Smooth surface reconstruction

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Smooth reconstruction

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.. just a portion

Classification:

- Computational Geometry vs. Implicit Surfaces
- Structured vs. Unstructured Data
- Oriented vs. Unoriented Points
- Watertight vs. Surface with Boundary Output
- Etc.

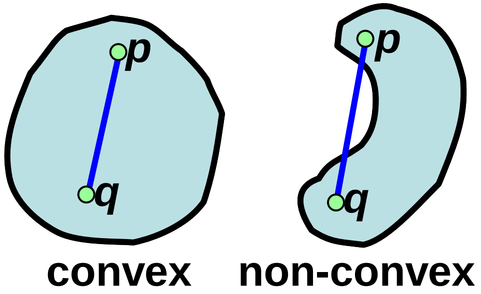
Classification:

- Computational Geometry
  - Uses input to partition space
  - Use a subset of the partition to define the shape
- Implicit Surfaces
  - Fit implicit function to the input
  - Extract iso-surface

- Computational geometry (briefly)
  - Convex hulls & Alpha-shapes
  - Delaunay triangulations
  - Voronoi diagrams
  - Medial axes
- A sampling of smooth methods
- A sampling of piecewise-smooth methods

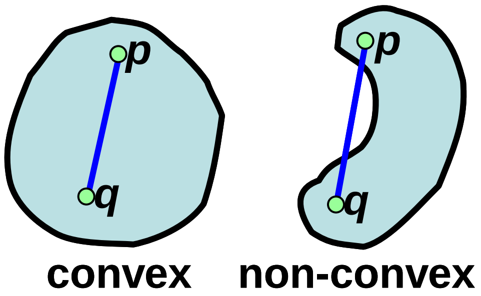
Convex Hulls:

a set S is *convex* if for any two points $a, b \in S$, the line segment between a and b is also in S.

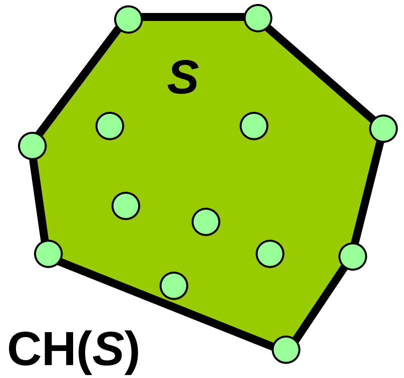


Convex Hulls:

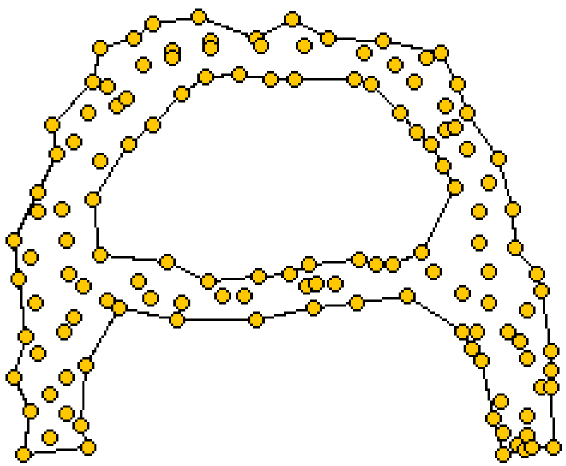
A set S is *convex* if for any two points $a,b \in S$, the line segment between a and b is also in S.

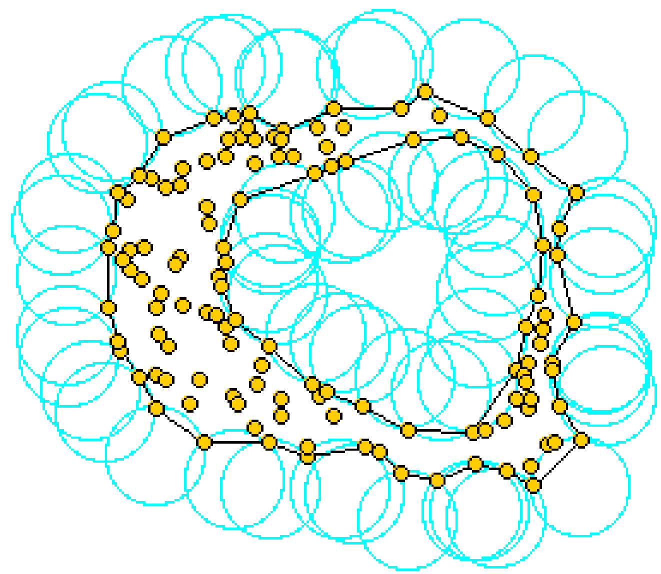


The *convex hull* of a set of points is the smallest convex set containing S.



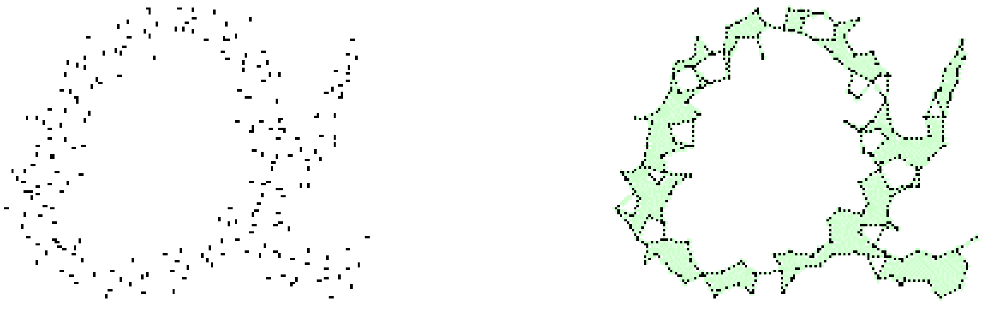
Alpha shapes:



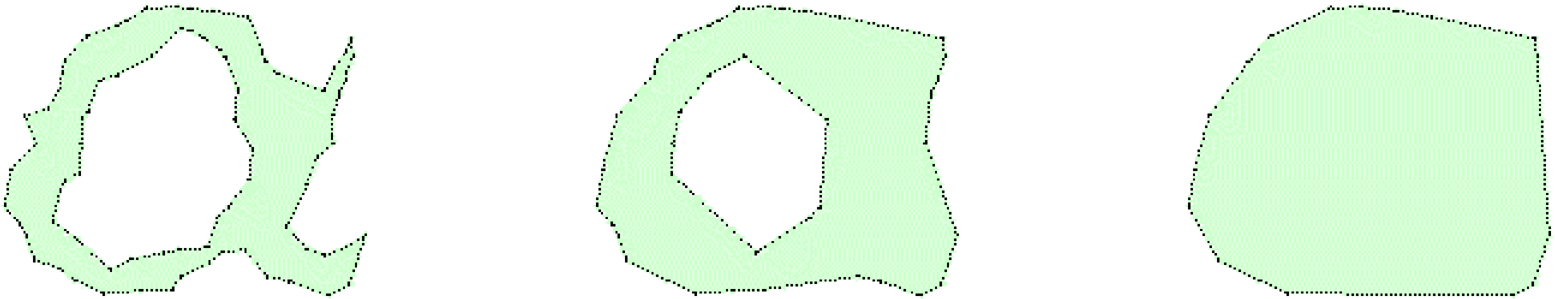


The space generated by point pairs that can be touched by an empty disc of radius alpha.

Alpha shapes:



$\alpha = 0$ Alpha Controls the desired level of detail.



$\alpha = \infty$

Delaunay Triangulation:

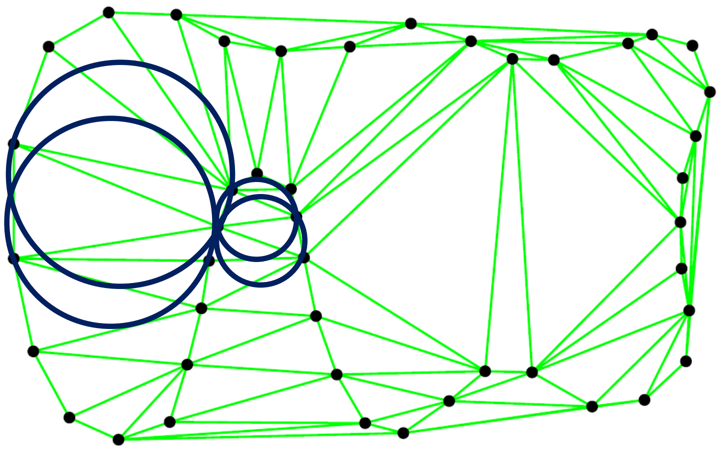
A *Delaunay Triangulation* of *S* is the set of all triangles with vertices in *S* whose circumscribing circle contains no other points in *S**.

