Vision par ordinateur 3-D
M2R IVR
Examen du 1 mars 2007
Tous les documents sont autorisés

1 Essential Matrix

Let E_{12} be the essential matrix between image 1 and image 2.
Give the essential matrix E_{21} for the reverse direction, i.e. between image 2 and image 1.

2 3D Reconstruction of a Point on a Plane

Consider a camera with known projection matrix P and a plane in 3D with known coordinate vector $\mathbf{M}$ (reminder : this is a 4-vector, defined such that $\mathbf{Q}^{\mathsf{T}}\mathbf{M} = 0$ for every point $\mathbf{Q}$ on the plane).
Let $\mathbf{q}$ be an image point (in homogeneous coordinates).
The goal is to reconstruct the 3D point $\mathbf{Q}$ associated with $\mathbf{q}$, using the information that it lies on the plane $\mathbf{M}$.
It is possible to compute $\mathbf{Q}$ by solving a system of the following form :

$$\mathsf{A}\mathbf{Q} \sim \mathbf{b}$$

where A is a 4×4 matrix and $\mathbf{b}$ a 4-vector.
Give A and $\mathbf{b}$.

3 Matching (mise en correspondance)

Knowledge of the epipolar geometry between two images helps the matching, as seen during the lecture : let $\mathbf{q}_1$ be a point in the first image ; to find the corresponding point $\mathbf{q}_2$ in the second image, it is sufficient to search along the epipolar line associated with $\mathbf{q}_1$.

In practice, one can restrict the problem even further :
- First, the 3D point associated with $\mathbf{q}_1$ has to lie **in front** of the first camera, i.e. on the half-line shown in the left part of figure figure 1.
- Second, one can often make a reasonable assumption on the minimal distance between 3D points in the scene and the cameras.

Questions :

(a) The constraint related to the above mentioned half-line allows to constrain the possible positions of $\mathbf{q}_2$ on the epipolar line : they are restricted to lie within a line **segment** (instead of an infinite line). Which are the two extremal points of this line segment ?

(b) Describe, using words (no equations), how much this constraint is beneficial, for each of the two scenarios shown in figure 1.

(c) Suppose that the projection matrices of the two cameras are given by :

$$\mathsf{P}_1 = \begin{pmatrix} \mathtt{I} & \mathbf{0} \end{pmatrix} \qquad \mathsf{P}_2 = \mathsf{R} \begin{pmatrix} \mathtt{I} & -\mathbf{t} \end{pmatrix}$$

Show how to compute the two extremal points of the line segment described in question (a).

(d) Consider now the second constraint mentioned above, i.e. the assumption on the minimal distance between 3D points and the cameras. So, suppose that the 3D point associated with $\mathbf{q}_1$ has to be at a distance from the first camera's optical center that is larger than d. This constraint allows to shrink the line segment of question (a). Show how to compute the new extremal points of the line segment, given this constraint.

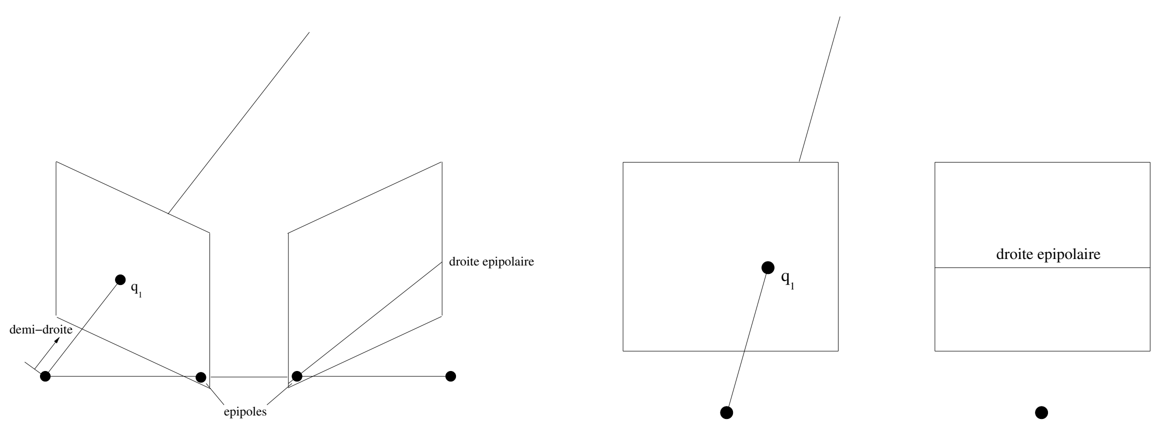

FIG. 1 – Two configurations of image pairs.

