# Queueing process with excluded-volume effect

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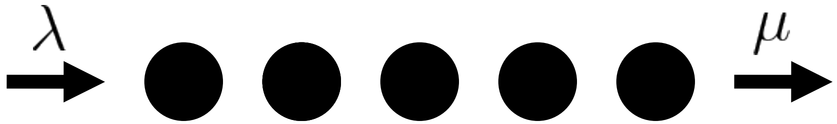
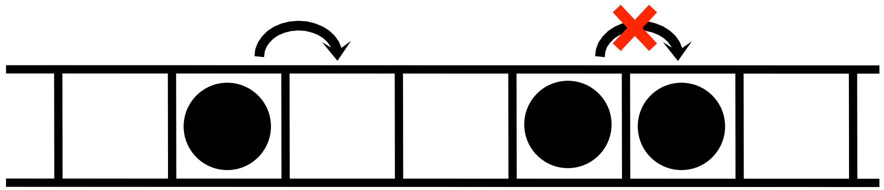
TRAM2 21.03.2013

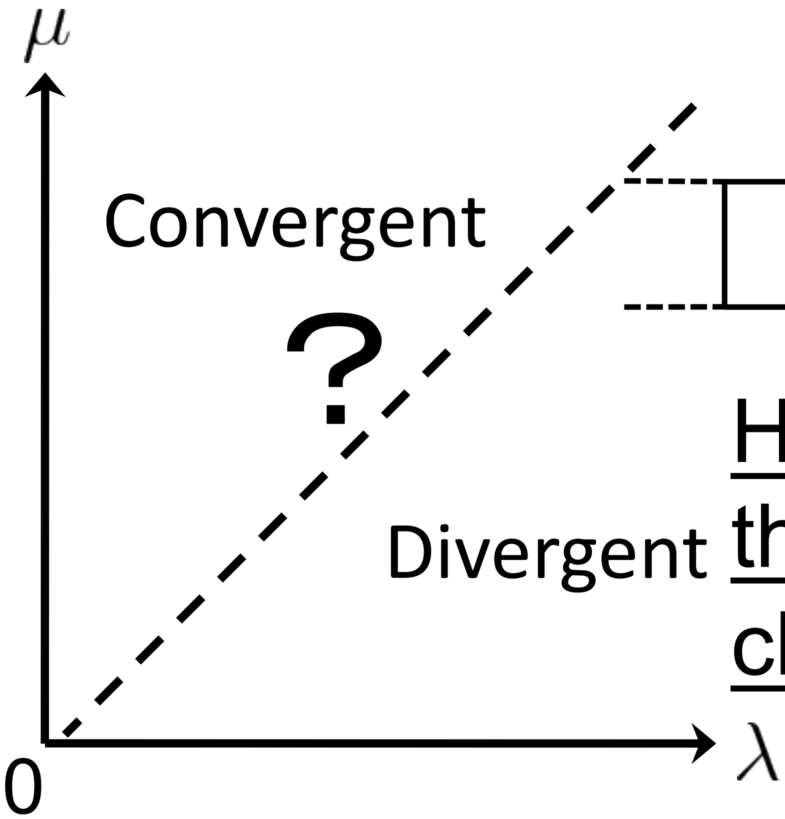
[1] CA, Physical Review E (2009)
[2] CA and D Yanagisawa, Journal of Statistical Physics (2010)
[3] CA and A Schadschneider, Physical Review E (2011)
[4] CA and A Schadschneider, Physical Review E (2011)
[5] CA and A Schadschneider, Journal of Statistical Mechanics (2012)

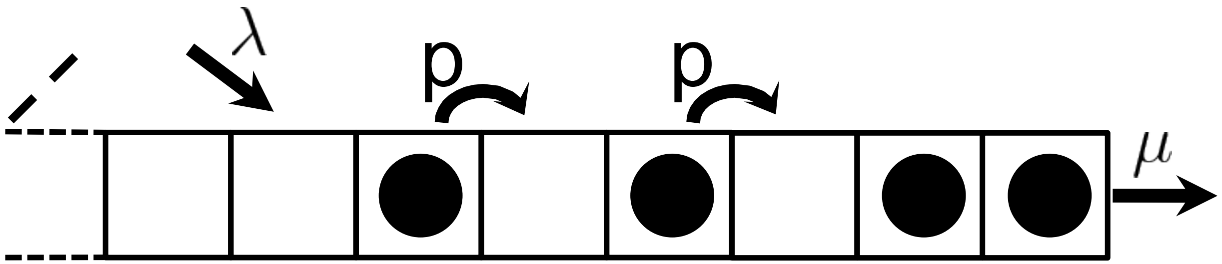
# Queueing process with excluded-volume effect