Compactness Property:

This is a triangulation that maximizes the min angle of all

The angles of the triangles

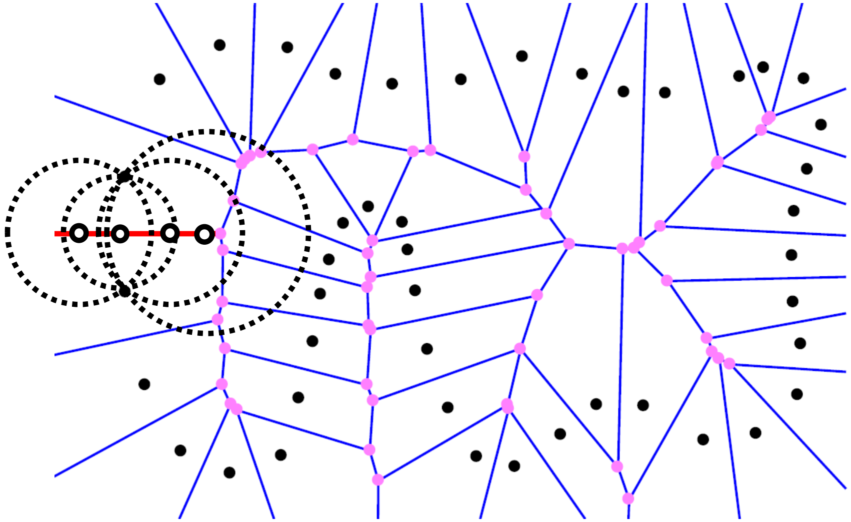
(avoid skinny triangles)



<u>Voronoi Diagrams</u>:

The *Voronoi Diagram* of *S* is a partition of space into regions *V*(*p*) ($p \in S$) such that all points in *V*(*p*) are closer to *p* than any other point in *S*.

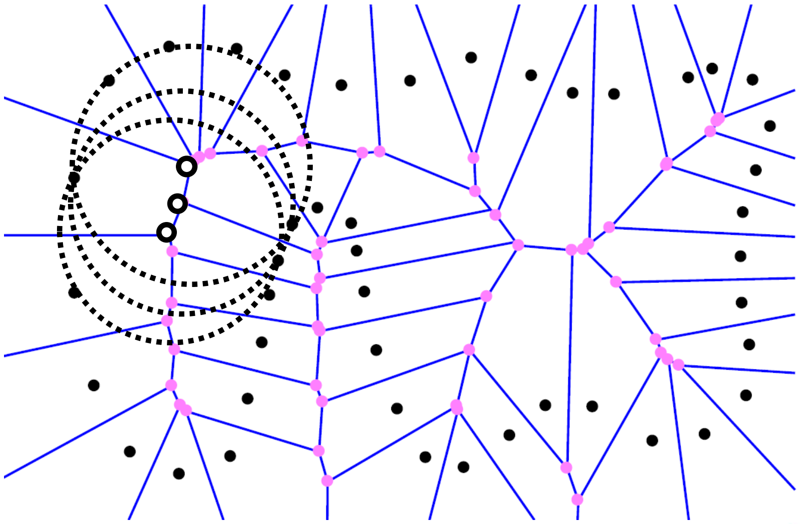
For a point on an edge, we can draw an empty circle that only touches the points in *S* separated by the edge.



<u>Voronoi Diagrams</u>:

The *Voronoi Diagram* of S is a partition of space into regions $V(p)$ ($p \in S$) such that all points in $V(p)$ are closer to p than any other point in S.

For a vertex, we can draw an empty circle that just touches the three points in S around the vertex.

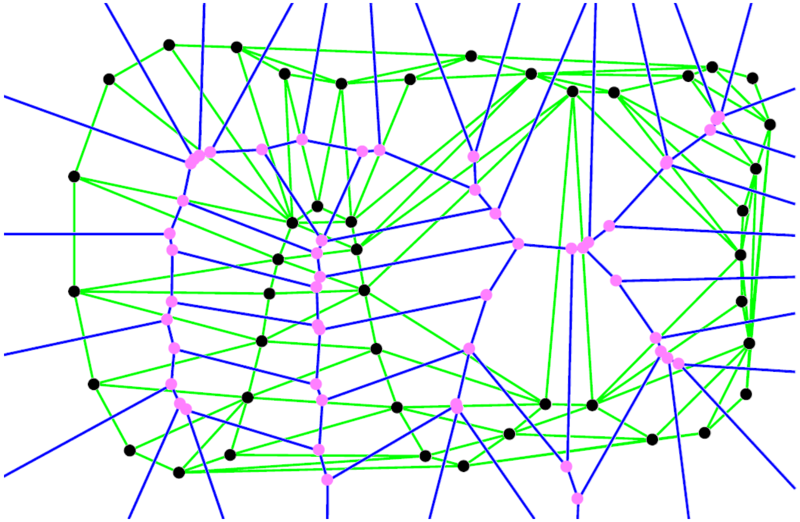


Voronoi Diagrams:

The *Voronoi Diagram* of S is a partition of space into regions $V(p)$ ($p \in S$) such that all points in $V(p)$ are closer to p than any other point in S.

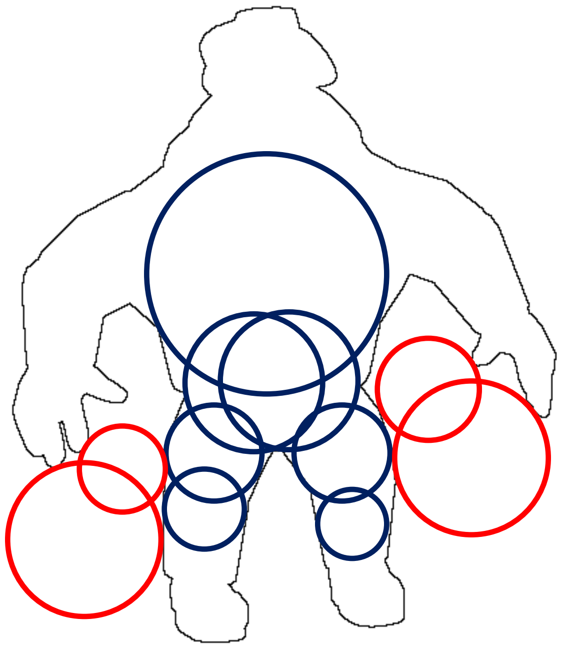
Duality:

Each Voronoi vertex is in one-to-one correspondence with a Delaunay triangle.



Medial Axis:

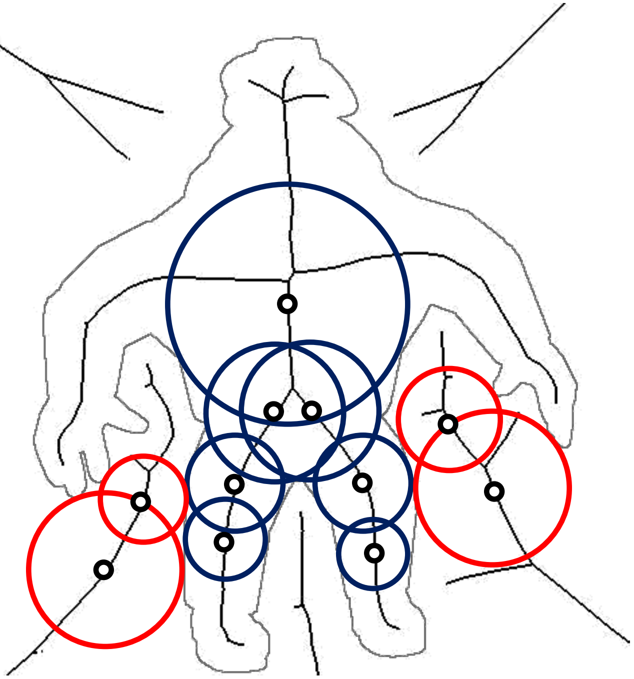
For a shape (curve/surface) a *Medial Ball* is a circle/sphere that only meets the shape tangentially, in at least two points.



