

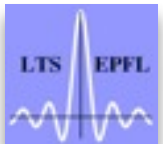
Emerging Applications of Sparse Representations

Pierre Vandergheynst

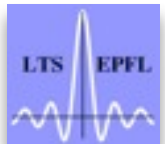
Signal Processing Lab

Ecole Polytechnique Fédérale de Lausanne (EPFL)

Workshop Sparse Models and Machine Learning, INRIA, 15-16 Oct. 2012



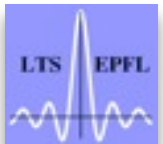
Do we need a new paradigm ?



Do we need a new paradigm ?

2010: 980 exabytes of new digital information

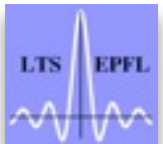
Big Data



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48 hrs video/minute on
Youtube (6 years/day)



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Hyperspectral imaging

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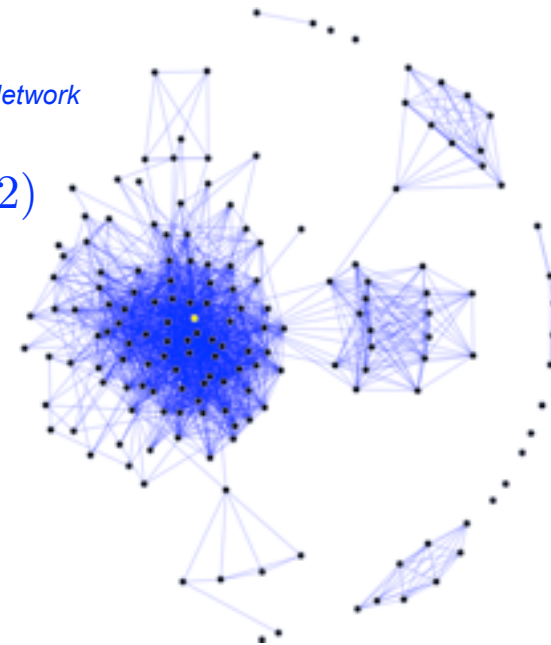
Hyperspectral imaging



Electrical Network

1 billion users on
Facebook (Oct. 2012)

Social Network



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Ubiquitous sensing



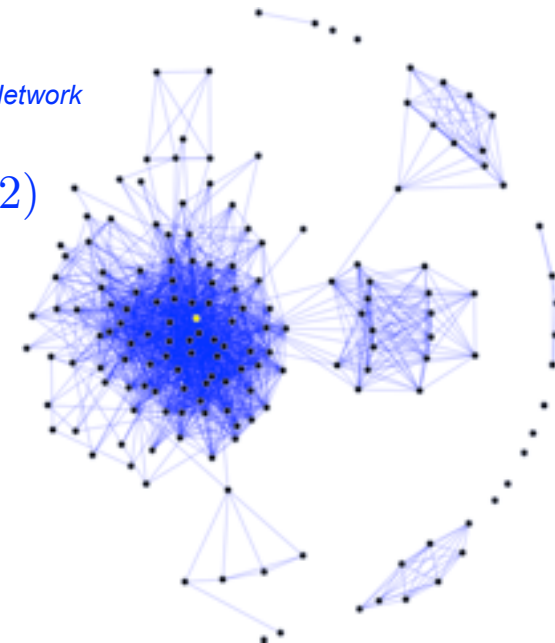
Hyperspectral imaging



United States
transmission grid
Source: FERA

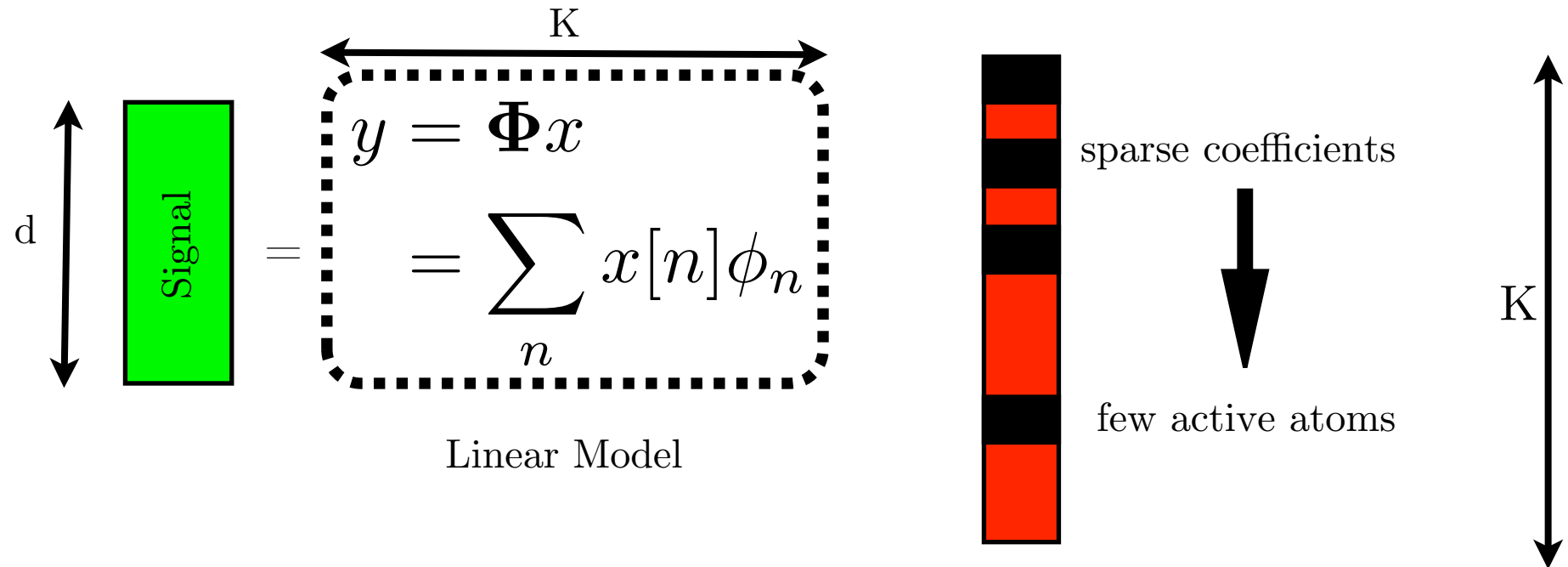
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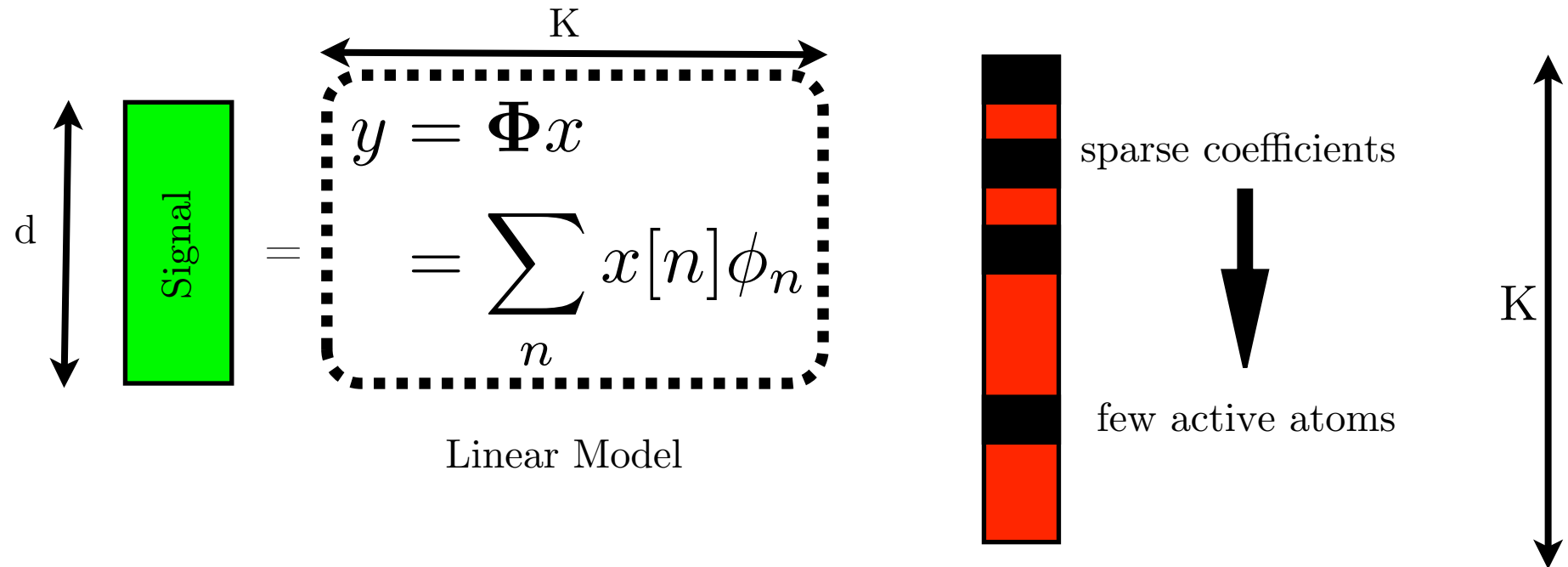


Electrical Network

Some notations



Some notations



$\Phi \in \mathbb{R}^{d \times K}$
 {

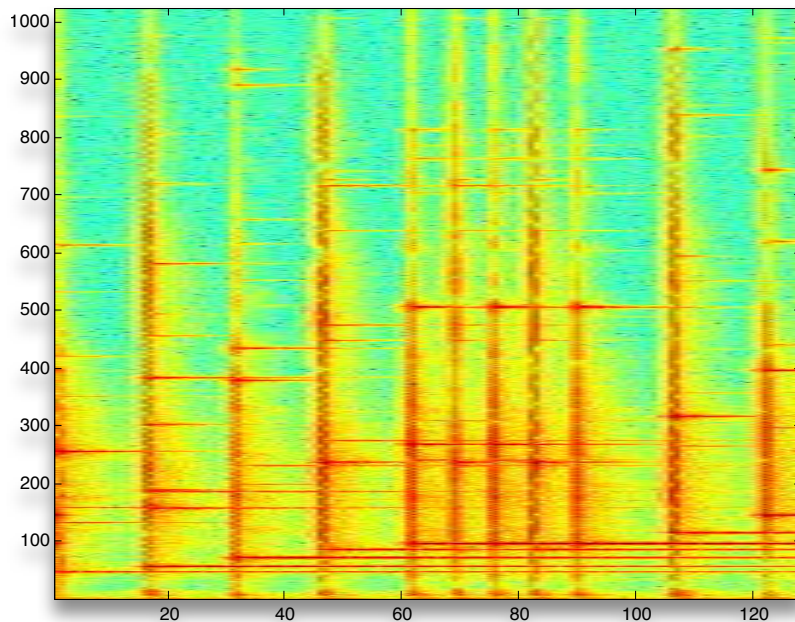
- A convenient ortho basis
- Redundant dictionary, fixed by signal properties (ex: union of wavelets, curvelets, gabor ...) or data driven
- Linear measurement system (compressed sensing)

Sparse Signal Models

$$y = \Phi x + n$$

Efficient model: coefficient vector is very sparse
OR contains few big entries

Example: MDCT/Gabor for audio signals



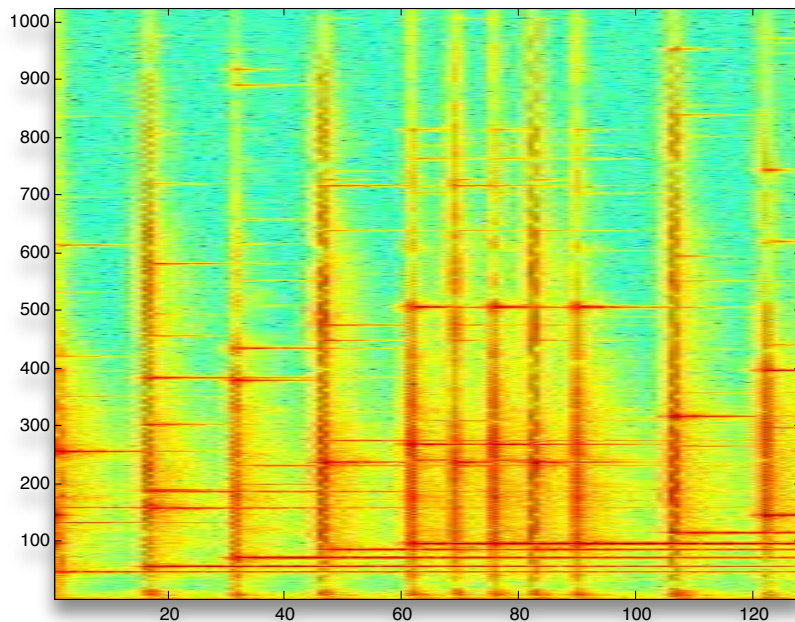
Challenge: Find “good” coefficients for the model