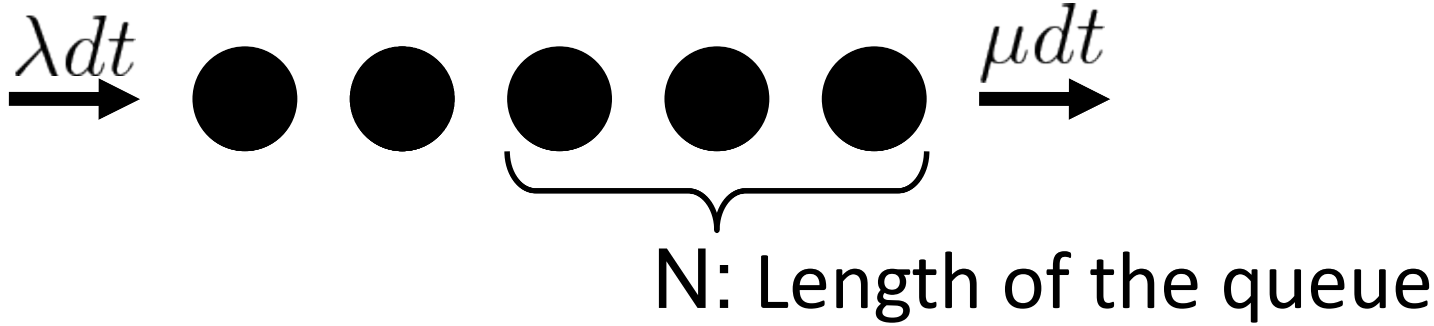
MM1 Queueing Process + Exclusion Process




<u>How does the excluded-volume effect change the phase diagram?</u>

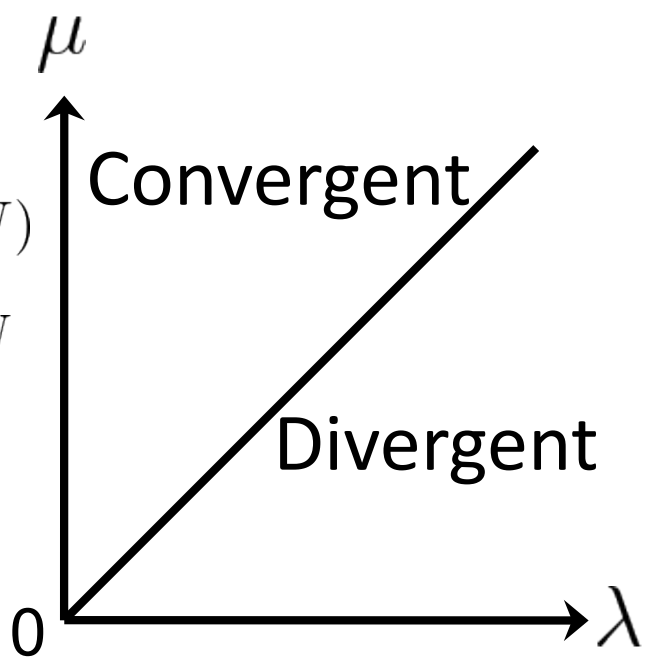
# MM1 Queueing Process




Master equation

$$\frac{d}{dt}P(0) = \mu P(1) - \lambda P(0)$$

$$\frac{d}{dt}P(N) = \lambda P(N-1) + \mu P(N+1) - (\lambda + \mu)P(N)$$



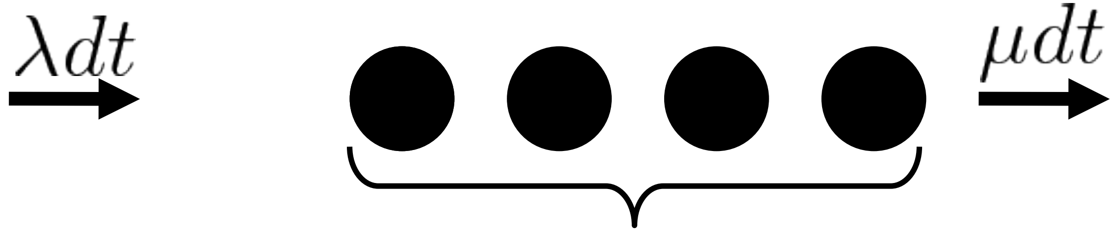

Stationary state
(the time independent
solution to the master eq.)

$$P(N) = \frac{1}{Z}\left(\frac{\lambda}{\mu}\right)^N$$

Normalization $$Z = \sum_{N \geq 0} \left(\frac{\lambda}{\mu}\right)^N = \frac{\mu}{\mu - \lambda}$$

When $\lambda < \mu$ , the stationary state actually exits.

