

# Topics in Causal Inference and Policy Learning with Applications to Precision Medicine

PhD defense

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September 4, 2024

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# Policy learning

Learning a treatment assignment policy is pivotal across various domains, for instance:

- individualized treatment rule in precision medicine
- personalized advertising in marketing
- educational/training programs in public policy

Basic causal setup:<sup>1</sup>

- data  $O = (X, A, Y) \sim P$  with covariates  $X \in \mathcal{X}$ , treatment  $A$  and outcome  $Y$
- complete data  $\mathbb{O} = (X, A, Y(0), Y(1)) \sim \mathbb{P}$   
w/ potential outcomes  $Y(0), Y(1)$
- policy  $d : \mathcal{X} \rightarrow \mathcal{A} = \{0, 1\}$

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<sup>1</sup>Athey, S., & Wager, S. (2021). Policy learning with observational data. *Econometrica*, 89(1), 133-161.

# Main approaches

(A) Heterogeneous treatment effects estimation:

$$\begin{aligned}x &\mapsto \text{CATE}_{\mathbb{P}}(x) = E_{\mathbb{P}}[Y(1) - Y(0) \mid X = x] \\&\rightsquigarrow d^{\text{opt}}(x) = I\{\text{CATE}_{\mathbb{P}}(x) > 0\}\end{aligned}$$

(B) Direct policy search:

define value function  $d \mapsto V_{\mathbb{P}}(d) = E_{\mathbb{P}}[Y(1)d(X) + Y(0)(1 - d(X))]$

$$\begin{aligned}d^{\text{opt}} &= \arg \max_{d \in \mathcal{D}} V_{\mathbb{P}}(d) \\&= \arg \max_{d \in \mathcal{D}} E_{\mathbb{P}}[(Y(1) - Y(0))d(X) + Y(0)] \\&= \arg \max_{d \in \mathcal{D}} E[\text{CATE}_{\mathbb{P}}(X)d(X)]\end{aligned}$$

Possibly subject to application-specific constraints, such as budget, fairness, simplicity

# Direct policy search - identification

Under consistency, unconfoundedness and positivity:

- inverse probability weighting (IPW):

$$V_{\mathbb{P}}(d) = E_{\mathcal{P}} \left[ \frac{Y I\{A = d(X)\}}{Pr_{\mathcal{P}}(A = d(X) | X)} \right]$$

- outcome regression (OR):

$$V_{\mathbb{P}}(d) = E_{\mathcal{P}} \{E_{\mathcal{P}}[Y | A = d(X), X]\}$$

- Augmented IPW (AIPW):

$$V_{\mathbb{P}}(d) = E \left\{ \frac{I\{A = d(X)\}}{Pr_{\mathcal{P}}(A | X)} (Y - E_{\mathcal{P}}[Y | A = d(X), X]) + E_{\mathcal{P}}[Y | A = d(X), X] \right\}$$

Consistency, excess risk bound, (minimax) regret bound etc. can be established<sup>2</sup>

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<sup>2</sup>Zhao, Y., Zeng, D., Rush, A. J., & Kosorok, M. R. (2012). Estimating individualized treatment rules using outcome weighted learning. Journal of the American Statistical Association, 107(499), 1106-1118.

## Main Contributions

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# Research articles & projects

## Publication and preprints:

- A Semiparametric Instrumented Difference-in-Differences Approach to Policy Learning. Major revision at Biometrika. **IMS Hannan Graduate Student Award**
- Positivity-free Policy Learning with Observational Data. Proceedings of The 27th International Conference on Artificial Intelligence and Statistics, PMLR 238:1918-1926, 2024.
- Efficient and robust transfer learning of optimal individualized treatment regimes with right-censored survival data. R & R at Journal of Machine Learning Research.
- Learning, Evaluating and Analysing An Individualized Decision Support Rule with Application to Early Intervention in Intensive Care Unit. In preparation.