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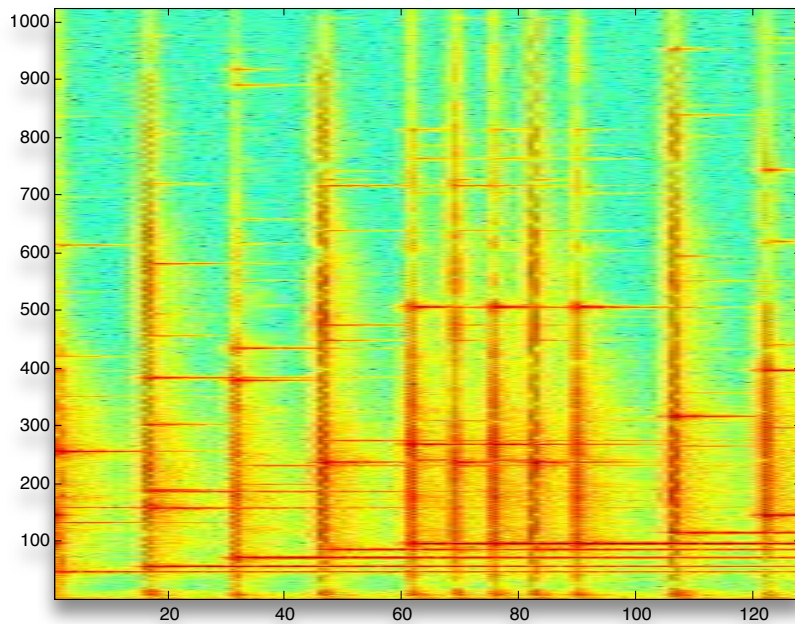
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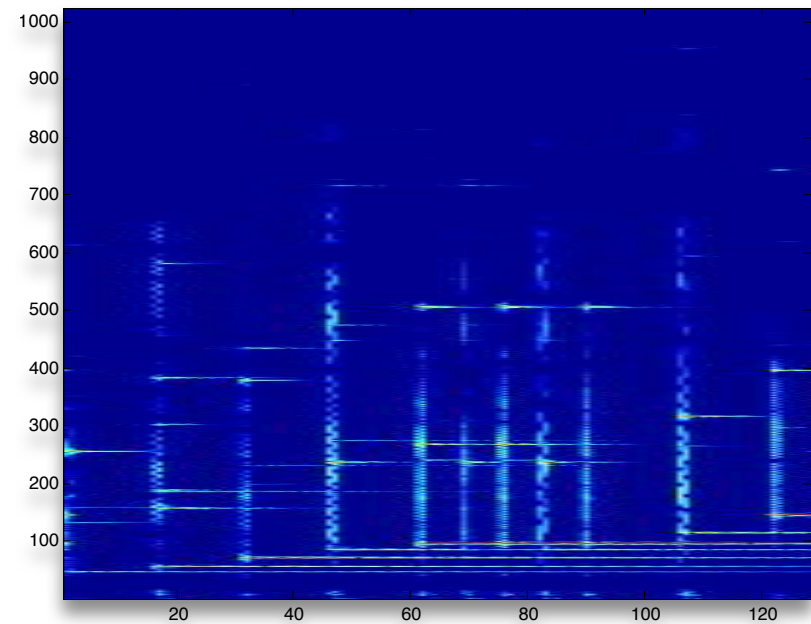
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10 % coefficients



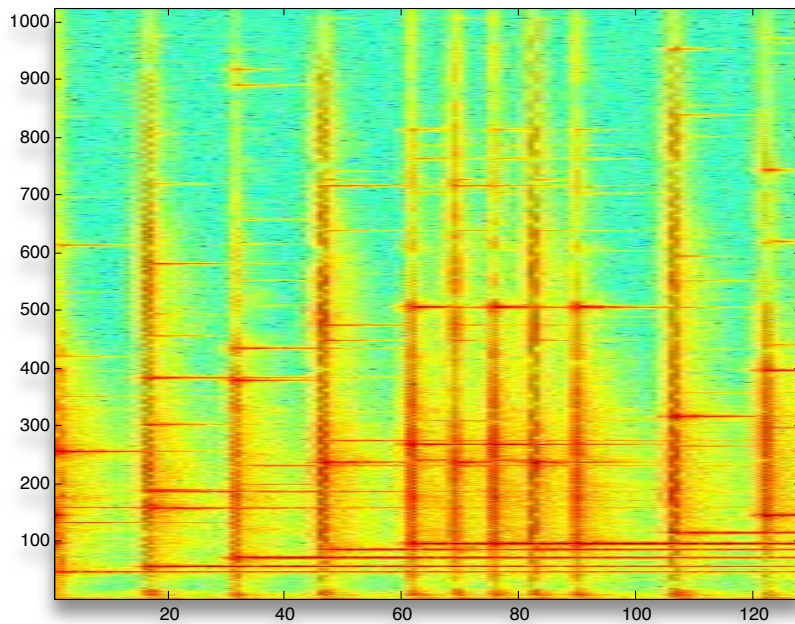
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Sparse Signal Models

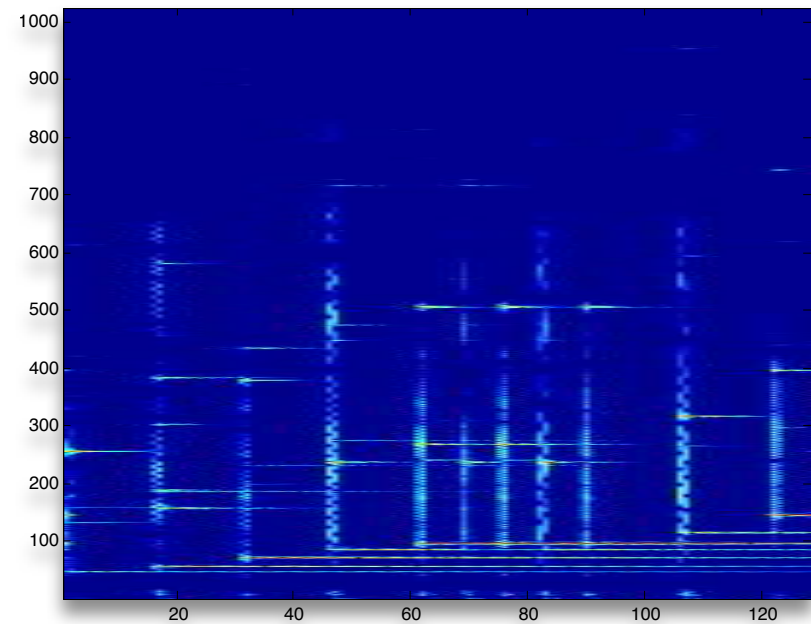
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Sparse Signal Models - solving

Recover x by promoting sparsity:

Sparse Signal Models - solving

Recover x by promoting sparsity: $\begin{matrix} \nearrow \|x\|_0 \\ \searrow \|x\|_1 \end{matrix}$ cfr. R. Gribonval

Sparse Signal Models - solving

Recover x by promoting sparsity:

- $\|x\|_0$
cfr. R. Gribonval
- $\|x\|_1$
- $\|x\|_{\text{Struct}}$
cfr. F. Bach

Sparse Signal Models - solving

Recover x by promoting sparsity:

- $\|x\|_0$
- $\|x\|_1$ cfr. R. Gribonval
- $\|x\|_{\text{Struct}}$ cfr. F. Bach

Ex:
$$x^* = \arg \min_x \frac{1}{2} \|y - \Phi x\|^2 + \mu \|x\|?$$

If the model is an ortho basis it is easy to solve

In general models are embodied by dictionaries

Model coefficients can be recovered by families of algorithms, often solving an optimization problem that promotes sparsity:
iterative shrinkage, greedy algorithms etc ...

Sparsity Constrained Inverse Problems

Sparsity constrained recovery and inverse problems:

$$x^* = \arg \min_x \frac{1}{2} \|s - \Phi x\|^2 + \mu \|x\|_1$$

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observed signal
degrading operator

$\tilde{y} = \mathbf{U}s$

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New perspective: Sparsity as a regularizer (ex: TV - sparsity of gradients)

How to find a needle in a haystack ?



You were right: There's a needle in this haystack...

See tutorials of Rémi Gribonval and Francis Bach on how to find a needle in a haystack

Take Home Messages So Far

- Many signals are sparse on some basis or dictionary
 - *zoology of fixed “optimal” bases*
 - *bases/dictionary learning*
 - *data driven representation (link with Machine Learning)*
- Sparsity offers a lot of flexibility
 - *dimensionality reduction*
 - *compression*
- Algorithms to handle sparsity (provably correct)
 - *greedy, convex relaxation ...*
- Applications !
 - *in particular “compressive sensing”*

Sad Realization and Hopeful Wish

Sparse recovery techniques are great for processing data but ...
... you acquire the whole signal, i.e dimension N and then ...

you trash most of it because you know it is sparse on some good basis !

$$y = \Phi x$$

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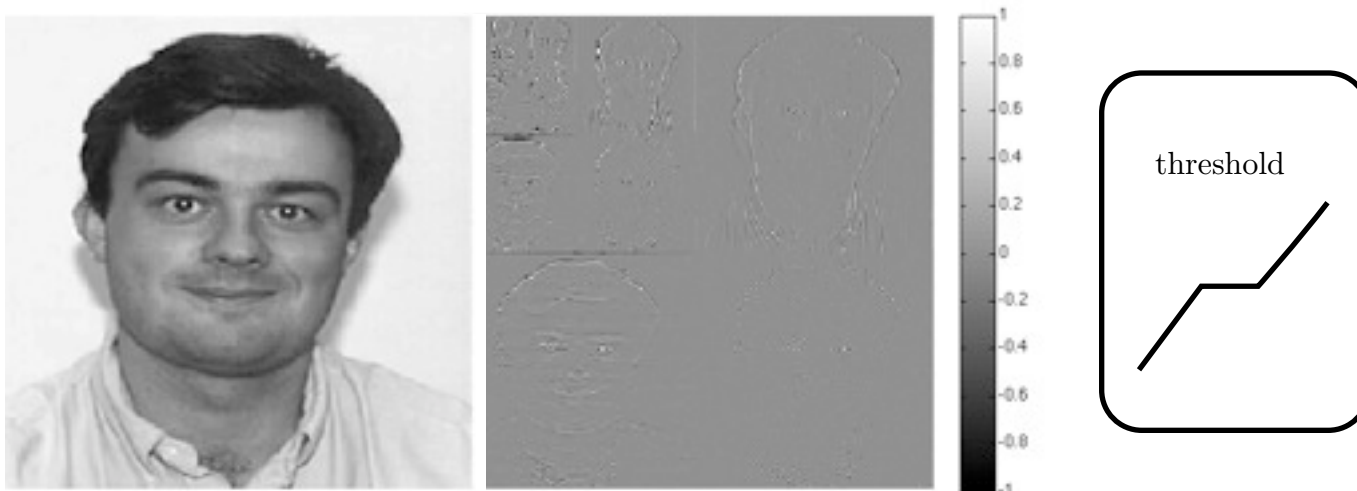


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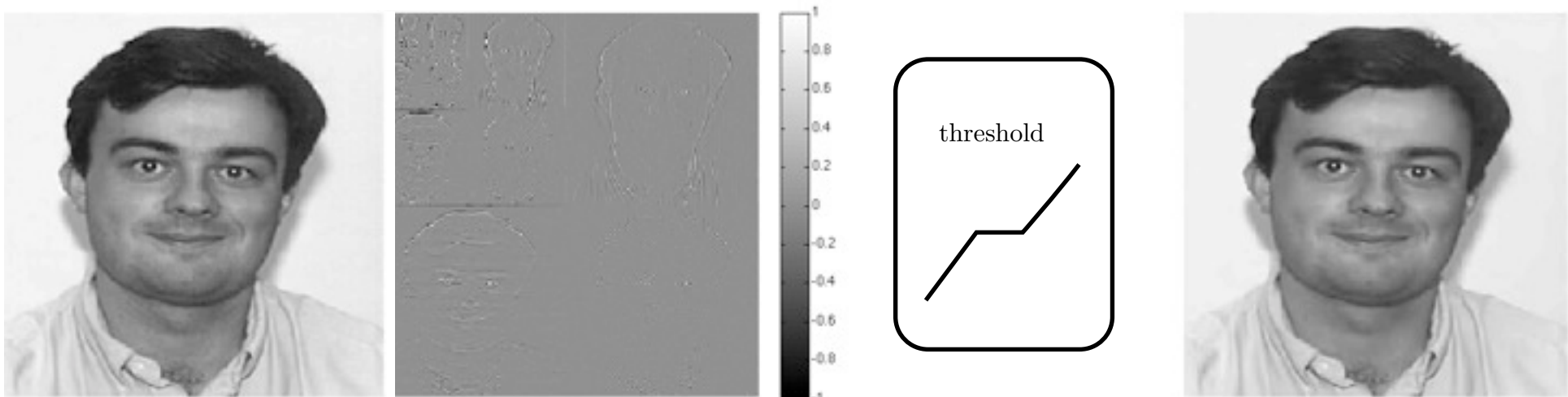


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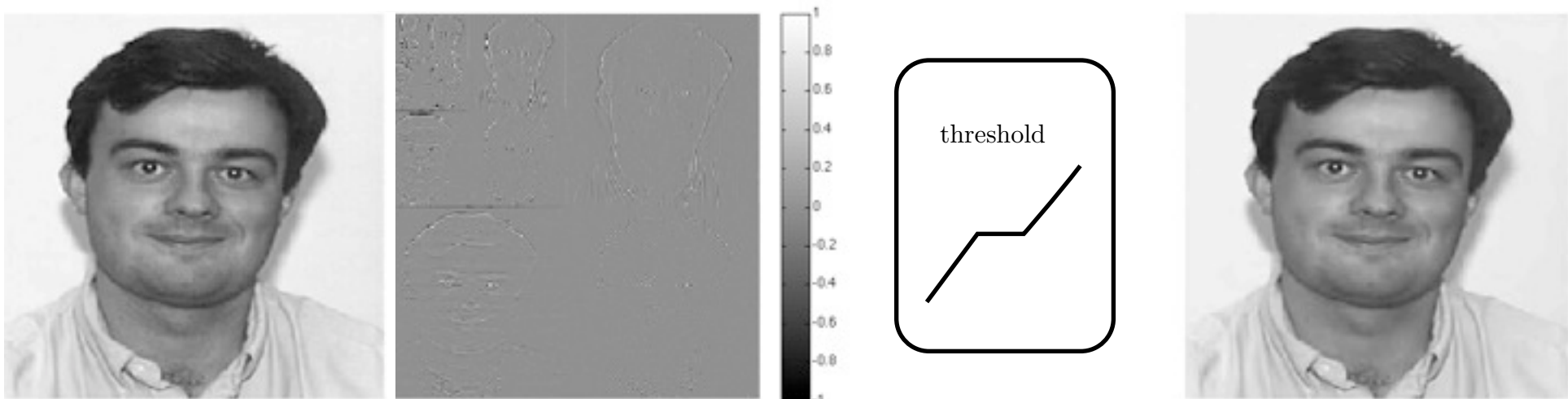
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Sad Realization and Hopeful Wish

Sparse recovery techniques are great for processing data but ...
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Can we estimate the sparse components from few measurements ?



