Stability analysis and control design for weakly hard real-time systems

How can richer interfaces, e.g. m-k guarantees, be used for the functional verification of control systems

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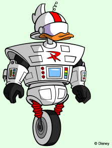

Standing upright | Moving to locations | . . . | Speaking with others

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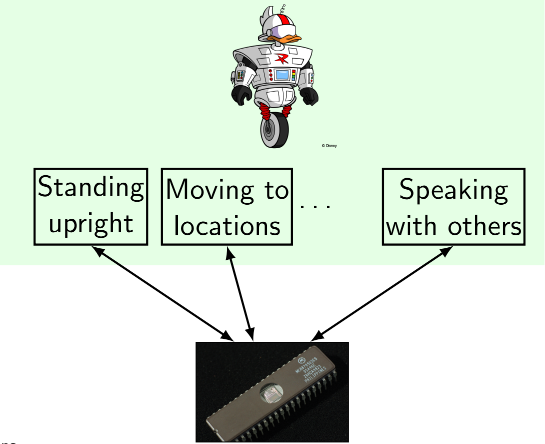
Embedded control systems - Relevant properties



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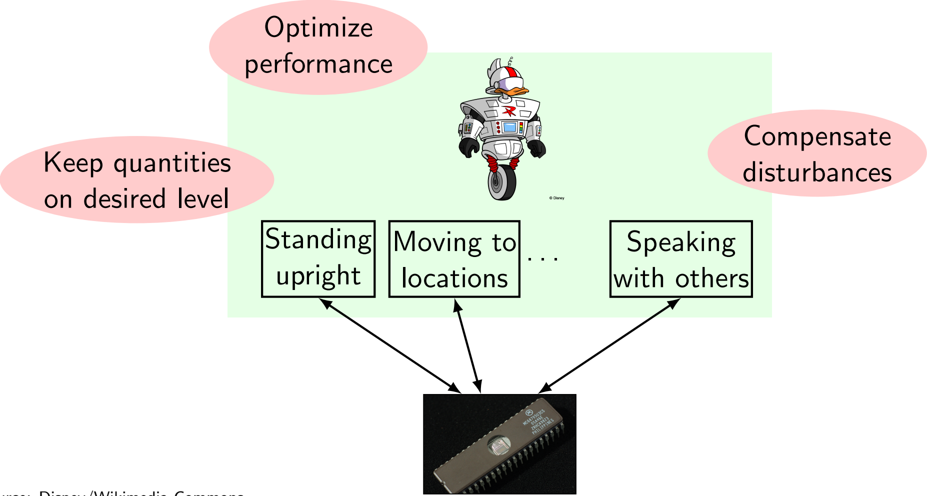

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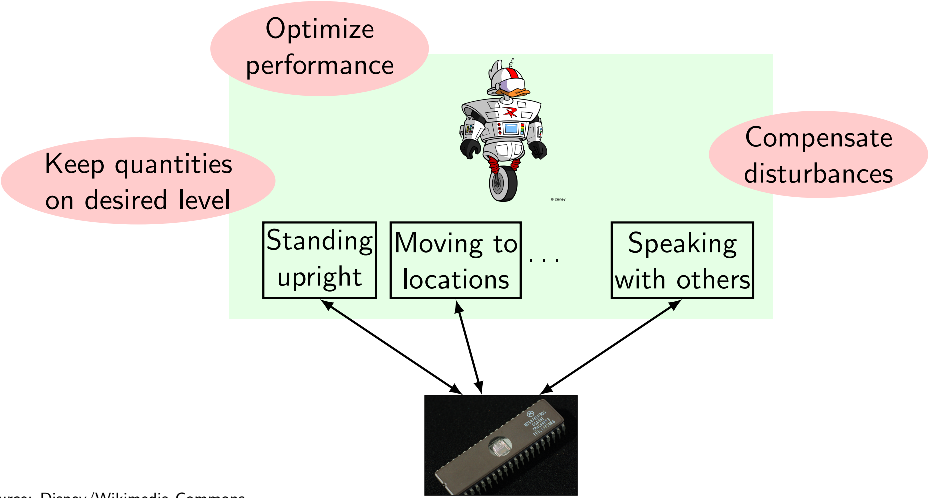
Embedded control systems - Relevant properties



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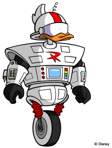
Is it necessary that all deadlines are met to guarantee properties of the control system?

