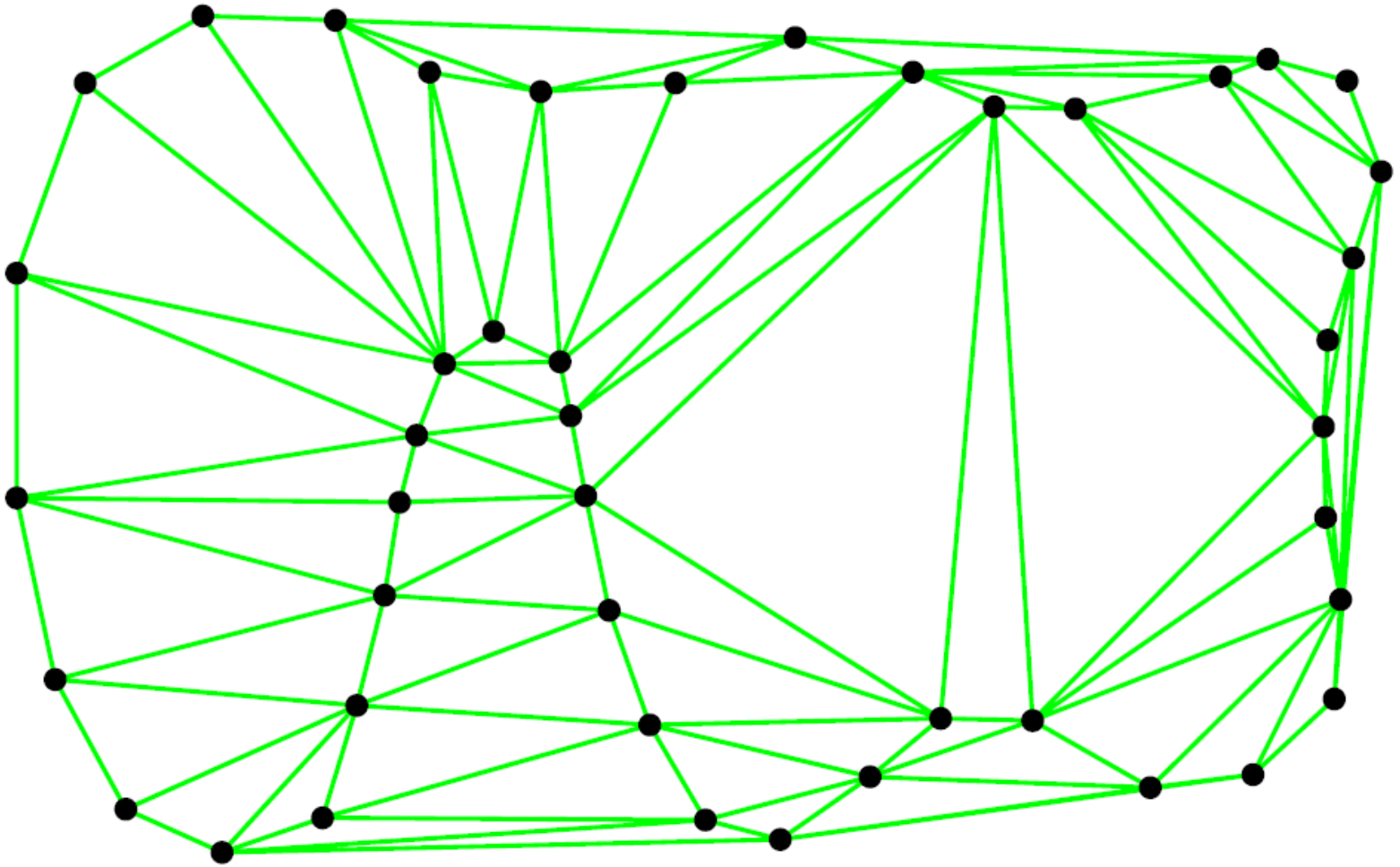
be near the Voronoi vertices.



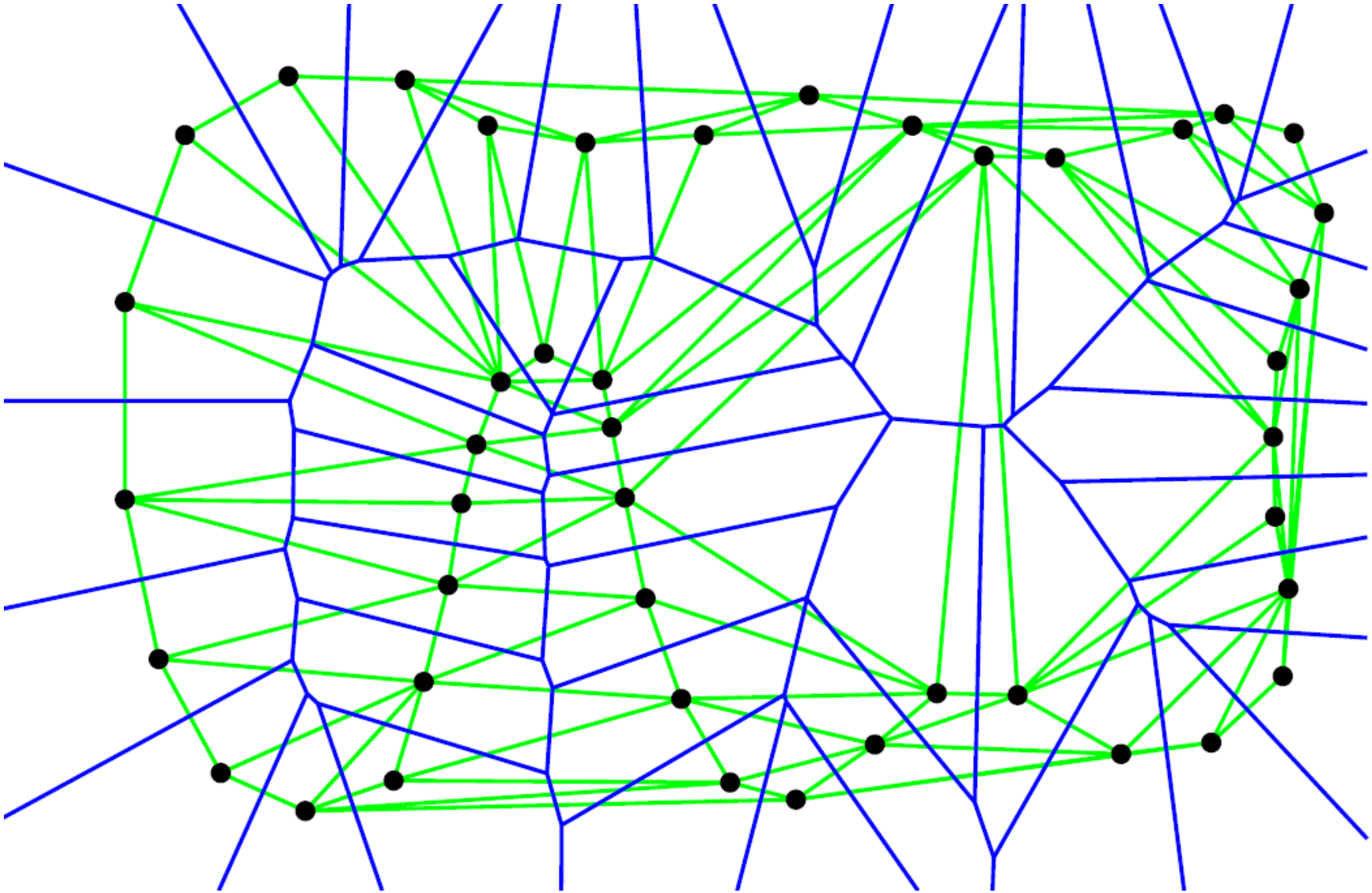
Crust [Amenta et al.]



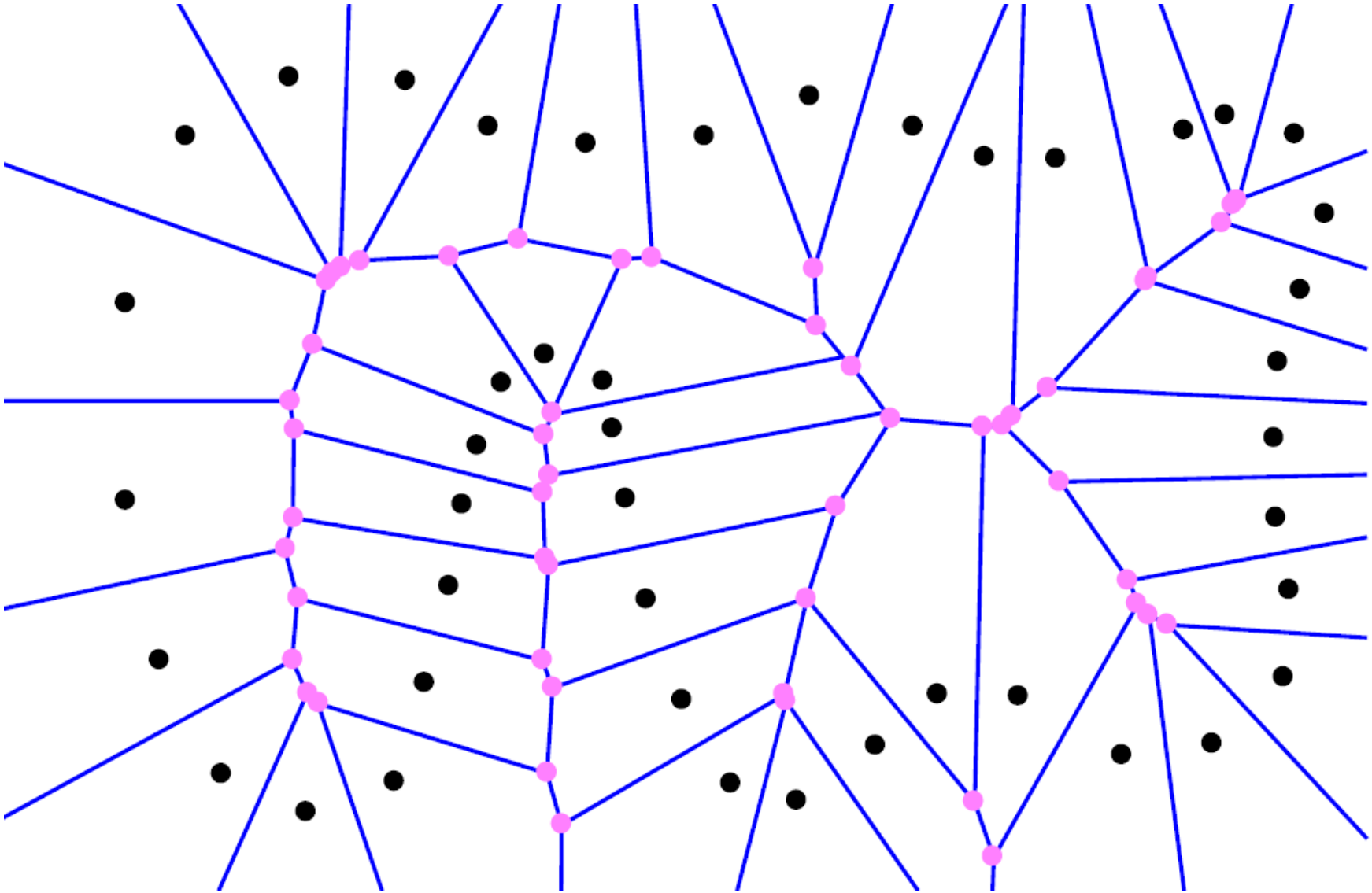
Delaunay Triangulation



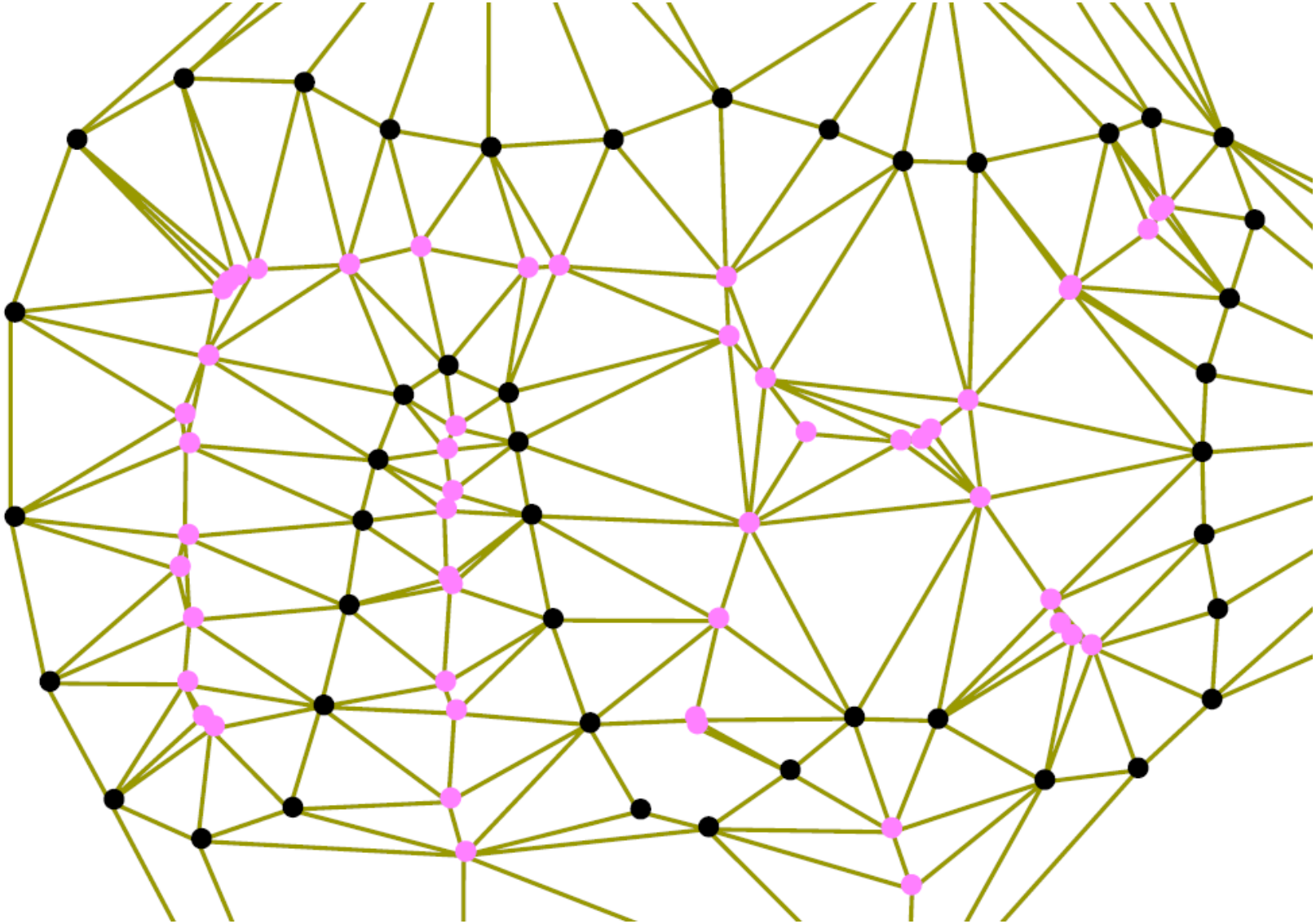
Delaunay Triangulation & Voronoi Diagram