Sparse Recovery and Compressive Sensing

We measure an unknown signal y : $s = \mathbf{A}y$ $\mathbf{A} \in \mathbb{R}^{M \times N}$ $M \ll N$

But we know it comes from a model

$$\arg \min_x \|s - \mathbf{A}\Phi x\|_2^2$$



Try to fit the model to
the observations

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But we know it comes from a model

$\arg \min_x \|s - \mathbf{A}\Phi x\|_2^2 \quad \longrightarrow \quad \text{Try to fit the model to the observations}$

The fit maybe hard to find BUT we know the model is sparse

$$\arg \min_x \|s - \mathbf{A}\Phi x\|_2^2 + \lambda \|x\|_1$$

$$\arg \min_{x \in \mathbb{R}^N} \|x\|_1 \text{ subject to } \|y - \mathbf{A}\Phi x\|_2^2 \leq \epsilon$$

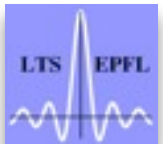
Sparsity constrained inverse problem

Bring Home Key Concepts

- Sparsity / Compressibility
 - large dimension but few degrees of freedom
- Linear (non adaptive !) measurements
 - $M = \mathcal{O}(K \log N/K)$
- Incoherence / Randomness
 - each measurement counts !
 - universality, robustness, scalability
- Recovery
 - provably correct algoS to solve inverse problem

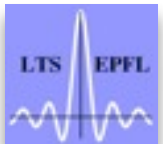
See 2nd talk of
R. Gribonval

[Candès, Romberg, Tao, Donoho]



Applications

well, applications @EPFL really...



Spread-Spectrum Compressive Sensing

A common sensing model: projections onto an ONB

$$y = \Psi x \quad \text{Generative sparse model}$$

$$s = \Phi_{\Omega}^* x \quad \text{Sensing model: randomly select few projections onto an ONB}$$

Spread-Spectrum Compressive Sensing

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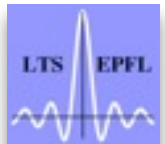
$$y = \Psi x \quad \text{Generative sparse model}$$

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Recovery performance driven by the incoherence between the two bases:

$$\mu = \max_{i,j} |\langle \phi_i, \psi_j \rangle|$$

Spread-Spectrum Compressive Sensing



Spread-Spectrum Compressive Sensing

Sparsity/Sensing in ONBs: CS results driven by mutual coherence

Introduce modulation aimed at decoherence

Spread-Spectrum Compressive Sensing

Sparsity/Sensing in ONBs: CS results driven by mutual coherence

Introduce modulation aimed at decoherence

Theoretical Analysis of Recovery (joint with R. Gribonval):

$$A_{\Omega} = \Phi_{\Omega}^* C \Psi \in \mathbb{C}^{m \times N} \quad \beta(\Phi, \Psi) = \max_{1 \leq i, j \leq N} \sqrt{\sum_{k=1}^N |\phi_{ki}^* \psi_{kj}|^2}$$

modulation

$$\mu \leq \beta(\Phi, \Psi) \sqrt{2 \log(2N^2/\epsilon)} \quad \text{with probability at least } 1 - \epsilon$$

$$m \geq C' N \beta^2(\Phi, \Psi) s \log^8(N)$$

Spread-Spectrum Compressive Sensing

Sparsity/Sensing in ONBs: CS results driven by mutual coherence

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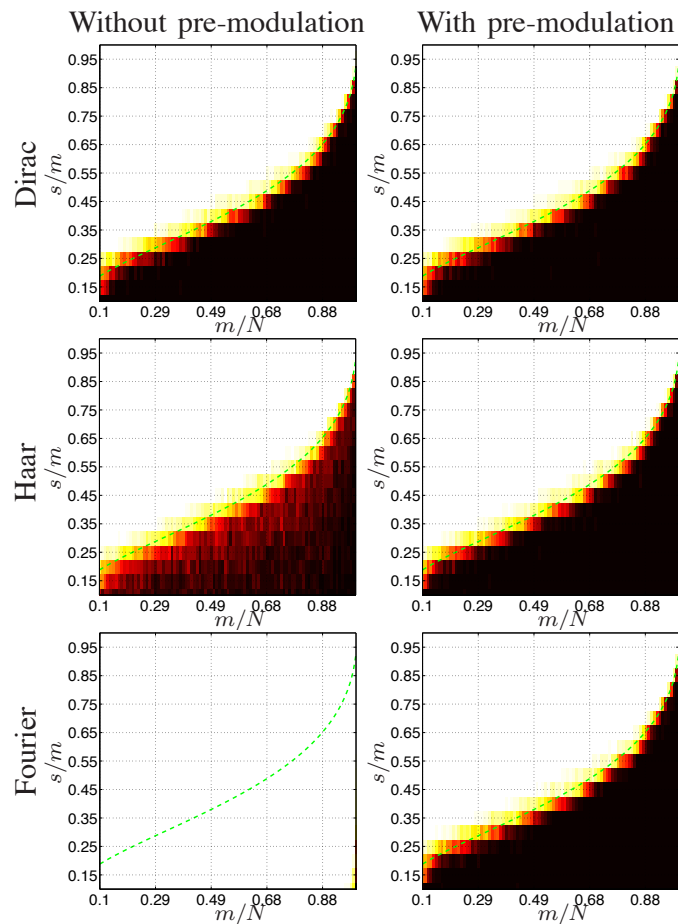
Universal Sensing Basis (Fourier, Hadamard, Noiselet ...)

$$|\phi_{ki}| = N^{-1/2} \implies \beta(\Phi, \Psi) = N^{-1/2} \text{ and } \mu \simeq N^{-1/2}$$

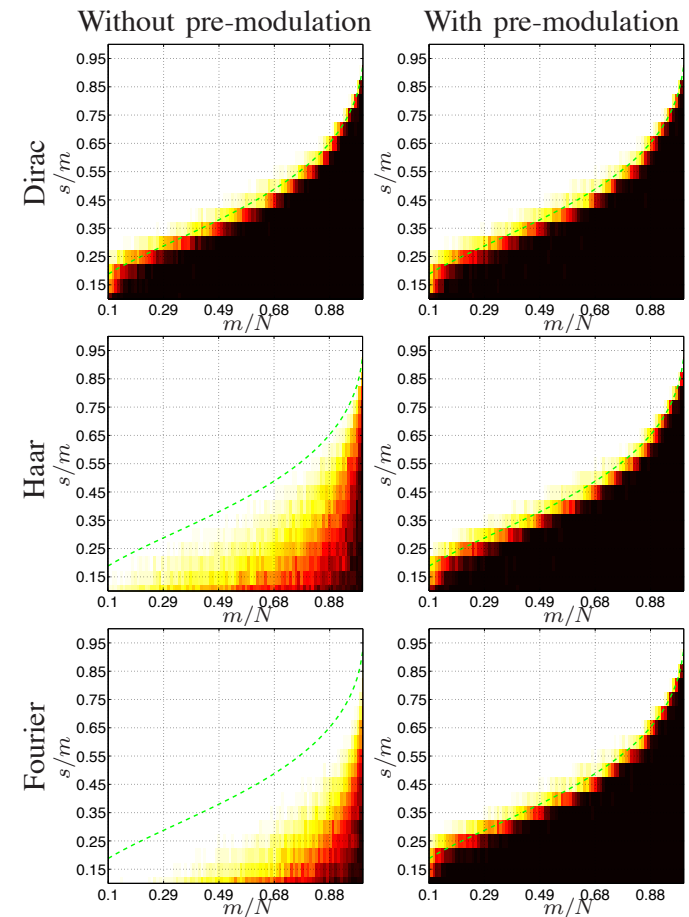
Spread-Spectrum Compressive Sensing

Universal sensing bases give optimal sampling, independently of sparsity basis !

Sensing = Fourier

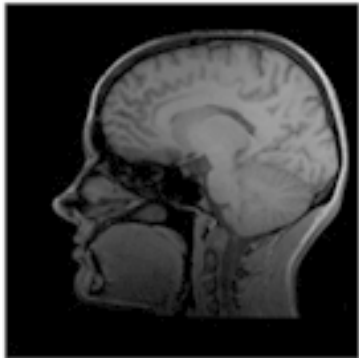


Sensing = Hadamard

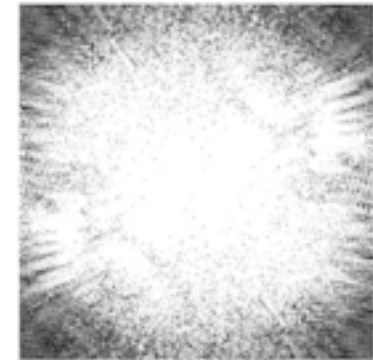


Compressed Sensing in MRI

Problem: accelerate MRI acquisition



$\rho(\mathbf{x})$



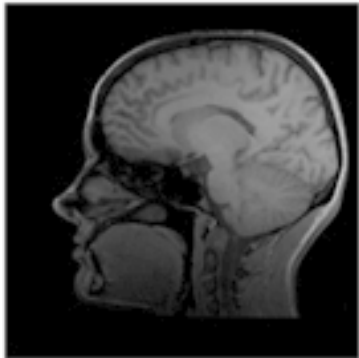
$$\nu(\mathbf{k}) = \int_{\mathbb{R}^2} \rho(\mathbf{x}) e^{-2i\pi \mathbf{k} \cdot \mathbf{x}} d^2 \mathbf{x}.$$

Sensing Model: wide-band modulation

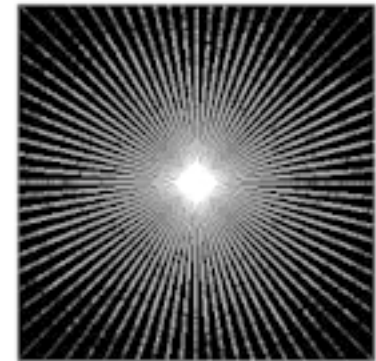
$$y(t) = p(t) \cdot s(t) \Leftrightarrow \hat{y}(\omega) = \hat{p}(\omega) \star \hat{s}(\omega)$$

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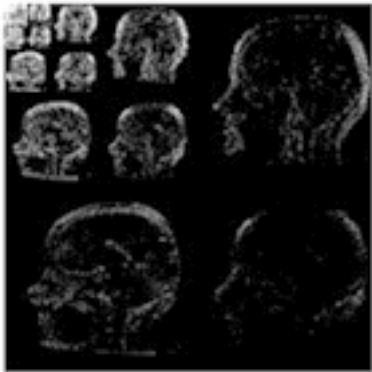
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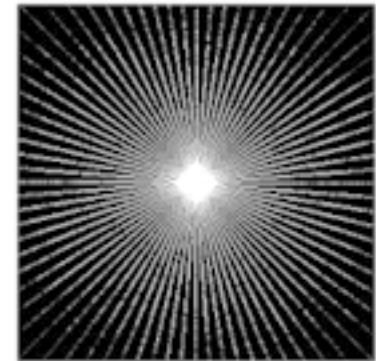
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Spread Spectrum in MRI

CS has already been applied to MR [Lustig]

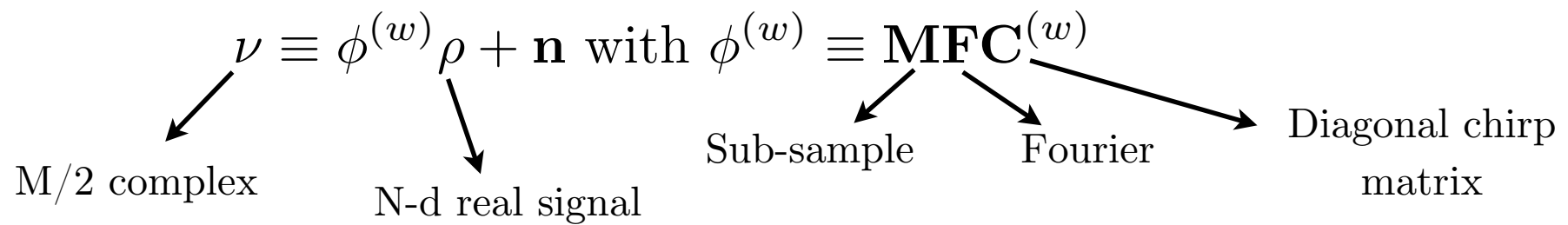
Here: Explore potential of spread-spectrum “conditioning”

$$\nu(\mathbf{k}) = \int_{\mathbb{R}^2} \rho(\mathbf{x}) e^{i\pi w |\mathbf{x}|^2} e^{-2i\pi \mathbf{k} \cdot \mathbf{x}} d^2 \mathbf{x}$$

Phase Scrambling

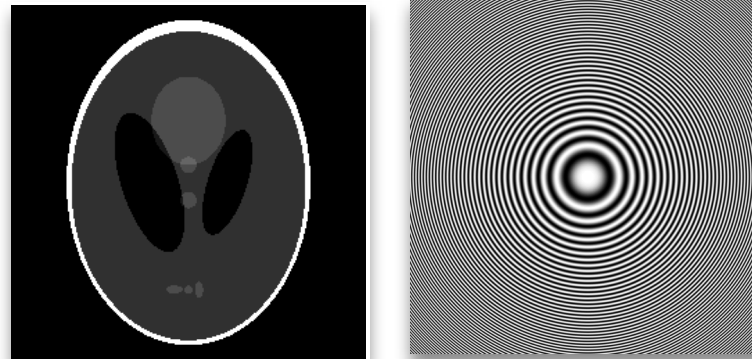
- well-known in MRI (high Dynamic, reduce aliasing)
- obtained through dedicated coils or RF pulses

Measurement model:

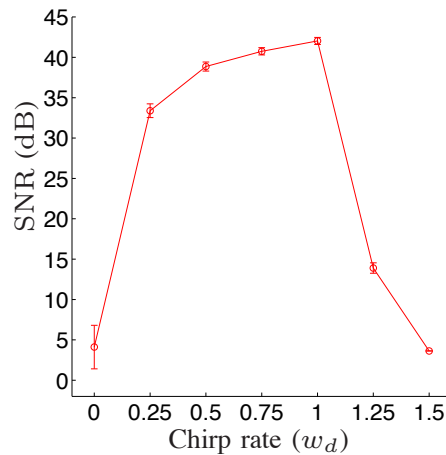


Simulations - leakage

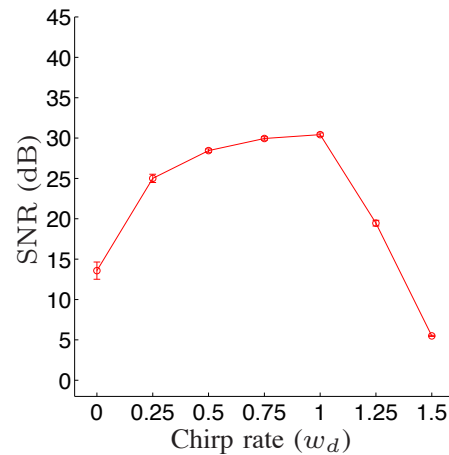
Shepp-Logan phantom and chirp



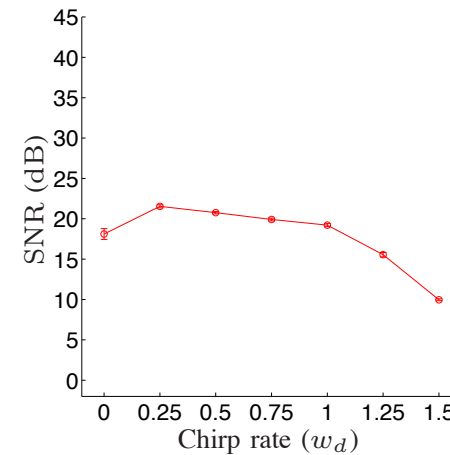
Input SNR = 30dB, coverage 20%, sparsity basis = wavelets, 30 simulations



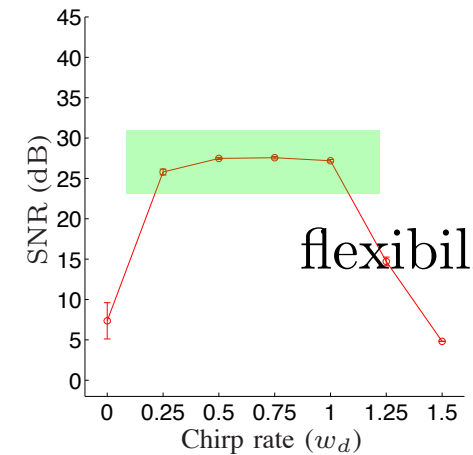
$s=5$



$s=3$



$s=1$



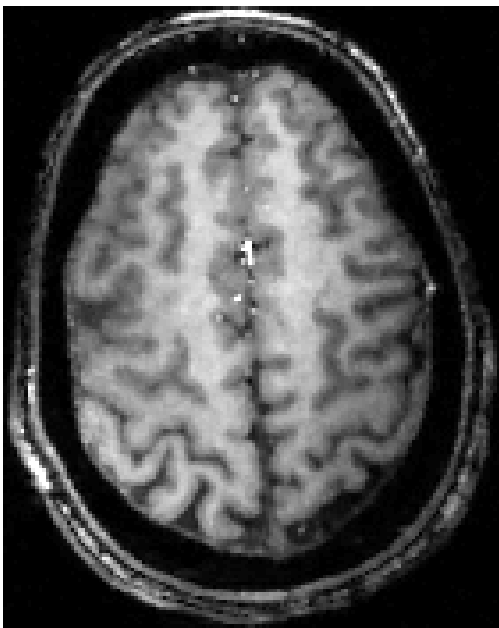
overall

flexibility

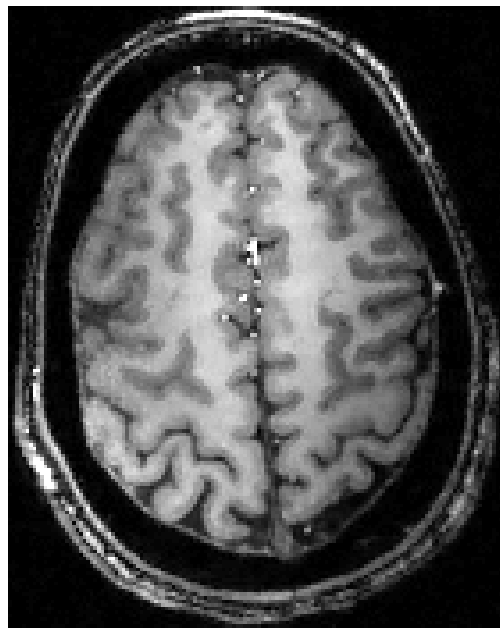
Application: Fast MR Imaging

Real data acquisition, 7T MRI@EPFL
chirp pre-modulation implemented with a dedicated shim coil

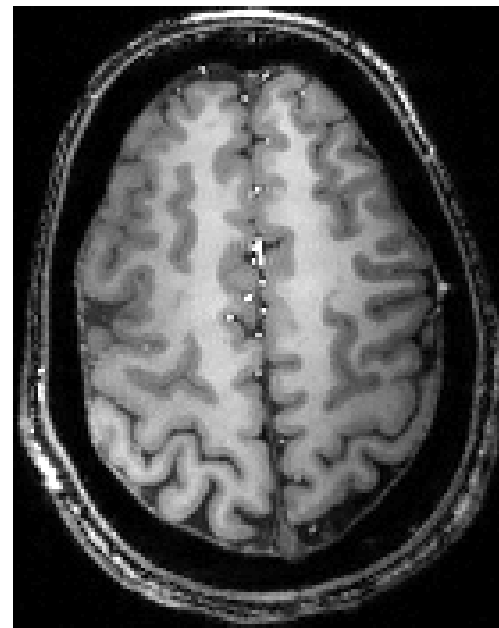
Acceleration
x 4



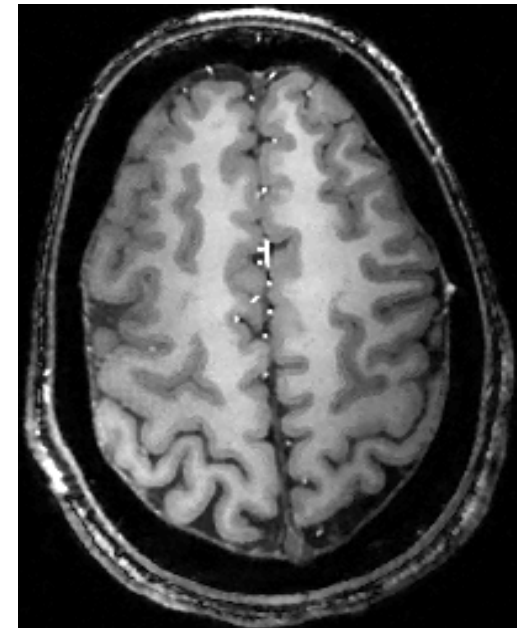
Acceleration
x 2

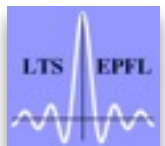


Acceleration
x 1

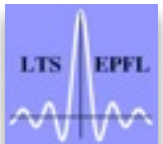


Enhanced
resolution





Application: ULP bio-sensing



Application: ULP bio-sensing

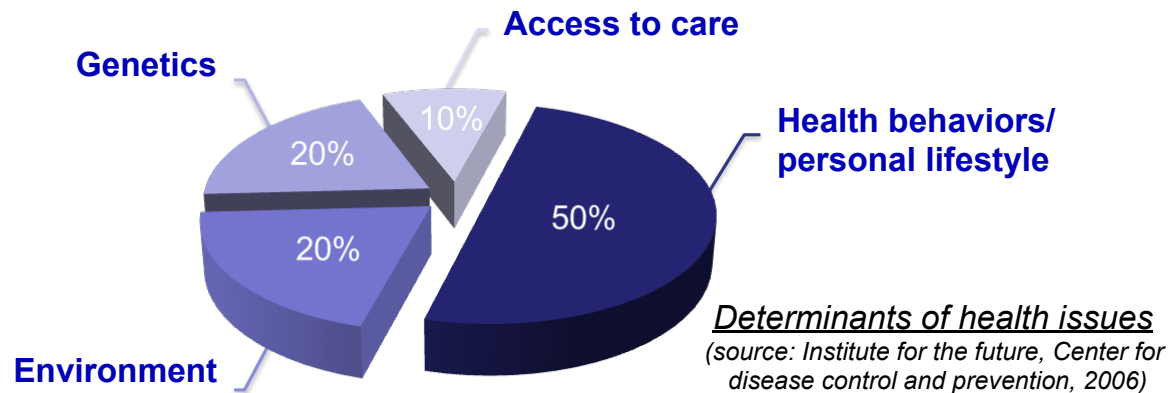
Societal motivation:

demography and life style conspire toward a global health care crisis (lifestyle-induced diseases, NCDs)

Application: ULP bio-sensing

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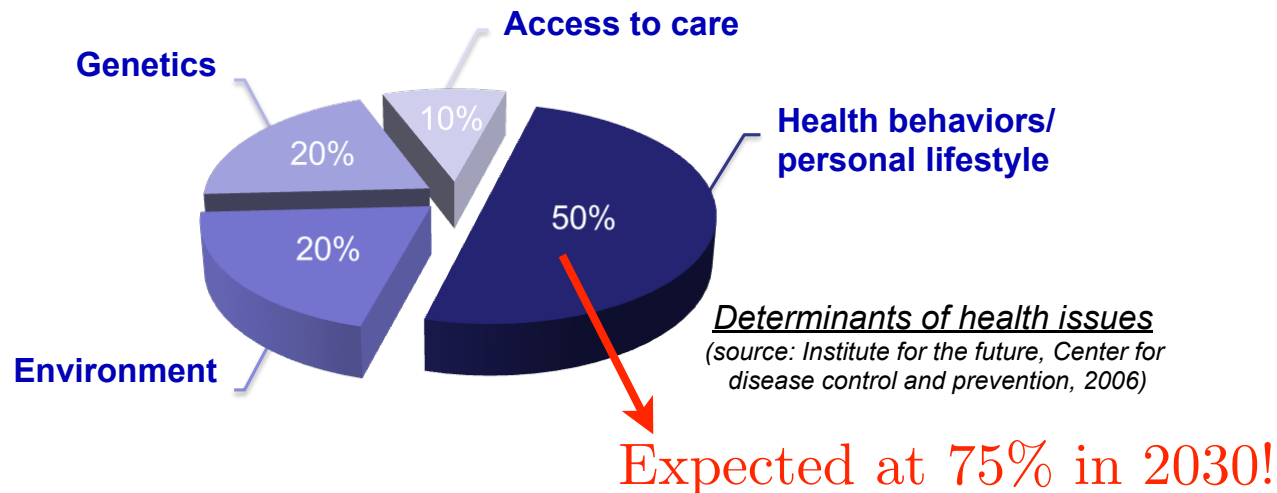
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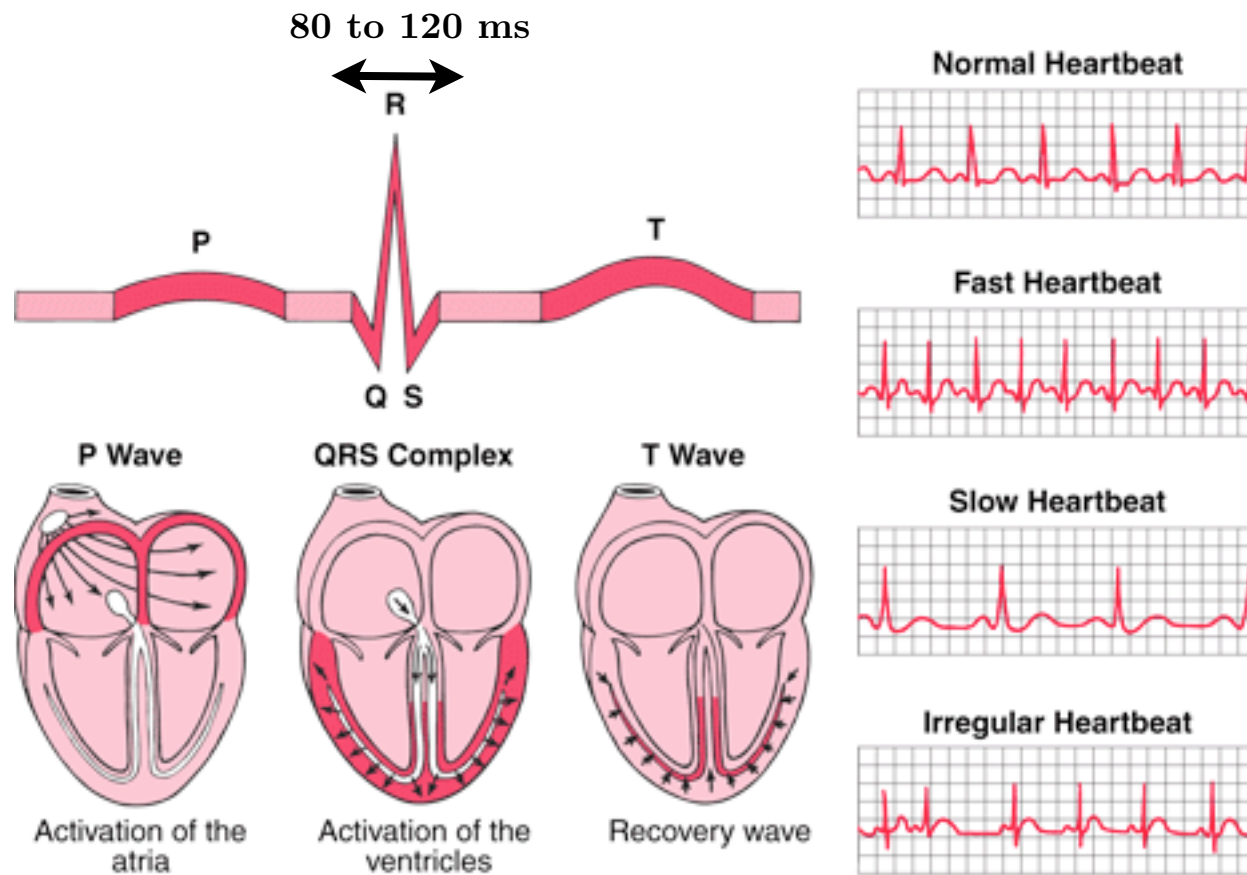
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Societal motivation:

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Typical modality: ECG



Typical components,
could be learned
Often, wavelets are used

Sampling rate few hundred Hz

Low-power ECG ambulatory system

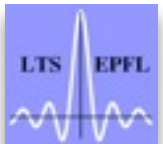
Problem: sense and transmit ECG (possibly multi-lead) from a low power body-area network

Compression ? Surely if we transmit less, we will waste less power in communication.