# Queueing model for Pedestrians?

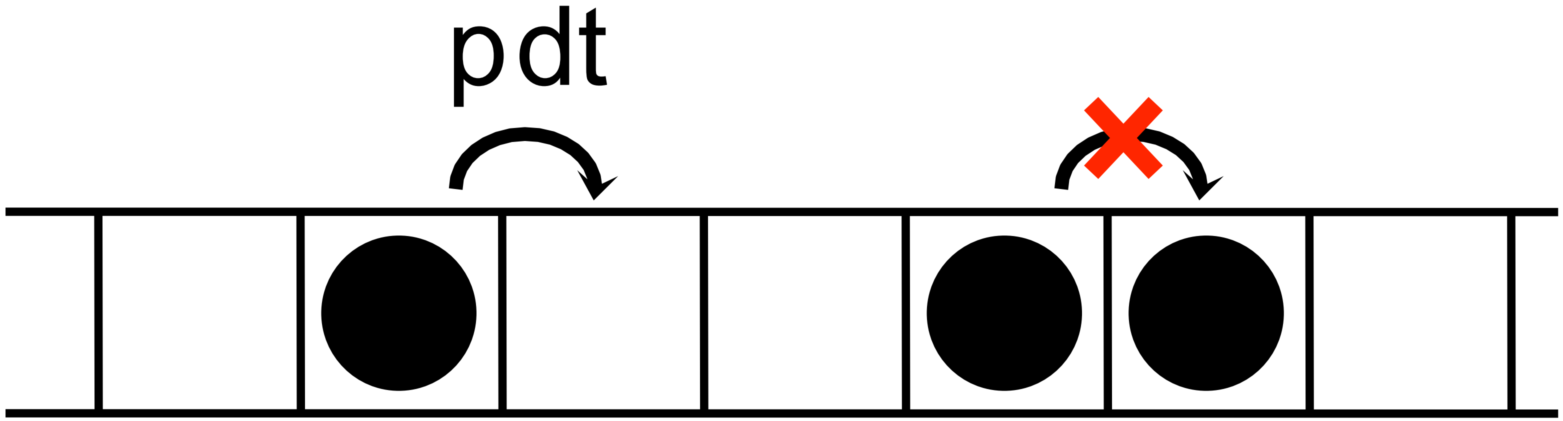



N: Length of the queue

In the queueing process,
all the customers are shifted simultaneously,
when the rightmost one gets the service.

But in our real life, we can proceed
if there is a space in front of us.
(Excluded-Volume Effect)

# Totally Asymmetric Simple Exclusion Process (TASEP)




A Typical Interacting Particle System

A Non-Equilibrium System

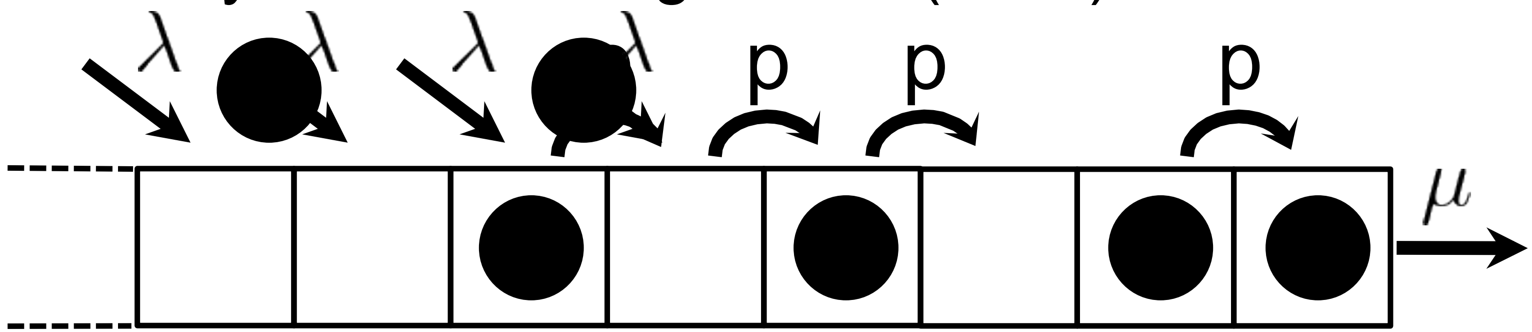
# “Exclusive Queueing Process” (EQP)