4 Distance Measurement

Consider the scenario shown on figure 2 : a calibrated camera observes a **planar** scene (this could for example be a camera on an airplane, observing the ground plane). The scene contains control points, whose coordinates are known in a 2D coordinate system attached to the ground plane.

The goal is to compute the distance between any two points $\mathbf{A}$ and $\mathbf{B}$ on the ground plane, from the information already described and the image points associated with the control points and $\mathbf{A}$ and $\mathbf{B}$.

Describe a method for doing this (it is not required to give equations, a verbal description is sufficient).

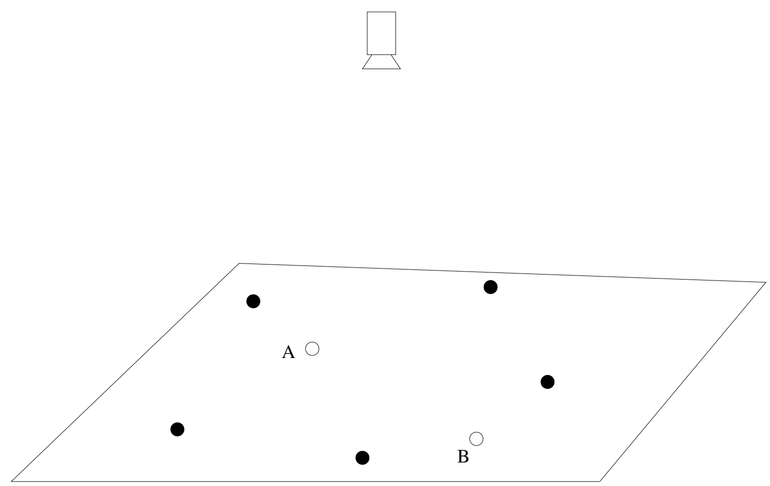

FIG. 2 – Scenario for question 4. The control points are shown in black.

5 Pose Computation

Consider a part of an object, consisting of a point and three line segments that meet in that point (see figure 3). I want to stress that this is a **3D** object, i.e. the three segments are not coplanar ! Suppose that we know the angles α_i between every pair of these segments. This scenario could for example concern the corner of a cube, in which case the three angles between pairs of segments are right angles ($90°$).

Suppose that we have acquired an image of this object, with a calibrated camera, and that in this image, we have extracted the image of the object point as well as the three (half-) lines corresponding to the three segments.

The goal is to compute, from these pieces of information, the pose of the object relative to the camera, i.e. its position and orientation.

(a) Is it possible to completely determine the **position** of the object ? If not, which partial information on the position can be determined ?

(b) Suppose we already know the 3D position of the object point, relative to the camera. What can we say about the possible positions of each of the object's segments ?

(c) Desribe a method for computing the object's pose and if possible, give associated equations. Hint : the "position" part of the pose is easily represented by the position of the object point ; the "orientation" (or, "rotation") part of the pose may be represented by the position of the three segments.

(d) Suppose now that the object is planar, i.e. that all its three segments are coplanar. Describe a method for computing the object's pose and if possible, give associated equations.

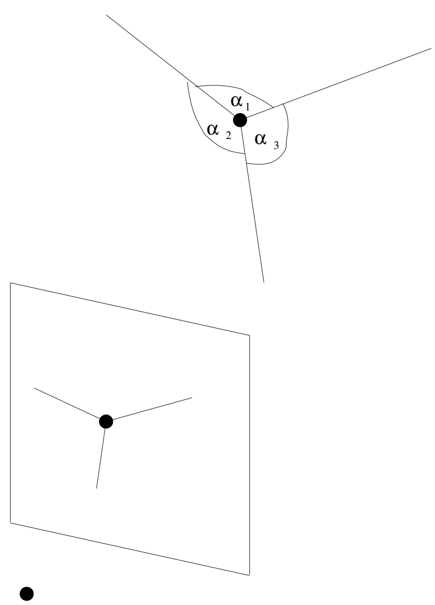

FIG. 3 – Scenario for question 5.