Medial Axis:

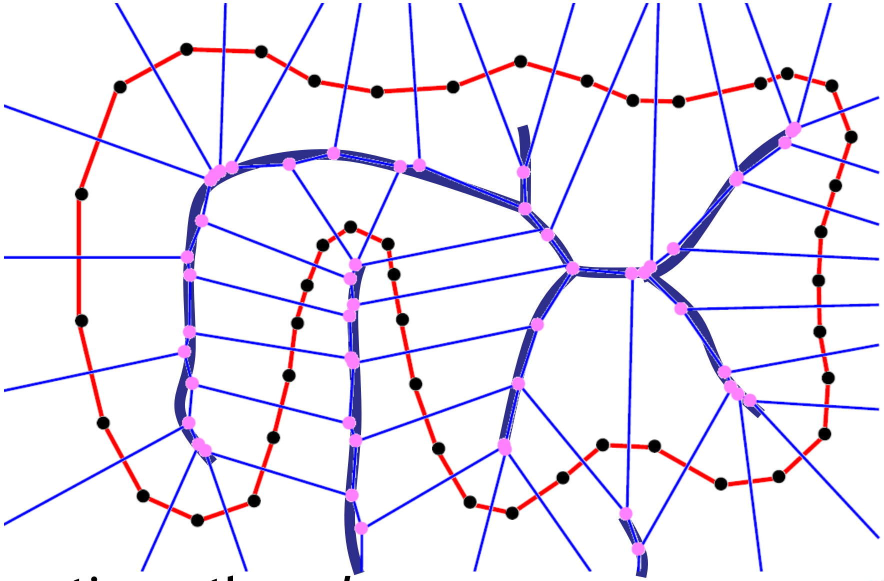
For a shape (curve/surface) a *Medial Ball* is a circle/sphere that only meets the shape tangentially, in at least two points.

The centers of all such balls make up the *medial axis/skeleton*.



Observation*:

For a reasonable point sample, the medial axis is well-sampled by the Voronoi vertices.



*In 3D, this is only true for a subset of the Voronoi vertices – the *poles*.

- Computational geometry
- A sampling of smooth methods
  - Space Partitioning
  - Crust
  - … from Unorganized Points
  - Poisson Reconstruction

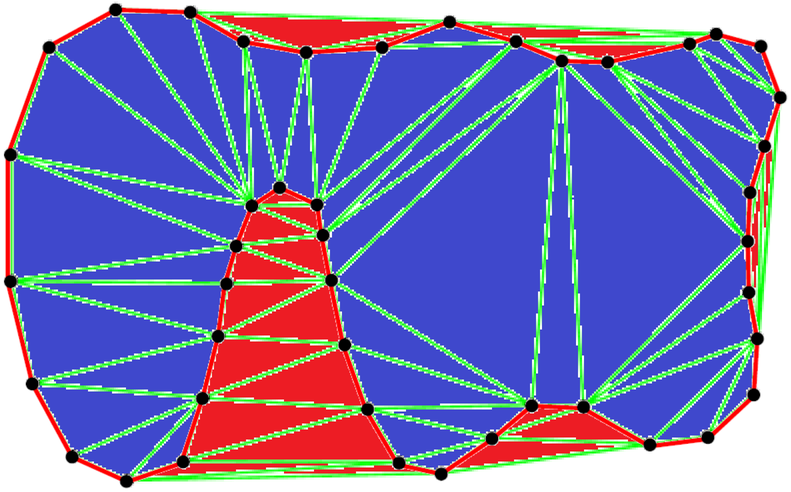
Computational Geometry

Implicit Surfaces

- A sampling of piecewise-smooth methods

Given a set of points, we can construct the Delaunay triangulation.

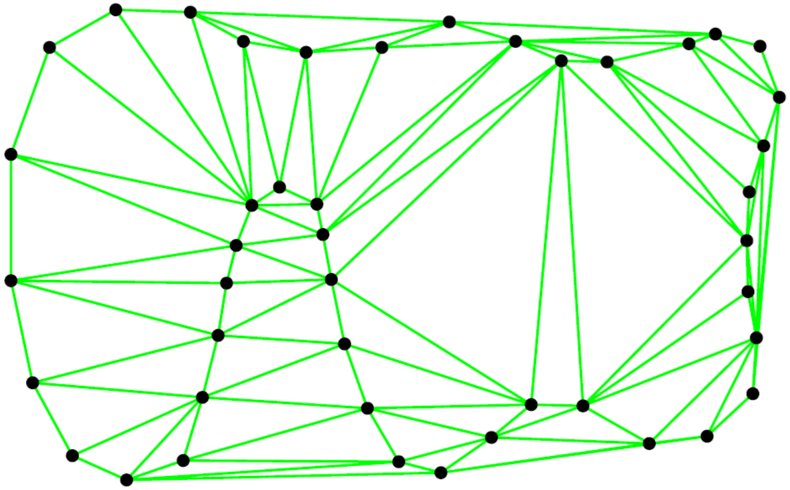
If we label each triangle as inside/outside, then the surface of interest is the set of edges that lie between inside and outside triangles.



Q: How to assign labels?

A: Spectral Partitioning

- Assign a weight to each edge indicating if the two triangles are likely to have the same label.

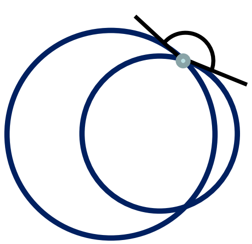


Assigning Edge Weights:

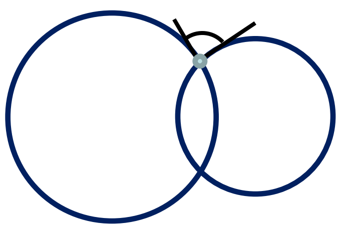
Q: When are triangles on opposite sides of an edge likely to have the same label?

A: If the triangles are on the same side, their circumscribing circles intersect deeply.

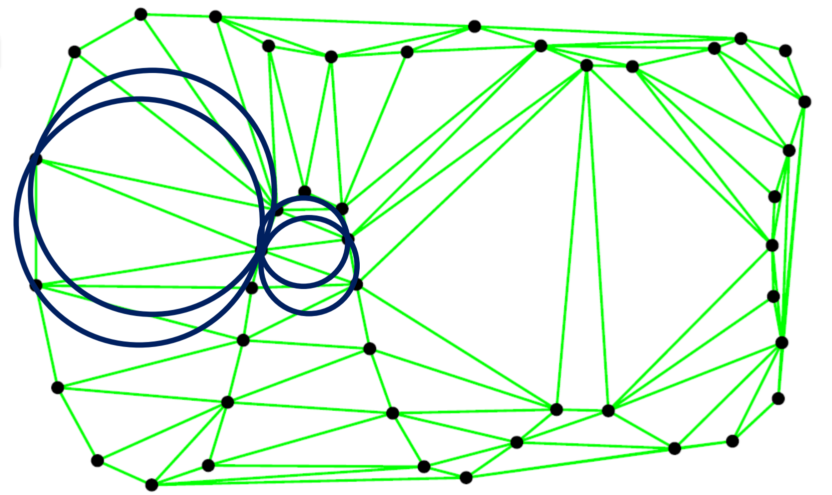
Use the angle of intersection to set the weight.



Large Weight



Small Weight



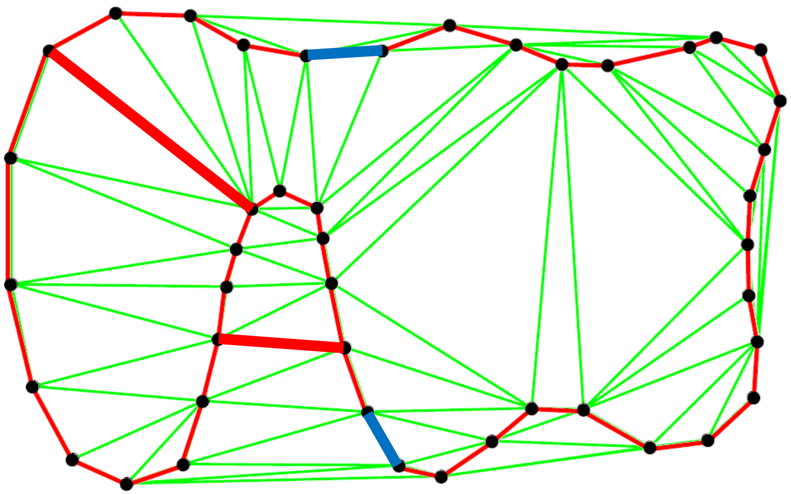
If we consider the Delaunay Triangulation of a point set, the shape boundary can be described as a subset of the Delaunay edges.