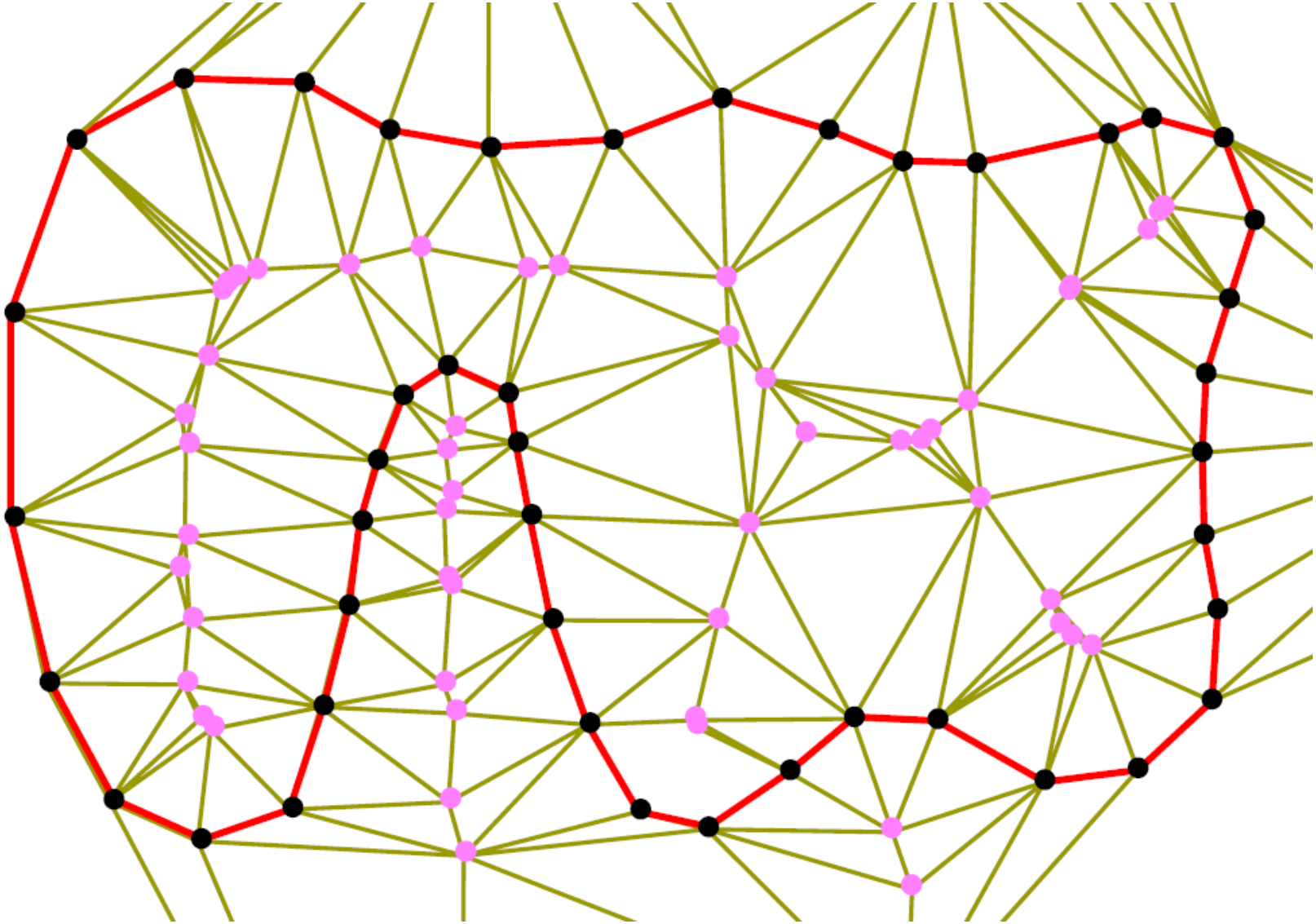
Voronoi Vertices



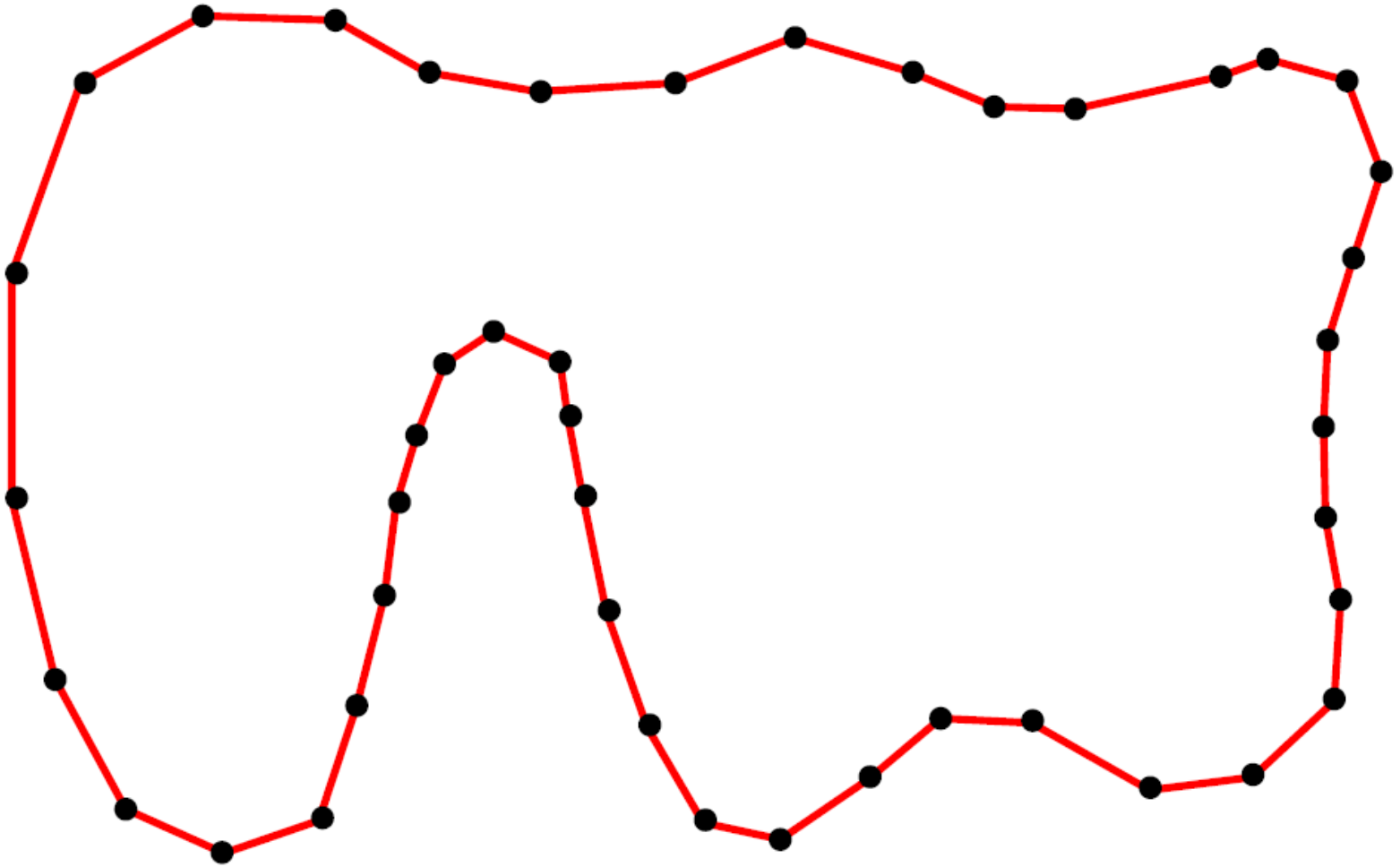
Refined Delaunay Triangulation



Crust



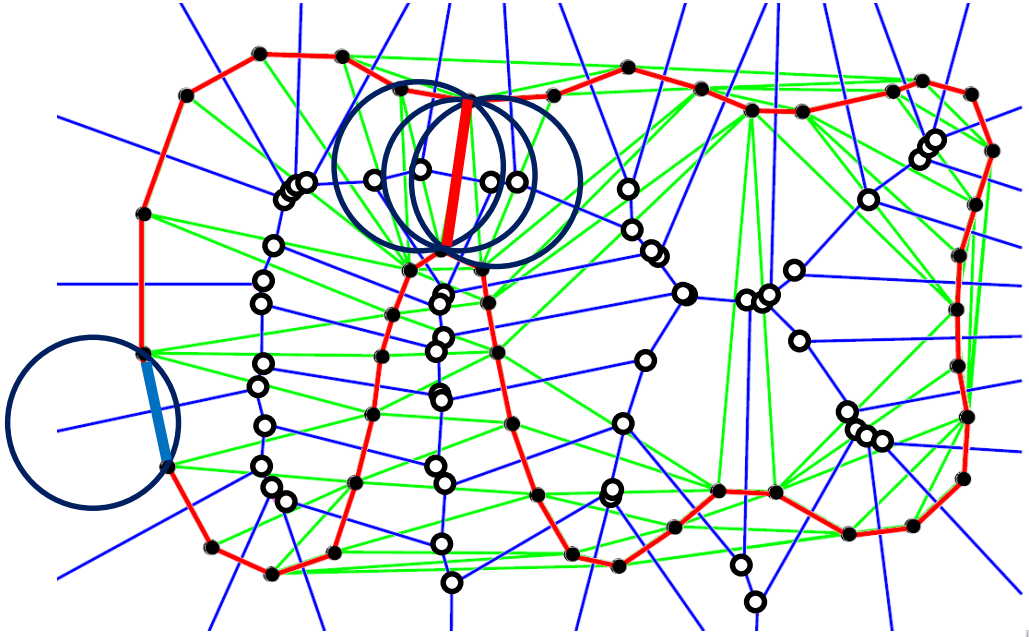
Crust



Crust (variant)

Algorithm:

1. Compute the Delaunay triangulation.
2. Compute the Voronoi vertices
3. Keep all edges for which there is a circle that contains the edge but no Voronoi vertices.

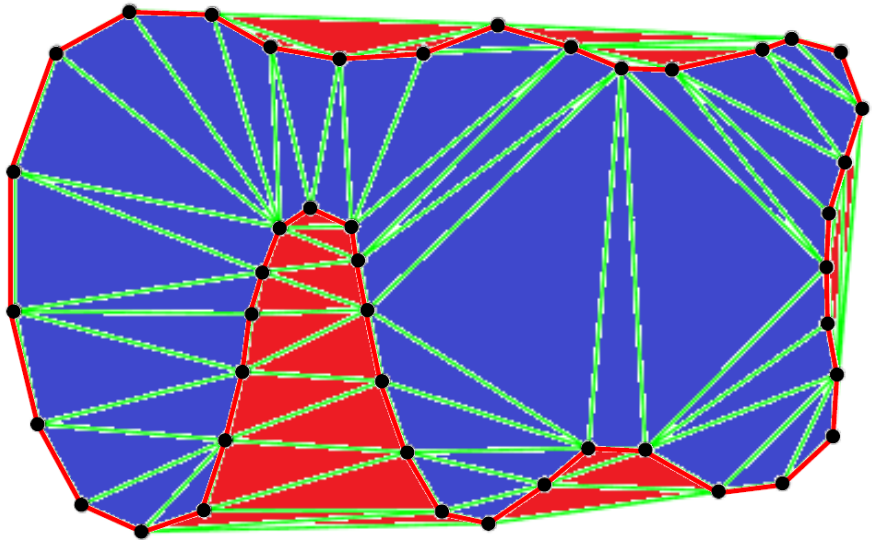


SPECTRAL « CRUST »

Space Partitioning

Given a set of points, construct the Delaunay triangulation.

If we label each triangle as inside/outside, then the surface of interest is the set of edges that lie between inside and outside triangles.

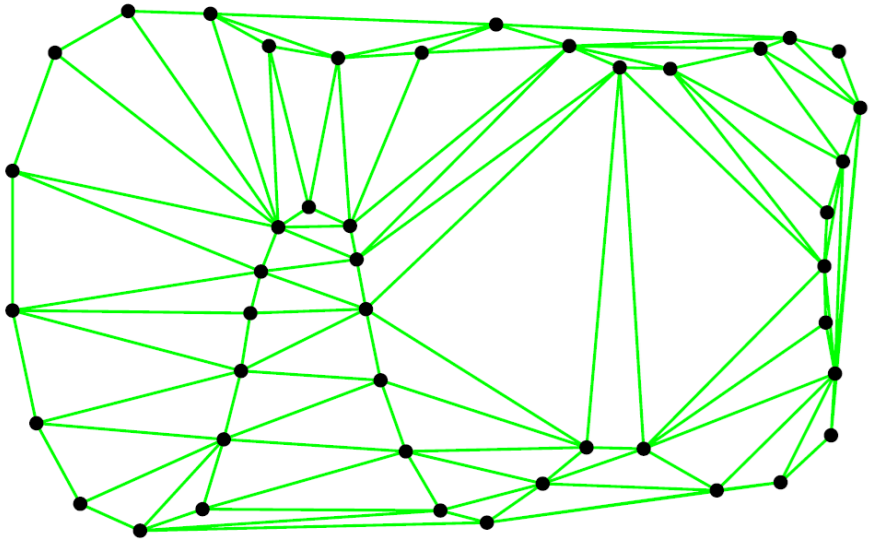


Space Partitioning

Q: How to assign labels?

A: Spectral Partitioning

Assign a weight to each edge indicating if the two triangles are likely to have the same label.



[Kolluri et al., 2004]

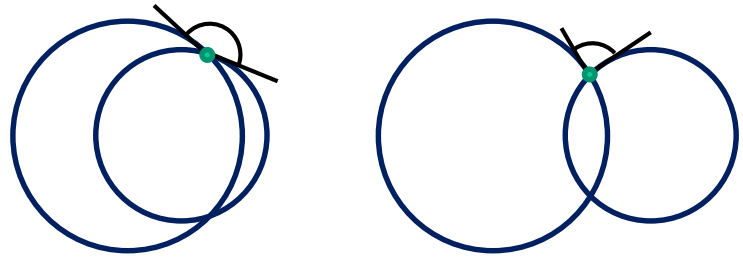
Space Partitioning

Assigning edge weights

Q: When are triangles on opposite sides of an edge likely to have the same label?

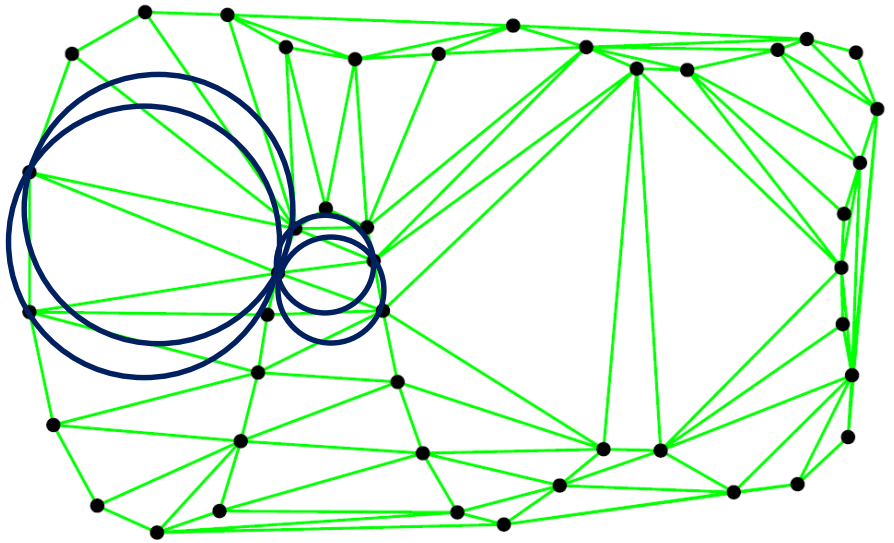
A: If the triangles are on the same side, their circumscribing circles intersect deeply.

Use the angle of intersection to set the weight.



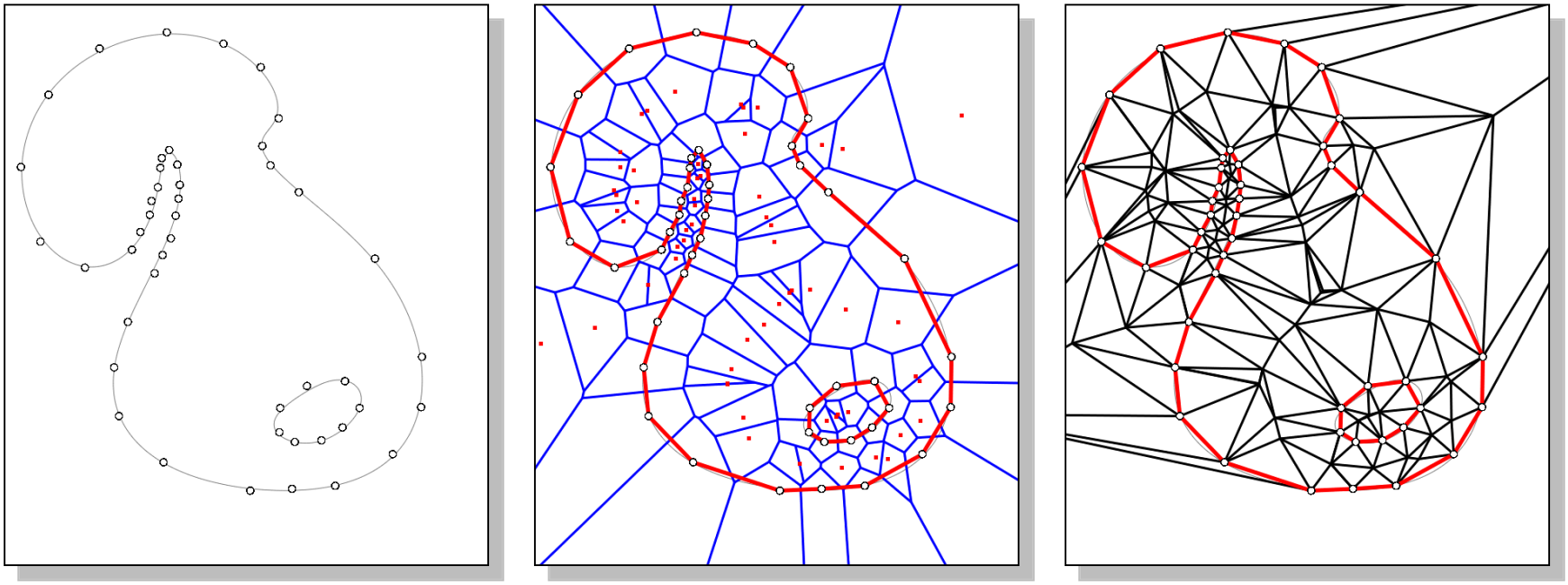
Large Weight

Small Weight



Crust

Several Delaunay algorithms provably correct

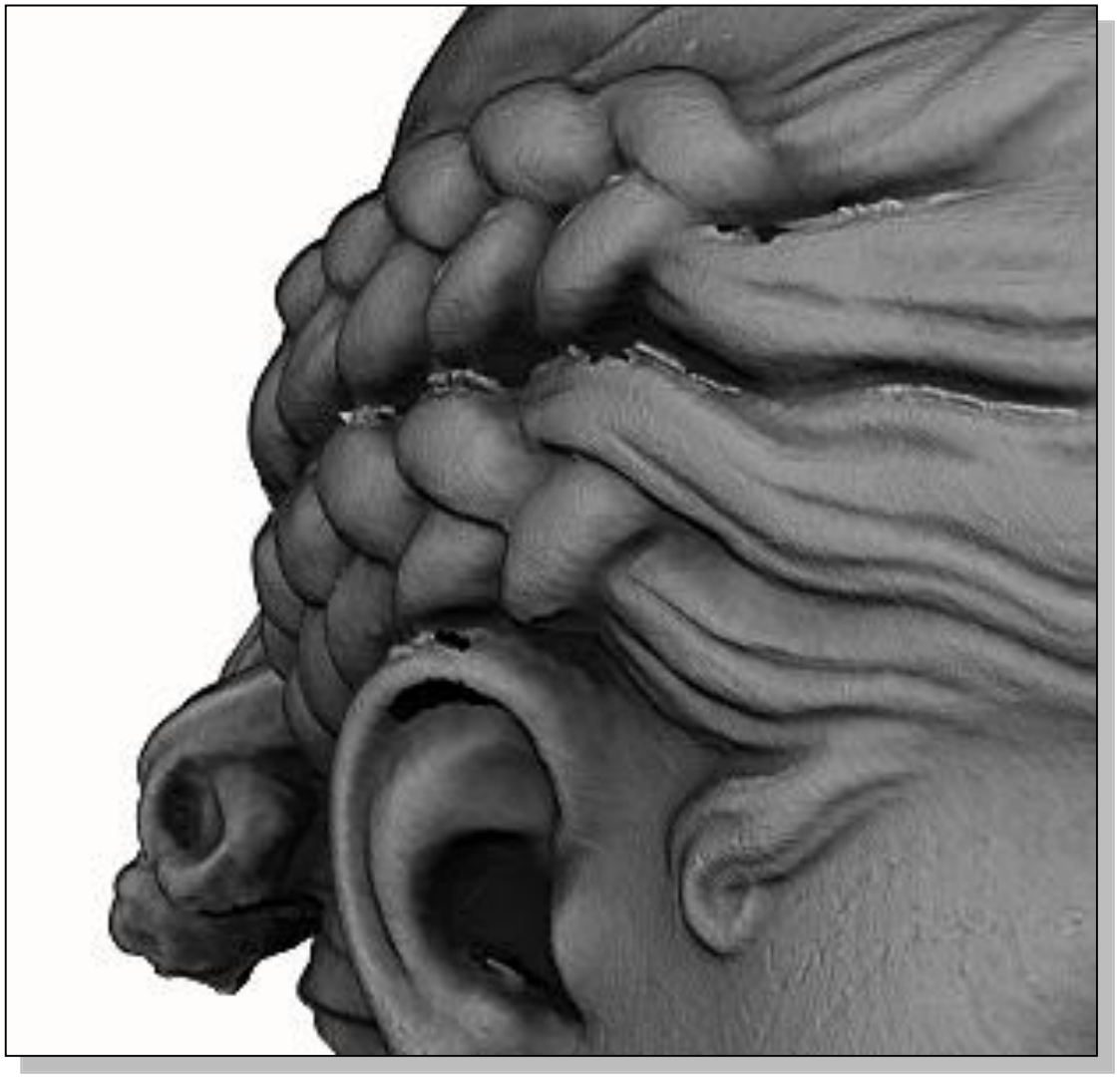


Delaunay-based

Several Delaunay algorithms are **provably correct**… in the absence of noise and undersampling.

perfect data ?