Outline

1. Functional verification of control systems
2. How to use weakly hard real-time constraints in control systems
3. Stability analysis with weakly hard real-time constraints
4. Controller design with weakly hard real-time constraints
5. Summary



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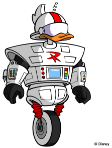

→ Standing upright

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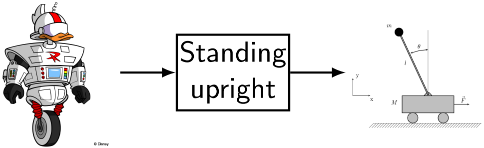

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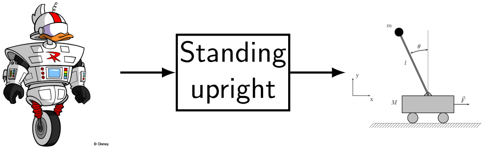

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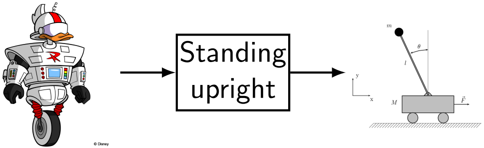
This can be used to motivate/illustrate a major property of control systems:
(Asymptotic) stability



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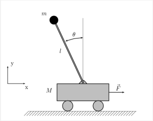
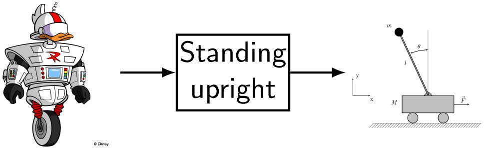
The upright equilibrium is unstable! $\Rightarrow$ One needs a controller to stabilize it.



Goal: Guarantee asymptotic stability of the upright equilibrium



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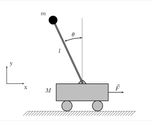
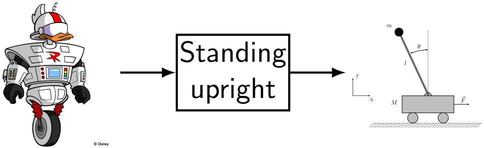


Goal: Guarantee asymptotic stability of the upright equilibrium *with richer interfaces*.



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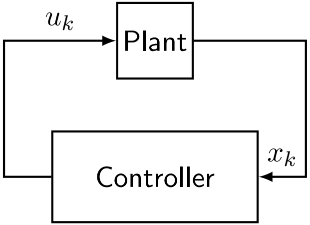
Asymptotic Stability

The equilibrium at the origin of $x_{k+1} = f(x_k)$ is asymptotically stable if it is stable and if δ can be chosen such that $\|x_{k_0}\| < \delta$ implies that $\|x_k\| \to 0$ when $k \to \infty$.

This talk: Linear time-invariant systems with full state available for feedback.

State-space representation of the plant:

$$x_{k+1} = Ax_k + Bu_k$$



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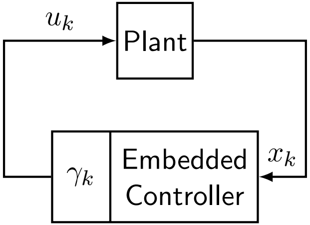
Representation of the embedded controller:

$$u_k = f(\gamma_k, x_k, (u_{k-1})), \gamma_k \in \{0, 1\}$$



Stabilization

Can we derive a (static state feedback) controller that renders the origin of the closed loop asymptotically stable?

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Can we derive a controller that minimizes a given cost functional?

Robustness

Can we derive a controller that is robust to uncertainties and disturbances?

Stabilization

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Up to now:

- Stability and Stabilization as important properties and tasks for functional verification of control systems.

Open tasks:

- Show that we can include richer interfaces in control system models.
- Analyze stability with richer interfaces.
- Design controllers with richer interfaces.

Control Problem

$x_{k+1} = Ax_k + Bu_k$

A, B stabilizable

Control Problem	*Embedded Controller*
$x_{k+1} = Ax_k + Bu_k$	$u_k = f(\gamma_k, x_k, (u_{k-1}))$
A, B stabilizable	model of the process γ_k

Control Problem	*Embedded Controller*
$x_{k+1} = Ax_k + Bu_k$	$u_k = f(\gamma_k, x_k, (u_{k-1}))$
A, B stabilizable	max. # of successive deadline misses m

- Worst-case consecutive specification $\Rightarrow$ introduces conservativity

Control Problem	*Embedded Controller*
$x_{k+1} = Ax_k + Bu_k$	$u_k = f(\gamma_k, x_k, (u_{k-1}))$
A, B stabilizable	deadline miss probability p%

- Worst-case consecutive specification $\Rightarrow$ introduces conservativity
- Statistical description $\Rightarrow$ leads to statements for mean and variance

Motivation for using richer interfaces

Control Problem	*Embedded Controller*
$x_{k+1} = Ax_k + Bu_k$	$u_k = f(\gamma_k, x_k, (u_{k-1}))$
A, B stabilizable	max. m deadline misses out of k transmissions

- Worst-case consecutive specification $\Rightarrow$ introduces conservativity
- Statistical description $\Rightarrow$ leads to statements for mean and variance
- Window based description $\Rightarrow$ promising idea to bridge the gap

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- Statistical description ⇒ leads to statements for mean and variance
- Window based description ⇒ promising idea to bridge the gap

Goal

Include window based descriptions in the model of embedded control systems.