Q: How do we determine which edges to keep?

A: Two types of edges:

1. Those connecting adjacent points on the boundary
2. Those traversing the shape.

Discard those that traverse.



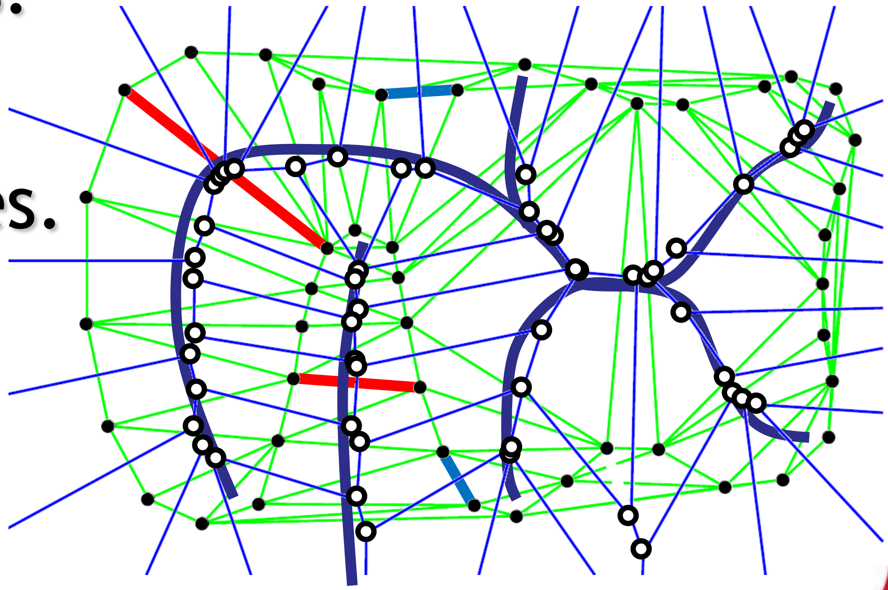
Observation:

Edges that traverse cross the medial axis.

Although we don't know the axis, we can sample it with the Voronoi vertices.

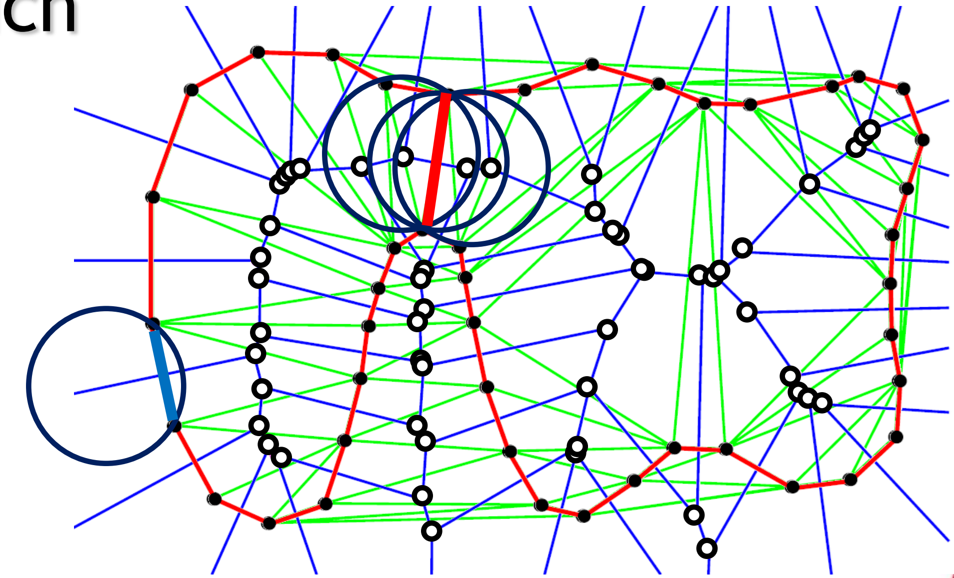
Edges that traverse must

be near the Voronoi vertices.



Algorithm:

1. Compute the Delaunay triangulation.
2. Compute the Voronoi vertices
3. Keep all edges for which there is a circle that contains the edge but no Voronoi vertices.



- Computational geometry
- A sampling of smooth methods
  - Space Partitioning
  - Crust
  - … from Unorganized Points
  - Poisson Reconstruction

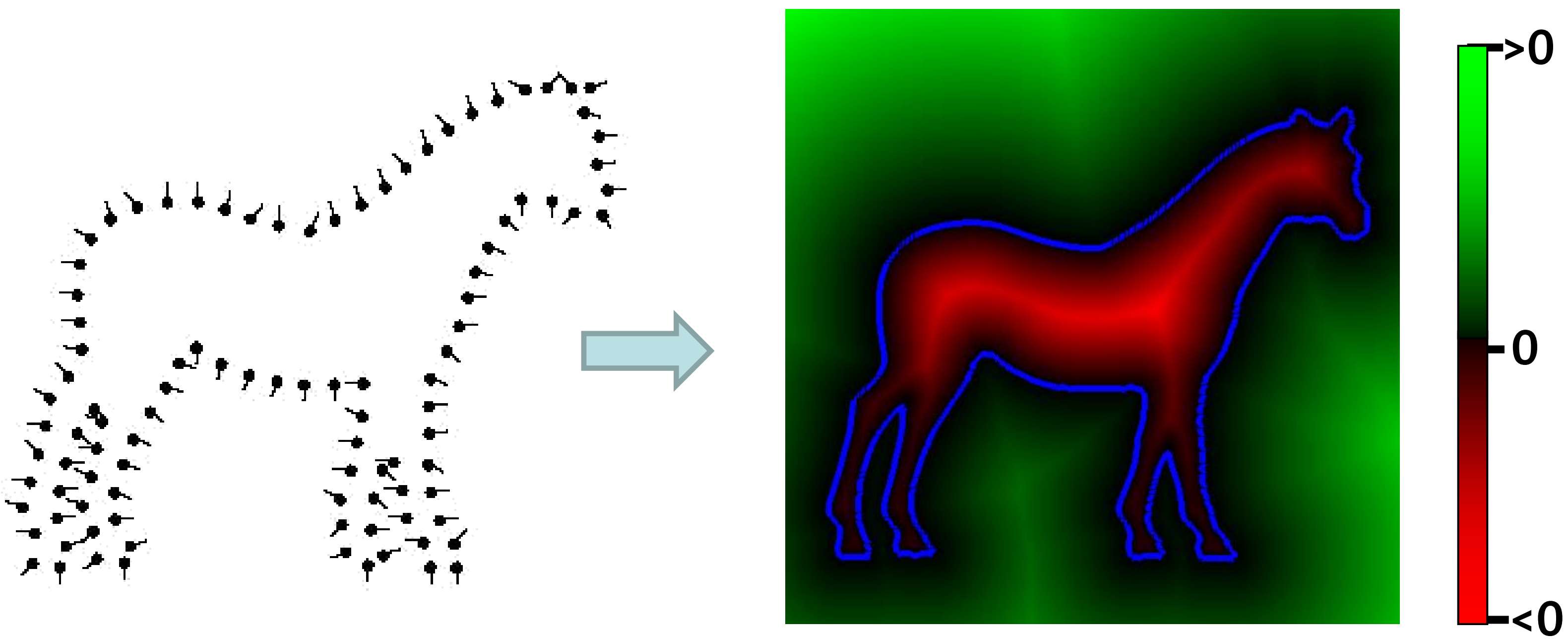
Computational Geometry

Implicit Surfaces

- A sampling of piecewise-smooth methods

Key Idea:

- Use the point samples to define a function whose values at the sample positions are zero.

