Preferences, Utilities and Identity Economics

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The focus on a single player

To rigorously analyze behavior in interactions (e.g., humans, firms, countries) we need to define

Preferences: what does each individual strive for in the interaction

If we can express these preferences through a real-valued function we gain analytical tractability:

Utilities: a real-valued function expressing a player's preferences

Preferences

Let X be the set of decision alternatives for a player

A *binary relation* $\succeq$ on a set X is a non-empty subset $P \subset X \times X$.
We write $x \succeq y$ if and only if $(x, y) \in P$.

$x \succeq y$: "the player weakly prefers x over y"

$x \succ y$: "the player strictly prefers x over y"

Common assumptions on preferences

1. Completeness: $\forall x, y \in X : x \succeq y$ or $y \succeq x$ or both
2. Transitivity: $\forall x, y, z \in X :$ if $x \succeq y$ and $y \succeq z$, then $x \succeq z$
3. Continuity
4. Independence of irrelevant alternatives $\forall x, y, z \in X :$ if $x \succ y$ then $x + z \succ y + z$

Definition. A **utility function** for a binary relation $\succeq$ on a set X is a function $u : X \to \mathbb{R}$ such that

$$u(x) \geq u(y) \iff x \succeq y$$

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Theorem. There exists a utility function for every transitive and complete preference ordering on any countable set.

Completeness: Choices over Chinese vegetables (for a European)

Transitivity: Choices over cars

 $\succ$  (because it is faster)

 $\succ$  (because it carries many people)

 $\succ$  (because it is easier to park)

Transitivity: Choices over cars

Let's play a game!

A fair coin is tossed until head shows for the first time:

- If head turns up first at 1^{st} toss you win 1 Euro
- If head turns up first at 2^{nd} toss you win 2 Euro
- If head turns up first at 3^{rd} toss you win 4 Euro
- ...
- If head turns up first at k^{th} toss you win 2^{k-1} Euro

You have a ticket for this lottery. For which price would you sell it?

Utility $\neq$ Payoff

If you only care about expected gain:

$$\begin{aligned} \mathbb{E}[\text{lottery}] &= \frac{1}{2}\cdot 1 + \frac{1}{4}\cdot 2 + \frac{1}{8}\cdot 4 + \ldots \\ &= \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \ldots \\ &= \infty \end{aligned}$$

- Bernoulli suggested in 1738 the theory of diminishing marginal utility of wealth.
- Further, the need for utility characterization under uncertainty arose.

This laid the foundation for ***expected utility theory***.

Expected-utility theory

Let $T = \{\tau_1, ...\tau_m\}$ be a finite set and let X consist of all probability distributions on T:

$$X = \Delta(T) = \{x = (x_1, ..., x_m) \in \mathbb{R}^m_+ : \sum_{k=1}^{m} x_k = 1\}$$

That is X is the unit simplex in $\mathbb{R}^m$.

Can we define a utility function in this setting?

Existence of von Neumann-Morgenstern utility function

- Axiom 1: Completeness
- Axiom 2: Transitivity
- Axiom 3: Continuity
- Axiom 4: Independence of irrelevant alternatives

Theorem (von Neumann-Morgenstern) Let $\succeq$ be a complete, transitive and continuous preference relation on $X = \Delta(T)$, for any finite set T.
Then $\succeq$ admits a utility function u of the expected-utility form if and only if $\succeq$ meets the axiom of independence of irrelevant alternatives.

Translation invariance

Given an expected utility function u for given preferences $\succeq$ let:

$$u' = \alpha + \beta u$$

where $\alpha \in \mathbb{R}$ and $\beta \in \mathbb{R}^+$.
Then u' is also an expected utility function for $\succeq$.

- Statements like ‘She likes x five times more than y’ are not representable
- Measuring welfare is not possible (no interpersonal comparability)
- Fairness cannot be defined

... additional, strong assumptions are needed!

Standard vs. non-standard preferences ...
... or what we are maximizing

Standard	Non-standard
• Money	• Pro-social preferences
• Time	• Altruism
• Risk	• **Identity-dependent preferences** which may evolve
...	...

Max Weber's (1914 [1978], pp. 958–959) view of successful bureaucracies, where "an office is a vocation" and "entrance into an office ... is considered an acceptance of a specific duty of fealty to the purpose of the office."

What is identity?

- Pareto (1920) distinguishes between *tastes* (normally seen as only input into preferences / utilities) and *norms*
  - *How should I behave?*
  - *Who do I want to be?*

- Sociologists and psychologists have long argued that people's decisions depend on the situation and who interacts with whom – *social category* describes types of people, e.g., black/white, female/male, manager/worker
- **Identity** is used to describe a person's
  - social category (with associated norms)
  - self-image

Akerlof & Kranton (2000, 2005, 2010)

A standard utility model

Agent i chooses to participate in an economic activity ($e_i = 1$) or not ($e_i = 0$).