Weakly hard real-time (WHRT) constraints to model task executions

Characterization of binary sequences γ by satisfaction of constraints λ "$(\gamma \vdash \lambda)$":

Bernat et al. (2001). Weakly hard real-time systems.

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In any window of m consecutive executions, ...

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...there are at least n consecutive successful executions.

$\gamma \vdash \overline{\left\langle {n \atop m} \right\rangle}$

...there are never n consecutive unsuccessful executions.

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Satisfaction Set

The *satisfaction set* of length N of a constraint λ, denoted $\mathcal{S}^N(\lambda)$, is the set of all sequences $\alpha \in \{0,1\}^N$ that satisfy λ. Formally,

$$\mathcal{S}^N(\lambda) := \left\{\alpha \in \{0,1\}^N : \alpha \vdash \lambda\right\}.$$

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Partial Order

Given two constraints λ', λ. We say that λ' is *harder than* λ (λ is *weaker than* λ'), denoted $\lambda' \preceq \lambda$ if all sequences that satisfy λ' also satisfy λ. Formally,

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Example:
At least 3 *succ. executions in a row* is harder than *at least* 2 *succ. executions in a row.*

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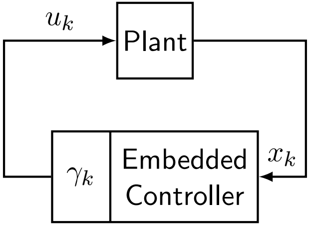

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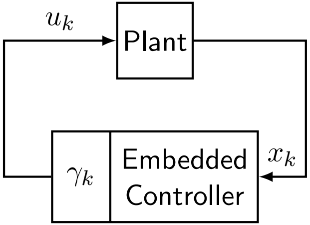

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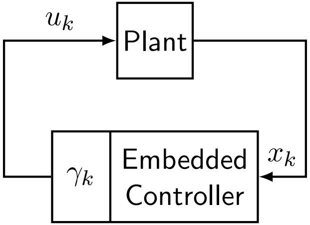

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Actuation strategies:

- zero strategy

$$u_k = \gamma_k K x_k$$

- hold strategy

$$u_k = \gamma_k K x_k + (1 - \gamma_k) u_{k-1}$$

Model of the closed loop with richer interfaces

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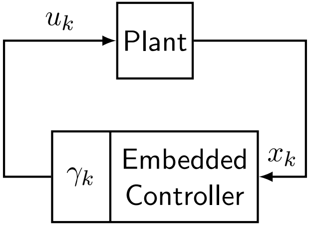

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- We can include WHRT constraints in control system models.

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Plant:

$$x_{k+1} = ax_k + bu_k, \quad x_k \in \mathbb{R}, a > 1 \Rightarrow \textbf{unstable}$$

Embedded Controller:

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Blind, Allgöwer (2015). Towards networked control systems with guaranteed stability: Using weakly hard real-time constraints to model the loss process.

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Probabilistic description of γ:

Stabilizable in the mean square sense if and only if the success probability is greater than $1 - \frac{1}{a^2}$.

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The system is asymptotically stable for all sequences $\gamma \vdash \lambda \preceq \binom{1}{m}$, where m might be arbitrarily large but finite.

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The closed loop can be modeled as a linear discrete-time switched system,

$$\xi_{k+1} = \mathcal{A}_{\gamma_k} \xi_k, \quad \xi_k \in \mathbb{R}^{n_\xi}, \gamma_k \in \{0, 1\}, \gamma \vdash \lambda$$

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Zero control:

- $u_k = \gamma_k K x_k$
- $\xi_k = x_k \rightarrow n_\xi = n$
- $\mathcal{A}_0 = A$
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Hold control:

- $u_k = \gamma_k K x_k + (1-\gamma_k)u_{k-1}$
- $\xi_k = [x_k, u_{k-1}] \rightarrow n_\xi = n + n_u$
- $\mathcal{A}_0 = \begin{bmatrix} A & B \\ 0 & I \end{bmatrix}, \mathcal{A}_1 = \begin{bmatrix} A+BK & 0 \\ K & 0 \end{bmatrix}$

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Notation

Given a sequence $w \in \{0,1\}^m$, the state evolves as $x_{k+|w|} = \prod_{i=|w|}^{1} \mathcal{A}_{w_i} x_k$.
Hence, we use $\mathcal{A}^w := \prod_{i=|w|}^{1} \mathcal{A}_{w_i}$.
Example: $\mathcal{A}^{1100101} = \mathcal{A}_1\mathcal{A}_0\mathcal{A}_1\mathcal{A}_0\mathcal{A}_0\mathcal{A}_1\mathcal{A}_1$.