## Ongoing projects:

- w/ Yifan Cui (Zhejiang University): Variable Importance for Heterogeneous Treatment Effects with Survival Data and Nonparametric Inference at the Parameter Space Boundary.
- w/ Oliver Dukes & Stijn Vansteelandt (Ghent University): Orthogonal Statistical Learning for Nonparametric Instrumental Variables.
- w/ Oliver Dukes & Bo Zhang (Fred Hutch): Estimating the risk and relative vaccine efficacy of updated vaccine regimens using historical phase 3 clinical trials and immunobridging data.

# Other activities

## Software:

- CRAN Task View: Causal Inference
- R package **missSuperLearner**
- R implementation of all projects available on GitHub:  
<https://github.com/panzhaooo>

Academic visit at Ghent University w/ Oliver Dukes & Stijn Vansteelandt.

## Talks:

- contributed: IDESP 2021, JDS 2022, IMS ICSDS 2023
- invited: JSM 2023, Ghent Causal Meeting, IMS APRM 2024, AISTATS 2024

# Introduction to Instrumental Variable

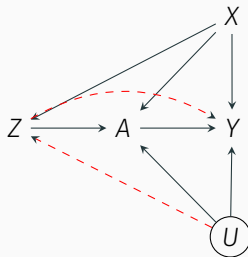
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# IV setup and DAG

Basic setup:

- observed data  $O = (X, Z, A, Y) \sim P$ : binary instrument  $Z$  and treatment  $A$ , covariates  $X$  and outcome  $Y$
- unmeasured confounder  $U$
- complete data  $\mathbb{O} = (X, U, Z, A(0), A(1), Y(0), Y(1)) \sim \mathbb{P}$   
w/ potential outcomes:  $Y = Y(1)A + Y(0)(1 - A)$ ,  
 $A = A(1)Z + A(0)(1 - Z)$



**Figure 1:** DAG for instrumental variable setup (red: not allowed).

Causal assumptions for IV: under  $\mathbb{P}$ ,

- exclusion:  $Y(a) = Y(a, z)$  for  $a, z \in \{0, 1\}$
- independence:  $Z \perp \{Y(0), Y(1), A(1), A(0)\}$
- relevance:  $E[A \mid Z = 1] > E[A \mid Z = 0]$
- monotonicity:  $A(1) \geq A(0)$ 
  - “Always taker”  $A(1) = A(0) = 1$
  - “Complier”  $A(1) = 1, A(0) = 0$
  - ~~“Defier”  $A(1) = 0, A(0) = 1$~~
  - “Never taker”  $A(1) = A(0) = 0$

## Local average treatment effect

$$\text{Wald}_P \stackrel{\text{def}}{=} \frac{E_P[Y \mid Z = 1] - E_P[Y \mid Z = 0]}{E_P[A \mid Z = 1] - E_P[A \mid Z = 0]} = E_{\mathbb{P}}[Y(1) - Y(0) \mid A(1) > A(0)]$$

Simple proof:

- by independence,  $E_P[A \mid Z = 1] - E_P[A \mid Z = 0] = E_{\mathbb{P}}[A(1) - A(0)]$
- similarly,

$$\begin{aligned} & E_P[Y \mid Z = 1] - E_P[Y \mid Z = 0] \\ &= E_{\mathbb{P}}[(Y(1) - Y(0))A(1) + Y(0) \mid Z = 1] \\ &\quad - E_{\mathbb{P}}[(Y(1) - Y(0))A(0) + Y(0) \mid Z = 0] \\ &= E_{\mathbb{P}}[(Y(1) - Y(0))(A(1) - A(0))] \end{aligned}$$

- by monotonicity

$$\frac{E_{\mathbb{P}}[(Y(1) - Y(0))(A(1) - A(0))]}{E_{\mathbb{P}}[A(1) - A(0)]} = E_{\mathbb{P}}[Y(1) - Y(0) \mid A(1) > A(0)]$$

$$\begin{aligned}\text{CATE}_{\mathbb{P}}(x) &= \text{def } E_{\mathbb{P}}[Y(1) - Y(0) \mid X = x] \\ &= \frac{E_P[Y \mid Z = 1, X = x] - E_P[Y \mid Z = 0, X = x]}{E_P[A \mid Z = 1, X = x] - E_P[A \mid Z = 0, X = x]}\end{aligned}$$