= Queueing process with exluded-volume effect

= TASEP with a new boudary condition

In semi-infinite chain, each customer

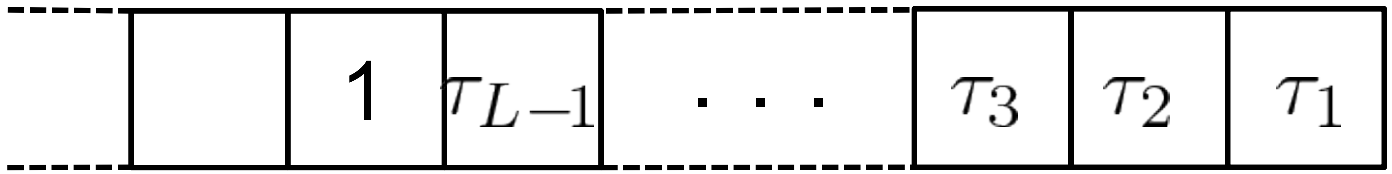
- enters the system at the left site next to the leftmost customer,
- hops to its right neighbor site if it is empty,
- leaves the system at the rightmost (fixed) site.




We consider not only the number of customers but also configurations (spatial structure).

EQP ! Usual Queueing process as “p!1”

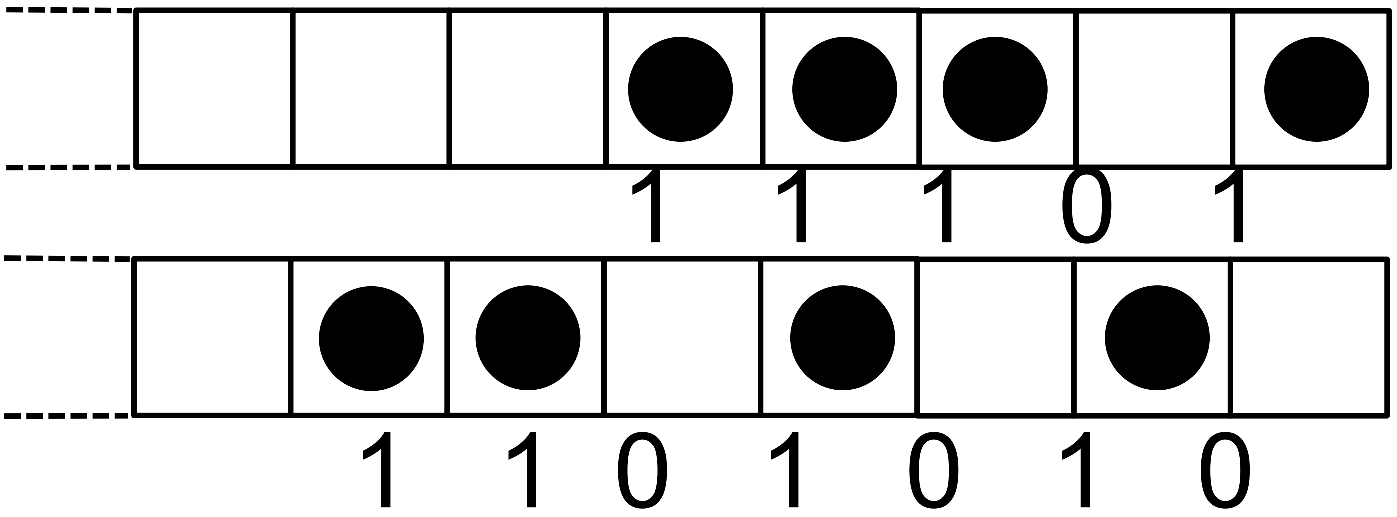
# State space of the EQP



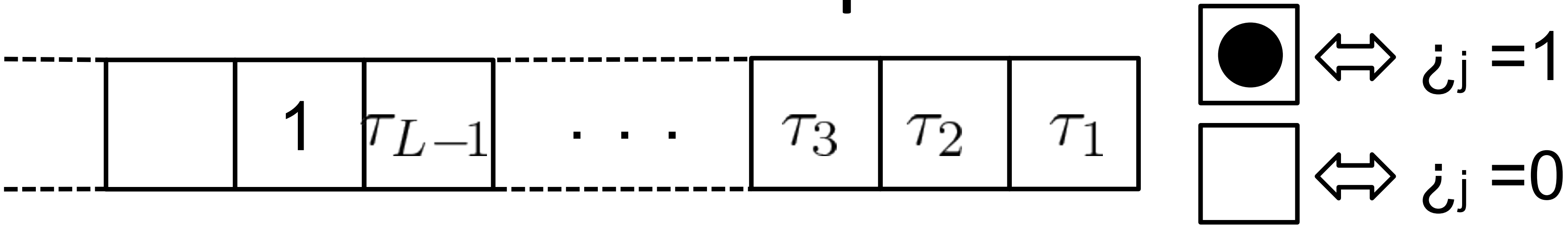


S= $\{\emptyset, 1, 10, 11, 100, 101, 110, 111, 1000 \ldots\}$

No customer (empty chain)



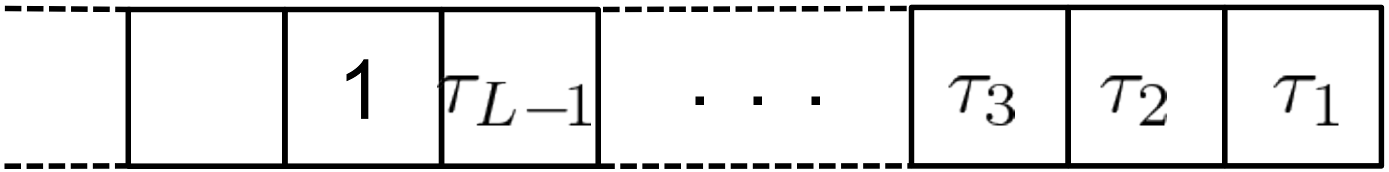
# Master equation




For a general configuration $\tau_L \tau_{L-1} \cdots \tau_2 \tau_1 \;\; (\tau_L = 1)$

