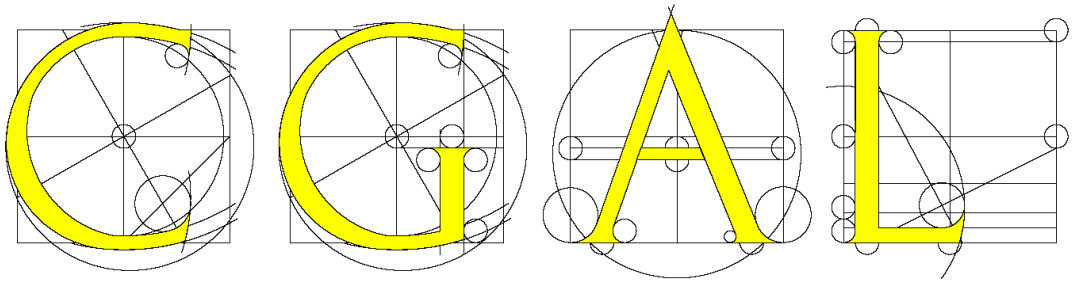
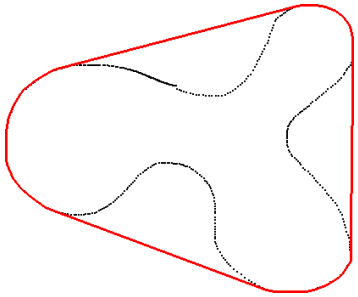
2D Convex Hull in





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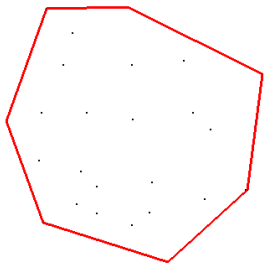
Outline

- Definitions
- Convex hull in CGAL
  - Interface
  - Extreme points
  - Subsequences of hull points
  - Convexity Checking
- Exercise

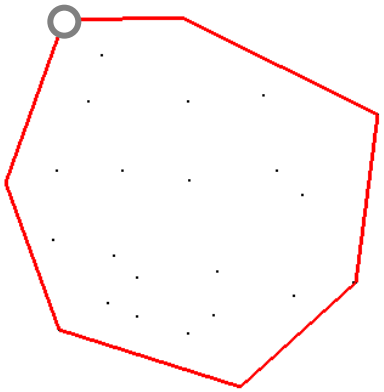
Definitions

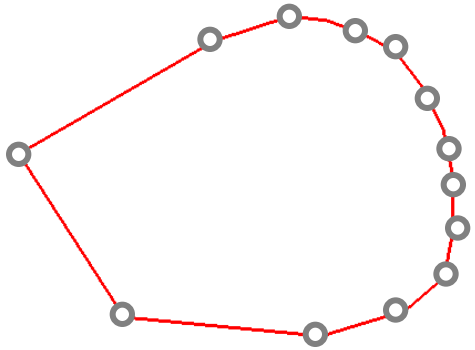
- A subset $S \subset \mathbb{R}^2$ is convex if for any two points p and q in the set the line segment with endpoints p and q is contained in S.
- The convex hull of a set S is the smallest convex set containing S.
- The convex hull of a set of points P is a convex polygon with vertices in P.



Definitions

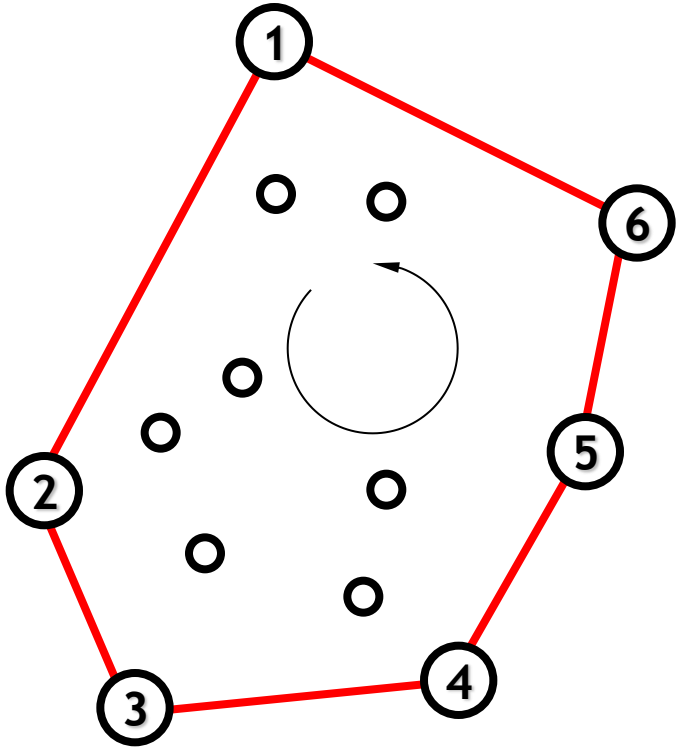
- A point in *P* is an extreme point (with respect to *P*) if it is a vertex of the convex hull of *P*.
- A point set is said to be strongly convex if it consists of only extreme points.





Convex Hull in CGAL

5 algorithms to compute the counterclockwise sequence of extreme points for a 2D point set.



Convex Hull in CGAL

n input points with h extreme points

Algorithms:

- [Bykat 78] O(*nh*) output-sensitive
- [Akl & Toussaint] O(*nlogn*) worst case
- [Graham-Andrew] O(*nlogn*)
- [Jarvis 73] O(*nh*)
- [Eddy] O(*nh*)

Interface

```
template <class InputIterator, class OutputIterator>

OutputIterator convex_hull_2(InputIterator first,
                             InputIterator beyond,
                             OutputIterator result,
                             Traits ch_traits =
                             Default_traits)
```

generates the counterclockwise sequence of extreme points of the points in the range [*first*,*beyond*). The resulting sequence is placed starting at position *result*, and the past-the-end iterator for the resulting sequence is returned (it is not specified at which point the cyclic sequence of extreme points is cut into a linear sequence).