Causal assumptions: under  $\mathbb{P}$ ,

- exclusion:  $Y(a) = Y(a, z)$  for  $a, z \in \{0, 1\}$
- independence:  $Z \perp U \mid X$
- relevance:  $Z \not\perp A \mid X$
- $Y(A) \perp \{A, Z\} \mid \{X, U\}$
- **either** no additive  $U-Z$  interaction

$$E_P[A \mid Z = 1, X, U] - E_P[A \mid Z = 0, X, U] = E_P[A \mid Z = 1, X] - E_P[A \mid Z = 0, X]$$

**or** no additive  $U-a$  interaction

$$E_{\mathbb{P}}[Y(1) - Y(0) \mid X, U] = E_{\mathbb{P}}[Y(1) - Y(0) \mid X]$$

Regression, IPW and efficient multiply robust estimators are provided

## IV for policy learning - Cui & Tchetgen Tchetgen 2018

Let  $\delta_P(X) = Pr_P(A = 1 \mid Z = 1, X) - Pr_P(A = 1 \mid Z = 0, X)$

Causal assumptions: under  $\mathbb{P}$ ,

- exclusion, independence, relevance
- no unmeasured common effect modifier:

$$Cov_P\{Pr_P(A = 1 \mid Z = 1, X, U) - Pr_P(A = 1 \mid Z = 0, X, U), E_{\mathbb{P}}[Y(1) - Y(0) \mid X, U] \mid X\} = 0$$

→ identification of the optimal policy:

$$d^{\text{opt}} = \arg \max_{d \in \mathcal{D}} E_P \left[ \frac{(2Z-1)(2A-1)YI\{A=d(X)\}}{\delta_P(X)Pr_P(Z|X)} \right] = \arg \max_{d \in \mathcal{D}} E_P \left[ \frac{YI\{Z=d(X)\}}{\delta_P(X)Pr_P(Z|X)} \right]$$

- independent compliance type:

$$\delta_P(X) = Pr_P(A = 1 \mid Z = 1, X, U) - Pr_P(A = 1 \mid Z = 0, X, U)$$

→ identification of the value function:

$$V_{\mathbb{P}}(d) = E_P \left[ \frac{(2Z-1)(2A-1)YI\{A=d(X)\}}{\delta_P(X)Pr_P(Z|X)} \right]$$

# Introduction to Difference-in-Differences

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# Difference-in-Differences

Basic setup:

- two time points  $T \in \{0, 1\}$
- covariates  $X$ , treatments  $A \in \{0, 1\}$  or  $(A_0, A_1) \in \{0, 1\}^2$ , outcomes  $Y$  or  $(Y_0, Y_1)$
- potential outcomes  $Y_t(a), t, a \in \{0, 1\}$

Two observed data structures:

- repeated cross-section data:  $O = (X, A, Y, T)$ , with  $Y = Y_T(A)$
- panel data:  $O = (X, A_0, Y_0, A_1, Y_1)$ , with  $Y_t = Y_t(A_t), t \in \{0, 1\}$

The complete and observed laws are  $\mathbb{P}$  and  $P$

# DiD illustration

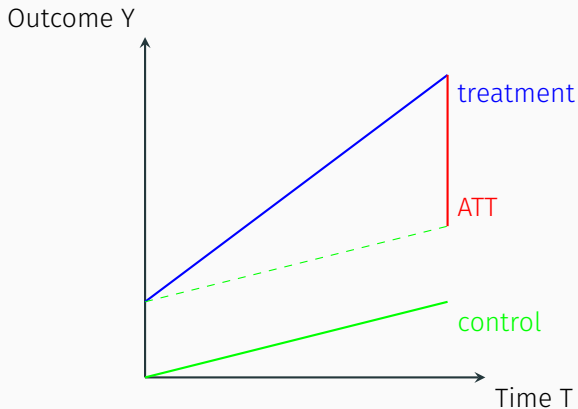


Figure 2: A simple illustration of DiD identification.