Sure, but if we compress more we will waste energy using a complex encoder !

Can CS offer an interesting trade-off ?

Can everything be real-time ?

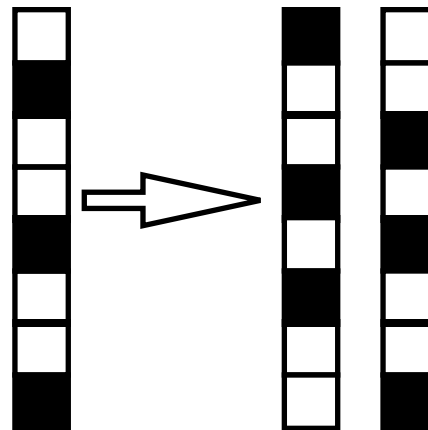


-
- State-of-the-art
 - Wavelet transform, followed by thresholding, quantization and entropy coding
 - Pros: excellent compression results, signals nicely sparse (at least ventricular part)
 - Cons: Full wavelet transform must be implemented on the sensing node
 - Can a light compressive sensing encoder achieve a good trade-off compression/power consumption ?

What is a good sensing matrix for low-power sensing ?

Surely not gaussian ! (dense, complex to apply to signal and even complex to generate ...)

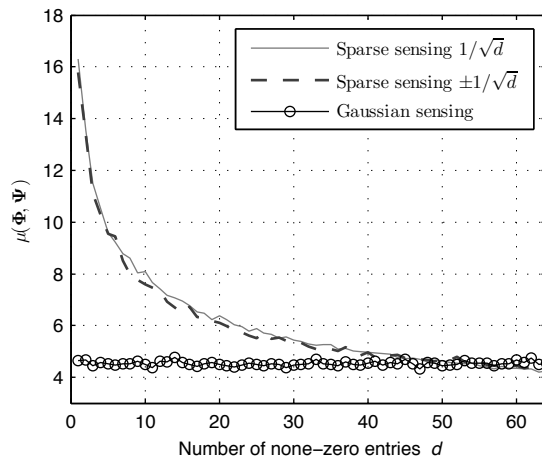
Sparse matrices, binary entries (ex: expander graphs)



generate binary vector
with d non-zero elements

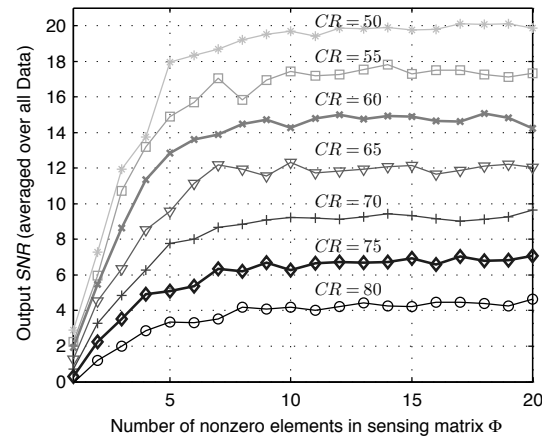
generate full matrix by
random permutations

Performance indexes for sensing mechanism

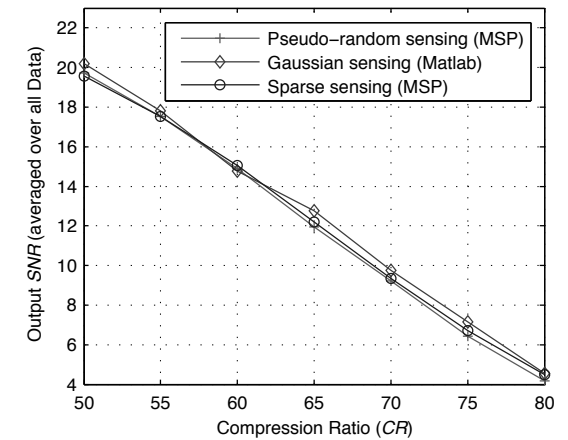


Mutual coherence $\mu(\Phi, \Psi)$ vs. d

coherence quickly approaches “optimal case”



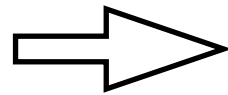
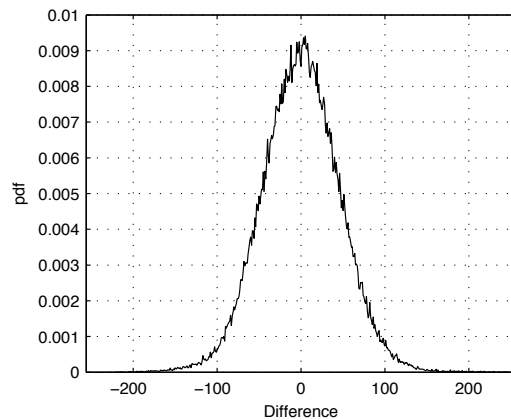
SNR saturates after $d=12$ non-zero elements



Hardly any difference between proposed sensing and gaussian sensing

2 s of signal are sensed in 82 ms

Coding: simple predictive scheme



Gaussian RD theory

9 bits quantizer

Huffman coding

1.5 kB codebook stored on platform

difference between successive sensing vectors

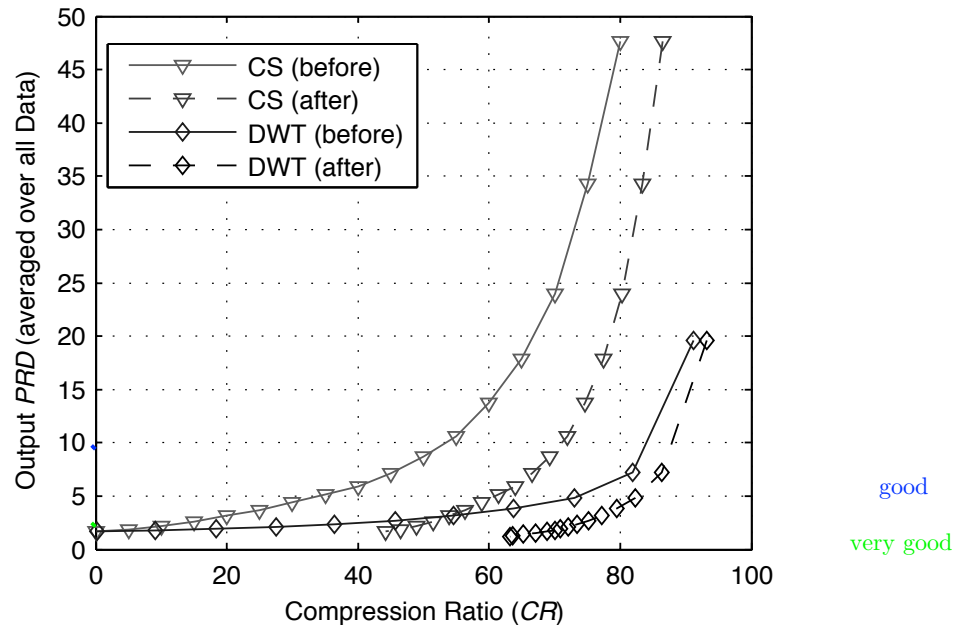
Compression Ration: 20%

Total memory footprint of CS implementation:

6.5 kB of RAM for computations

7.5 kB of Flash

Comparisons - Quality vs Compression



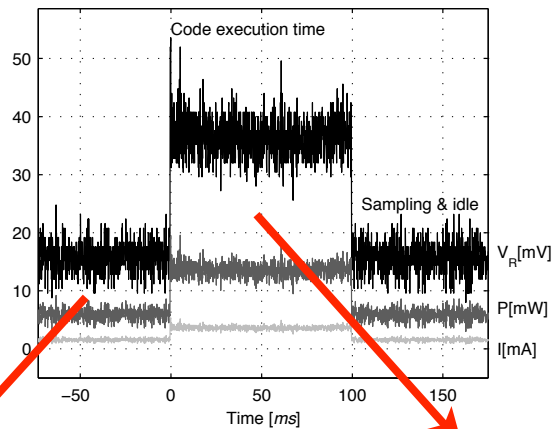
In terms of pure compression performance, an optimized DWT encoder is clearly (and obviously) better than the non-adaptive CS scheme

Comparisons - Power consumption

Consumption *measured* on the platform in real-life use

Code execution time: 95ms for CS, 580ms for DWT

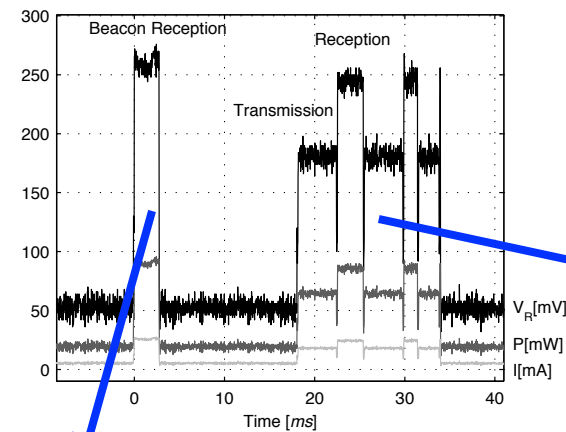
Node consumption



MSP430 idle and
sampling till buffer full

CS code running

Radio consumption



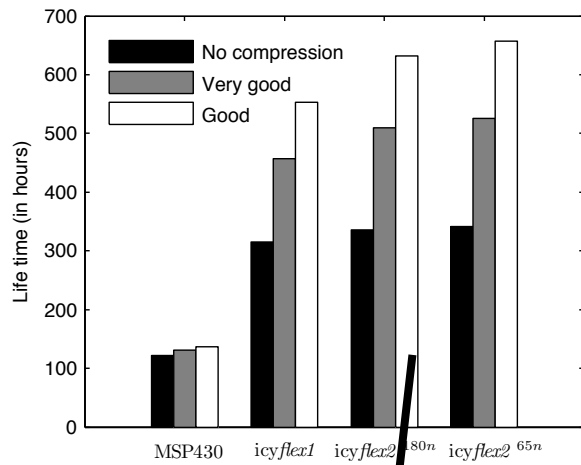
periodic "ping"

transmission
& reception

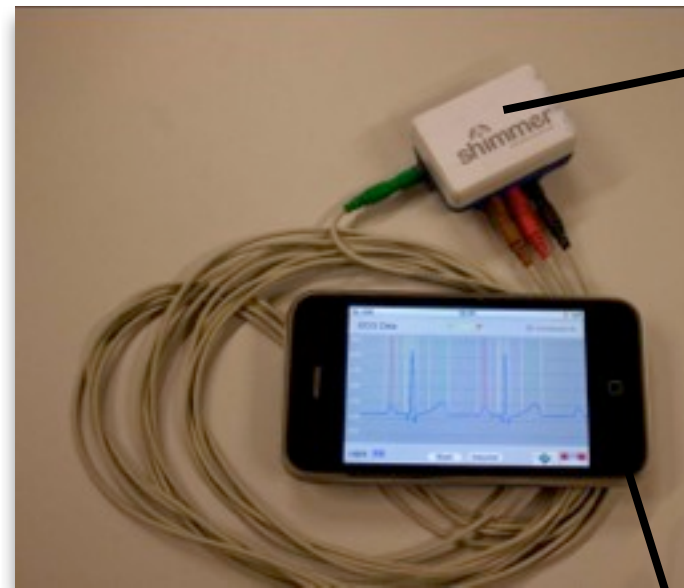
Note: contrary to what is usually assumed in literature, optimized antennas are really low power !