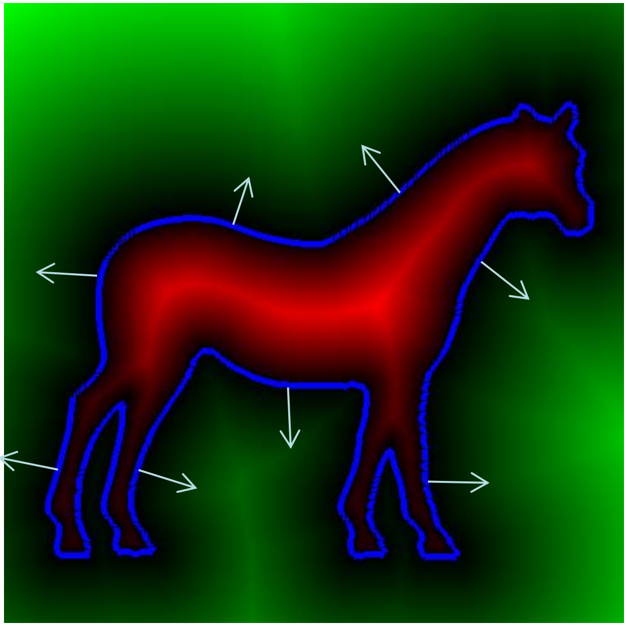


Sample Points

Observation:

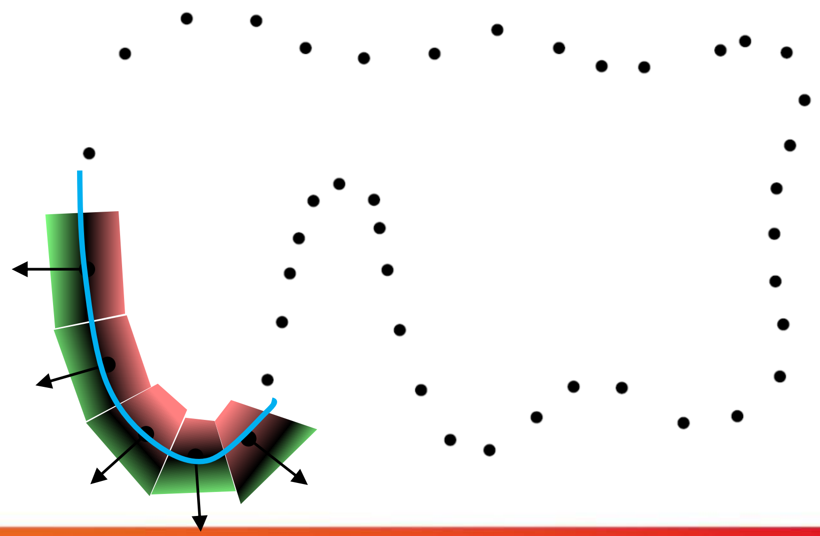
For points on the the surface, the signed (Euclidean) distance transform has a gradient that equal to the normal.

$$EDT(p) = \pm \min_{q \in S} \| p - q \|$$

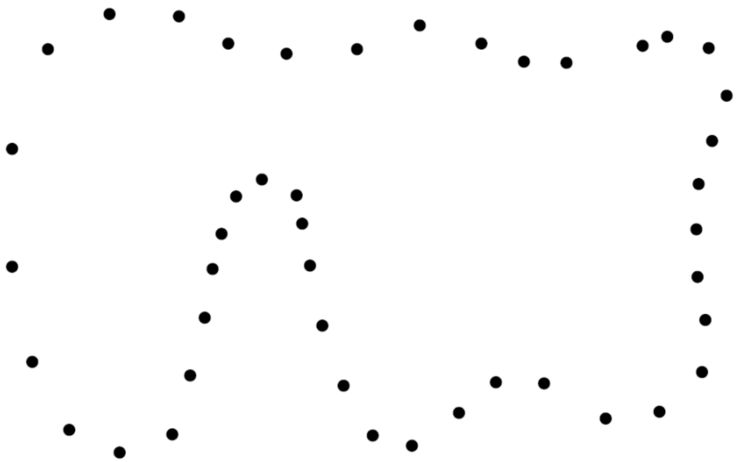


Computes a local signed distance transform by using the sample normals to define a linear approximation to the function.

Extracts the zero level (where defined).

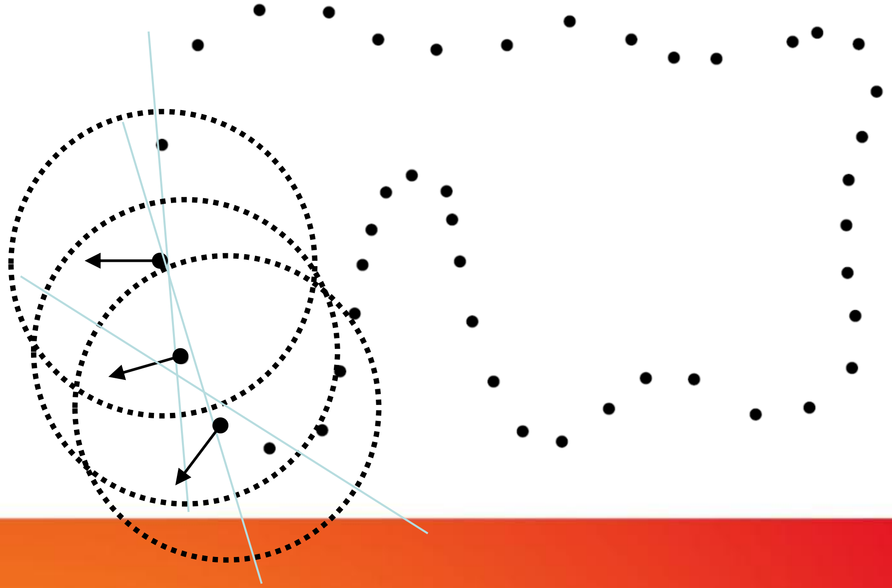


Q: How do we get the normals?



Q: How do we get the normals?

A1: Fit a line to the neighbors of each point.

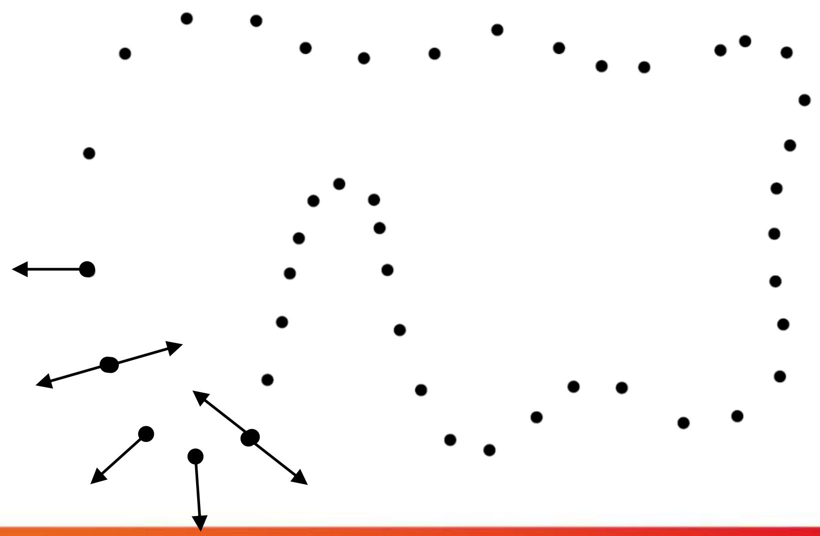


Q: How do we get the normals?

A1: Fit a line to the neighbors of each point.

This doesn't guarantee a consistent orientation!

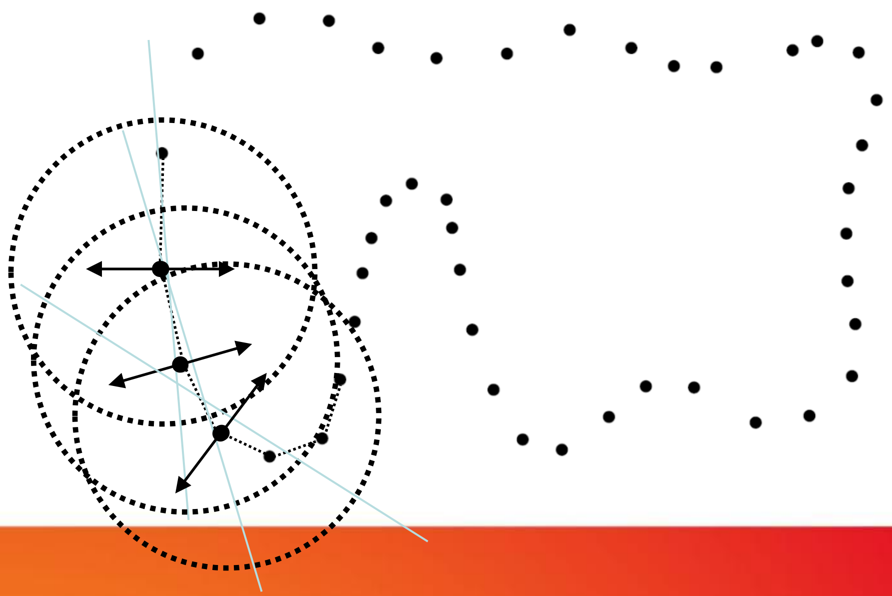
For the orientation to be consistent, neighboring points should point in the same direction.