# Parallel trends & average treatment effect on the treated

(Conditional) parallel trends assumption:

$$E_{\mathbb{P}}[Y_1(0) - Y_0(0) \mid A = 1, X] = E_{\mathbb{P}}[Y_1(0) - Y_0(0) \mid A = 0, X]$$

Simple proof:

$$\begin{aligned} \text{ATT}_{\mathbb{P}} &\stackrel{\text{def}}{=} E_{\mathbb{P}}[Y_1(1) - Y_1(0) \mid A = 1] \\ &= E_{\mathbb{P}}[Y_1(1) - Y_1(0) + Y_0(0) - Y_0(0) \mid A = 1] \\ &= E_{\mathbb{P}}[Y_1(1) \mid A = 1] - E_{\mathbb{P}}[Y_1(0) - Y_0(0) \mid A = 0] - E_{\mathbb{P}}[Y_0(0) \mid A = 1] \\ &= E_{\mathbb{P}}[Y_1 - Y_0 \mid A = 1] - E_{\mathbb{P}}[Y_1 - Y_0 \mid A = 0] \end{aligned}$$

# Instrumented Difference-in-Differences

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# Instrumented DiD setup

First introduced by Ye et al. 2022 for (conditional) ATE, also structural mean models by Vo et al. 2023

Basic setup:

- two time points  $T \in \{0, 1\}$
- binary instrument  $Z$  and treatment  $A \in \{0, 1\}$  or  $(A_0, A_1) \in \{0, 1\}^2$
- covariates  $X$  and unmeasured confounder  $U = (U_0, U_1)$
- potential outcomes  $Y_t(a), t, a \in \{0, 1\}$

Two observed data structure:

- repeated cross-section data  $O = (X, Z, A, Y, T)$ , with  $Y = Y_T(A)$
- panel data  $O = (X, Z, A_0, Y_0, A_1, Y_1)$ , with  $Y_t = Y_t(A_t), t \in \{0, 1\}$

The complete and observed laws are  $\mathbb{P}$  and  $P$

## Instrumented DiD DAG: trend scale

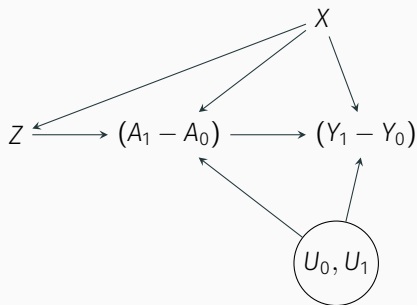


Figure 3: DAG for instrumented DiD on the trend scale.

## Instrumented DiD DAG: two time points

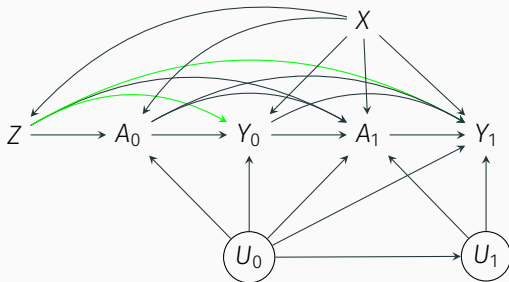


Figure 4: DAG for instrumented DiD over two time points.

- IV for DiD: e.g. haphazard encouragement targeted at a subpopulation toward faster uptake of the exposure or a surrogate of such encouragement (Ye et al. 2022)
- Longitudinal randomized experiment: after a baseline period, some individuals are randomly selected to be encouraged to take the treatment regardless of treatment history
- See Ye et al. 2022 for an analysis of the effect of cigarette smoking on lung cancer mortality

# Instrumented DiD to Policy Learning

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Optimal policy given by

$$d_t^{\text{opt}} = \arg \max_{d \in \mathcal{D}} V_{\mathbb{P},t}(d) = \arg \max_{d \in \mathcal{D}} E_{\mathcal{P}}[\text{CATE}_{\mathbb{P},t}(X)d(X)].$$