Examples:

- *Group contribution.* $e_i = 1$ is high effort
- *Education choice.* $e_i = 1$ is college education
- *Labor force participation.* $e_i = 1$ is joining labor force
- *Occupational choice.* $e_i = 1$ is high-valued (e.g. STEM)

$$U(e_i) = y_i(e_i) - c_i(e_i)$$

where y_i is profit from action e_i and c_i is cost from action e_i.

Incorporating identity into a utility model

Agent i has identity $\Theta_i \in \{0, 1\}$.
Suppose that for $\Theta = 1$ the 'default' action is $e = 1$ and for $\Theta = 0$ it is $e = 0$.

Examples:

- female / male
- black / white
- manager / worker

$$U(e_i) = y_i(e_i) - c_i(e_i) + \hat{y}(\Theta_i) - \hat{c} \cdot |\Theta_i - e_i|$$

where $\hat{y}$ is her identity utility from being in the category and $\hat{c}$ is the cost from diverging from her 'default' action.

Examples

Using ‘worth’ of identity

- Academic occupation: feeling of purpose, superiority, ...
- Private sector incentives: group activities / travel, ‘unique culture’, etc.
- Military, sports, ...

Basing decisions on identity

- Which hobby to choose? Ballet versus football
- Which career choice? ‘Goldman’ vs. ‘public sector’
- ...

Identity and Underrepresentation

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The Representation Model

Large, but finite population N.

Partitioned into two groups, N_A and N_B:

- m_k is share of group $k \in \{A, B\}$
- group sizes fixed for all time

Discrete time $t = 0, 1, 2, \ldots$

- New cohort in each period

In every $t \geq 1$, each i chooses to participate in an economic activity ($e_i = 1$) or not ($e_i = 0$).

Economic Incentives

Economic return (net benefit) to participation: y

- Independent draw from F with associated density f
- Unless otherwise stated, groups have the same F
- All results hold for exponential, power-law, uniform, Beta (for certain parameters), and many other distributions

Social Identity

Members of group A have identity $\theta = 1$; for group B, $\theta = 0$.

Individuals care about their group's economic representation.

The **representation** of group A in period t

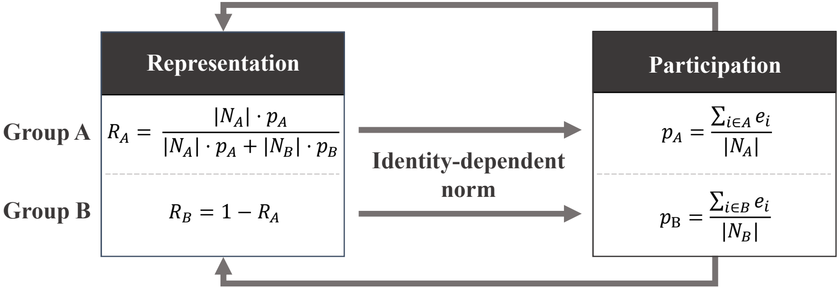
$$R^t = \frac{\sum_{i \in N_A} e_i^{t-1}}{\sum_{i \in N_A} e_i^{t-1} + \sum_{i \in N_B} e_i^{t-1}}$$

Group B's representation is $1 - R^t$.

The Representation Dynamic

Two groups: N_A and N_B.

Participation: $e_i = 1$. Non-participation: $e_i = 0$.



Retains increasing returns within groups and adds to it rivalry between groups. I.e. representation is a rival good.

Payoffs

Identity-based cost of participation is increasing in the other group's representation.

Participation ($e_i = 1$): payoff is

$$y - \alpha\left[\theta(1 - R^t) + (1 - \theta)R^t\right],$$

where $\alpha > 0$ is the (common) level of group identification.

Consistent with internalized and socially enforced identity-dependent norms.

Non-participation ($e_i = 0$): payoff is zero.

Representation Dynamics

Start from arbitrary initial representation $R^1 \in [0,1]$.

Study deterministic approximation of the stochastic dynamic:

$$r^{t+1} = \frac{m_A\left[1 - F\big(\alpha(1-r^t)\big)\right]}{m_A\left[1 - F\big(\alpha(1-r^t)\big)\right] + m_B\left[1 - F\big(\alpha r^t\big)\right]} \equiv G\big(r^t\big)$$

Equilibrium

An absorbing state or equillibrium r^* is a fixed point of G.

$G : [0, 1] \to [0, 1]$ is continuous, so there exists at least one fixed point by Brouwer's fixed point theorem.

As G is strictly increasing and continuous:

Proposition 1. The process r^t converges to an equilibrium from any initial state r^1.
Every equilibrium is interior, $r^* \in (0, 1)$.

Literature

+ Expected utility theory over view Wikipedia: https://en.wikipedia.org/wiki/Expected_utility_hypothesis

+ Akerlof Kranton 2000: https://academic.oup.com/qje/article-abstract/115/3/715/1828151

+ Akerlof Kranton 2005: https://www.aeaweb.org/articles?id=10.1257/0895330053147930

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