Q: How do we get the normals?

A1: Fit a line to the neighbors of each point.

A2: Build a (Euclidian) minimal spanning tree and propagate the orientation from a root.



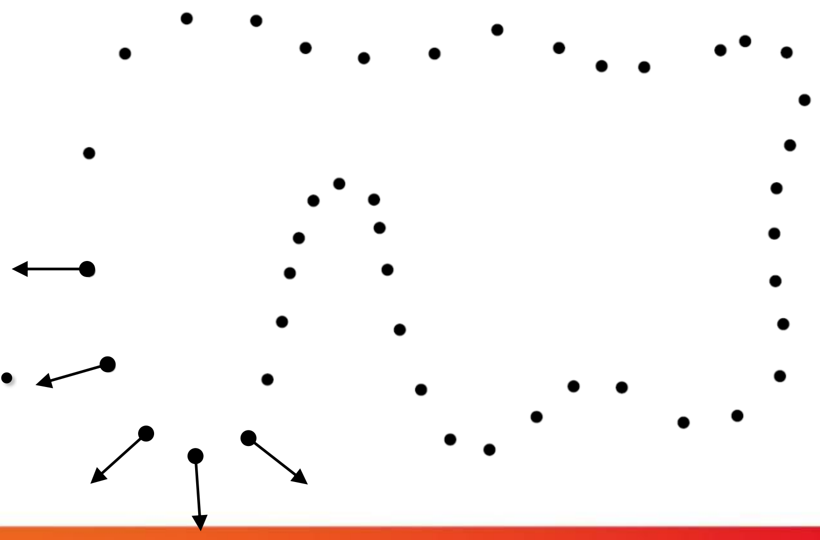
Q: How do we get the normals?

A1: Fit a line to the neighbors of each point.

A2: Build a (Euclidian) minimal spanning tree and propagate the orientation from a root.

<u>Note</u>:

In 2D, building a good spanning tree is almost the same as fitting a closed curve, it's not in 3D.



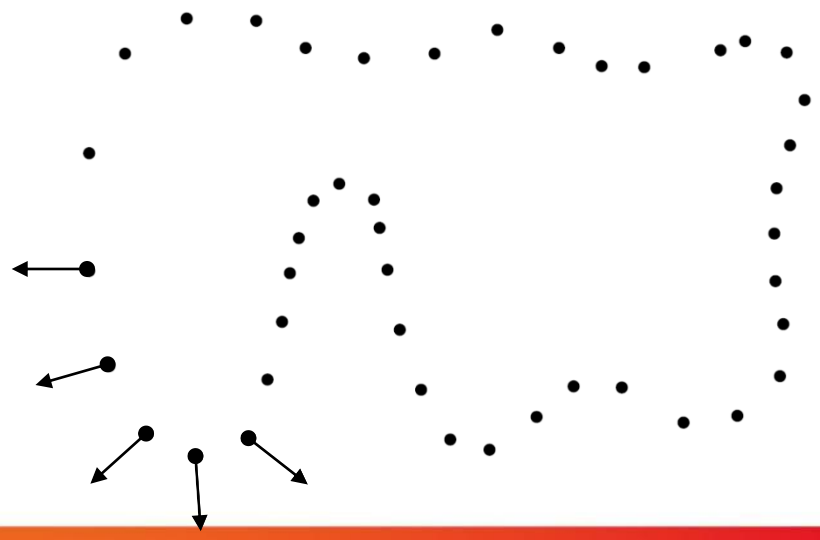
Q: How do we get the normals?

A1: Fit a line to the neighbors of each point.

A2: Build a (Euclidian) minimal spanning tree and propagate the orientation from a root.

Note:

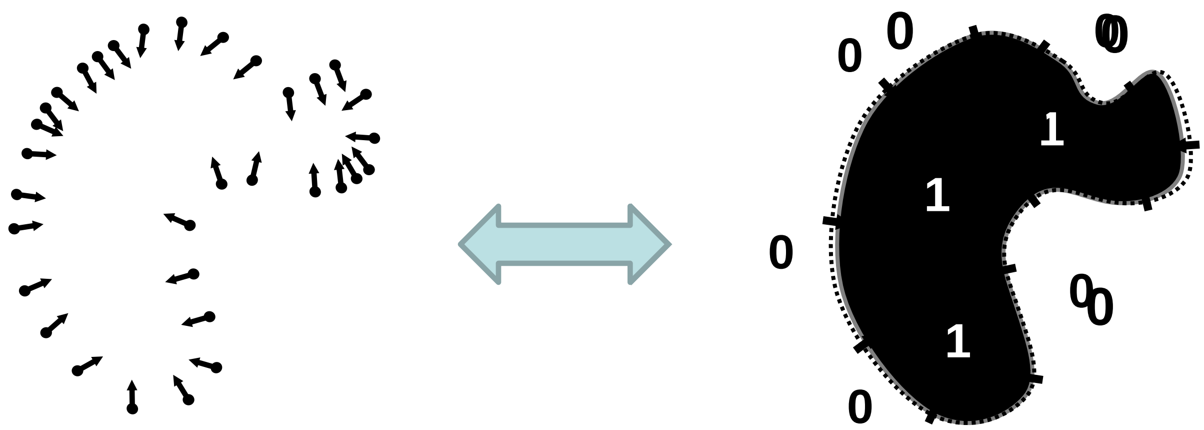
If the spanning graph is not a tree, can prioritize propagation based on confidence.



Reconstructs the indicator function of the surface and then extracts the boundary.

Q: How to fit the function to the samples?

A: Normals sample the function's gradients.



Oriented points

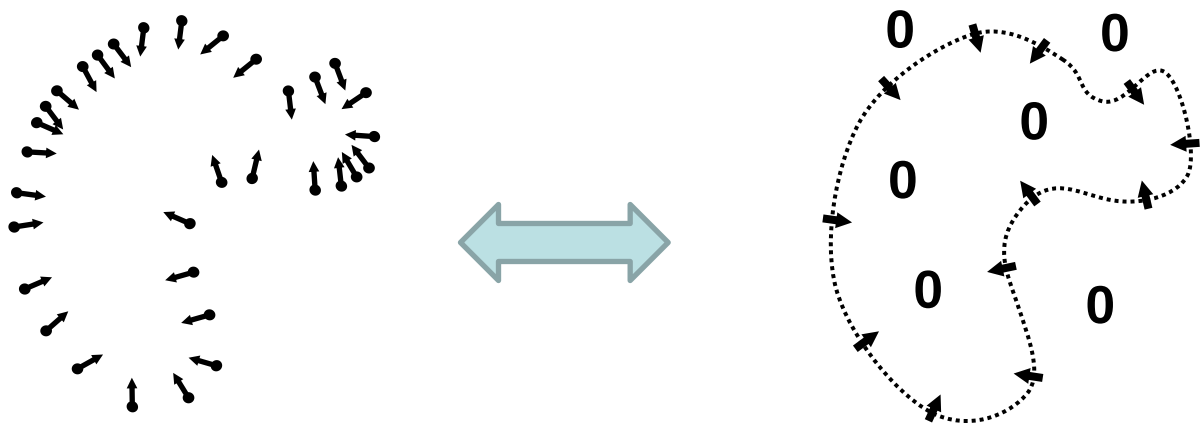
Indicator gradient

Indicator function

To fit a scalar field F to the gradients $\vec{V}$:

$$\min_F \left\| \nabla F - \vec{V} \right\|^2$$

we take the divergence of both sides, which results in a Poisson equation: $\nabla \cdot (\nabla F) - \nabla \cdot \vec{V} = 0 \quad \Leftrightarrow \quad \Delta F = \nabla \cdot \vec{V}$

