LM260 – TD Atelier:
Indefinite integrals and useful formulas

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1 Table of standard integrals

i. $\displaystyle\int x^n \, \mathrm{d}x = \frac{x^{n+1}}{n+1} + C$, where $n \neq 1$;

ii. $\displaystyle\int \frac{\mathrm{d}x}{x} = \ln|x| + C$;

iii. $\displaystyle\int \frac{\mathrm{d}x}{x^2+a^2} = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C = -\frac{1}{a}\mathrm{arccot}\left(\frac{x}{a}\right) + C$, where $a \in \mathbb{R}$ and $a \neq 0$;;

iv. $\displaystyle\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$, where $a \in \mathbb{R}$ and $a \neq 0$;
$\displaystyle\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + C$, where $a \in \mathbb{R}$ and $a \neq 0$;

v. $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{x^2+a^2}} = \ln\left|x + \sqrt{x^2+a}\right| + C$, where $a \in \mathbb{R}$ and $a \neq 0$;

vi. $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\left(\frac{x}{a}\right) + C = -\arccos\left(\frac{x}{a}\right) + C$, where $a > 0$;

vii. $\displaystyle\int a^x \, \mathrm{d}x = \frac{a^x}{\ln(a)} + C$, where $a > 0$;
$\displaystyle\int e^x \, \mathrm{d}x = e^x + C$;

viii. $\displaystyle\int \sin(x) \, \mathrm{d}x = -\cos(x) + C$;

ix. $\displaystyle\int \cos(x) \, \mathrm{d}x = \sin(x) + C$;

x. $\displaystyle\int \frac{\mathrm{d}x}{\cos^2(x)} = \tan(x) + C$;

xi. $\displaystyle\int \frac{\mathrm{d}x}{\sin^2(x)} = -\cot(x) + C$;

xii. $\displaystyle\int \frac{\mathrm{d}x}{\sin(x)} = \ln\left|\tan\left(\frac{x}{2}\right)\right| + C = \ln|\csc(x) - cot(x)| + C$;

xiii. $\int \frac{\mathrm{d}x}{\cos(x)} = \ln\left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right| + C = \ln|\tan(x)+\sec(x)| + C$;

xiv. $\int \sinh(x)\,\mathrm{d}x = \cosh(x) + C$;

xv. $\int \cosh(x)\,\mathrm{d}x = \sinh(x) + C$;

xvi. $\int \frac{\mathrm{d}x}{\cosh^2(x)} = -\tanh(x) + C$;

xvii. $\int \frac{\mathrm{d}x}{\sinh^2(x)} = -\coth(x) + C$;

2 Table of standard derivatives

i. $\frac{\mathrm{d}}{\mathrm{d}x} b^x = b^x \ln(b)$;

ii. $\frac{\mathrm{d}}{\mathrm{d}x} \sin(x) = \cos(x)$

iii. $\frac{\mathrm{d}}{\mathrm{d}x} \cos(x) = -\sin(x)$;

iv. $\frac{\mathrm{d}}{\mathrm{d}x} \csc(x) = \csc(x)\cot(x)$;

v. $\frac{\mathrm{d}}{\mathrm{d}x} \sec(x) = \sec(x)\tan(x)$;

vi. $\frac{\mathrm{d}}{\mathrm{d}x} \tan(x) = \sec^2(x)$;

vii. $\frac{\mathrm{d}}{\mathrm{d}x} \cot(x) = -\csc^2(x)$;

viii. $\frac{\mathrm{d}}{\mathrm{d}x} \arcsin(x) = \frac{1}{\sqrt{1-x^2}}$;

ix. $\frac{\mathrm{d}}{\mathrm{d}x} \arccos(x) = \frac{-1}{\sqrt{1-x^2}}$;

x. $\frac{\mathrm{d}}{\mathrm{d}x} \arctan(x) = \frac{1}{1+x^2}$;

xi. $\frac{\mathrm{d}}{\mathrm{d}x} \mathrm{arccot} = \frac{-1}{1+x^2}$;

xii. $\frac{\mathrm{d}}{\mathrm{d}x} \mathrm{arcsec} = \frac{1}{x\sqrt{x^2-1}}$;

xiii. $\frac{\mathrm{d}}{\mathrm{d}x} \mathrm{arccsc} = \frac{-1}{x\sqrt{x^2-1}}$;

3 Trigonometric functions

We list now few fundamental trigonometric functions.

The **tangent** ($\tan(A)$) is the ratio

$$\tan(A) = \frac{\sin(A)}{\cos(A)}.$$

The **cosecant** ($\csc(A)$) is the reciprocal of $\sin(A)$

$$\csc(A) = \frac{1}{\sin(A)}.$$

The **secant** ($\sec(A)$) is the reciprocal of $\cos(A)$

$$\sec(A) = \frac{1}{\cos(A)}.$$

The **cotangent** ($\cot(A)$) is the reciprocal of $\tan(A)$

$$\cot(A) = \frac{1}{\tan(A)}.$$

4 Trigonometric formulas

Pythagorean trigonometric identity:

$$\cos^2\theta + \sin^2\theta = 1 \tag{1}$$

from which we get

$$\sin\theta = \sqrt{1 - \cos^2\theta} \tag{2}$$

$$\cos\theta = \sqrt{1 - \sin^2\theta}. \tag{3}$$

Dividing the identity (1) by either $\cos^2\theta$ or $\sin^2\theta$, we obtain

$$1 + \tan^2\theta = \sec^2\theta = \frac{1}{\cos^2\theta} \tag{4}$$

$$1 + \cot^2\theta = \csc^2\theta = \frac{1}{\sin^2\theta}. \tag{5}$$

Therefore we have

$$a\sqrt{\sec^2\theta - 1} = a\tan\theta.$$

Addition/subtraction formulas:

$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta) \tag{6}$$

$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta) \tag{7}$$

$$\tan(\alpha \pm \beta) = \frac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)} \tag{8}$$

$$\cot(\alpha \pm \beta) = \frac{\cot(\alpha)\cot(\beta) \mp 1}{\cot(\beta) \pm \cot(\alpha)} \tag{9}$$

Bisection formulas:

$$\sin\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1-\cos(\alpha)}{2}} \tag{10}$$

$$\cos\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1+\cos(\alpha)}{2}} \tag{11}$$

$$\tan\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1-\cos(\alpha)}{1+\cos(\alpha)}} \qquad \text{with } \alpha \neq \pi + 2k\pi. \tag{12}$$

Parametric formulas:

$$\cos(\alpha) = \frac{1-t^2}{1+t^2} \tag{13}$$

$$\sin(\alpha) = \frac{2t}{1+t^2}, \tag{14}$$

where $t = \tan(\alpha/2)$, where $\alpha \neq \pi + 2k\pi$.