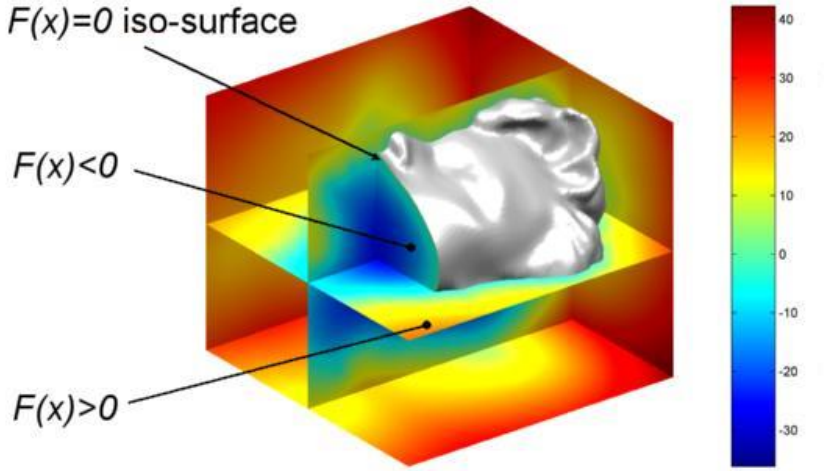
Noise & Undersampling



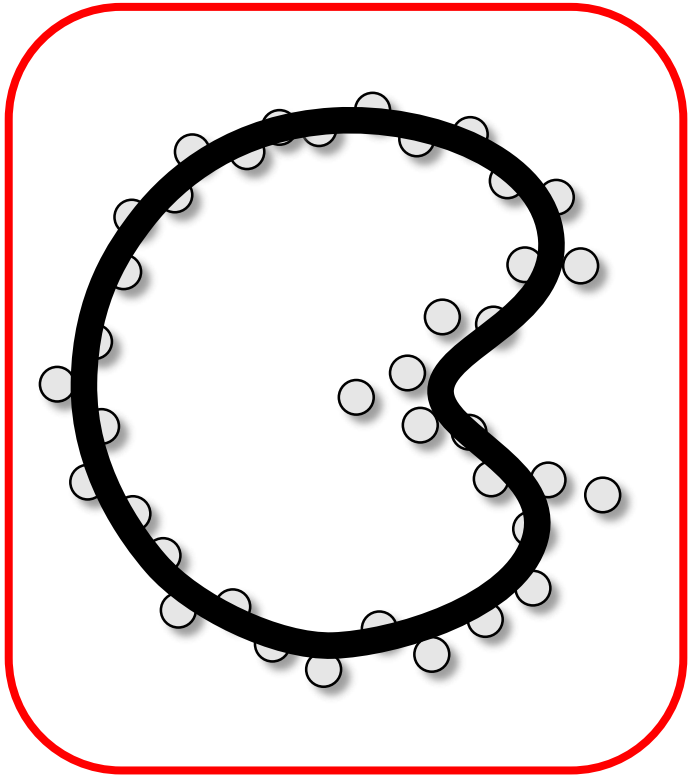
Delaunay-based

Several Delaunay algorithms are **provably correct**… in the absence of noise and undersampling.

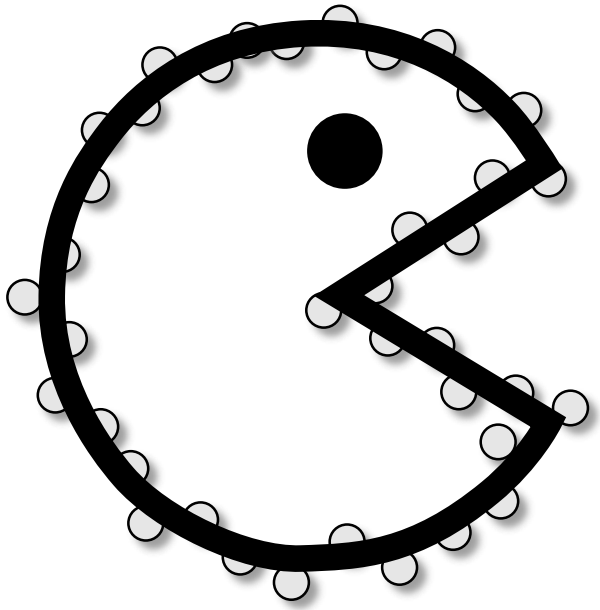
Motivates reconstruction by fitting **approximating** implicit surfaces



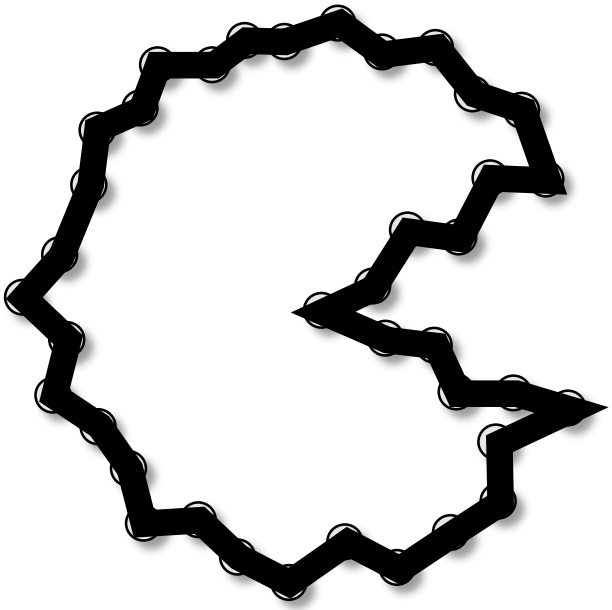
VARIATIONAL FORMULATIONS



Smooth



Piecewise Smooth



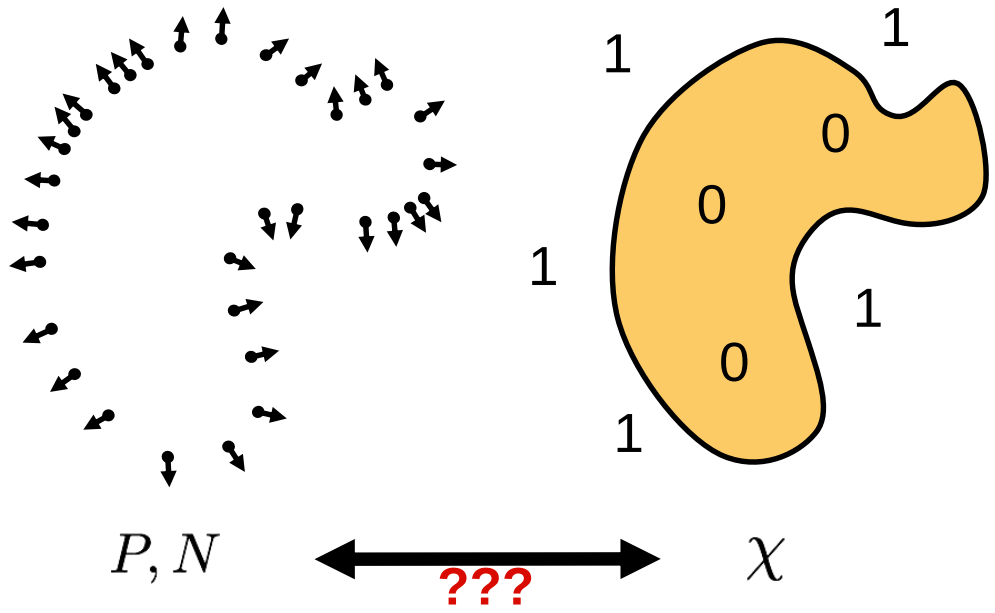
“Simple”

Poisson Surface Reconstruction

[Kazhdan et al. SGP‘06]

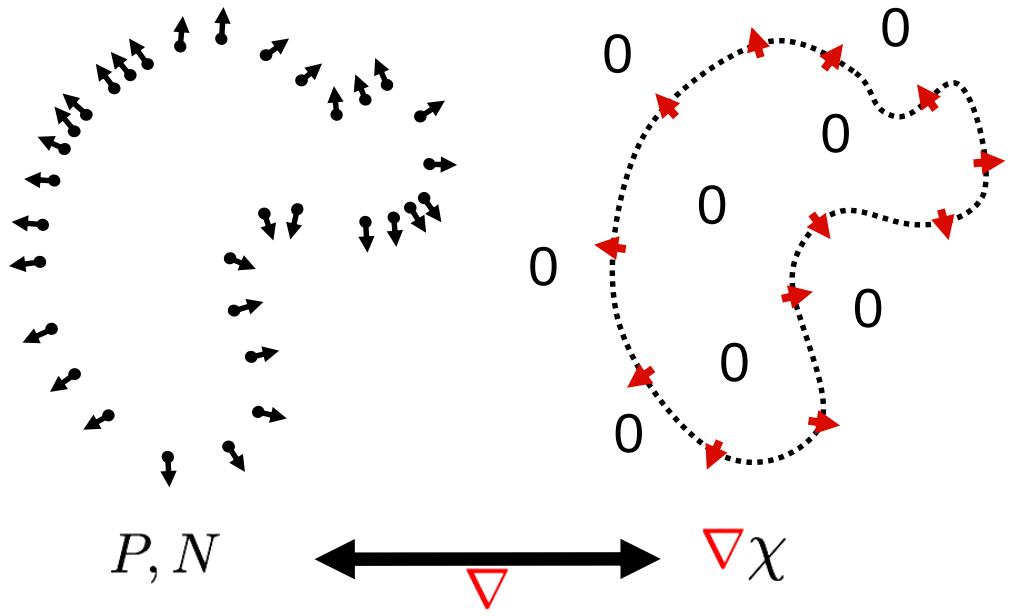
Indicator Function

Construct indicator function from point samples



Indicator Function

Construct indicator function from point samples



splatted normals

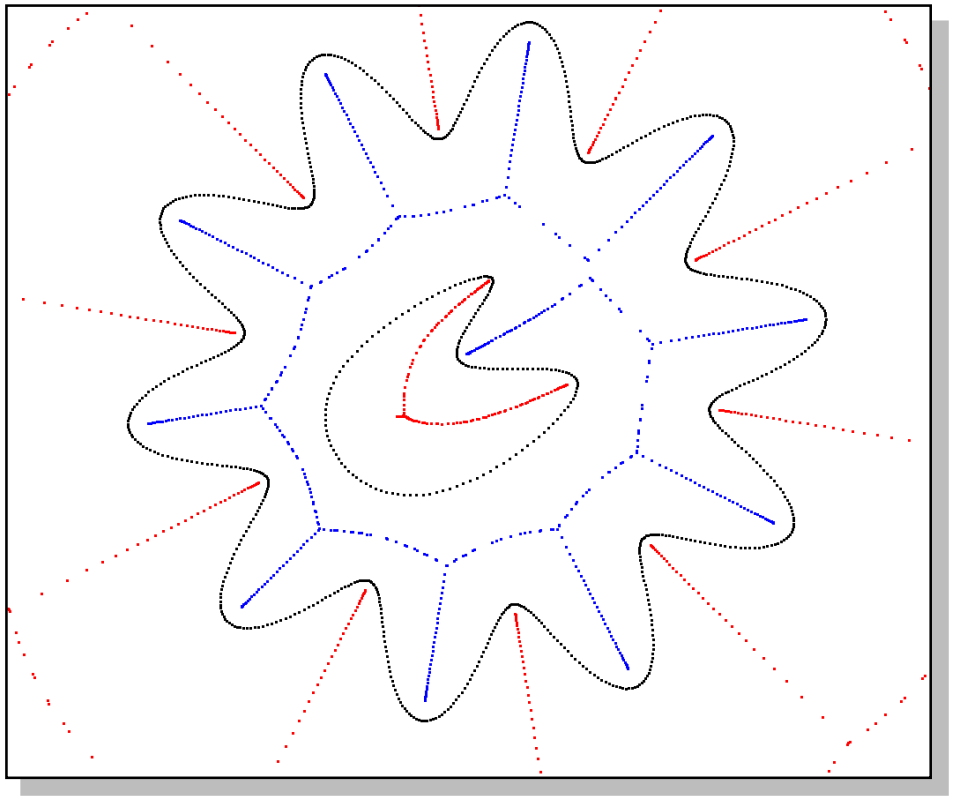
$$\min_{\chi} \int \|\nabla\chi(\mathbf{x}) - \mathcal{N}(\mathbf{x})\|_2^2 \, dx$$

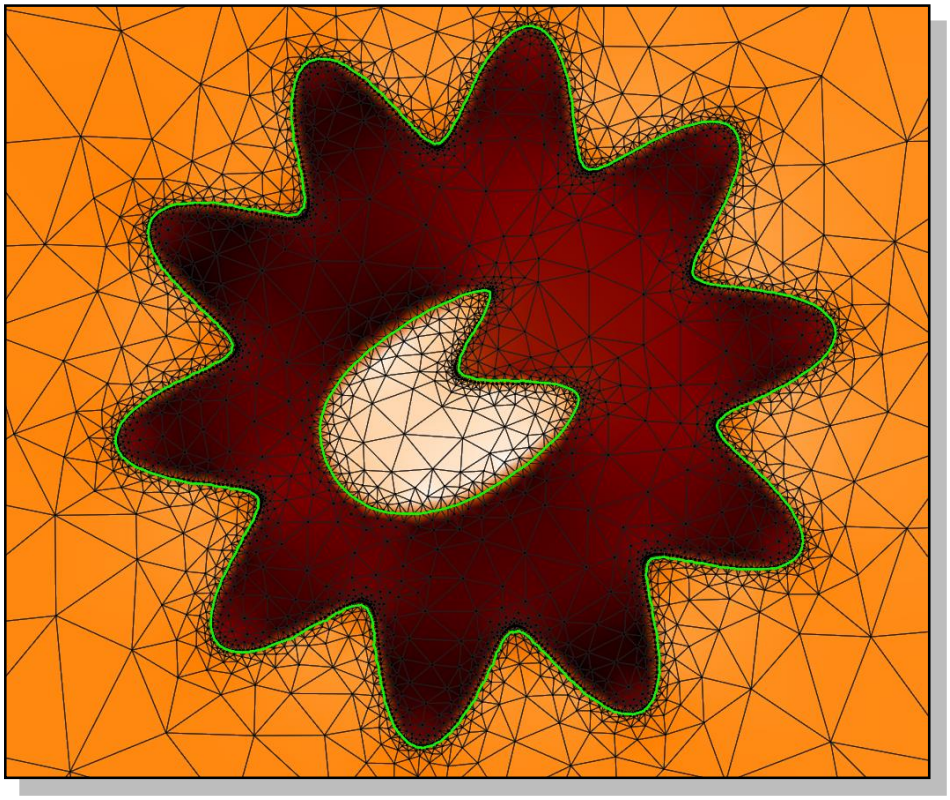
variational calculus

$$\Delta\chi = \nabla \cdot \mathcal{N}$$

sparse linear system

2D Poisson Reconstruction





Poisson Reconstruction

Requires oriented normals, as many other implicit approaches.

Poisson Reconstruction

Requires oriented normals, as many other implicit approaches.

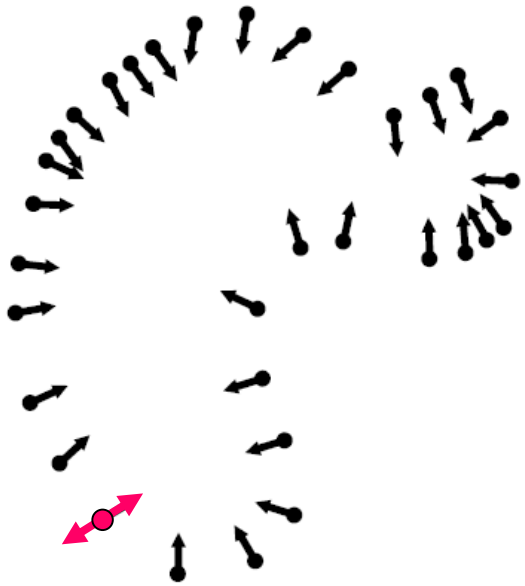
Normal estimation

Normal orientation

} ill-posed problems

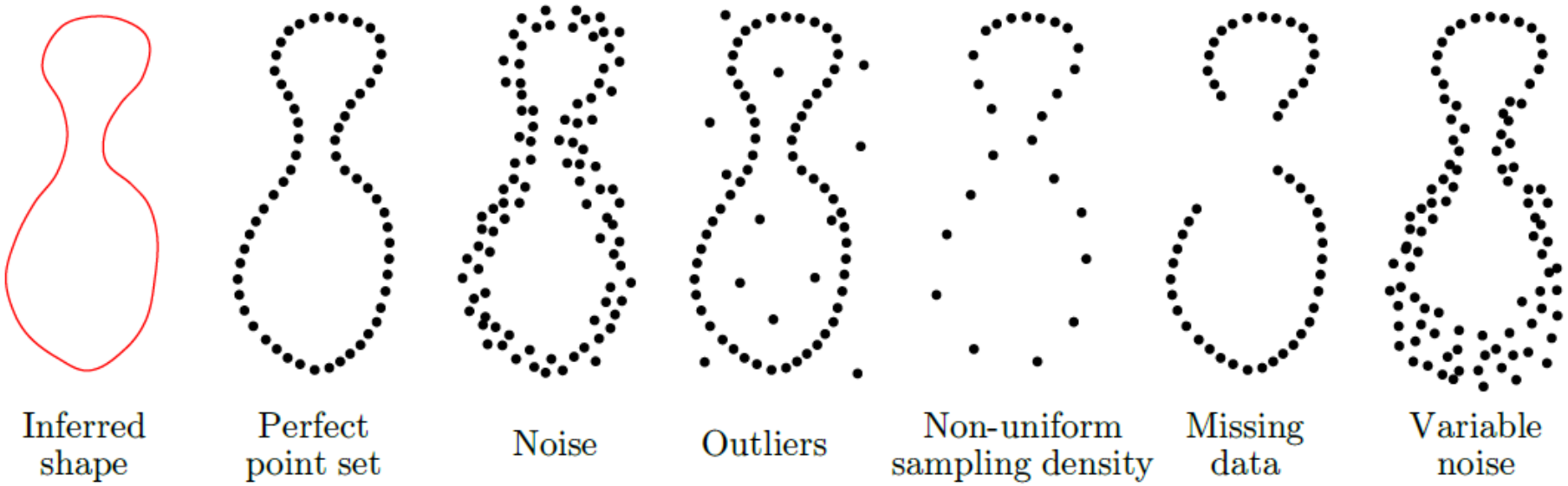
Poisson Reconstruction

Can we deal with unoriented normals?



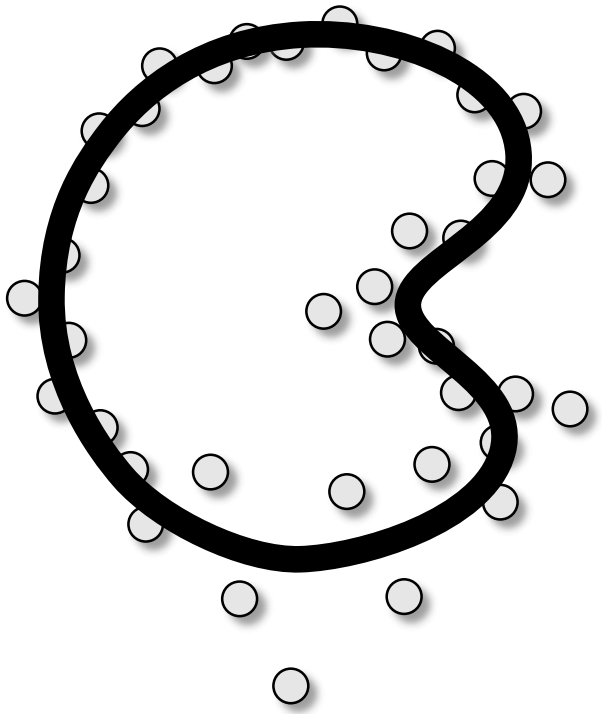
QUEST FOR ROBUSTNESS

Quest for Robustness



?