Comparisons - Power consumption

Final results: it is important to know your architecture VERY well



State-of-the-art low-power micro-controllers

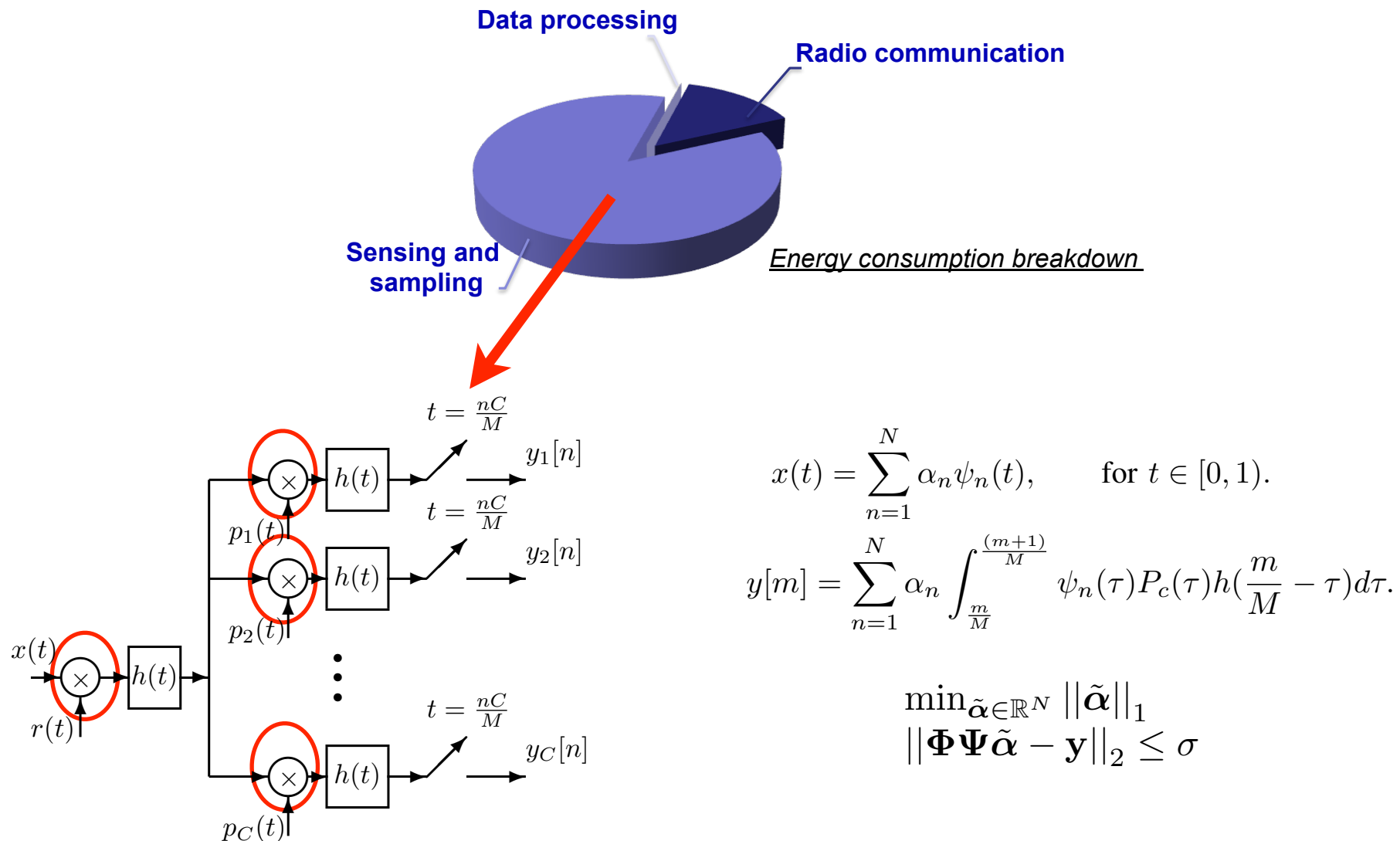


Light Compressive sensing encoder on embedded system

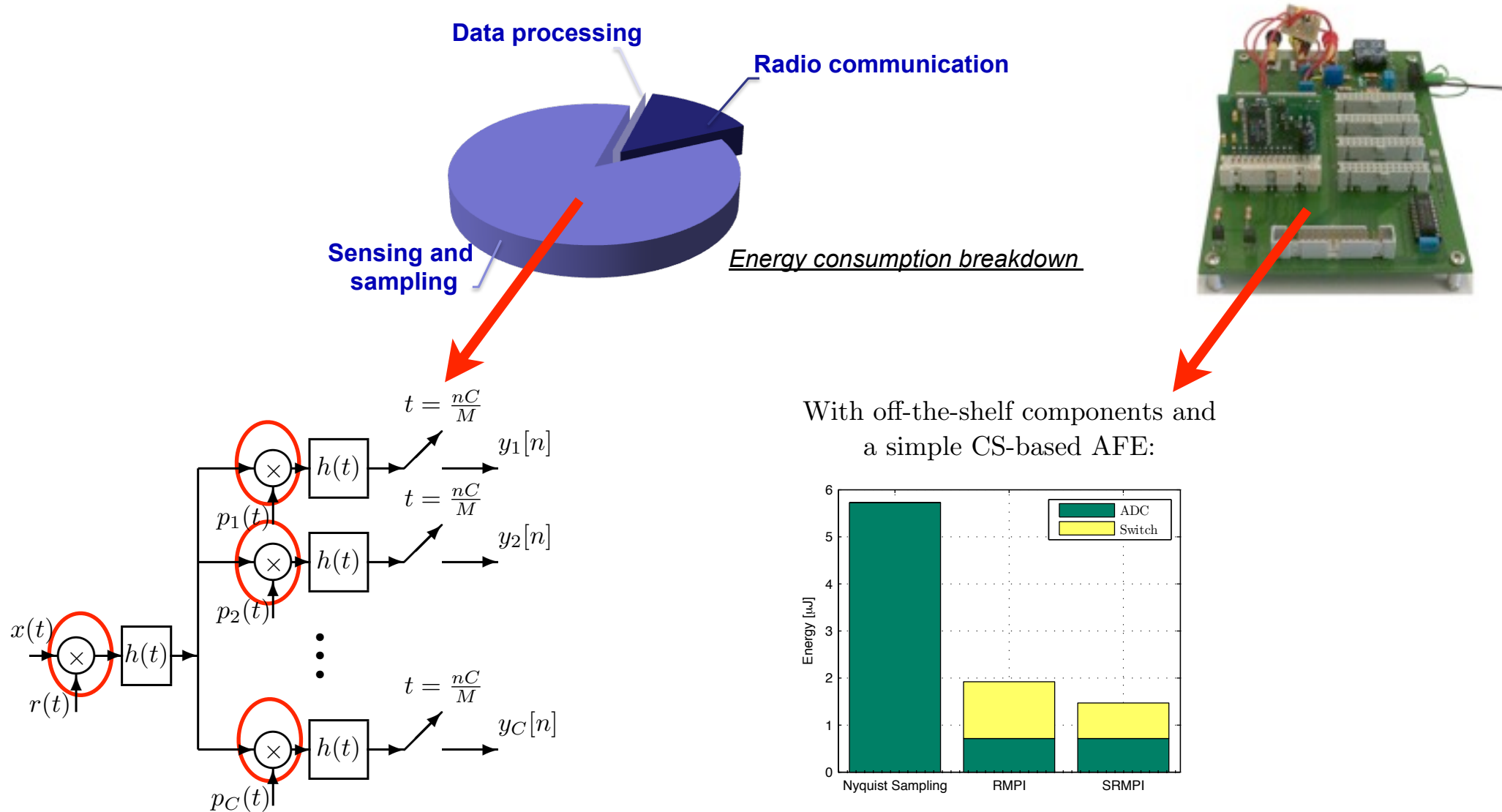
CS decoder (iterative soft thresholding) running real-time on an iphone

92% lifetime extension, 6 times better than MSP430

Going further: Full compressive DAC



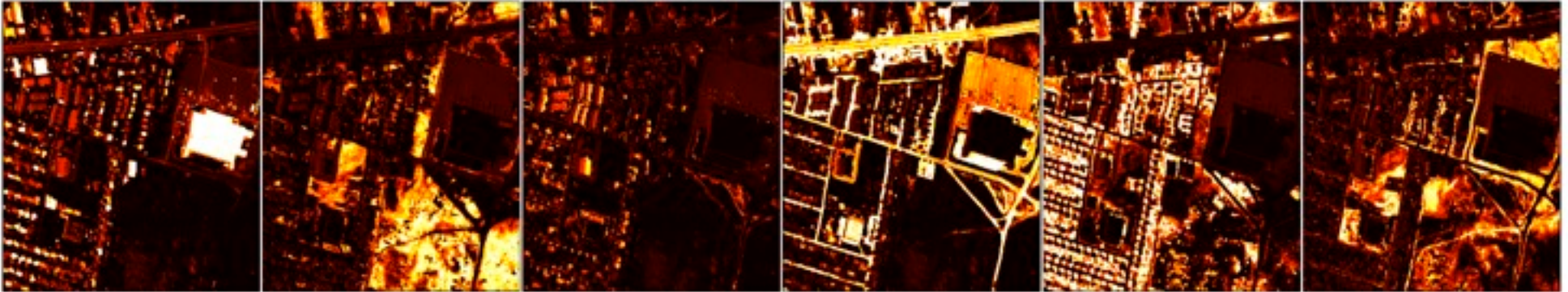
Going further: Full compressive DAC



-
- Big advantage of direct samples
 - they are easily interpreted
 - they can be processed
 - Can we **process signals in the compressed domain**?
 - detection, recognition, classification
 - segmentation

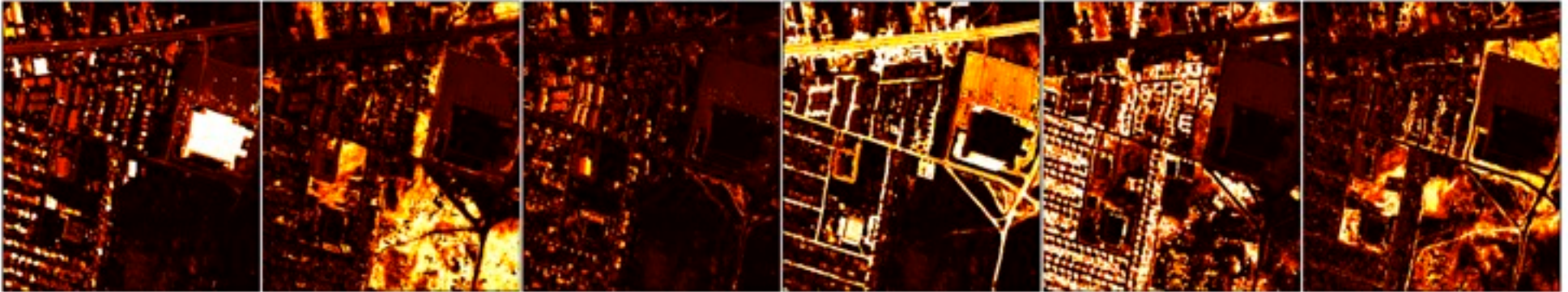
Hyper Spectral Imaging

S: Sources (element abundancies) $\mathbf{S} \in \mathbb{R}^{n_1 \times \rho}$

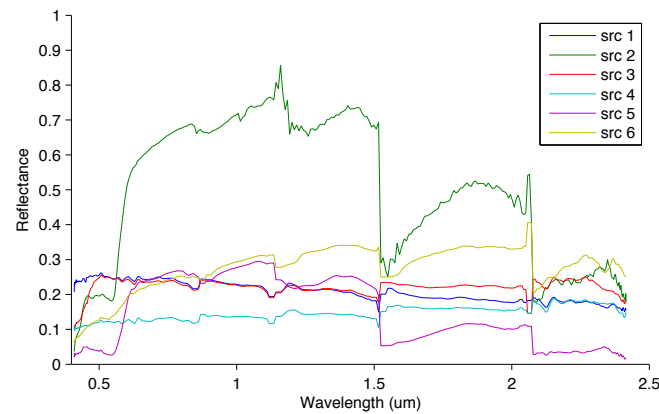


Hyper Spectral Imaging

S: Sources (element abundancies) $\mathbf{S} \in \mathbb{R}^{n_1 \times \rho}$

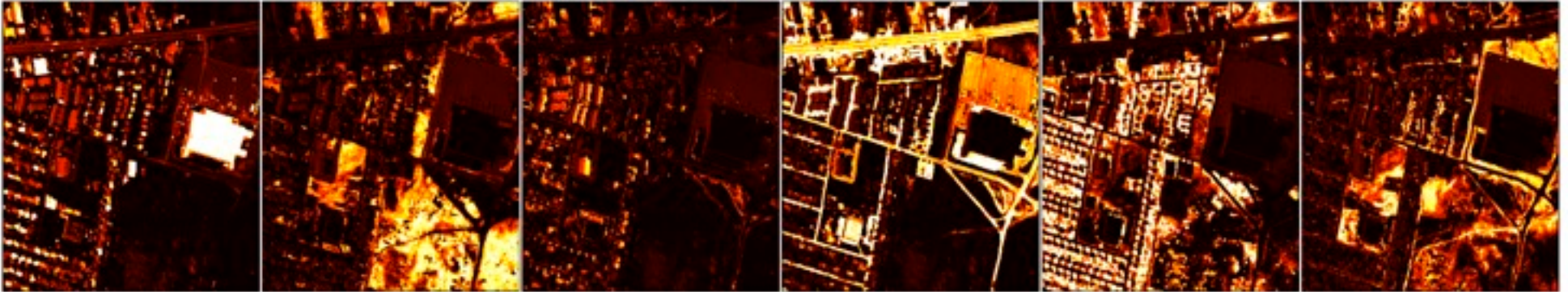


A: Spectra (depending on modality) $\mathbf{A} \in \mathbb{R}^{n_2 \times \rho}$

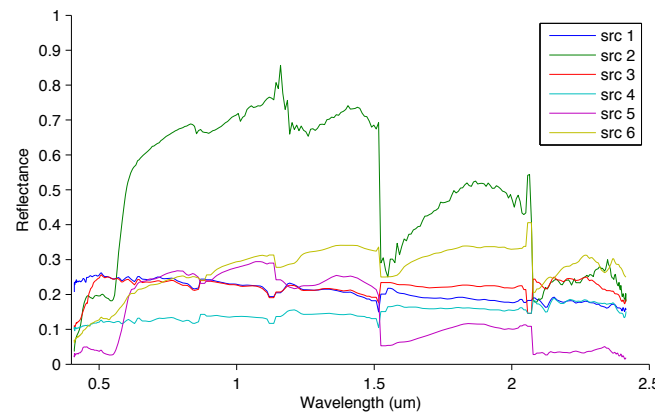


Hyper Spectral Imaging

S: Sources (element abundancies) $\mathbf{S} \in \mathbb{R}^{n_1 \times \rho}$



A: Spectra (depending on modality) $\mathbf{A} \in \mathbb{R}^{n_2 \times \rho}$



Each pixel is a weighted combination of source spectra: $y = \mathbf{S}\mathbf{A}^T$

