LM260 – TD Atelier: Power Series

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1 Power series and differential equations

Ex. 1 — Let $p \in \mathbb{N}$ and

$$f(x) = \sum_{n=0}^{+\infty} \binom{n+p}{p} x^n$$

a)What is the radius of convergence of the previous power series ?

b)Compute $f(x)$ by studying $(1-x)f'(x)$.

Ex. 2 — Let f be a function on $]-1,1[$ defined as

$$f(x) = \frac{\arcsin x}{\sqrt{1-x^2}}.$$

a)Why can f be developed in power series in $]-1,1[$?

b)Show that f is solution of the ODE

$$(1-x^2)y' - xy = 1.$$

c)Compute the development in power series of f on $]-1,1[$.

Ex. 3 — Let f be a function on $]-1,1[$ defined as

$$f(x) = \cos(\alpha \arcsin x)$$

where $\alpha \in \mathbb{R}$.

a)Find a second order differential equation which has f as solution.

b)Find the power development of the function f.

2 Various

Ex. 4 — Write in two different ways the power development in 0 of

$$f(x) = e^{-x^2} \int_0^x e^{t^2} \, \mathrm{d}t.$$

Deduce the relation

$$\sum_{k=0}^{n} \frac{(-1)^k}{2k+1} \binom{n}{k} \binom{2n}{n} = \frac{4^n}{2n+1}$$