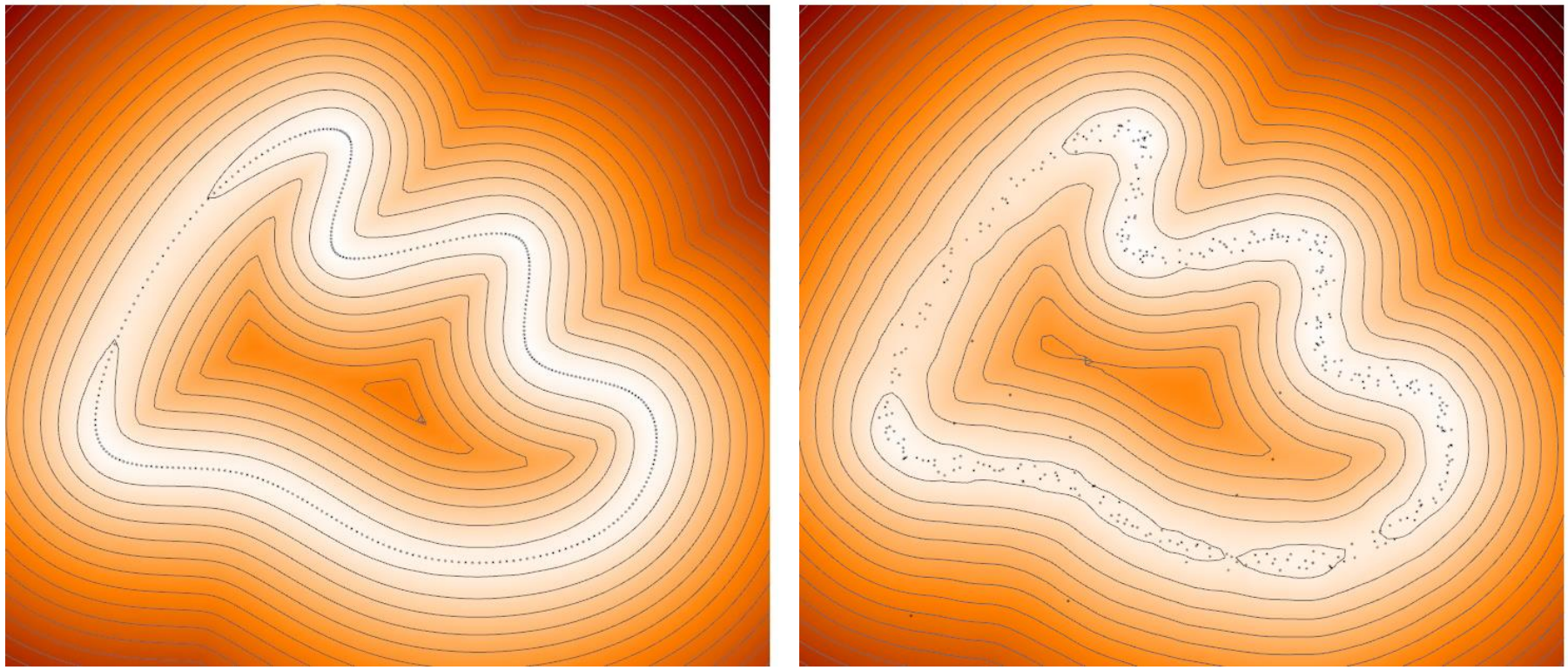
Noise-adaptive Reconstruction



Smooth

[Giraudot, Cohen-Steiner, A. SGP‘13]

Robust Distance Function

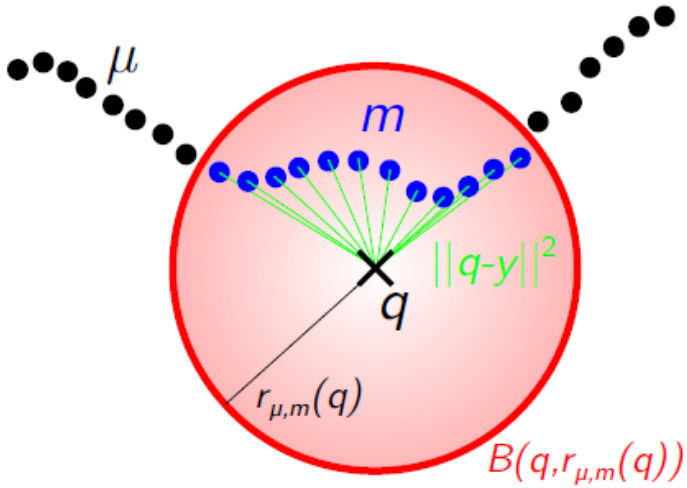


[Chazal, Cohen-Steiner, Mérigot 11] Noise and outlier robust. Based on optimal transport distance between geometric measures (Wasserstein-2 distance)

Robust Distance Function

Unsigned distance function to a measure [Chazal et al., 2011]

$$d^2_{\mu,m} : \mathbb{R}^n \to \mathbb{R},\ q \mapsto \frac{1}{m} \int_{B(q, r_{\mu,m}(q))} \|q - y\|^2 \mathrm{d}\mu(y)$$



Robust Distance Function

Unsigned distance function to a measure [Chazal et al., 2011]

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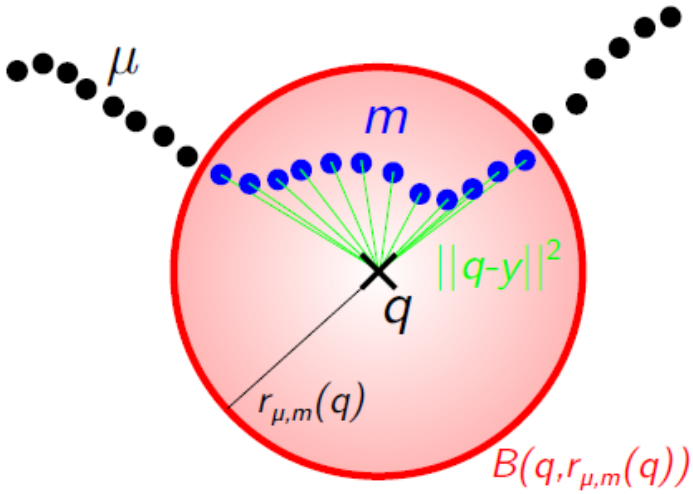
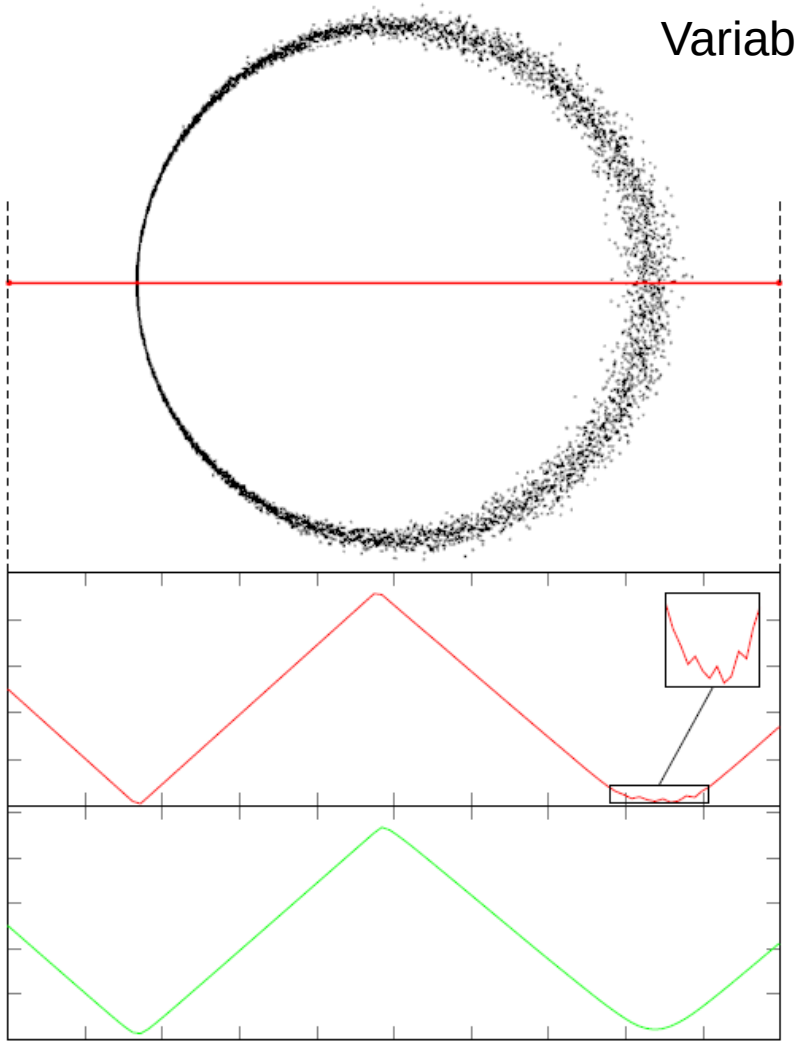


Note: scale parameter m

- User-specified
- Depends on point set properties
- Global: not noise-adaptive

Non-adaptive Distance Function

Variable noise

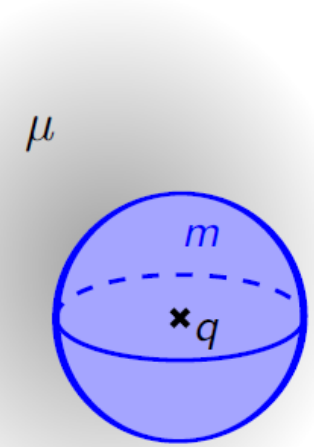


$K = 6$

$K = 70$

Case of Ambient Noise

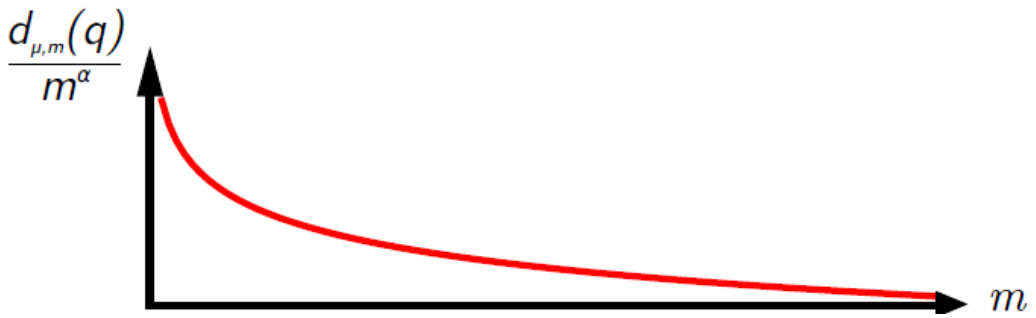
Uniform measure in d-dimensional space



$$d^2_{\mu,m}(q) = c \cdot m^{\frac{2}{d}}$$

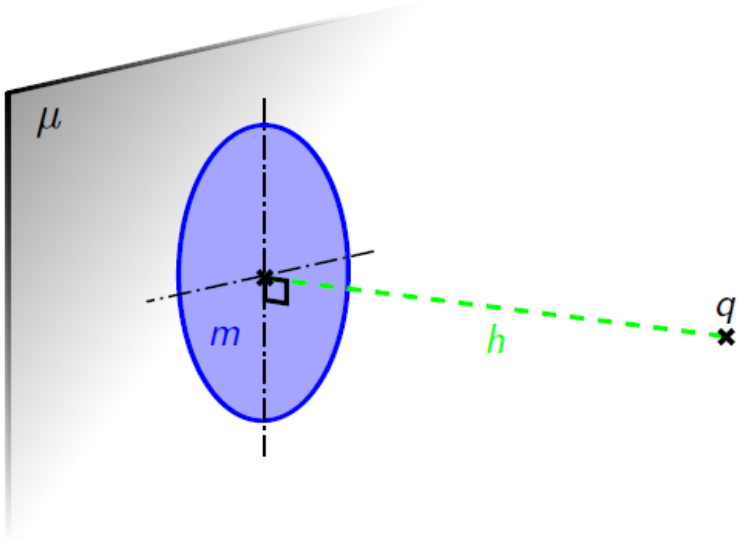
$$d_{\mu,m}(q) \propto m^{\frac{1}{d}} \text{ for } q \text{ fixed}$$

$$\frac{d_{\mu,m}(q)}{m^{\alpha}} \text{ decreasing for } \alpha > \frac{1}{d}$$



Case of Submanifold

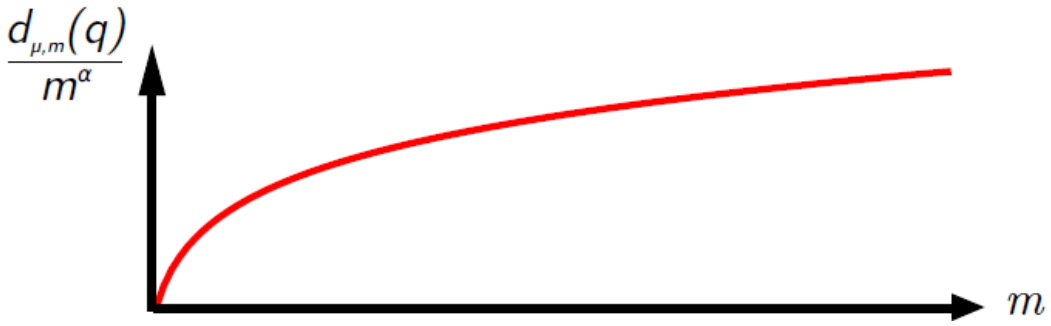
Uniform measure on k-submanifold



$$d^2_{\mu,m}(q) = c \cdot m^{\frac{2}{k}} + h^2$$

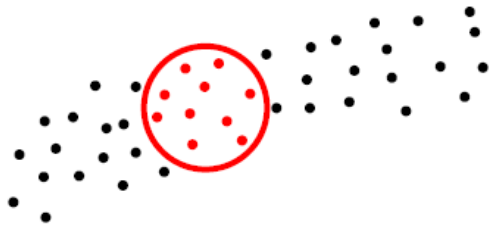
$$d_{\mu,m}(q) \propto m^{\frac{1}{k}} \text{ for } q \text{ fixed}$$

$$\frac{d_{\mu,m}(q)}{m^{\alpha}} \text{ increasing for } \alpha < \frac{1}{k}$$



Noisy Case

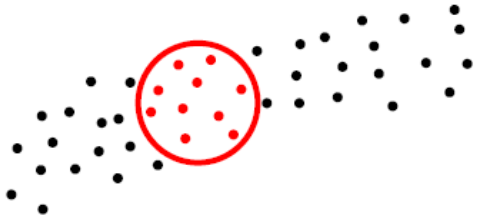
Scale $m = 10$ nearest neighbors



- Apparent dimension $= 2$
- Ambiant noise in $2D$
- $\frac{d_{\mu,m}(q)}{m^{\alpha}}$ decreasing for $\alpha > \frac{1}{2}$

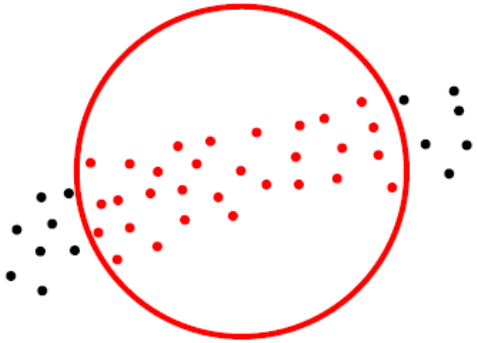
Noisy Case

Scale $m = 10$ nearest neighbors

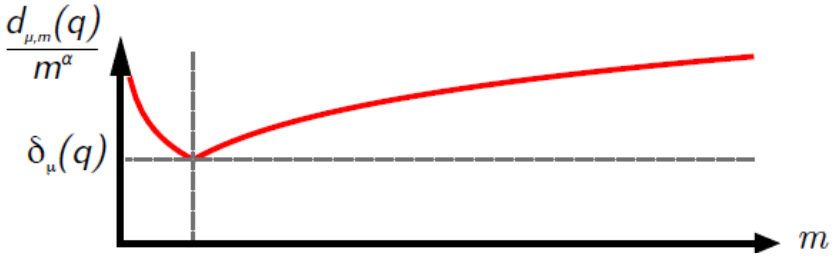


- Apparent dimension $= 2$
- Ambiant noise in $2D$
- $\frac{d_{\mu,m}(q)}{m^\alpha}$ decreasing for $\alpha > \frac{1}{2}$

Scale $m = 30$ nearest neighbors



- Apparent dimension $= 1$
- 1-submanifold in $2D$
- $\frac{d_{\mu,m}(q)}{m^\alpha}$ increasing for $\alpha < 1$