Hyper Spectral Imaging

Very high dimensional data (thousands of channels)

Typical problem: **Source Separation**

Given the dictionary of spectra \mathbf{A} and the data y

Recover the source abundances, factorizing $y = \mathbf{S}\mathbf{A}^T$

Hyper Spectral Imaging

Very high dimensional data (thousands of channels)

Typical problem: Source Separation

Given the dictionary of spectra \mathbf{A} and the data y

Recover the source abundances, factorizing $y = \mathbf{S}\mathbf{A}^T$

Can this be achieved even when:

$$y = \mathcal{A}(\mathbf{S}\mathbf{A}^T) \quad \text{Indirect/degraded observations}$$

The dictionary of spectra \mathbf{A} is unknown

Hyper Spectral Imaging

HSI Compressive Blind Source Separation (CS-BSS)

$$\begin{aligned}
 & \arg \min_{\mathbf{S}, \mathbf{A}} \quad \|\mathbf{y} - \mathcal{A}(\mathbf{S}\mathbf{A}^T)\|_{\ell_2}^2 \\
 & \text{subject to} \quad \sum_j^\rho \|\mathbf{S}_j\|_{TV} \leq \tau \\
 & \quad \sum_{i=1}^\rho [\mathbf{S}]_{i,j} = 1 \quad \text{Source image constraints} \\
 & \quad [\mathbf{S}]_{i,j} \geq 0 \\
 & \quad \|\Psi_{1D}^T \mathbf{A}\|_{\ell_1} \leq \gamma \quad \text{Spectral signature constraints} \\
 & \quad [\mathbf{A}]_{i,j} \geq 0.
 \end{aligned}$$

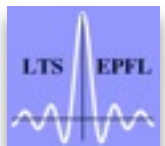
- Bi-convex minimization

Algorithm: alternating convex minimization

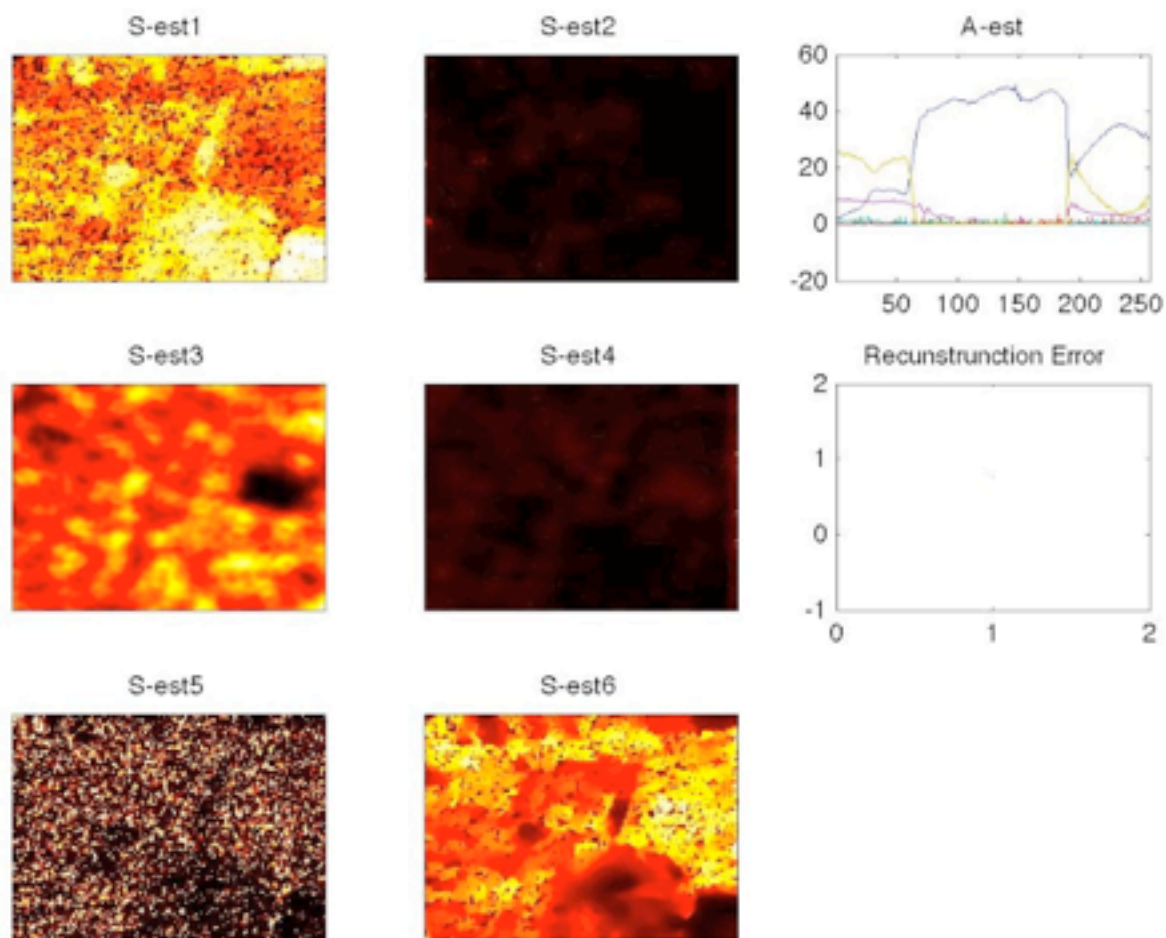
- 1- Initialize A at random
- 2- Source recovery given A
- 3- Mixture recovery given S
- 4- Repeat 2-3 until convergence

Goal: Compute source maps directly from compressed measurements
 Separation or Segmentation in the compressed domain

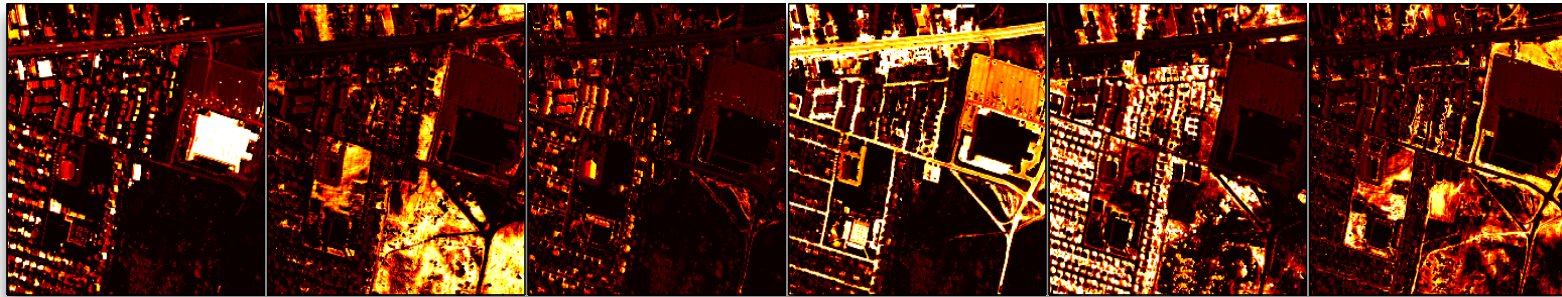
Hyper-Spectral Imaging



Hyper-Spectral Imaging

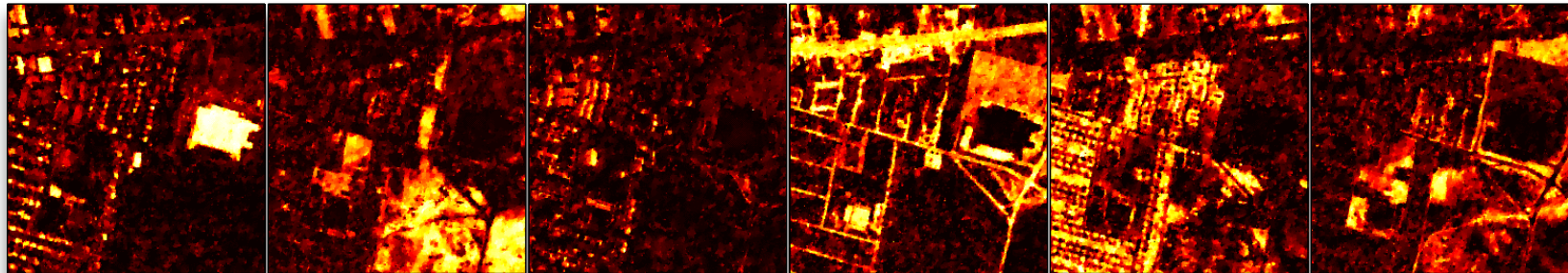


Hyper-Spectral Imaging



(a) Ground truth

From 3% of the original data:



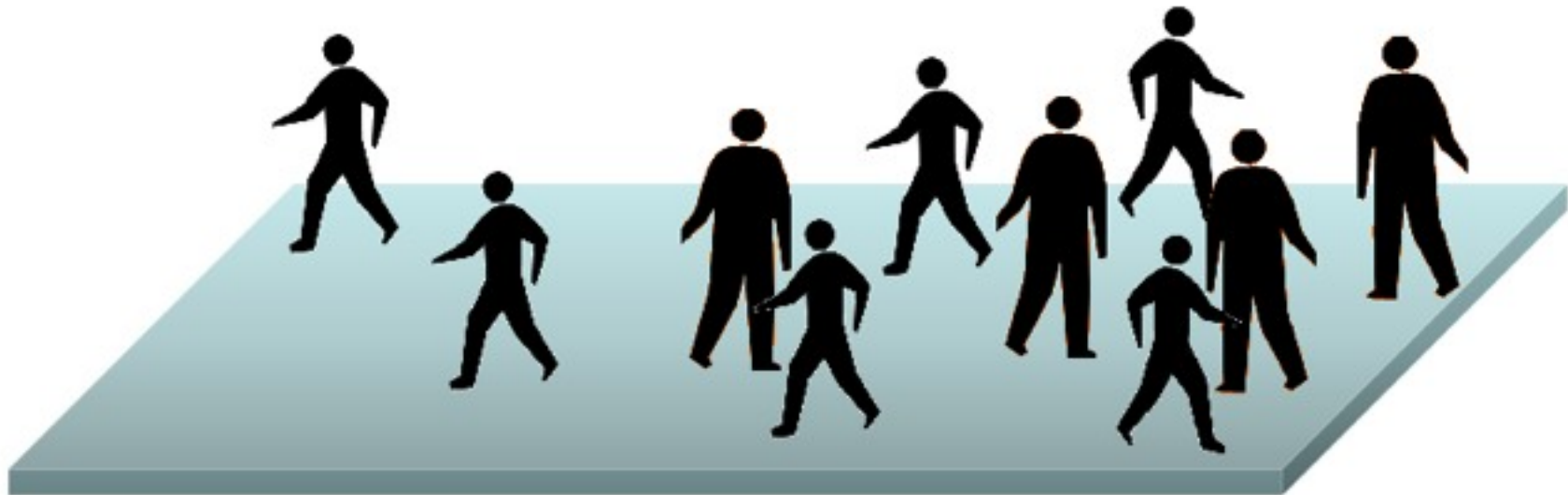
(e) SS-TV-decorr, source reconstruction SNR: 8.64 dB

Other applications: Mass spectrometry (MALDI), ...