Theorem 1

Given the constraint λ and the matrices $\mathcal{A}_0, \mathcal{A}_1 \in \mathbb{R}^{n_\xi \times n_\xi}$. If there exists a symmetric, positive definite matrix $\mathcal{P} \in \mathbb{R}^{n_\xi \times n_\xi}$ such that

$$(\mathcal{A}^w)^\intercal \mathcal{P} \mathcal{A}^w - \mathcal{P} < 0, \forall w \in \mathcal{S}^m(\lambda),$$

then the closed loop is asymptotically stable for all sequences $\gamma \vdash \lambda' \preceq \lambda$.

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Procedure to check whether the origin of the closed-loop is asy. stable:

- Compute $\mathcal{A}_0, \mathcal{A}_1$ according to the actuation strategy.
- Derive all sequences in $\mathcal{S}^m(\lambda)$.
- Compute all $\mathcal{A}^w$ for $w \in \mathcal{S}^m(\lambda)$.
- Find a positive definite matrix $\mathcal{P}$ such that all LMI's are satisfied.

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Theorem 2

Given the constraint λ and the matrices $\mathcal{A}_0, \mathcal{A}_1 \in \mathbb{R}^{n_\xi \times n_\xi}$. If there exist symmetric, positive definite matrices $\mathcal{P}_w \in \mathbb{R}^{n_\xi \times n_\xi}, w \in \mathcal{S}^m(\lambda)$ such that

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Procedure to check whether the origin of the closed-loop is asy. stable:

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- Compute all $\mathcal{A}^w$ for $w \in \mathcal{S}^m(\lambda)$.
- Find $|\mathcal{S}^m(\lambda)|$ positive definite matrices $\mathcal{P}_w$ such that LMI's are satisfied (combinations s.t. $w_1 w_2 \vdash \lambda$).

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- Find $|\mathcal{S}^m(\lambda)|$ positive definite matrices $\mathcal{P}_w$ such that LMI's are satisfied (combinations s.t. $w_1 w_2 \vdash \lambda$). $\Rightarrow$ increased possibility to find a solution

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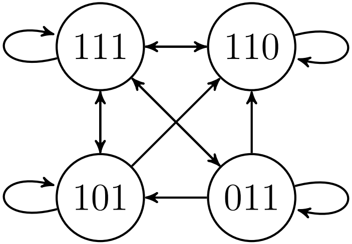
Example: $\lambda = \binom{2}{3}$



Plant:

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Up to now:

- Stability and Stabilization as important properties and tasks for functional verification of control systems.
- We can include WHRT constraints in control system models.
- We can analyze stability with WHRT constraints.

Open tasks:

- Design controllers with richer interfaces.

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In the remainder:

Static state feedback stabilization

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- and an actuation strategy (zero resp. hold),

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In the remainder:

Static state feedback stabilization

Given

- a weakly hard real-time constraint λ,
- a discrete-time linear control system (A, B stabilizable),
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find K s.t. the origin of the closed loop system with $u_k = \gamma_k K x_k$ resp. $u_k = \gamma_k K x_k + (1 - \gamma_k) u_{k-1}$ is asymptotically stable for all $\gamma \vdash \lambda$.

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$\Rightarrow$ This is covered by the introduction of an alternative discretization

New sequences are derived based on γ :

- $(\tau_k)_{k\in\mathbb{N}}$ contains all time instants when the computation is executed in time.
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- Example:

$$\gamma = (0, 1, 0, 0, 1, 0, 1, \dots)$$
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Those sequences are used for the restricted dynamical system

$$\tilde{\xi}_{k+1} = \tilde{A}_{\alpha_k}\tilde{\xi}_k$$

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- Stability of the original closed loop system can be deduced from the $\tilde{x}$ dynamics of the restricted system.
- The $\tilde{x}$ dynamics are in the form $A_l^x + B_l^x K$.