Noise-adaptive Distance Function

Assumption

Inferred shape is a submanifold of known dimension

For a k-submanifold in d-dimensional space:

$$\delta_\mu = \inf_{m>0} \frac{d_{\mu,m}}{m^\alpha},$$

with $\alpha \in [\frac{1}{d}; \frac{1}{k}]$

Noise-adaptive Distance Function

$$\delta_\mu = \inf_{m>0} \frac{d_{\mu,m}}{m^\alpha}$$

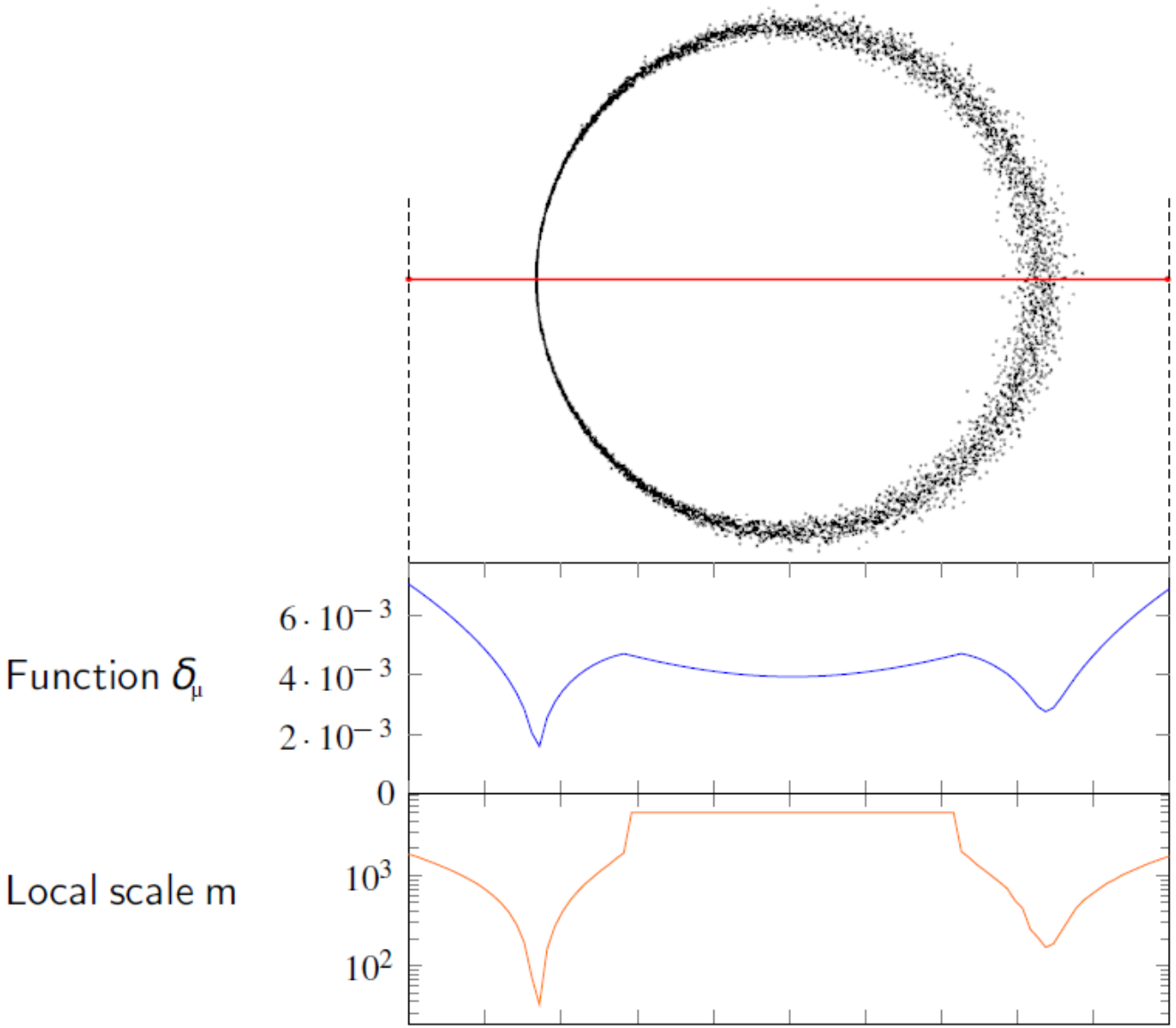
Infimum:

1. m as small as possible $\rightarrow$ no oversmoothing
2. m large enough $\rightarrow$ point subset appears as k-submanifold

Setting α ($\alpha \in [\frac{1}{d}; \frac{1}{k}]$)

- Curve in $2D$: $\alpha = \frac{3}{4}$ to satisfy $\alpha \in [\frac{1}{2}; 1]$
- Surface in $3D$: $\alpha = \frac{5}{12}$ to satisfy $\alpha \in [\frac{1}{3}; \frac{1}{2}]$

Noise-adaptive Distance Function



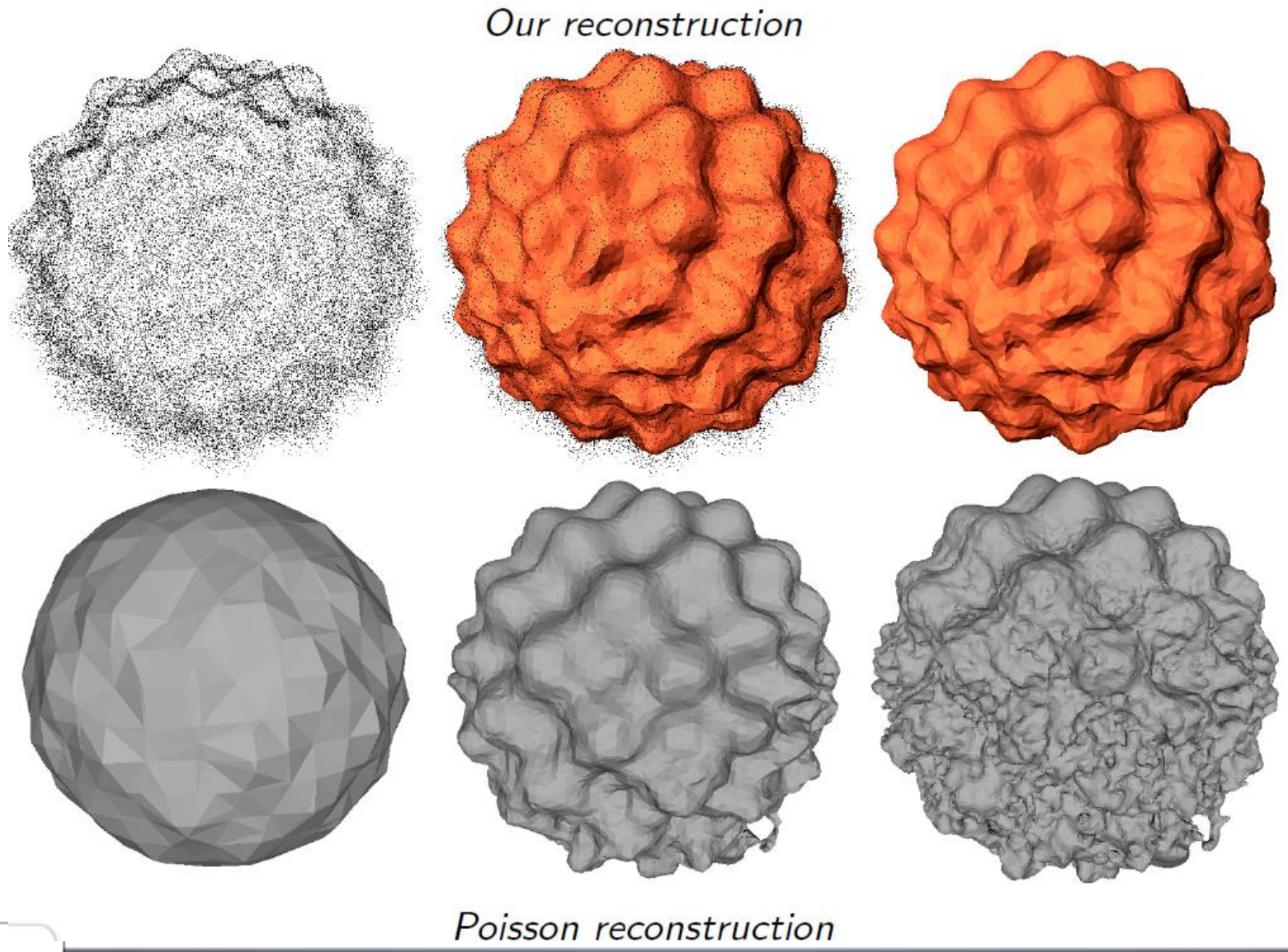
Noise-adaptive Distance Function

Wasserstein robustness & semiconcavity

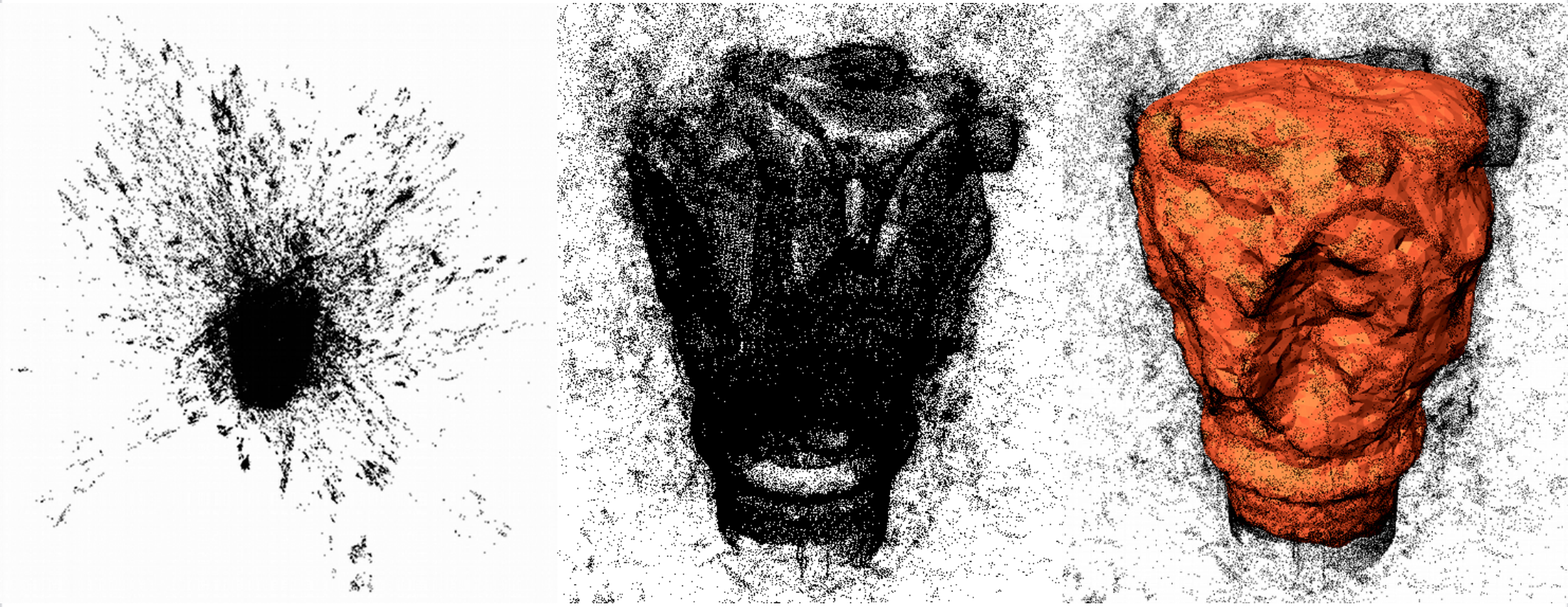
- $\forall m$, $d_{\mu,m}$ is $1/\sqrt{m}$-robust and 1-semiconcave
- Limiting infimum over values of m above m_0: properties preserved for δ_μ

$\rightarrow$ correct topological inference (*Geometric Inference for Probability Measures* [F. Chazal, D. Cohen-Steiner, Q. Mérigot, 2011])

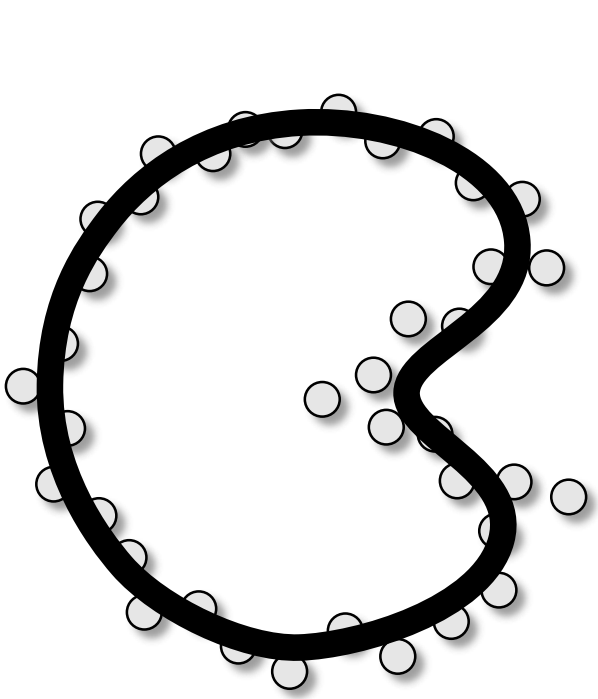
Experiment



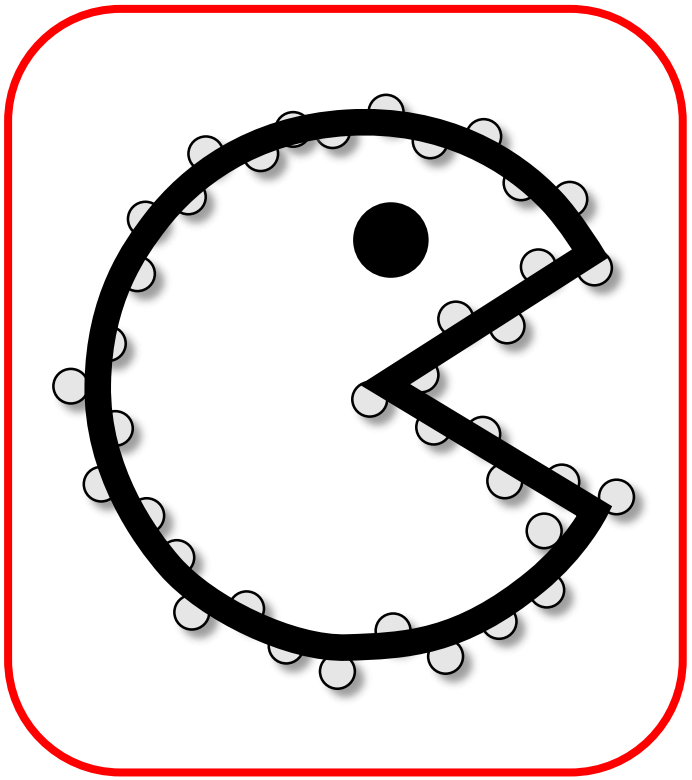
Outlier Robustness



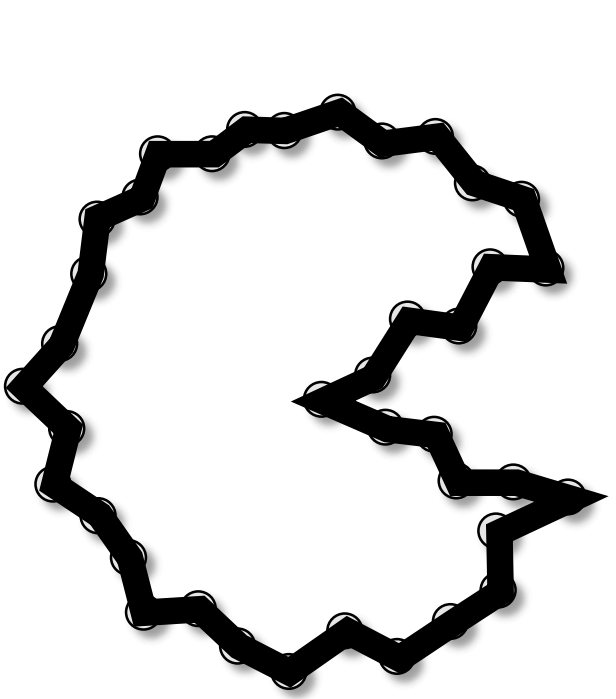
OPTIMAL TRANSPORTATION



Smooth



Piecewise Smooth



“Simple”

Motivations

Geometric data:

Defect-laden sampling

- Sparse
- Non-uniform
- Widely variable
- Missing data

Motivations

Geometric data:

Uncertain

- Measurement noise
- Registration noise
- Outliers
- Spurious topology

Heterogeneous

Calls for Robustness

Motivations

Complex shapes:

- Sharp features
- Non-manifold features
- Boundaries

Calls for feature preservation

Previous Work

Approach	Reference	Sampling				Noise		Outliers	Features	Boundaries
Robust Delaunay	Dey	•	•			•				
Spectral	O'Brien et al. 04	•	•		•	•		•		
Poisson	Kahzdan et al. 06	•	•	•	•	•				
Moving Least Squares	Alexa et al. 02	•	•	•		•				
Robust statistics	Fleishman et al. 05	•	•	•		•		•	•	
K-order alpha shapes	Krasnoshchekov 08		•			•		•		
Scale space	Digne et al. 10	•	•	•		•	•			•
L1-minimization (only denoising)	Avron et al. 10	•	•			•			•	•
Feature detection	Salman et al. 10		•	•	•	•			•	
Signing distance function	Mullen et al. 10		•		•	•		•		

Approach in 2D

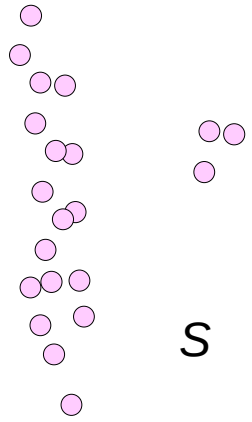
Given a point set S, find a coarse triangulation T such that S is well approximated by uniform measures on the 0- and 1-simplices of T.

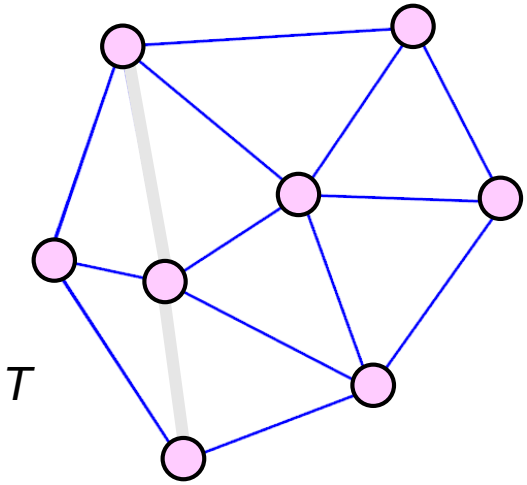
How to measure distance D(*S*,*T*)?