Interface

```
template <class InputIterator, class OutputIterator>

OutputIterator convex_hull_2(InputIterator first,
                             InputIterator beyond,
                             OutputIterator result,
                             Traits ch_traits =
                             Default_traits)
```

InputIterator::value_type

OutputIterator::value_type

} Traits::Point_2

Traits contains types and functions:

Traits::Point_2, Traits::Less_xy_2, Traits::Less_yx_2, Traits::Leftturn_2.

Default...

```
#include <CGAL/Cartesian.h>
#include <CGAL/convex_hull_2.h>
#include <list>

typedef CGAL::Cartesian<double> Kernel;
typedef Kernel::Point_2 Point;

std::list<Point> points;
std::list<Point> hull;
points.push_back(Point(0,0));
points.push_back(Point(0,1));
// ...
CGAL::convex_hull_2(points.begin(),
                    points.end(),
                    std::inserter(hull,hull.begin()));
```

Graham-Andrew

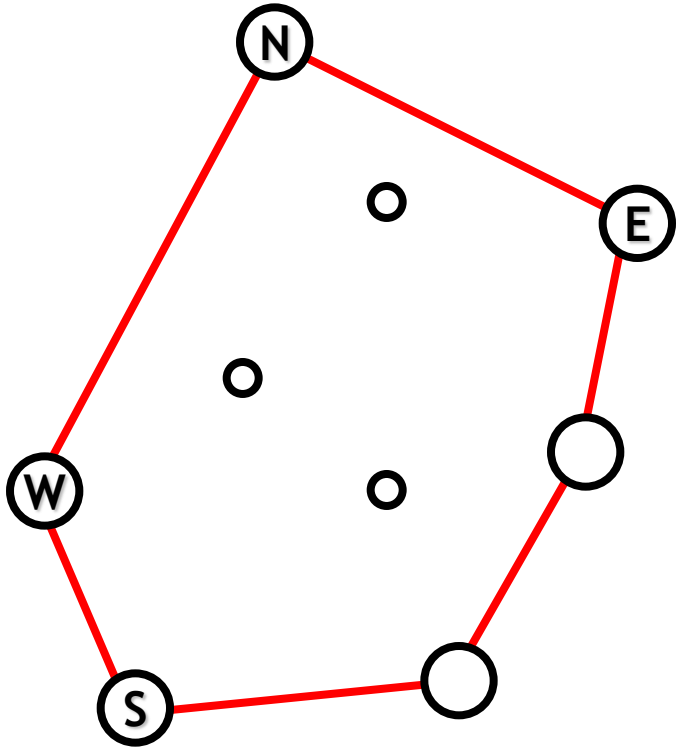
```
#include <CGAL/Cartesian.h>
#include <CGAL/graham_andrew.h>
#include <list>

typedef CGAL::Cartesian<double> Kernel;
typedef Kernel::Point_2 Point;

std::list<Point> points;
std::list<Point> hull;
points.push_back(Point(0,0));
points.push_back(Point(0,1));
// ...
CGAL::ch_graham_andrew(points.begin(),
                       points.end(),
                       std::inserter(hull,hull.begin()));
```

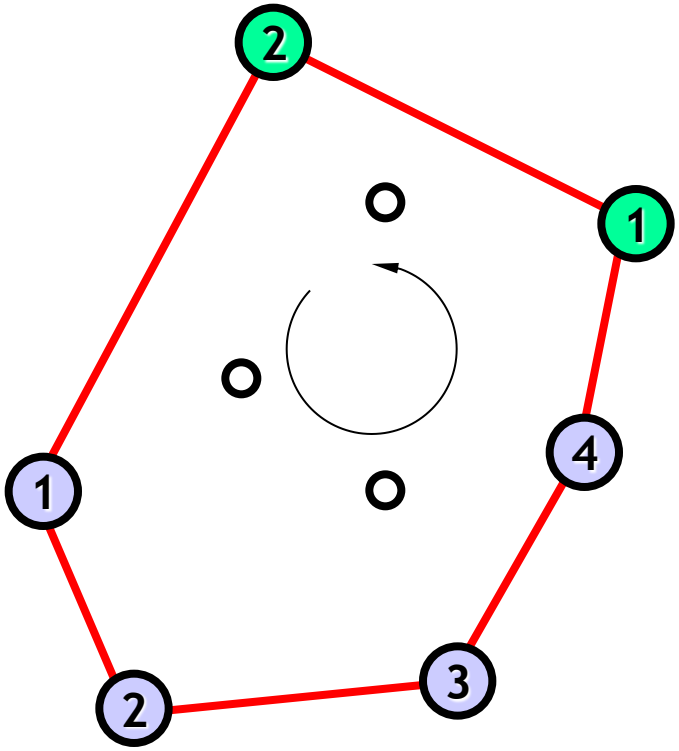
Extreme Points

- North, South, East, West and combinations.



SubSequences of Hull Points

- Lower Hull
- Upper Hull



Convexity Checking

```
#include <CGAL/convexity_check_2.h>


template <class ForwardIterator, class Traits>
bool is_ccw_strongly_convex_2 (
    ForwardIterator first,
    ForwardIterator beyond,
    Traits ch_traits = Default_traits)
```

returns true iff the point elements in [first,beyond) form a counterclockwise-oriented strongly convex polygon.

Exercise on Computer