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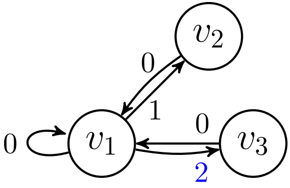

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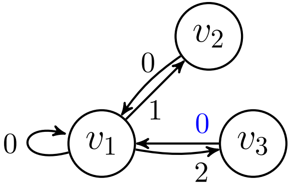

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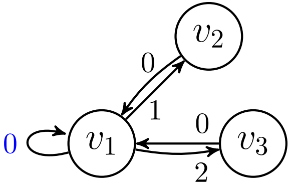

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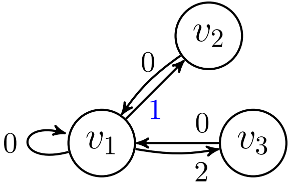

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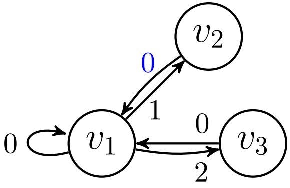

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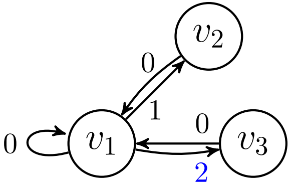

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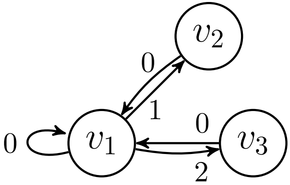

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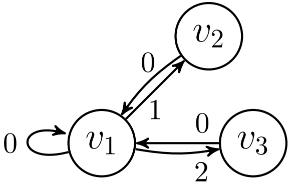

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This sequence does not satisfy the constraint!

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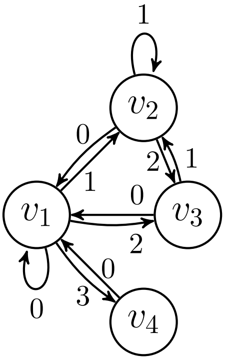

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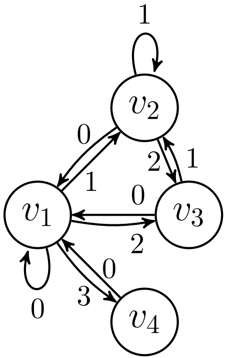

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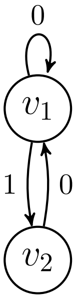

Theorem 3

Given a WHRT constraint λ and its graph representation $\mathcal{G}$,

Linsenmayer, Allgöwer (2017). Stabilization of Networked Control Systems with weakly hard real-time dropout description.

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Given a WHRT constraint λ and its graph representation $\mathcal{G}$, the discrete-time linear control system is state feedback stabilizable under the zero (resp. hold) strategy w.r.t. all $\lambda' \preceq \lambda$

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Given a WHRT constraint λ and its graph representation $\mathcal{G}$, the discrete-time linear control system is state feedback stabilizable under the zero (resp. hold) strategy w.r.t. all $\lambda' \preceq \lambda$, if there exist n_V symmetric matrices $S_1, \ldots, S_{n_V} \in \mathbb{R}^{n \times n}$ and matrices $G \in \mathbb{R}^{n \times n}$, $F \in \mathbb{R}^{n_u \times n}$, satisfying

$$\begin{bmatrix} G + G^\intercal - S_i & (A_l^x G + B_l^x F)^\intercal \\ A_l^x G + B_l^x F & S_j \end{bmatrix} > 0, \ \forall (i, j, l) \in E.$$

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A stabilizing state feedback controller is given by $K = FG^{-1}$.

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$\lambda = \binom{1}{3}$	stable with $K = \begin{bmatrix} -0.35 & -0.85 \end{bmatrix}$ ✓	stabilization with $K = \begin{bmatrix} -0.33 & -0.54 \end{bmatrix}$ ✓
$\lambda = \binom{1}{4}$	no stabilization with Thm 4 ✗	unstable with $K = \begin{bmatrix} -0.35 & -0.85 \end{bmatrix}$

Plant:

$$x_{k+1} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} x_k + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u_k$$

Embedded Controller:

$K = \begin{bmatrix} -0.35 & -0.85 \end{bmatrix}$ was used for zero and hold control

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$\lambda = \binom{1}{4}$	no stabilization with Thm 4 ✗	stabilization with $K = \begin{bmatrix} -0.29 & -0.48 \end{bmatrix}$ ✓

Outline

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Possible other directions:

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Thank you for your attention!

Stability analysis and control design for weakly hard real-time systems

How can richer interfaces, e.g. m-k guarantees, be used for the functional verification of control systems

Steffen Linsenmayer (partially work of and with R. Blind and F. Allgöwer)

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