⇒ optimal transport between measures

How to construct *T* that minimizes D(*S*,*T*)?

optimal location problem! ⇒ greedy decimation



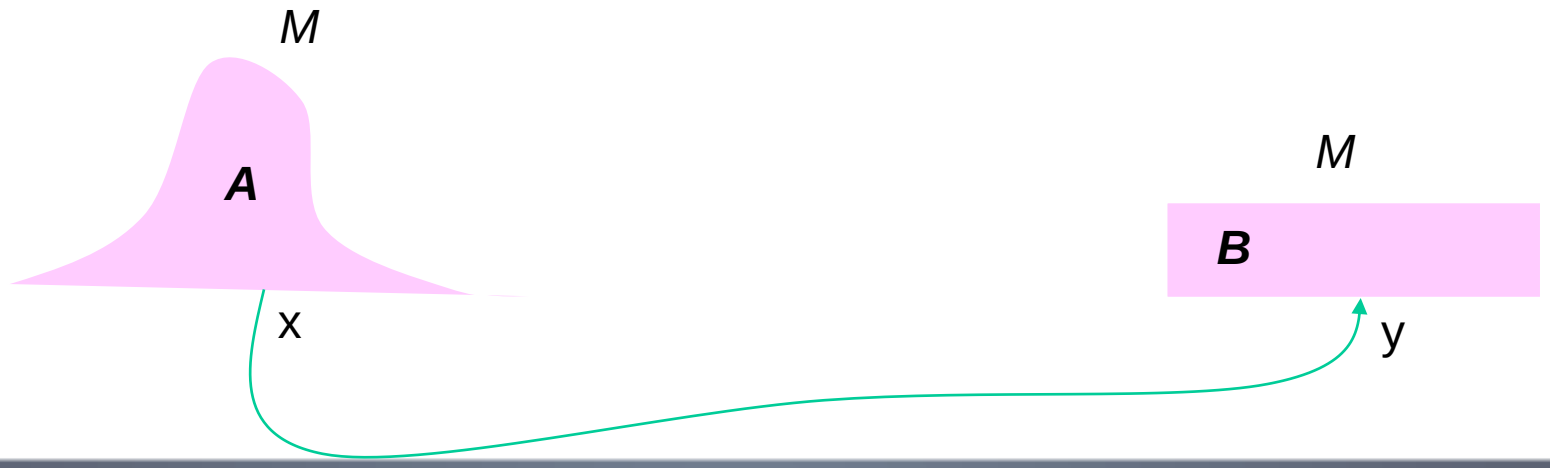


Distance between Measures (1D)

Transport plan: π on $\mathbb{R} \times \mathbb{R}$ whose marginals are A and B

Transport cost: $$W_2(A, B, \pi) = \left(\int_{\mathbb{R} \times \mathbb{R}} \|x - y\|^2 d\pi(x, y) \right)^{1/2}$$

Optimal transport: $$W_2(A, B) = \inf_{\pi} W_2(A, B, \pi)$$

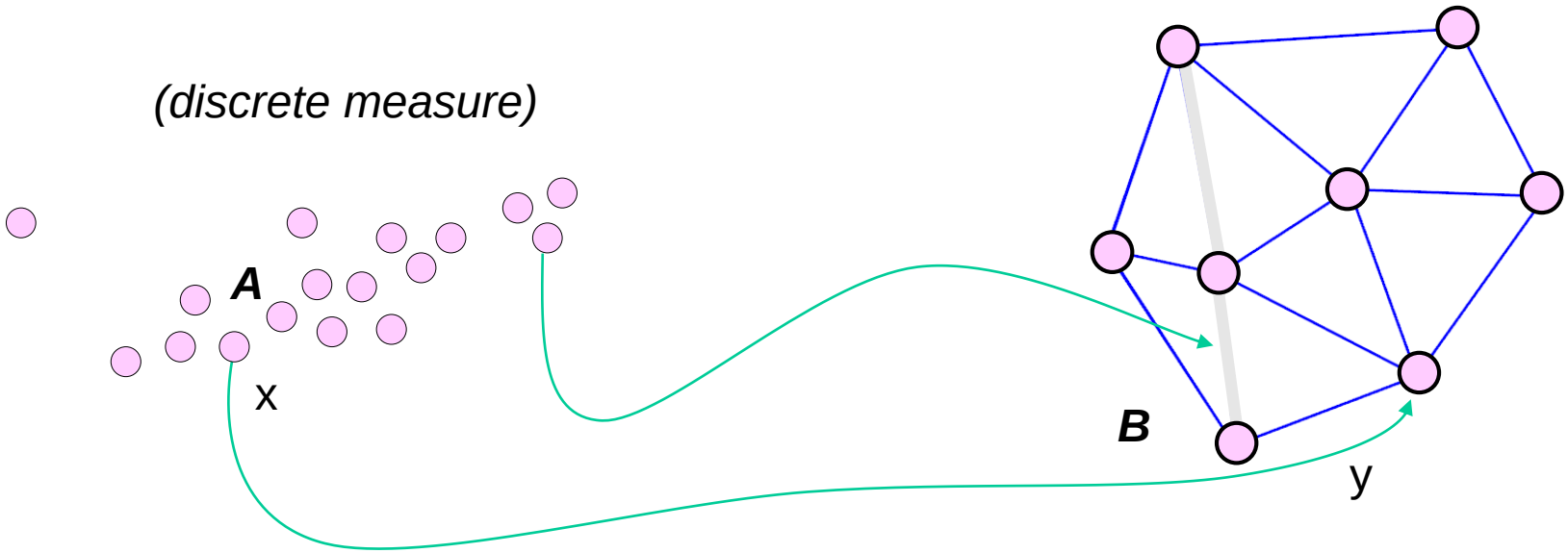


Distance between Measures

Transport plan: π on $\mathbb{R} \times \mathbb{R}$ whose marginals are A and B

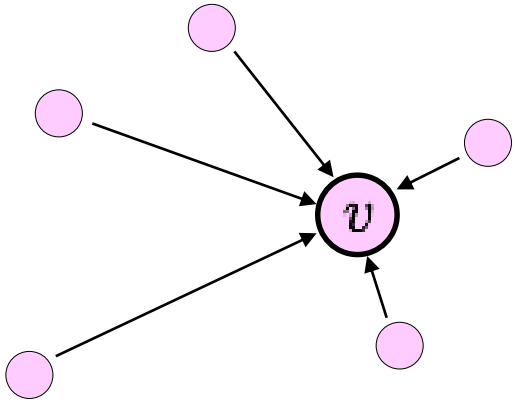
Transport cost: $$W_2(A, B, \pi) = \left(\int_{\mathbb{R} \times \mathbb{R}} \|x - y\|^2 d\pi(x, y) \right)^{1/2}$$

Optimal transport: $$W_2(A, B) = \inf_{\pi} W_2(A, B, \pi)$$



Optimal Transport on a Vertex

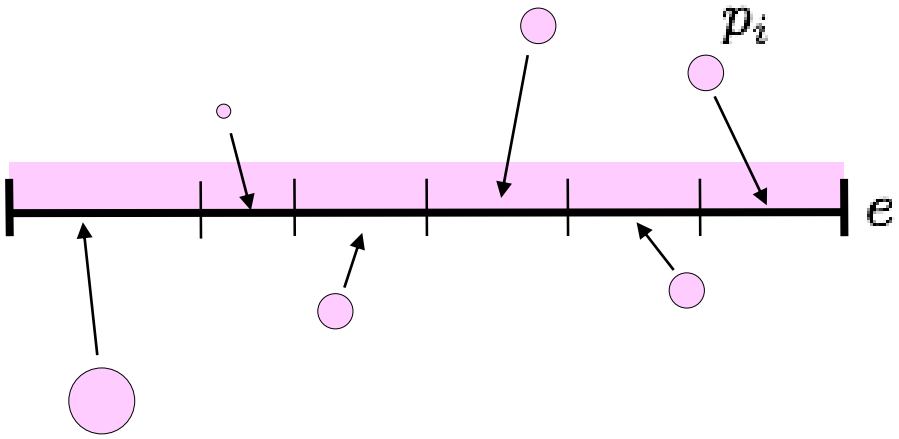
(assume given *binary* transport plan)

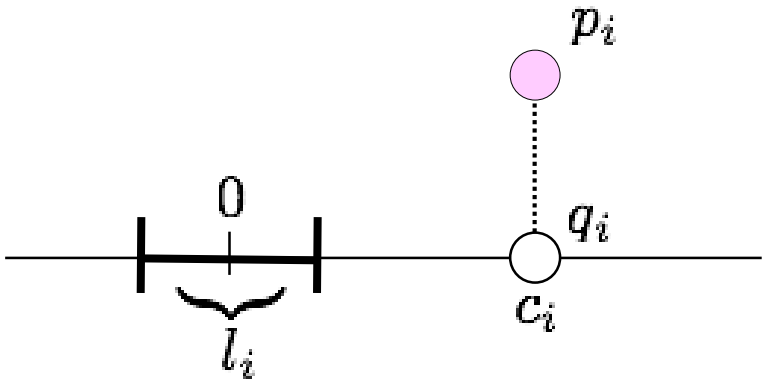


$$W_2(v, S_v) = \sqrt{\sum_{p_i \in S_v} m_i \|p_i - v\|^2}.$$

Optimal Transport on an Edge

(assume given *binary* transport plan)

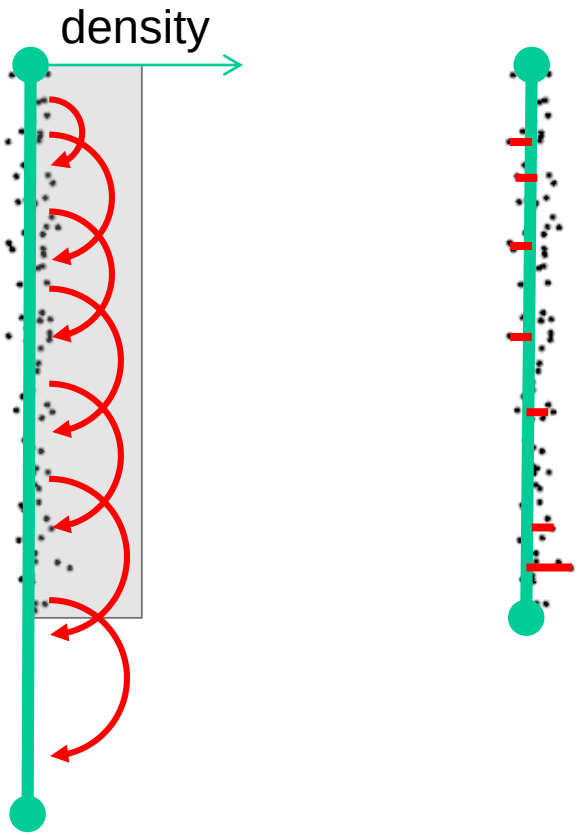




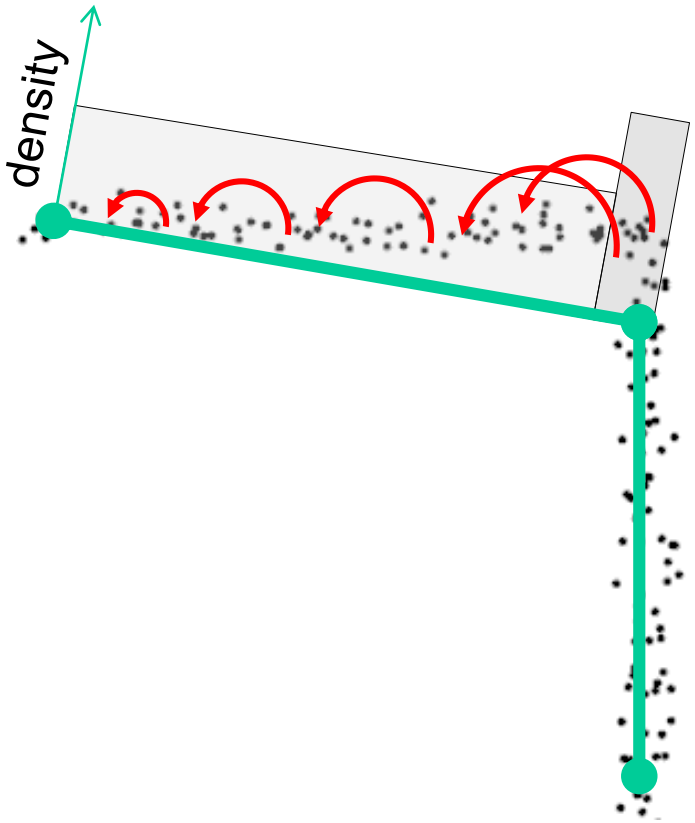
$$N(e, S_e) = \sqrt{\sum_{p_i \in S_e} m_i \|p_i - q_i\|^2}$$

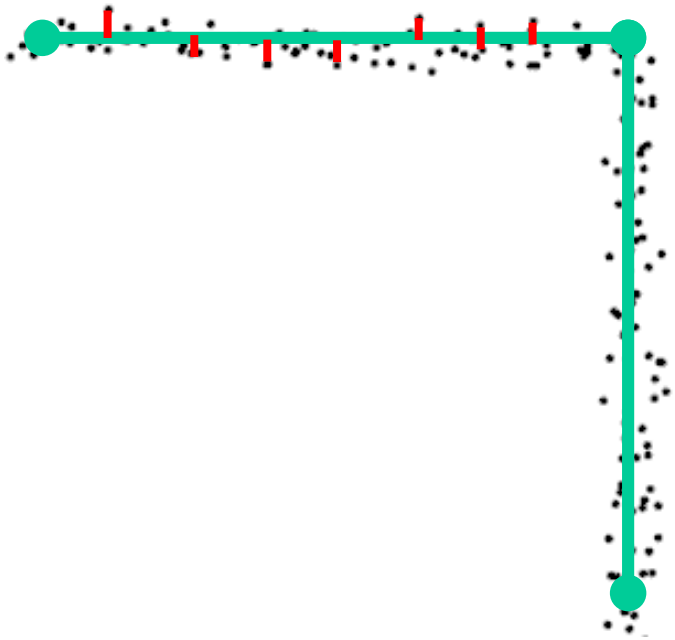
$$T(e, S_e) = \sqrt{\sum_{p_i \in S_e} \frac{M_e}{|e|} \int_{-l_i/2}^{l_i/2} (x - c_i)^2 \, dx} = \sqrt{\sum_{p_i \in S_e} m_i \left(\frac{l_i^2}{12} + c_i^2 \right)}$$

Boundary Preserving?

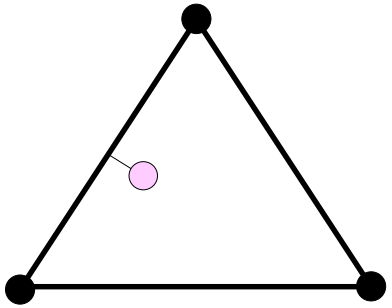


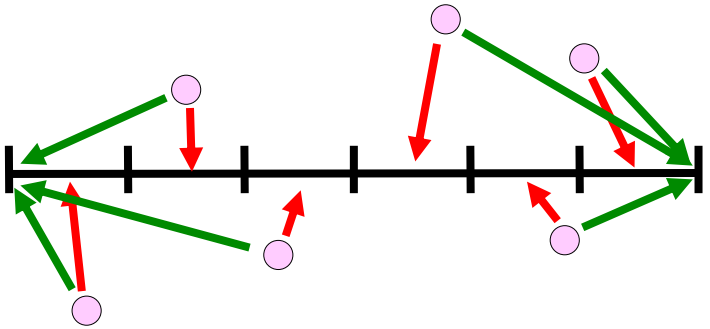
Feature Preserving?

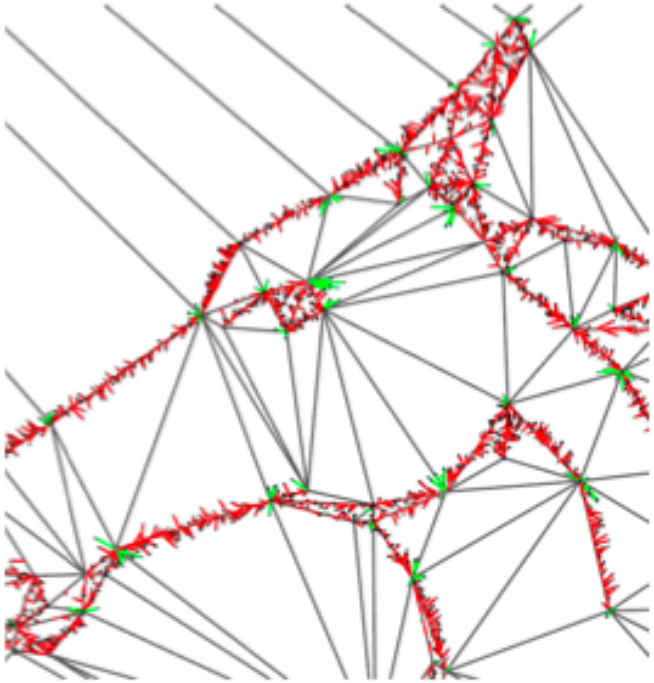


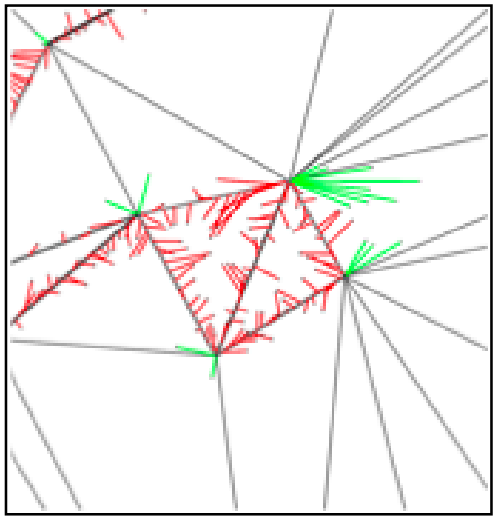


Current (Binary) Transport Plan





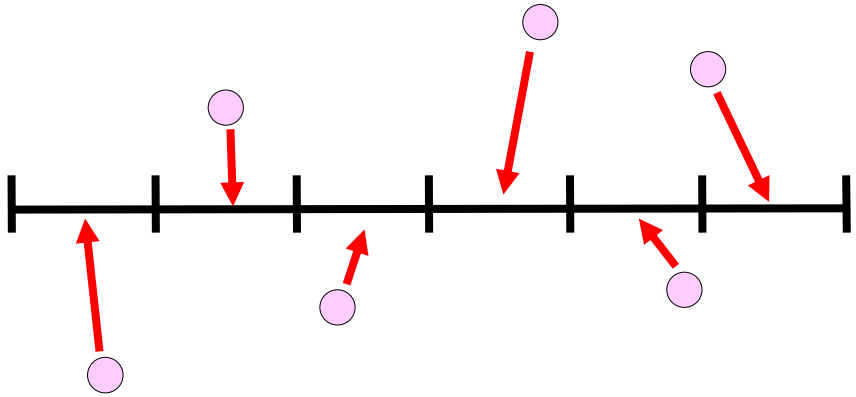




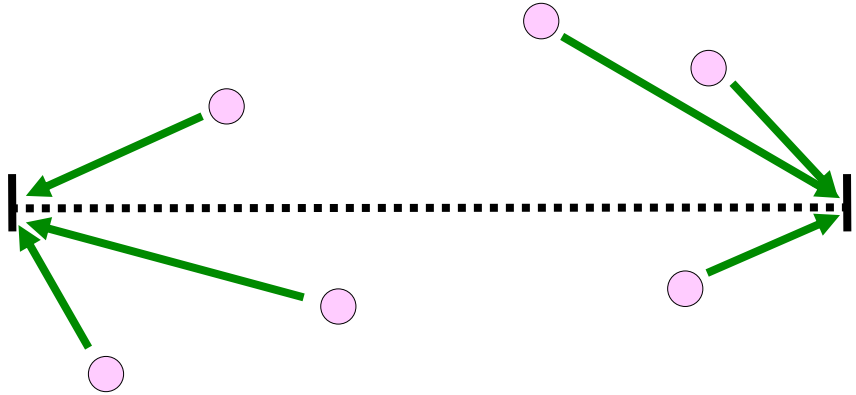
suboptimal but local

Total cost $W_2(\pi) = \sqrt{\sum_{e \in \mathcal{T}} [N(e,\mathcal{S}_e)^2 + T(e,\mathcal{S}_e)^2] + \sum_{v \in \mathcal{T}} W_2(v,\mathcal{S}_v)^2}$