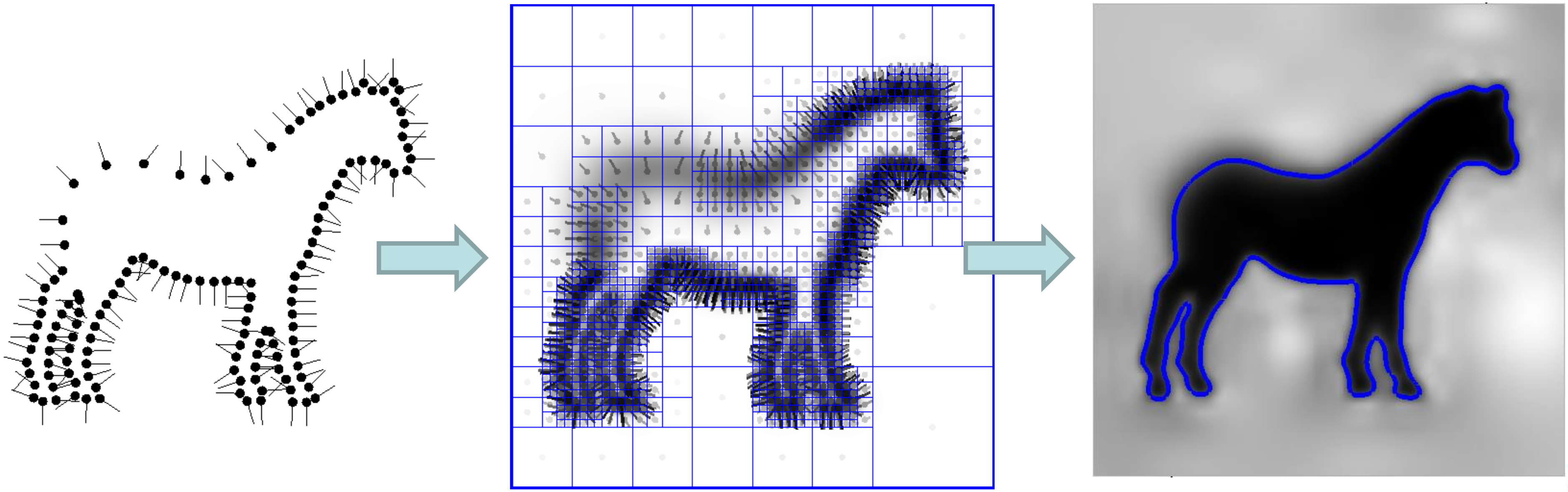


Oriented points **Indicator gradient**

[Kazhdan et al., 2006]

Algorithm:

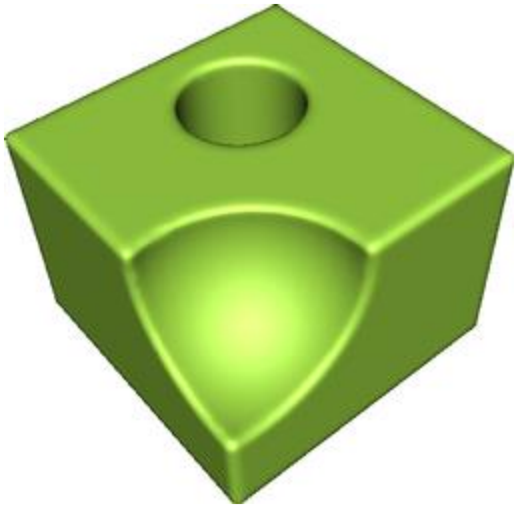
1. Transform samples into a vector field.
2. Fit a scalar-field to the gradients.
3. Extract the isosurface.



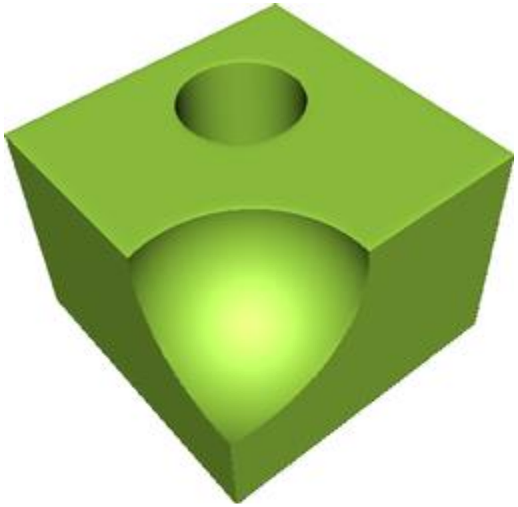
[Kazhdan et al., 2006]

- Computational geometry
- A sampling of smooth methods
- A sampling of piecewise-smooth methods
  - Feature extraction
  - Prior shape fitting

Piecewise-smooth



smooth

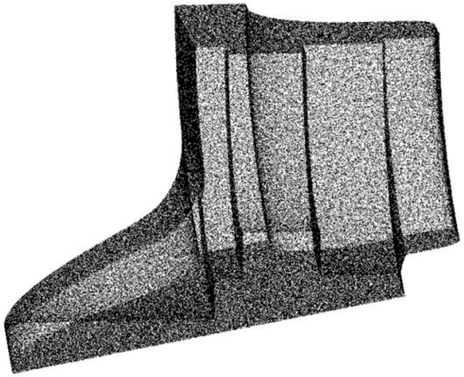


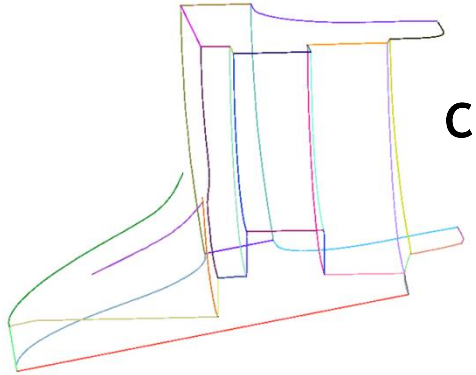
piecewise-smooth

feature detection:

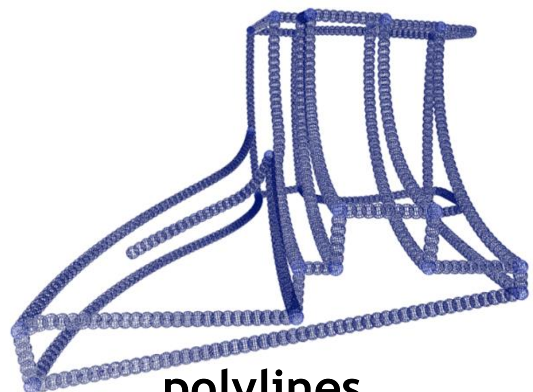
Goal: extract a set of sharp features from the point cloud in order to break the C_1 property of the surface at some locations during the reconstruction process