$$
\begin{aligned}
&\frac{d}{dt} P(1\tau_{L-1}\cdots\tau_1) = \\
&pP(10\tau_{L-1}\cdots\tau_1) \\
&\quad + \lambda\tau_{L-1} P(1\tau_{L-2}\cdots\tau_1) - \lambda P(1\tau_{L-1}\cdots\tau_1) \\
&\quad + p(\tau_{L-1} - 1) P(10\tau_{L-2}\cdots\tau_1) \\
&\quad + p\sum_{j=1}^{L-2} (\tau_j - \tau_{j+1}) P(1\tau_{L-1}\cdots \overset{j+1}{1}\overset{j}{0}\cdots\tau_1) \\
&\quad + \lambda(1 - 2\tau_1) P(1\tau_{L-1}\cdots\tau_2 1)
\end{aligned}
$$

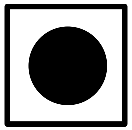
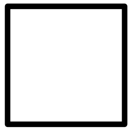
# Matrix Product Form




Probability of finding a configuration $1\tau_{L-1}\cdots\tau_2\tau_1$

$$P(1\tau_{L-1}\cdots\tau_1) = \frac{1}{Z} f(1\tau_{L-1}\cdots\tau_1)$$

$$= \frac{1}{Z}\frac{\lambda^L}{p^L}\langle W|X_{\tau_{L-1}}\cdots X_{\tau_1}|V\rangle$$