- assume stable treatment effect over time
- directly maximize some functional  $d \mapsto V_{\mathbb{P}}(d)$  similarly
- CATE-based approaches



# Causal assumptions

Let  $\pi_P(t, z, x) = \Pr_P(T = t, Z = z \mid X = x)$ . Under  $\mathbb{P}$ ,

- consistency:  $A = A_T(Z)$  and  $Y = Y_T(A)$
- positivity:  $c_1 < \pi_P(t, z, x) < 1 - c_1$  for some  $0 < c_1 < 1/2$
- random sampling:  
 $T \perp \{A_t(z), Y_t(a) : t = 0, 1, z = 0, 1, a = 0, 1\} \mid X, Z$
- stable treatment effect over time:  
 $E_{\mathbb{P}}[Y_0(1) - Y_0(0) \mid X] = E_{\mathbb{P}}[Y_1(1) - Y_1(0) \mid X]$
- trend relevance:  
 $E_{\mathbb{P}}[A_1(1) - A_0(1) \mid Z = 1, X] \neq E_{\mathbb{P}}[A_1(0) - A_0(0) \mid Z = 0, X]$
- independence & exclusion restriction:  
 $Z \perp \{A_t(1), A_t(0), Y_t(1) - Y_t(0), Y_1(0) - Y_0(0) : t = 0, 1\} \mid X$
- no unmeasured common effect modifier:  
 $\text{Cov}_{\mathbb{P}}\{A_t(1) - A_t(0), Y_t(1) - Y_t(0) \mid X\} = 0$  for  $t = 0, 1$

# Identification of optimal policy

For  $C \in \{A, Y\}$ , let  $\mu_{P,C}(t, z, x) = E_P[C \mid T = t, Z = z, X = x]$ , and  $\delta_{P,C}(x) = \mu_C(1, 1, x) - \mu_C(0, 1, x) - \mu_C(1, 0, x) + \mu_C(0, 0, x)$

- CATE-based approach:

$$d^{\text{opt}} = \arg \max_{d \in \mathcal{D}} E_P \left[ \frac{\delta_{P,Y}(X)}{\delta_{P,A}(X)} d(X) \right]$$

- novel IPW formula 1:

$$d^{\text{opt}} = \arg \max_{d \in \mathcal{D}} E_P \left[ \frac{(2Z - 1)(2T - 1)(2A - 1)YI\{A = d(X)\}}{\pi_P(T, Z, X)\delta_{P,A}(X)} \right]$$

- novel IPW formula 2:

$$d^{\text{opt}} = \arg \max_{d \in \mathcal{D}} E_P \left[ \frac{(2T - 1)YI\{Z = d(X)\}}{\pi_P(T, Z, X)\delta_{P,A}(X)} \right]$$

→ simple plug-in estimators can be constructed

# Semiparametric efficiency

Efficient influence function (Ye et al. 2022)

$$\Delta_P(O) = \frac{\delta_{P,Y}(X)}{\delta_{P,A}(X)} + \frac{(2Z-1)(2T-1)}{\pi_P(T,Z,X)\delta_{P,A}(X)} \left\{ Y - \mu_{P,Y}(T,Z,X) - \frac{\delta_{P,Y}(X)}{\delta_{P,A}(X)} (A - \mu_{P,A}(T,Z,X)) \right\},$$

Recall the optimization tasks:

$$\arg \max_{d \in \mathcal{D}} E_P[W_P^{(1)} I\{A = d(X)\}], \quad \arg \max_{d \in \mathcal{D}} E_P[W_P^{(2)} I\{Z = d(X)\}]$$

where

$$W_P^{(1)} = \frac{(2A-1)\delta_{P,Y}(X)}{\delta_{P,A}(X)} + \frac{(2A-1)(2Z-1)(2T-1)}{\pi_P(T,Z,X)\delta_{P,A}(X)} \left\{ Y - \mu_{P,Y}(T,Z,X) - \frac{\delta_{P,Y}(X)}{\delta_{P,A}(X)} (A - \mu_{P,A}(T,Z,X)) \right\}$$

and

$$W_P^{(2)} = \frac{(2Z-1)\delta_{P,Y}(X)}{\delta_{P,A}(X)} + \frac{2T-1}{\pi_P(T,Z,X)\delta_{P,A}(X)} \left\{ Y - \mu_{P,Y}(T,Z,X) - \frac{\delta_{P,Y}(X)}{\delta_{P,A}(X)} (A - \mu_{P,A}(T,Z,X)) \right\}$$