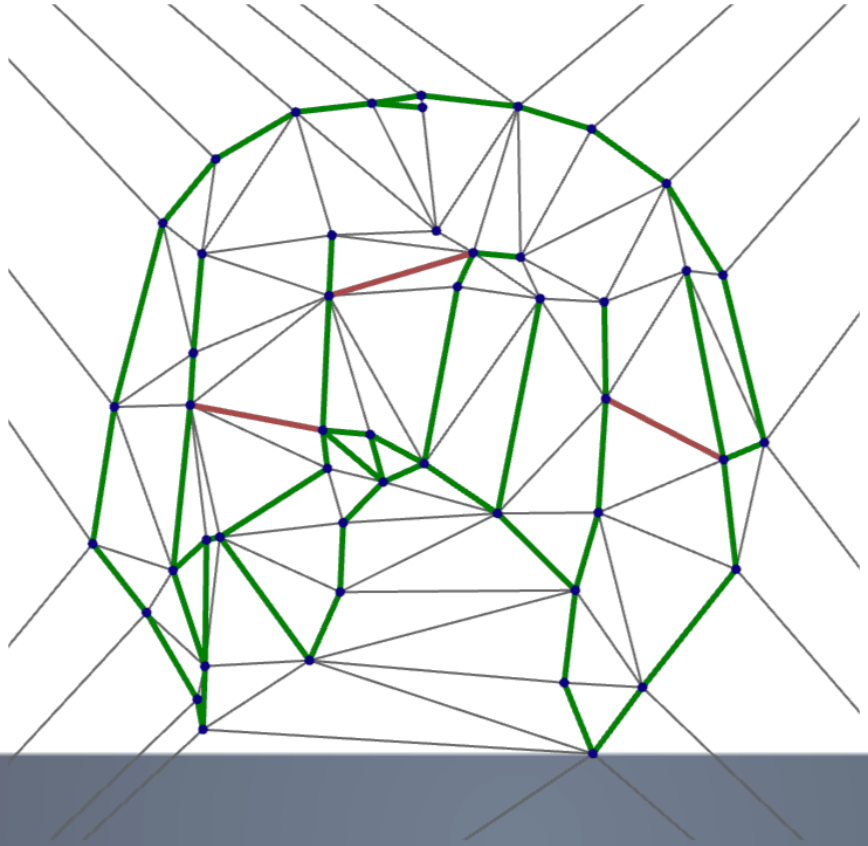
Solid vs Ghost Edge



Solid



Ghost



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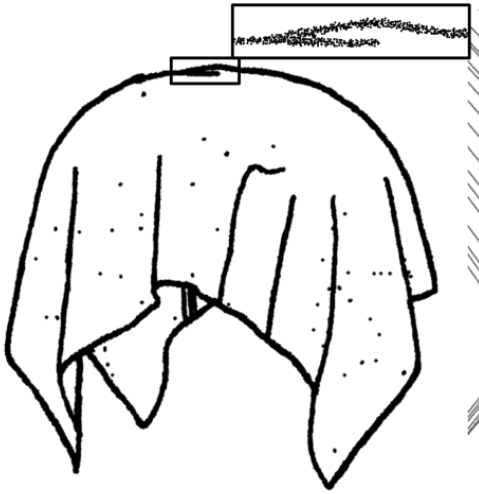
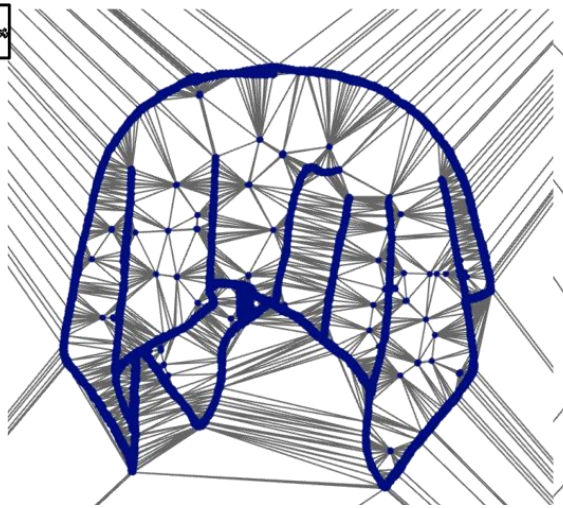
F. de Goes, D. Cohen-Steiner, P.A., and M. Desbrun.
Symposium on Geometry Processing, 2011.

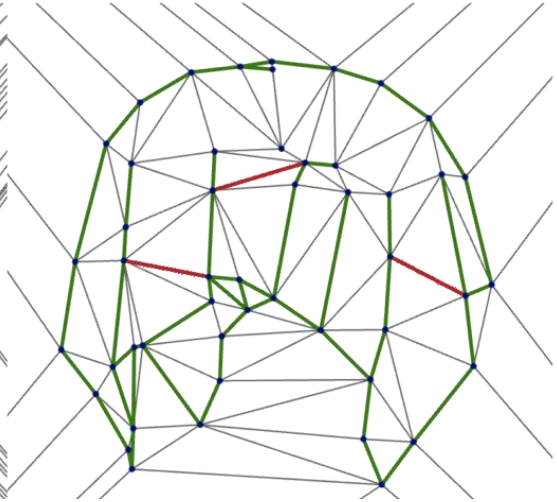
Algorithm Overview



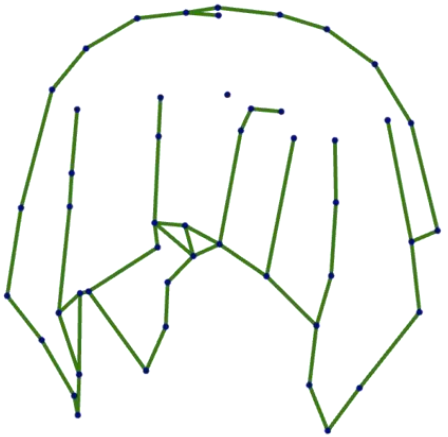
input point set (potentially variable mass)