Detecting and Following People

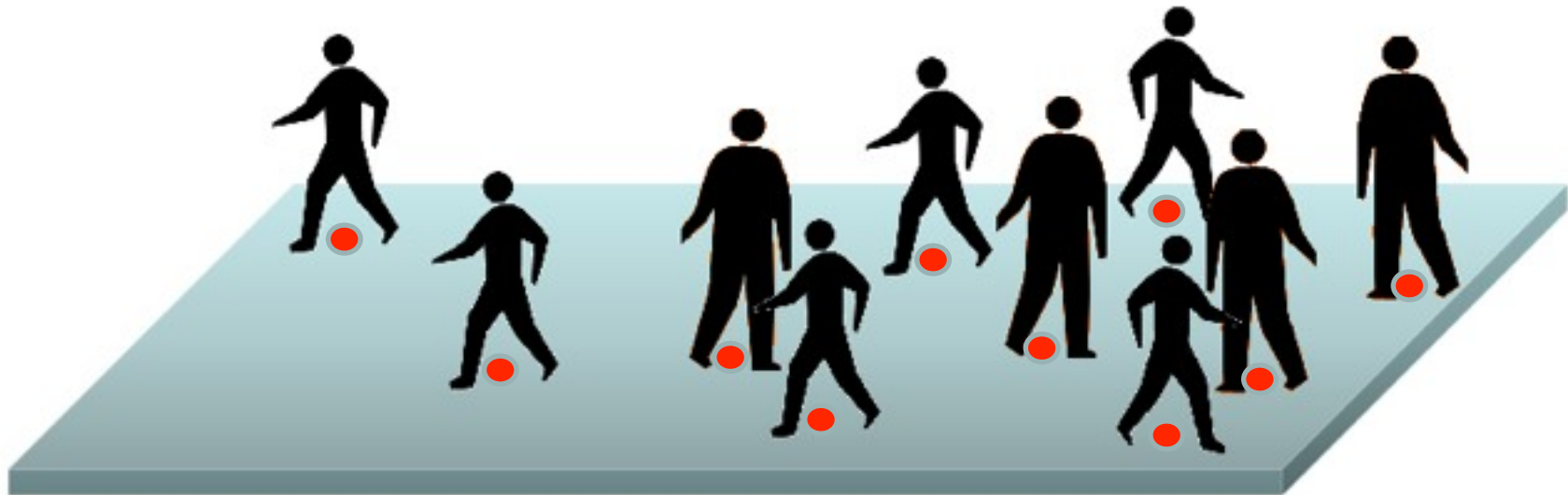


Detecting and Following People



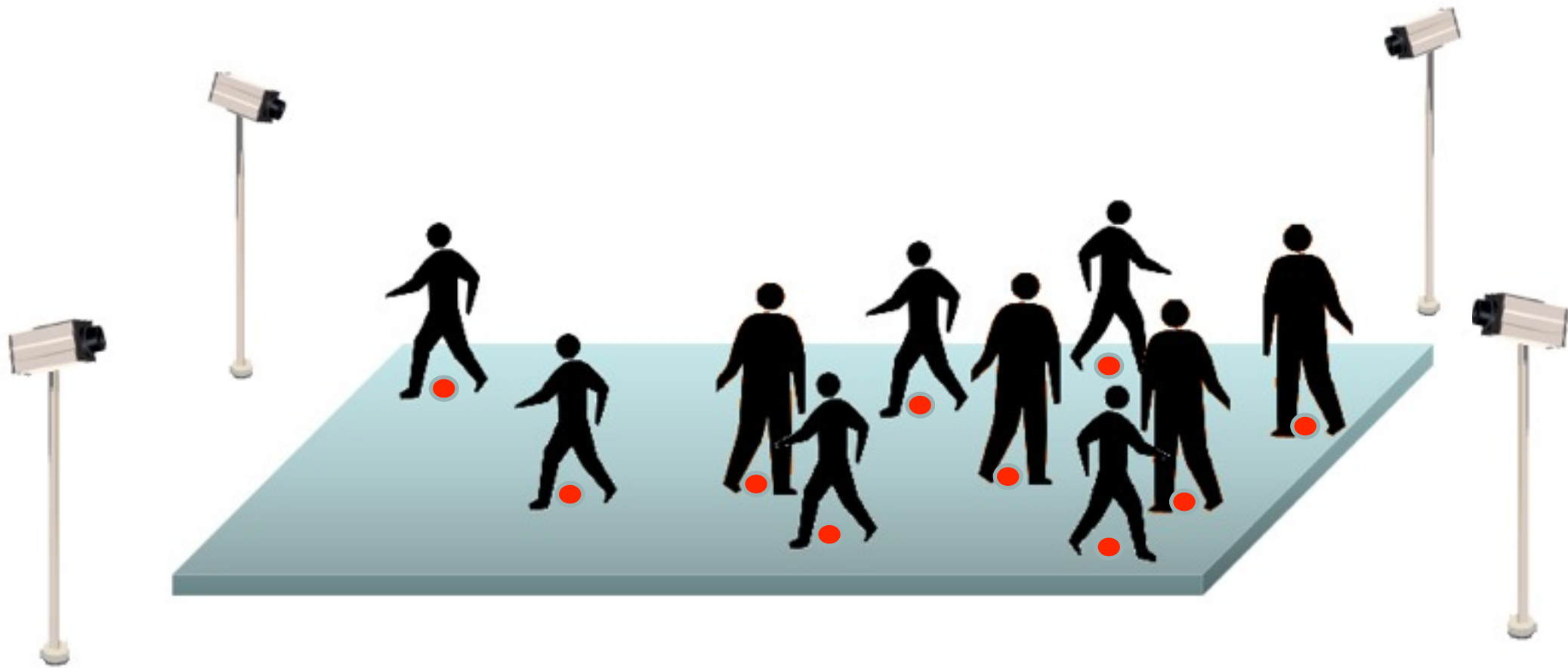
Detecting and Following People

- Locate group of people occluding each other



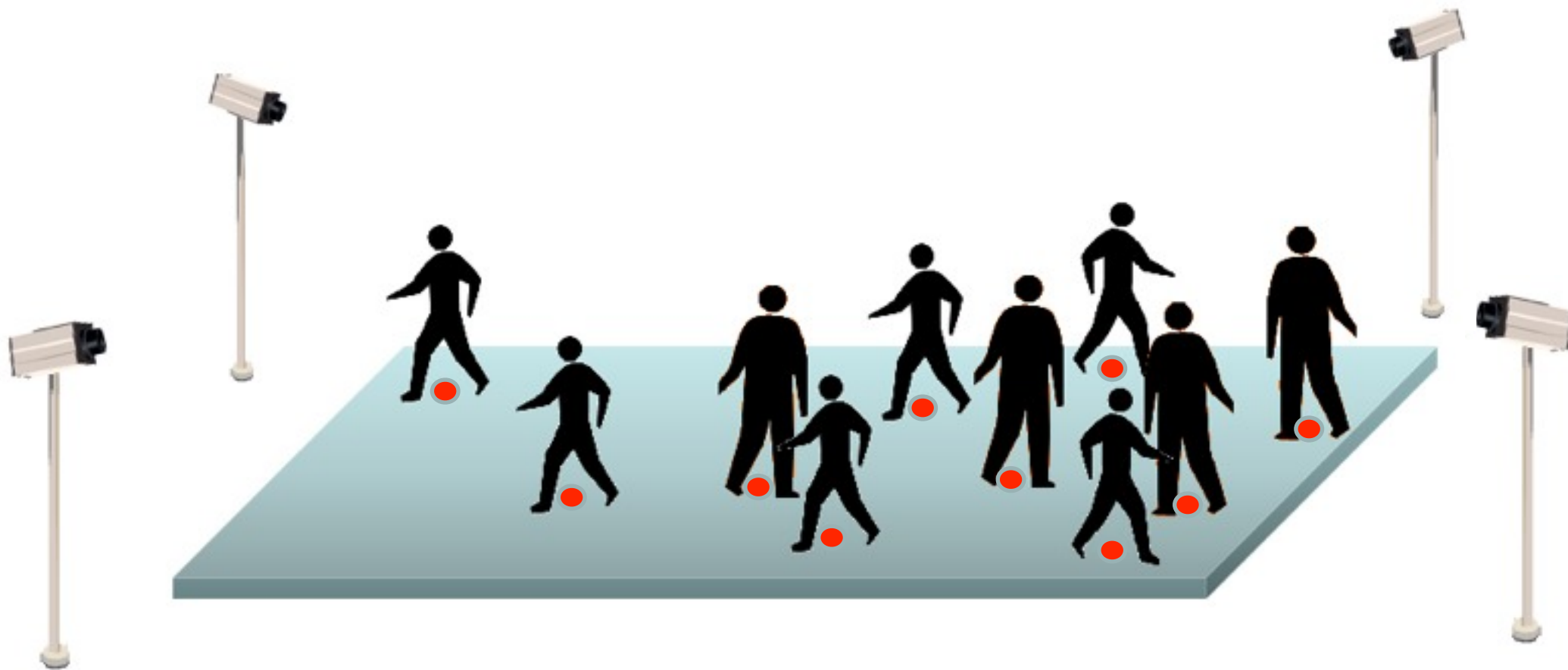
Detecting and Following People

- Locate group of people occluding each other
- With a network of cameras



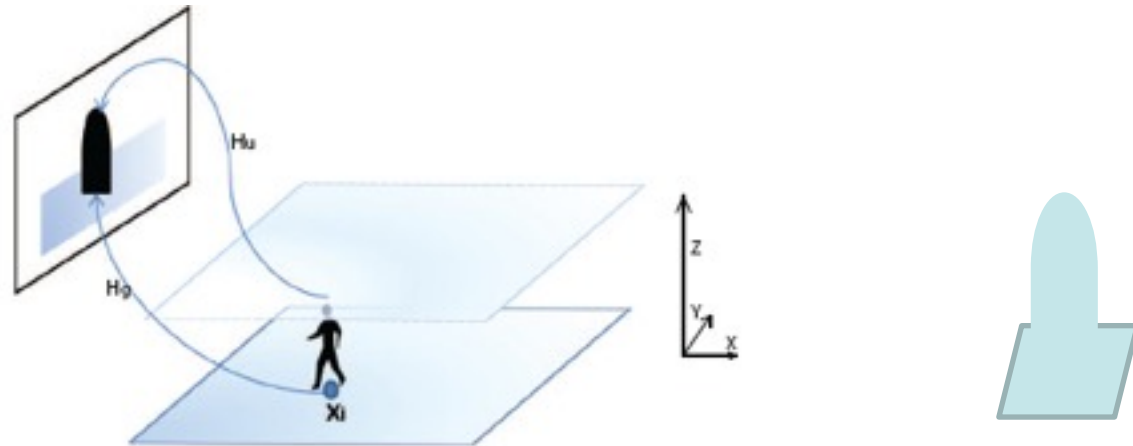
Detecting and Following People

- Locate group of people occluding each other
- With a network of cameras
- Given extracted foreground silhouettes only



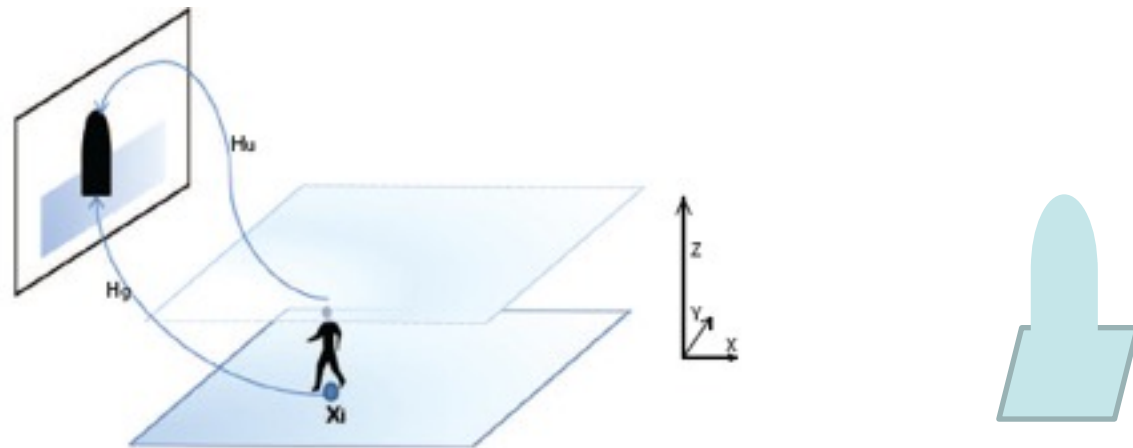
Problem Formulation

Use a dictionary D associating to x a certain configuration of silhouettes in y



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Use a dictionary D associating to x a certain configuration of silhouettes in y

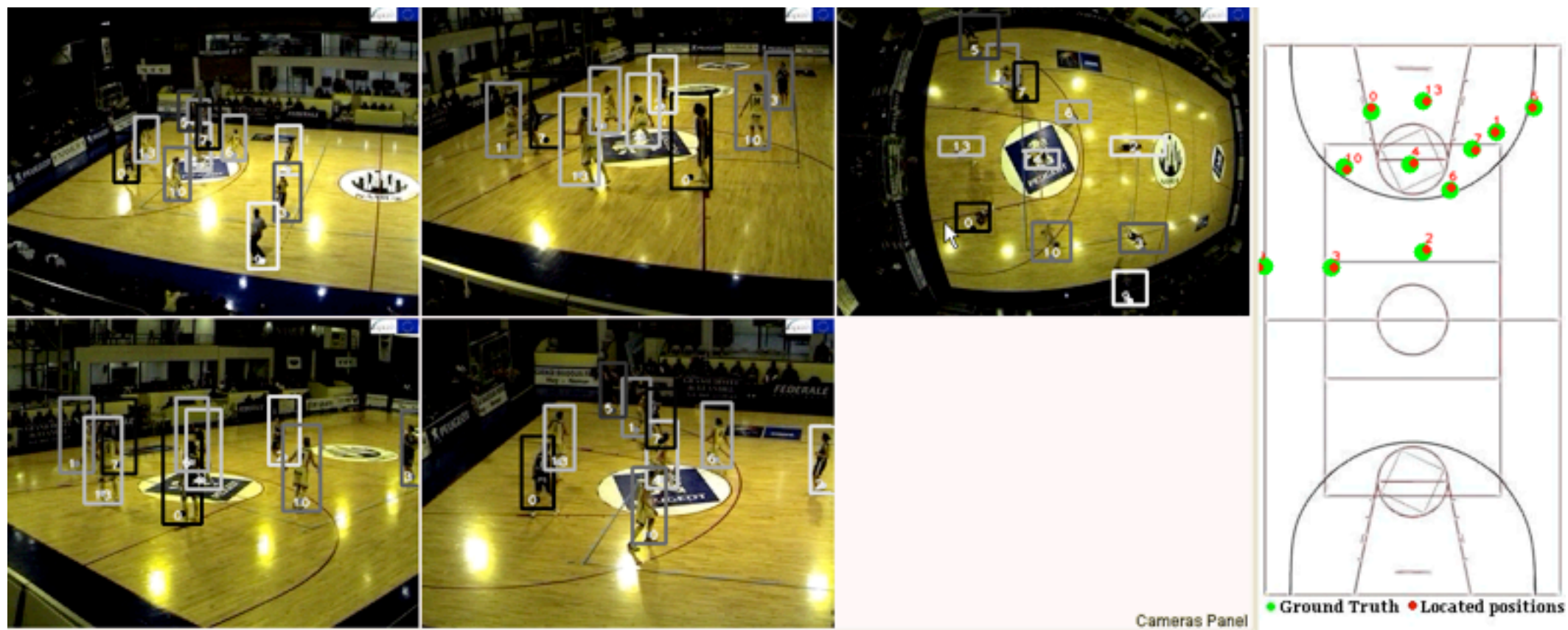


We want:

$$\arg \min_{x \in \{0,1\}^N} \|x\|_0 \quad \text{s.t.} \quad \|y - Q(Dx)\|_2^2 < \varepsilon$$



Results



Alexandre Alahi, Mohammad Golbabaee

Outlook

- Significant challenges ahead in signal processing
 - Big Data
 - Ubiquitous but Cheap Sensing (i.e dirty signals)
- Need new models, algorithms
 - Hardware/Software co-design
 - New sensor designs
- Where do we go from here in terms of applications ?
 - Structured notions of sparsity for specific applications
 - More data driven approaches
 - Non-linearities ?

Application focus

- Big Data
 - Too big for sparsity ?
- What can we hope from Sparsity+Machine Learning?
 - Better algos ?
 - Better models ?
- What new application fields ?
 - Beyond restoration ?
- What's missing ?
 - Time/Space variant operators
 - Non-Linearities