Optimal policy identified by

$$\begin{aligned}\arg \max_{\mathcal{D}} E_P \left[ W_P^{(1)} I\{A = d(X)\} \right] &= \arg \max_{\mathcal{D}} E_P \left[ W_P^{(2)} I\{Z = d(X)\} \right] \\ &= \arg \max_{\mathcal{D}} E_P [\Delta_P(X) d(X)]\end{aligned}$$

Under the union model  $\mathcal{M}_1 \cup \mathcal{M}_2 \cup \mathcal{M}_3$

- $\mathcal{M}_1$ : models for  $\pi_P$  and  $\delta_{P,A}$  are correct
- $\mathcal{M}_2$ : models for  $\pi_P$  and  $\delta_{P,Y}/\delta_{P,A}$  are correct
- $\mathcal{M}_3$ : models for  $\delta_{P,Y}/\delta_{P,A}$  and  $\mu_{P,C}(0, 0, \cdot)$ ,  $\mu_{P,C}(1, 0, \cdot)$ ,  $\mu_{P,C}(0, 1, \cdot)$  for  $C \in \{A, Y\}$  are correct

Cross-fitted estimator:

$$\hat{M}_{CF} = \frac{1}{K} \sum_{k=1}^K P_{n,k} \{ \Delta(O; \hat{\mu}_{A,-k}, \hat{\mu}_{Y,-k}, \hat{\pi}_{-k}) d(X) \},$$

1. randomly split the data into  $K$  folds;
2. for  $k = 1, \dots, K$ , learn the nuisance parameters  $\mu_{P,A}, \mu_{P,Y}, \pi_P$  with  $\hat{\mu}_{A,-k}, \hat{\mu}_{Y,-k}, \hat{\pi}_{-k}$  using data excluding the  $k$ -th fold; then evaluate the value on the  $k$ -th fold;
3. average the value estimates from the  $K$  folds.

# Asymptotic analysis of policy learning

Focus on a class of feasible policies  $\mathcal{D} = \{x \mapsto I\{\eta^\top x > 0\} : \eta \in \mathbb{H}\}$

Policy estimator:

$$\hat{\eta} = \arg \max_{\eta \in \mathbb{H}} \hat{M}(\eta) = \arg \max_{\eta \in \mathbb{H}} \frac{1}{n} \sum_{i=1}^n \hat{\Delta}(O_i) I\{\eta^\top X_i > 0\},$$

where  $\hat{\Delta}$  is the estimator of  $\Delta_p$  obtained by substitution.

Under certain regularity and rate of convergence conditions:

- $\|\hat{\eta} - \eta^*\|_2 = O_p(n^{-1/3})$
- $\sqrt{n}\{M(\hat{\eta}) - M(\eta^*)\} = o_p(1)$
- $\sqrt{n}\{\hat{M}(\hat{\eta}) - M(\eta^*)\} \rightsquigarrow \mathcal{N}(0, \sigma^2)$

## Extension to panel data: identification

- Analog causal assumptions for panel data
- Alternatively, Vo et al. 2023 consider sequential ignorability for structural mean model
- We also prove identification if, under  $\mathbb{P}$ ,
  - sequential ignorability:  $Y_t(a) \perp A_t \mid U, X, Z$  for  $t, a = 0, 1$
  - no additive interaction of **either**

$$E_{\mathbb{P}}[A_1 - A_0 \mid X, U, Z = 1] - E_{\mathbb{P}}[A_1 - A_0 \mid X, U, Z = 0] = E_{\mathbb{P}}[A_1 - A_0 \mid X, Z = 1] - E_{\mathbb{P}}[A_1 - A_0 \mid X, Z = 0]$$

or

$$E_{\mathbb{P}}[Y_t(1) - Y_t(0) \mid U, X] = E_{\mathbb{P}}[Y_t(1) - Y_t(0) \mid X]$$