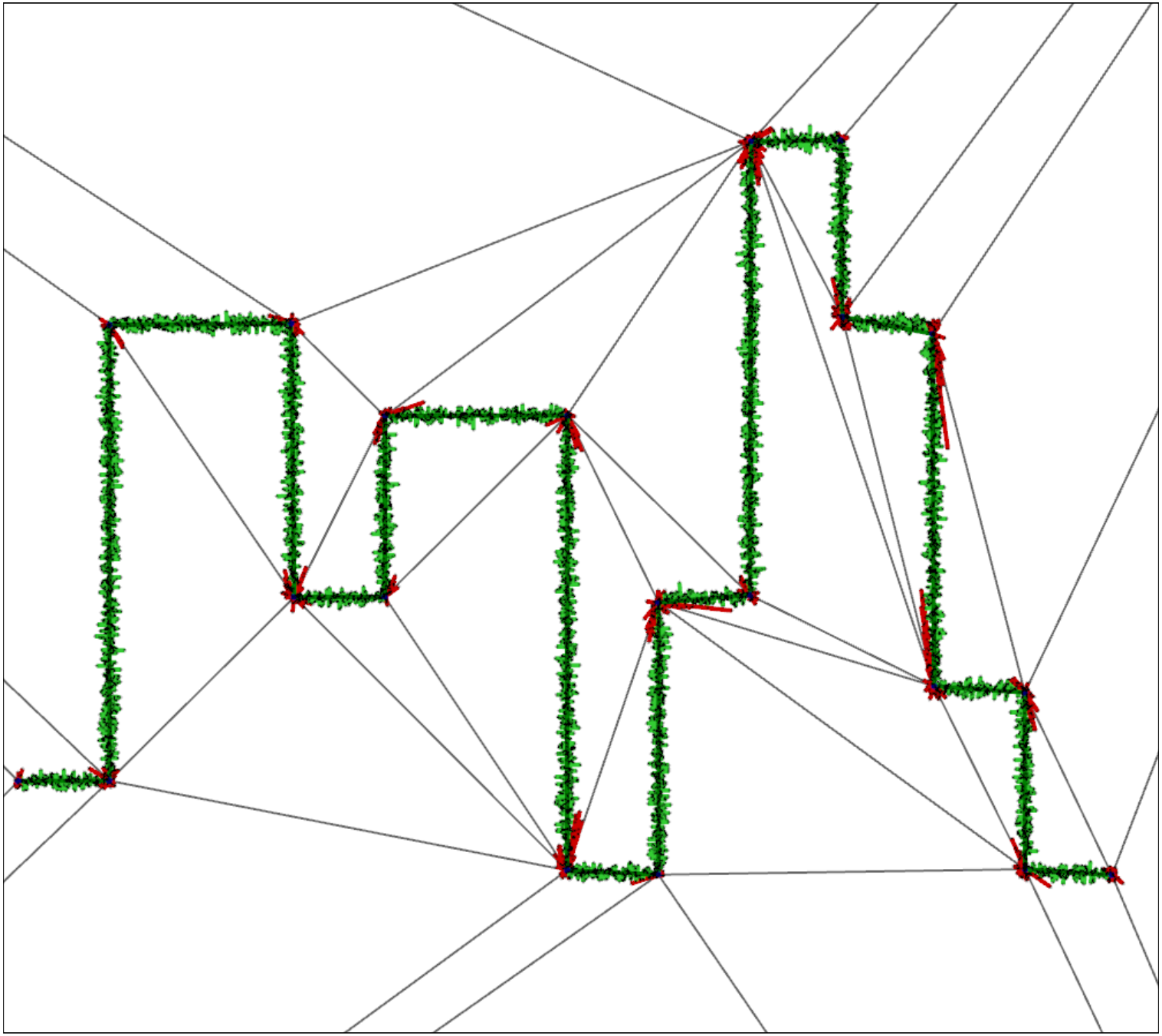
Delaunay triangulation



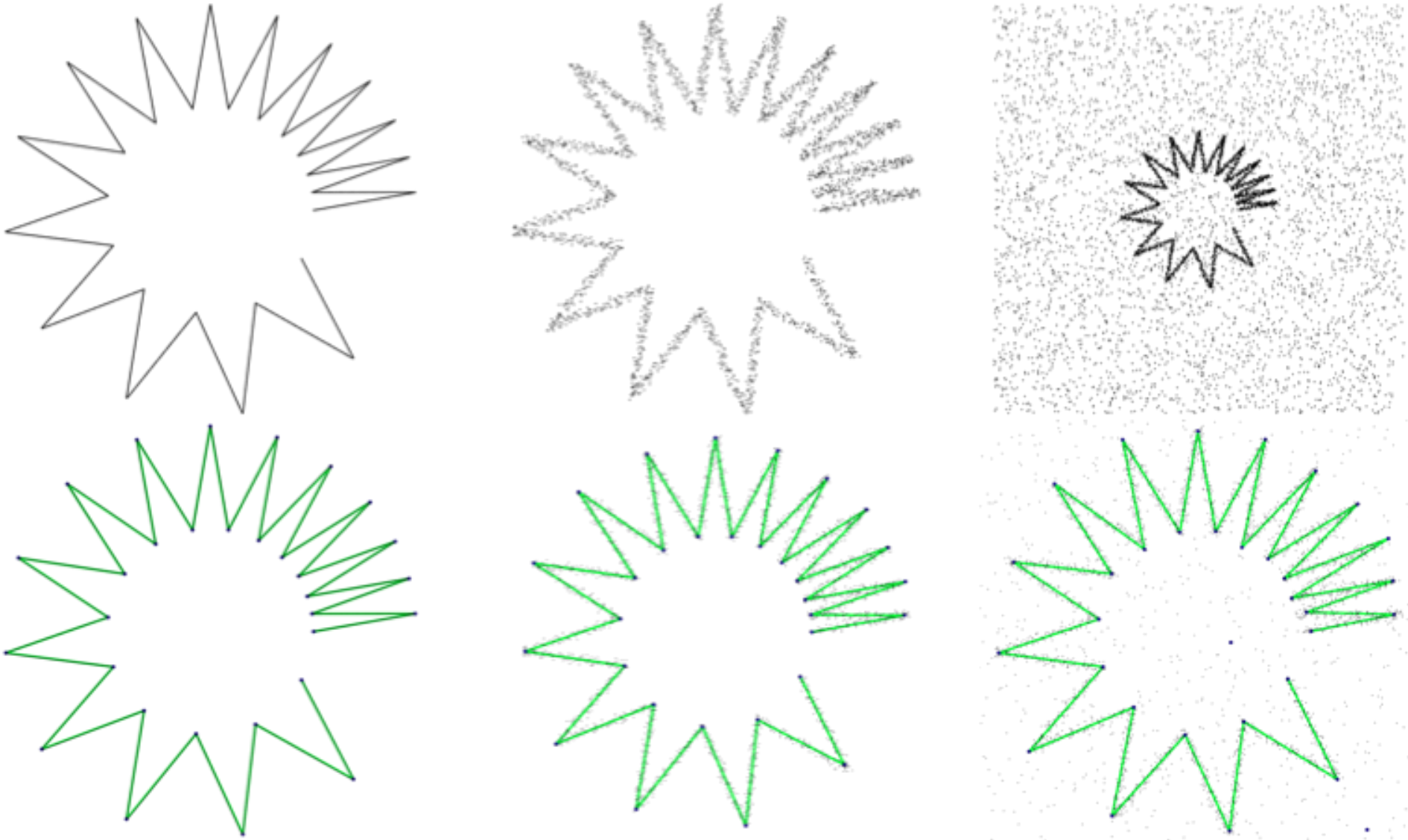
after decimation



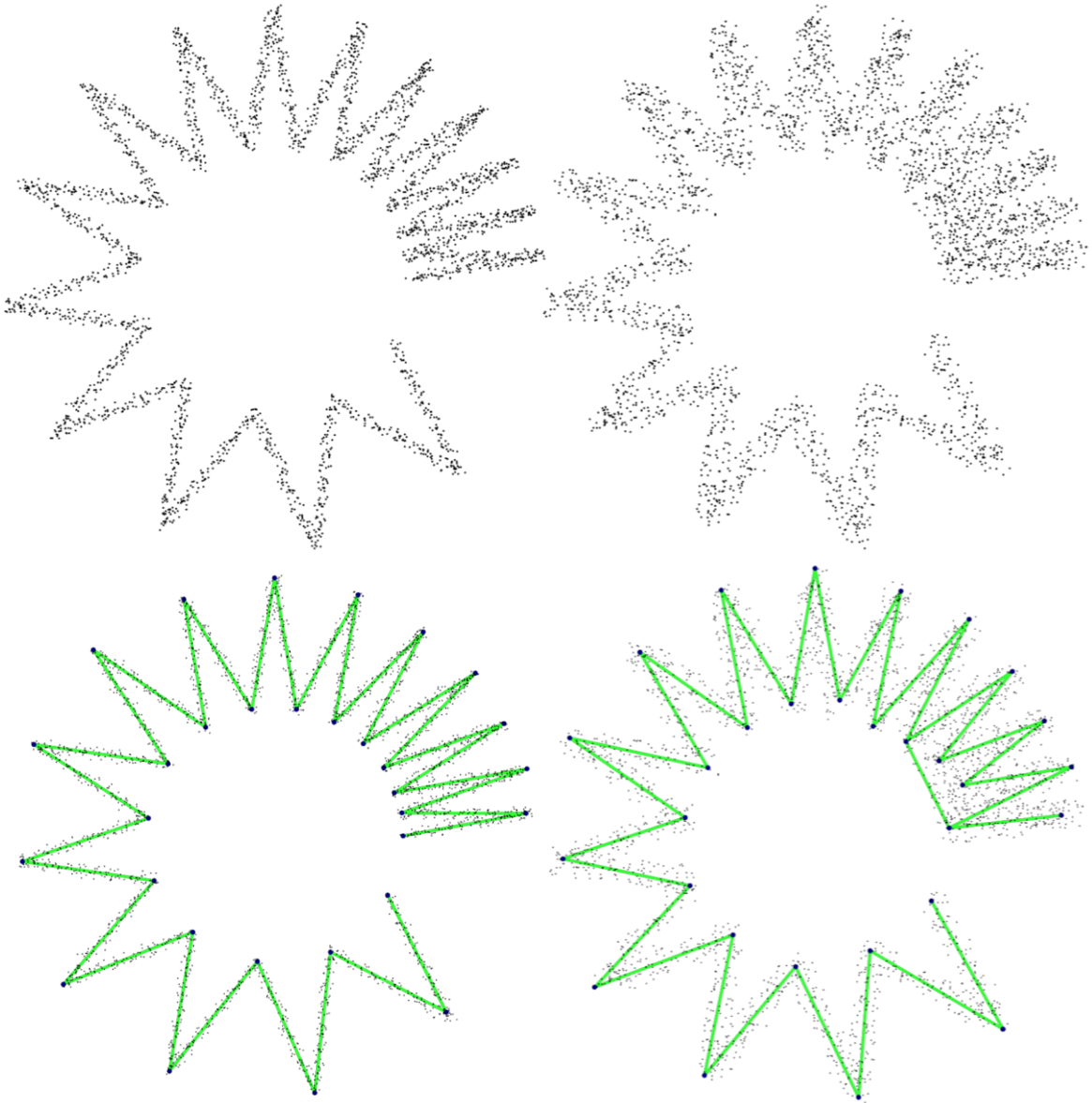
output after edge filtering



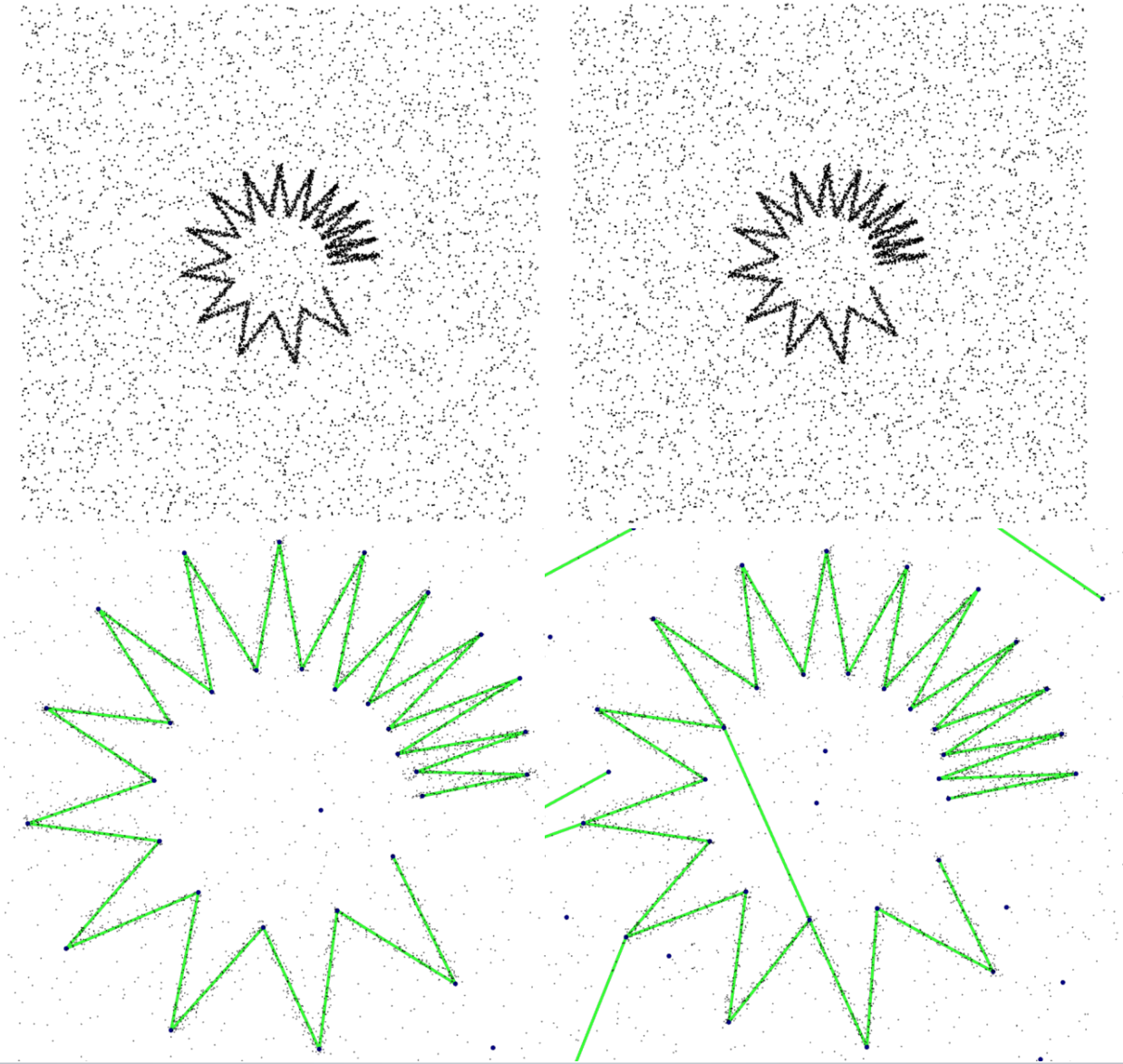
Robustness



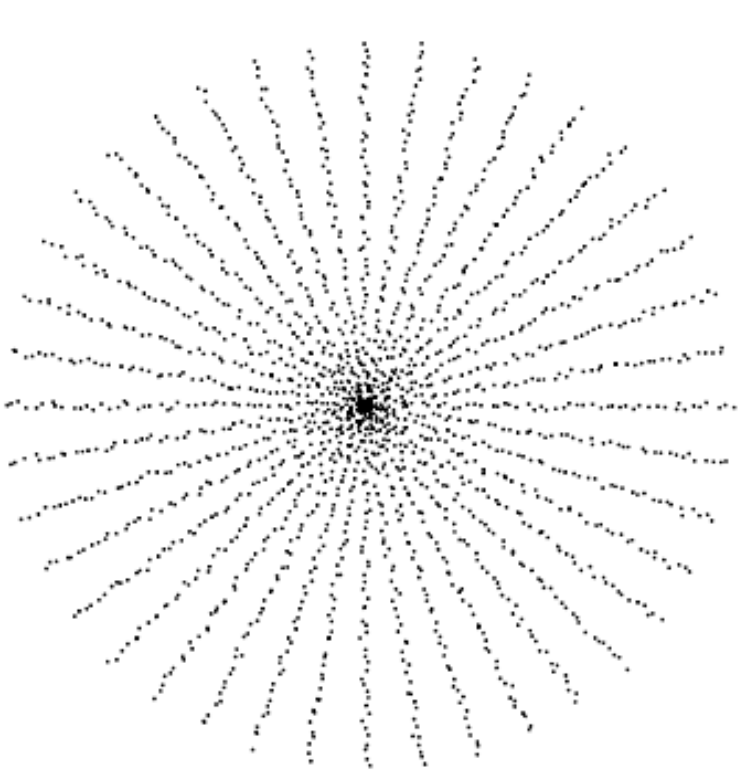
More Noise

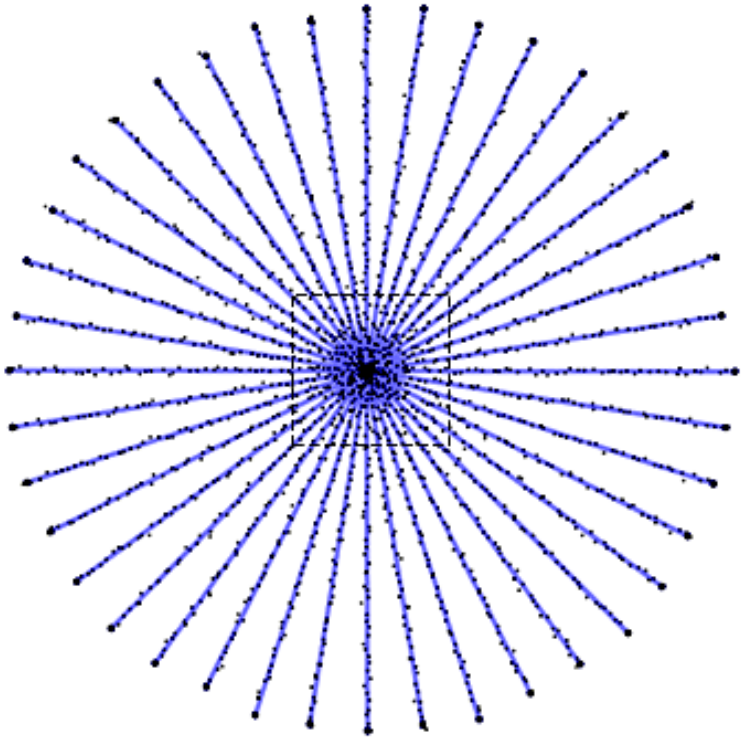


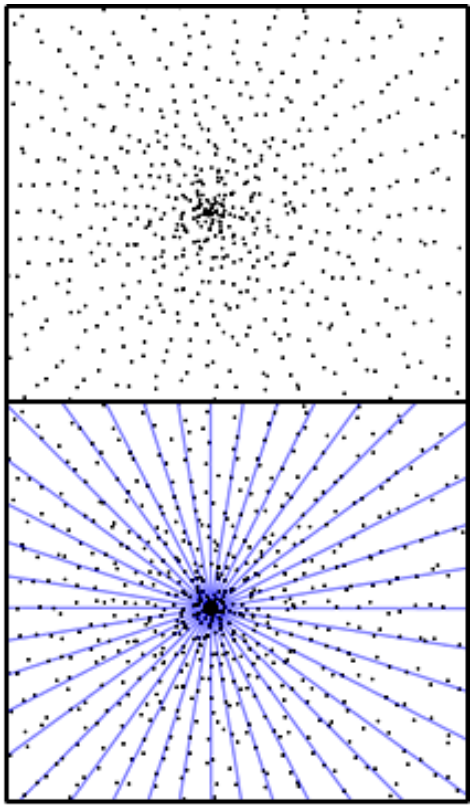
More Outliers



Features and Robustness







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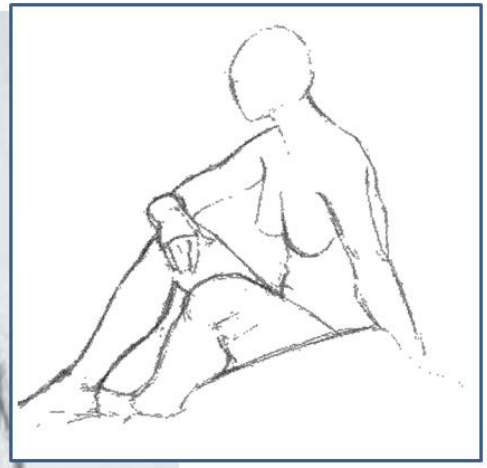
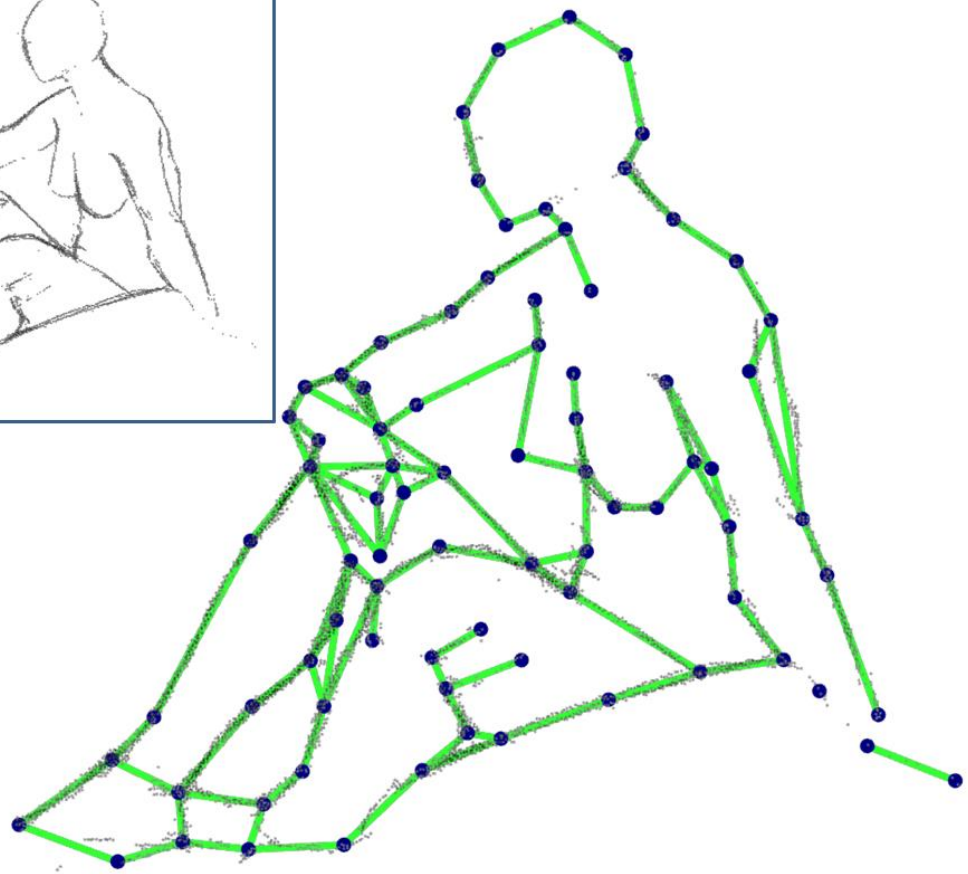
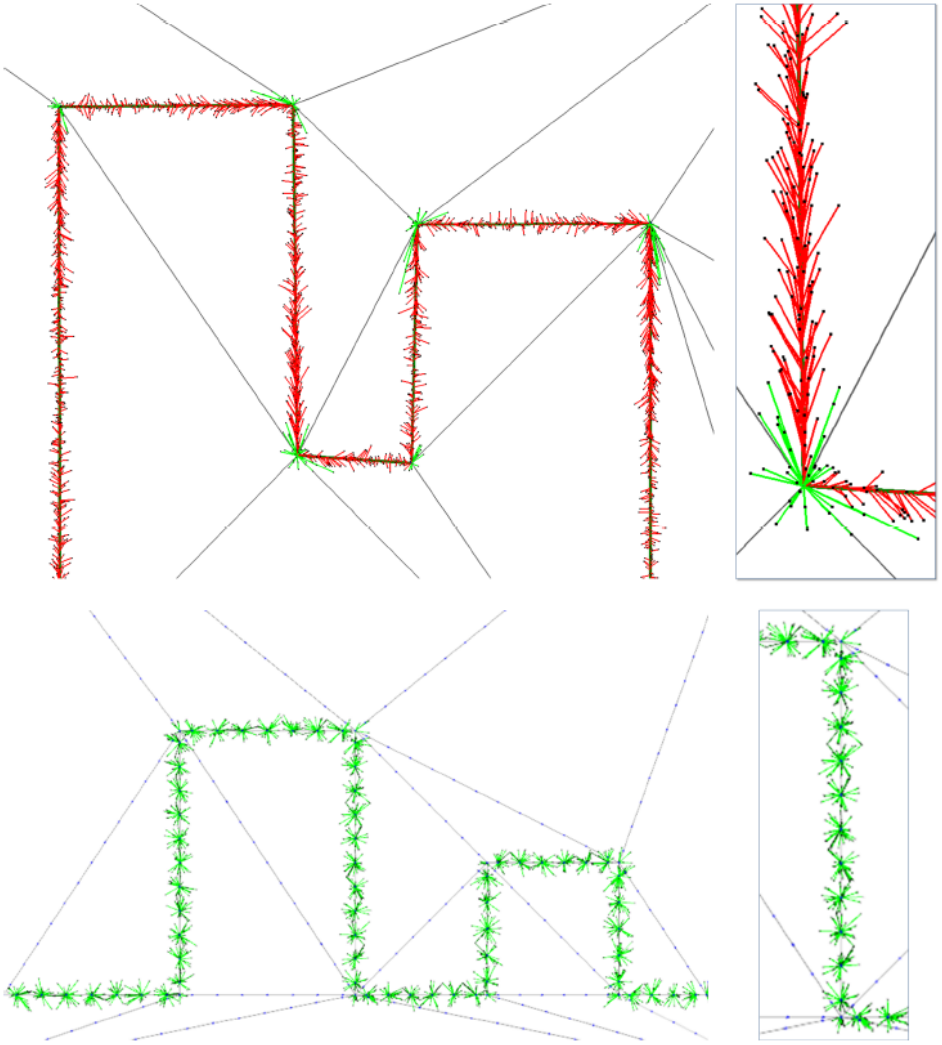
Symposium on Geometry Processing, 2011.

Variable M





Limitation: Binary Transport Plan



WHAT NEXT

What Next

Robustness

- Structured outliers
- Heterogeneous data

New acquisition paradigms

- Super-resolution
- Community data

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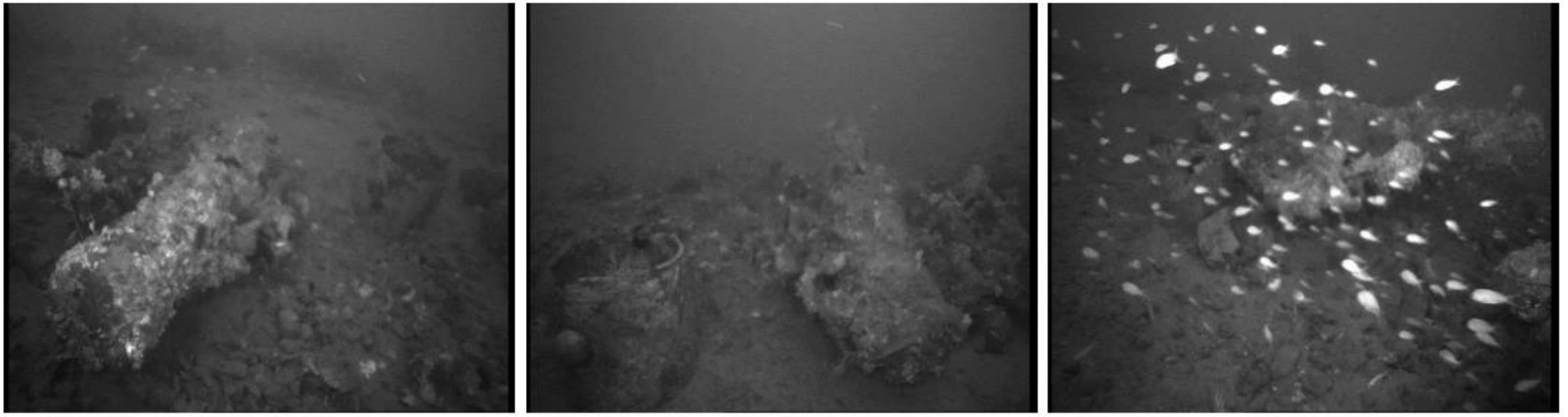
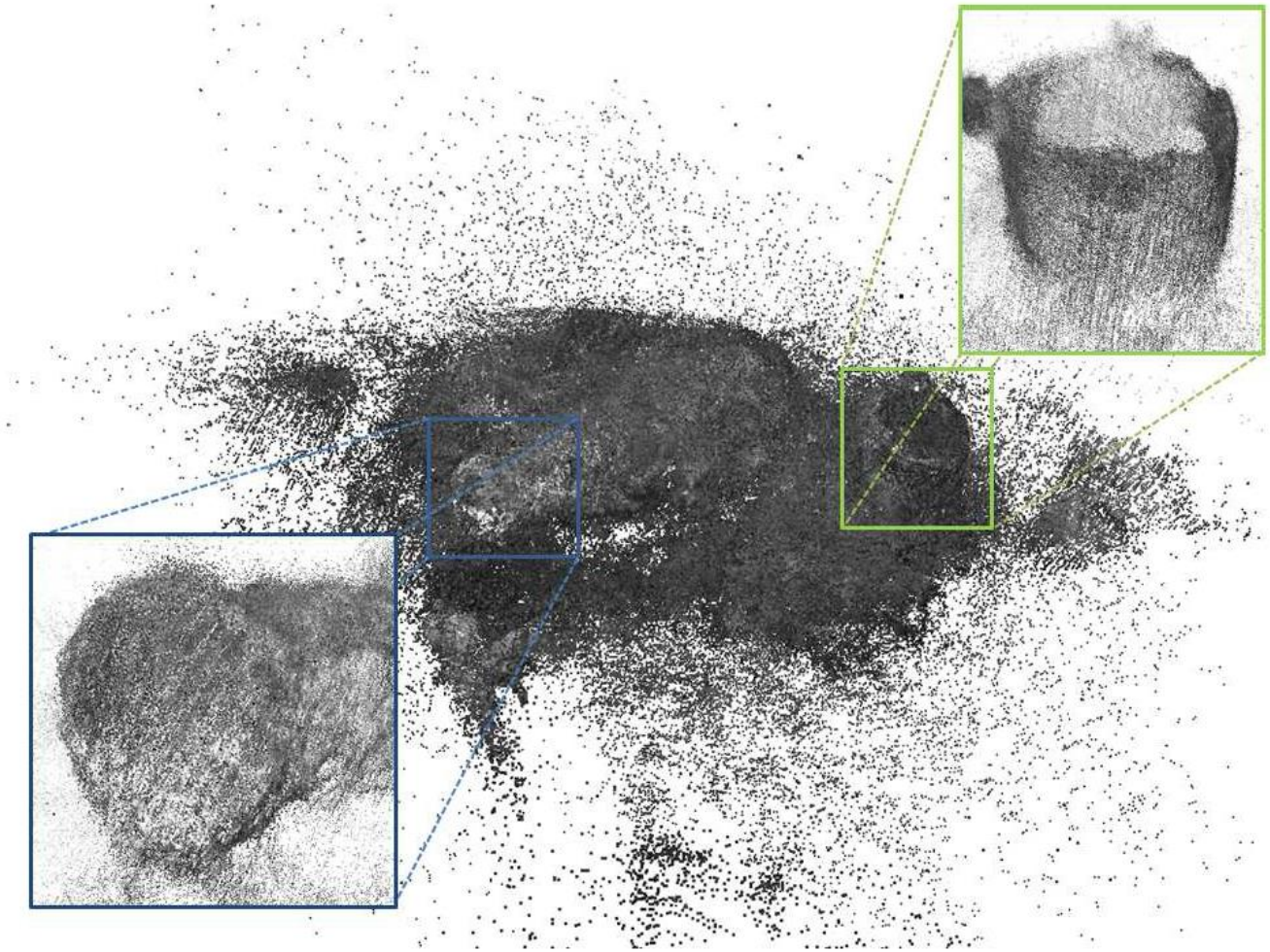
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