feature detection

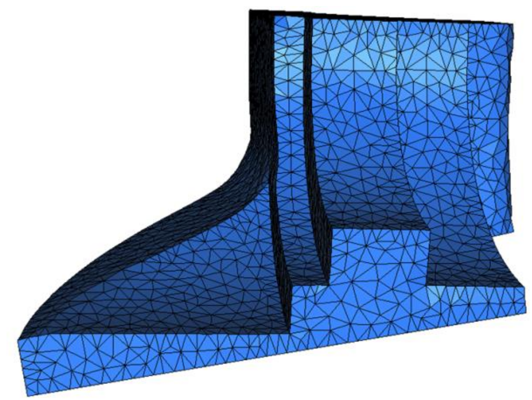




Clusters of points



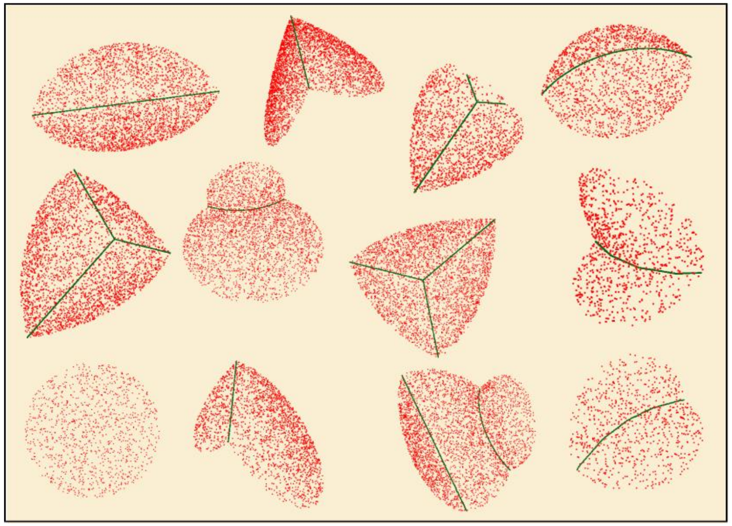
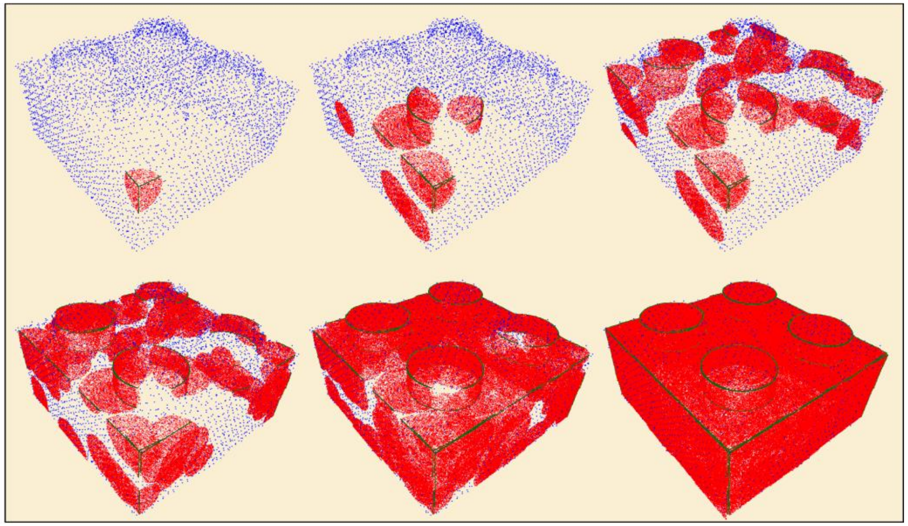
polylines



[Salman et al., 2011]

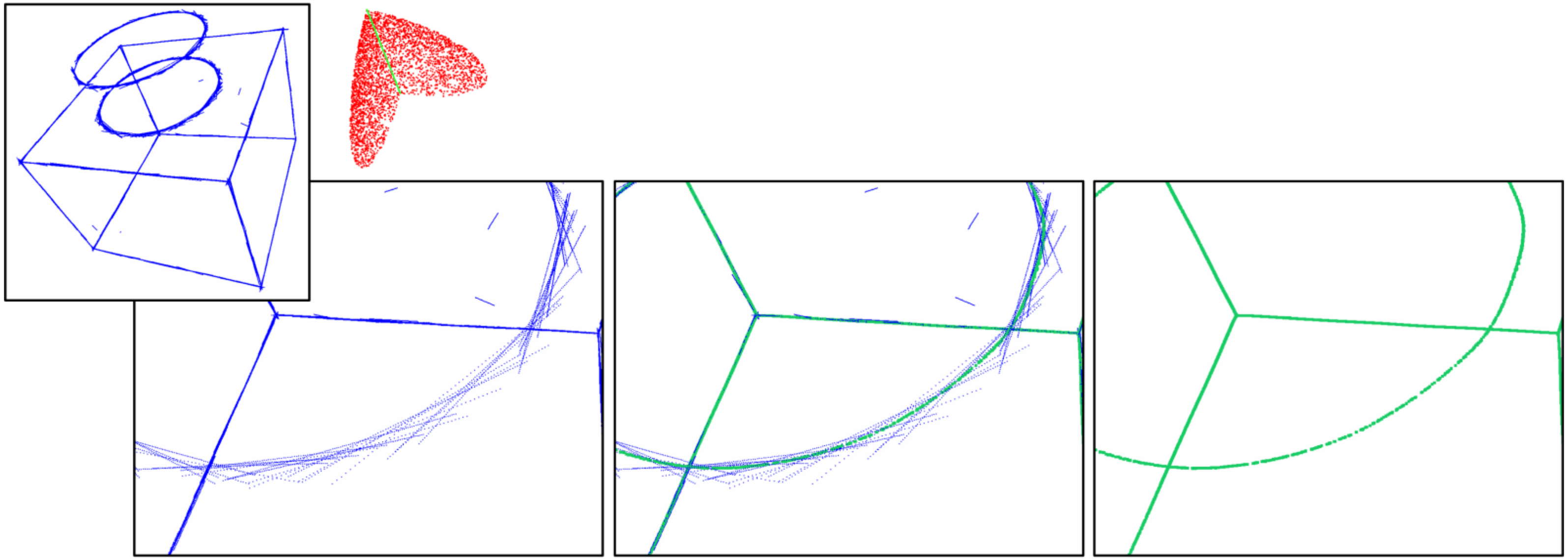
Prior shape fitting

Idea: fit a collection of shapes with sharp edges to the point cloud

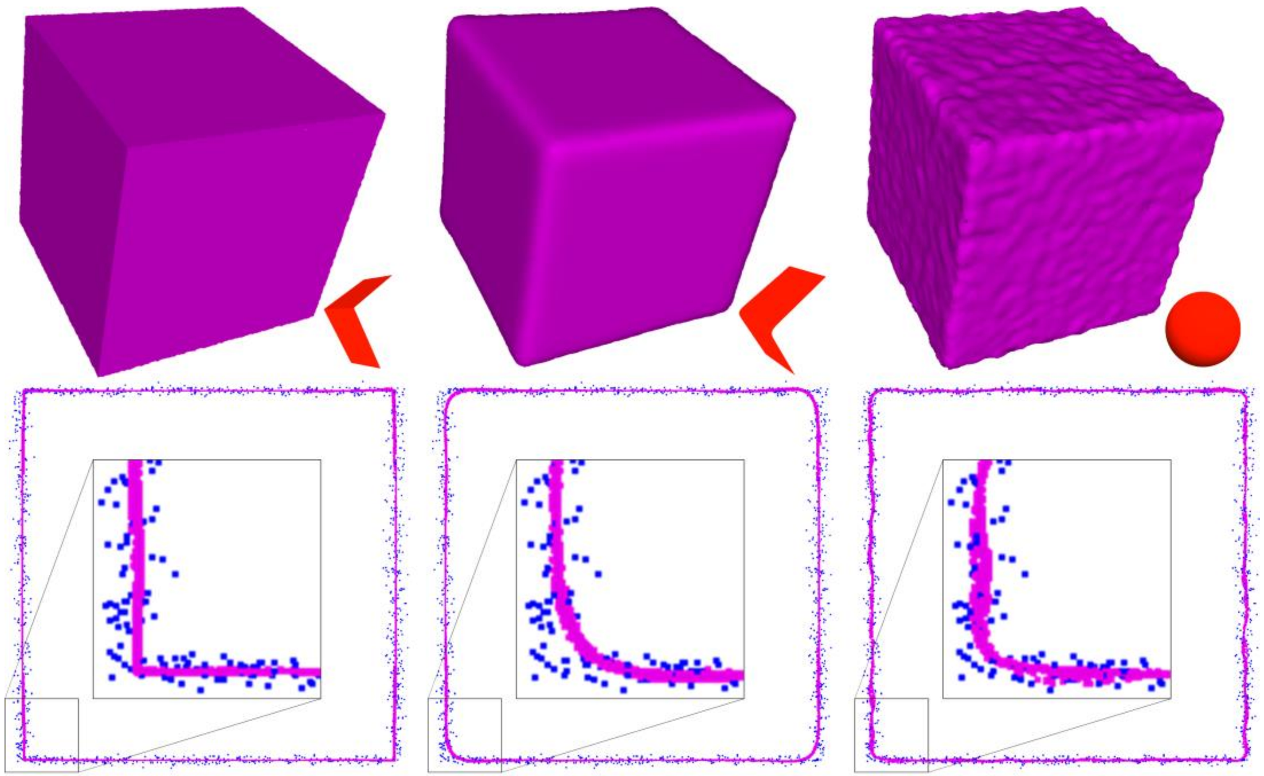
[Gal et al., 2007]

Prior shape fitting



[Gal et al., 2007]

Prior shape fitting



[Gal et al., 2007]