→ CATE identified by

$$\begin{aligned} & \text{CATE}_{\mathbb{P}}(x) \\ &= \frac{E_P[Y_1 - Y_0 \mid X = x, Z = 1] - E_P[Y_1 - Y_0 \mid X = x, Z = 0]}{E_P[A_1 - A_0 \mid X = x, Z = 1] - E_P[A_1 - A_0 \mid X = x, Z = 0]} \stackrel{\text{def}}{=} \tau_P(x) \end{aligned}$$

# Semiparametric efficiency

EIF given by

$$\begin{aligned} & \phi_{\text{panel},P}(0) \\ &= \frac{\delta_{P,Y,1}(X) - \delta_{P,Y,0}(X)}{\delta_{P,A,1}(X) - \delta_{P,A,0}(X)} \\ & \quad - \frac{Z - \pi_{P,Z}(X)}{\pi_{P,Z}(X)(1 - \pi_{P,Z}(X))(\delta_{P,A,1}(X) - \delta_{P,A,0}(X))^2} \{ (y_1 - y_0)(\delta_{P,A,1}(X) - \delta_{P,A,0}(X)) \\ & \quad - (a_1 - a_0)(\delta_{P,Y,1}(X) - \delta_{P,Y,0}(X)) + \delta_{P,Y,1}(X)\delta_{P,A,0}(X) - \delta_{P,Y,0}(X)\delta_{P,A,1}(X) \} - \tau_P(X) \end{aligned}$$

Optimal policy:

$$\arg \max_{\mathcal{D}} E_P \left[ \frac{\delta_{P,Y,1}(X) - \delta_{P,Y,0}(X)}{\delta_{P,A,1}(X) - \delta_{P,A,0}(X)} d(X) \right] = \arg \max_{\mathcal{D}} E_P [\Delta_{\text{panel},P}(X) d(X)],$$

Asymptotic results can be obtained similarly



# Simulation

Data-generation process:

$X_1, X_2 \sim \mathcal{N}(0, 1)$ ,  $U_0, U_1 \sim \text{Bridge}(0.5)$ ,  $T \sim \text{Bernoulli}(0.5)$  independently

$\Pr(A_0 = 1 \mid Z, U, X) = \text{expit}(2 - 7Z + 0.2U_0 + 2X_1)$ ,

$\Pr(A_1 = 1 \mid Z, U, X) = \text{expit}(-1.5 + 5Z - 0.15U_1 + 1.5X_2)$ ,

$(Y_0 \mid Z, U, X, A_0) \sim \mathcal{N}(\mu_0, 1)$ ,  $(Y_1 \mid Z, U, X, A_1) \sim \mathcal{N}(\mu_1, 1)$

where

$$\mu_0 = 200 + 10(A_0(1.5X_1 + 2X_2 - 0.5) + 0.5U_0 + \textcolor{red}{2Z} + 1.5X_1 + 2X_2)$$

$$\mu_1 = 240 + 10(A_1(1.5X_1 + 2X_2 - 0.5) + 0.5U_1 + \textcolor{red}{2Z} + 2X_1 + 1.5X_2)$$

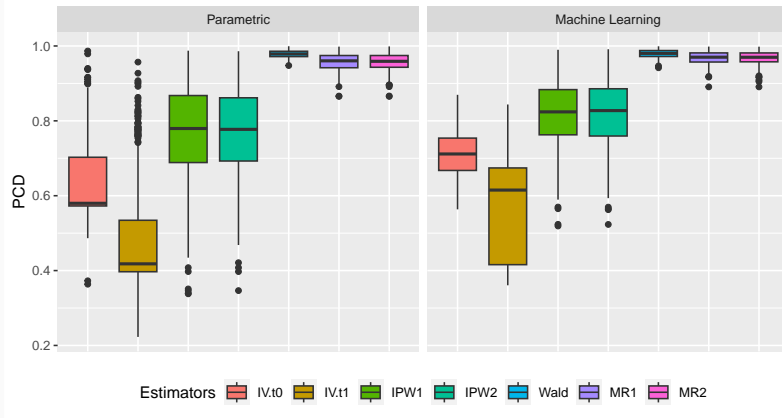
Evaluate by percentage of correct decisions (PCD) of estimated  $\hat{d}(x)$

$$1 - N^{-1} \sum_{i=1}^N |\hat{d}(X_i) - d^{\text{opt}}(X_i)|$$

Compare with standard IV methods (Cui & Tchetgen Tchetgen 2018)

Correctly specified parametric models, or random forests (**grf**)

# Results



**Figure 5:** Results of the estimated optimal policies, using parametric models (left) or machine learning (right).

# Data application – Australian Longitudinal Survey

- Conducted annually since 1984, mainly on the dynamics of the youth labour market, including basic demographic, labour market and background variables, and topics related to the main labour market theme
- Card 2001: endogeneity of education might partially explain the continuing interest “in this very difficult task of uncovering the causal effect of education in labor market outcomes”
- Cross-section data from 1984 and 1985 waves (Vella 1994)

Policies	intercept	born_australia	married	uni_mem	gov_emp	age	year_expe
IVt0	0.4442	−0.4547	0.1311	−0.1179	−0.5181	0.0080	−0.5444
IVt1	−0.2518	−0.3103	0.2445	−0.6157	−0.1406	0.2015	−0.5840
IPW1	−0.4203	−0.0847	0.5454	−0.3941	−0.5690	0.0299	0.1969
IPW2	−0.2503	−0.0529	0.6051	−0.4384	−0.5801	0.0207	0.1980
Wald	0.5032	0.3891	0.4738	0.5755	−0.1656	−0.0772	0.0793
MR1	−0.0513	0.1341	−0.6039	0.4127	0.5861	−0.0226	−0.3168
MR2	0.5480	−0.3937	−0.4072	0.4393	0.4167	−0.0302	−0.1064

- Coefficients should be interpreted with caution
- Majority vote from Wald, MR1, MR2 estimators

- By monotonicity assumption  $A_t(1) \geq A_t(0)$  for  $t = 0, 1$   
→ optimal policy for compliers
- Fuzzy DiD in econometrics (De Chaisemartin & d'Haultfoeuille 2018)
- DiD on multiple time points, or continuous time
- Weak IV, continuous IV

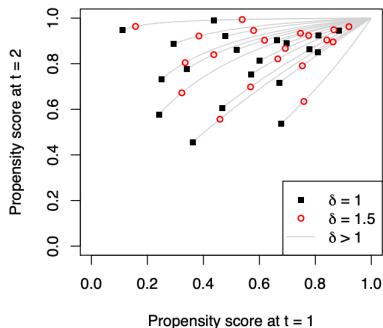
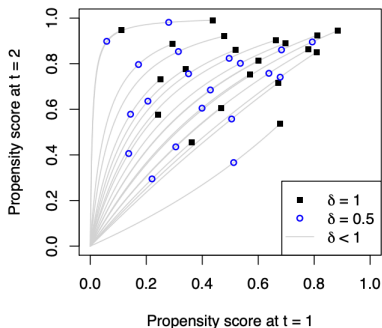
Thank you! & Questions?

# Backup slides I

## Positivity-free Policy Learning with Observational Data

Assign treatment 1 with probability

$$d(x) = \frac{\delta(x) \pi(x)}{\delta(x) \pi(x) + 1 - \pi(x)}$$



**Figure 1:** Observational propensity scores for  $n = 20$  simulated units in a study with  $T = 2$  timepoints, and their values under incremental interventions based on different  $\delta$  values ( $\delta \leq 1$  in the left plot,  $\delta \geq 1$  in the right).

## Backup slides II

Efficient and robust transfer learning of optimal individualized treatment regimes with right-censored survival data