 ⇔ $X_1$  ⇔ $X_0$ :matrices

$\langle W|$ :row vector $|V\rangle$ :column vector

Algebra $\langle W|X_0 = \langle W|,\ X_1|V\rangle = \frac{p}{\mu}|V\rangle$

$$X_1X_0 = X_0 + X_1,\ \langle W|V\rangle = \frac{p}{\mu}$$

# Condition for convergence

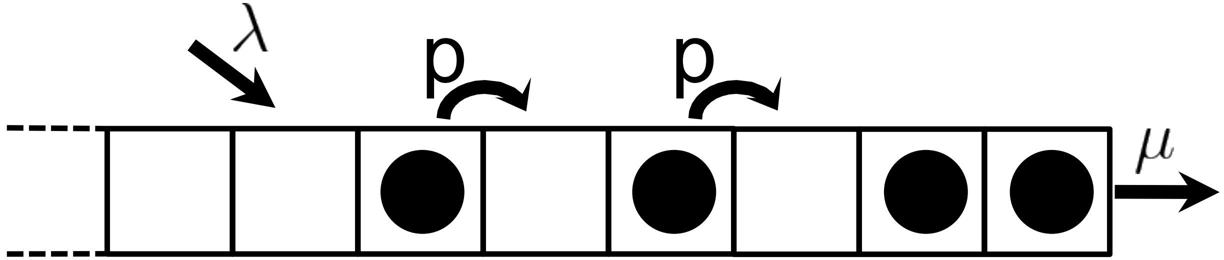
$$Z = 1 + \sum_{L \geq 1} \frac{\lambda^L}{p^L} \underbrace{\sum_{1 \leq j \leq L} \frac{j(2L-1-j)!}{L!(L-j)!} \frac{p^j}{\mu^j}}$$

$$\sim \begin{cases} \frac{1}{\sqrt{L^3}} 4^L & \mu > \frac{p}{2} \\ \frac{1}{\sqrt{L}} 4^L & \mu = \frac{p}{2} \\ \left( \frac{1}{(1-\mu/p)\mu/p} \right)^L & \mu < \frac{p}{2} \end{cases}$$

Thus, when $\lambda \begin{cases} \leq \frac{p}{4} & \mu > \frac{p}{2} \\ < \mu \left(1 - \frac{\mu}{p}\right) & \mu \leq \frac{p}{2} \end{cases}$

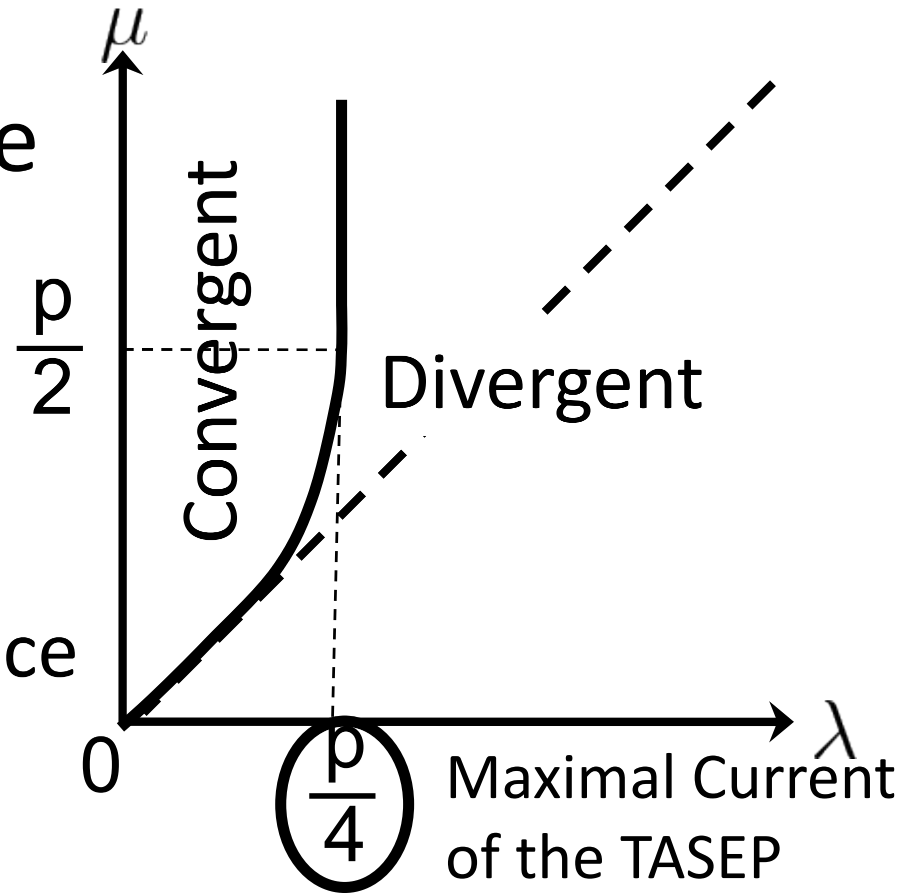
the stationary state exists.

# Phase Diagram




## Condition for convergence

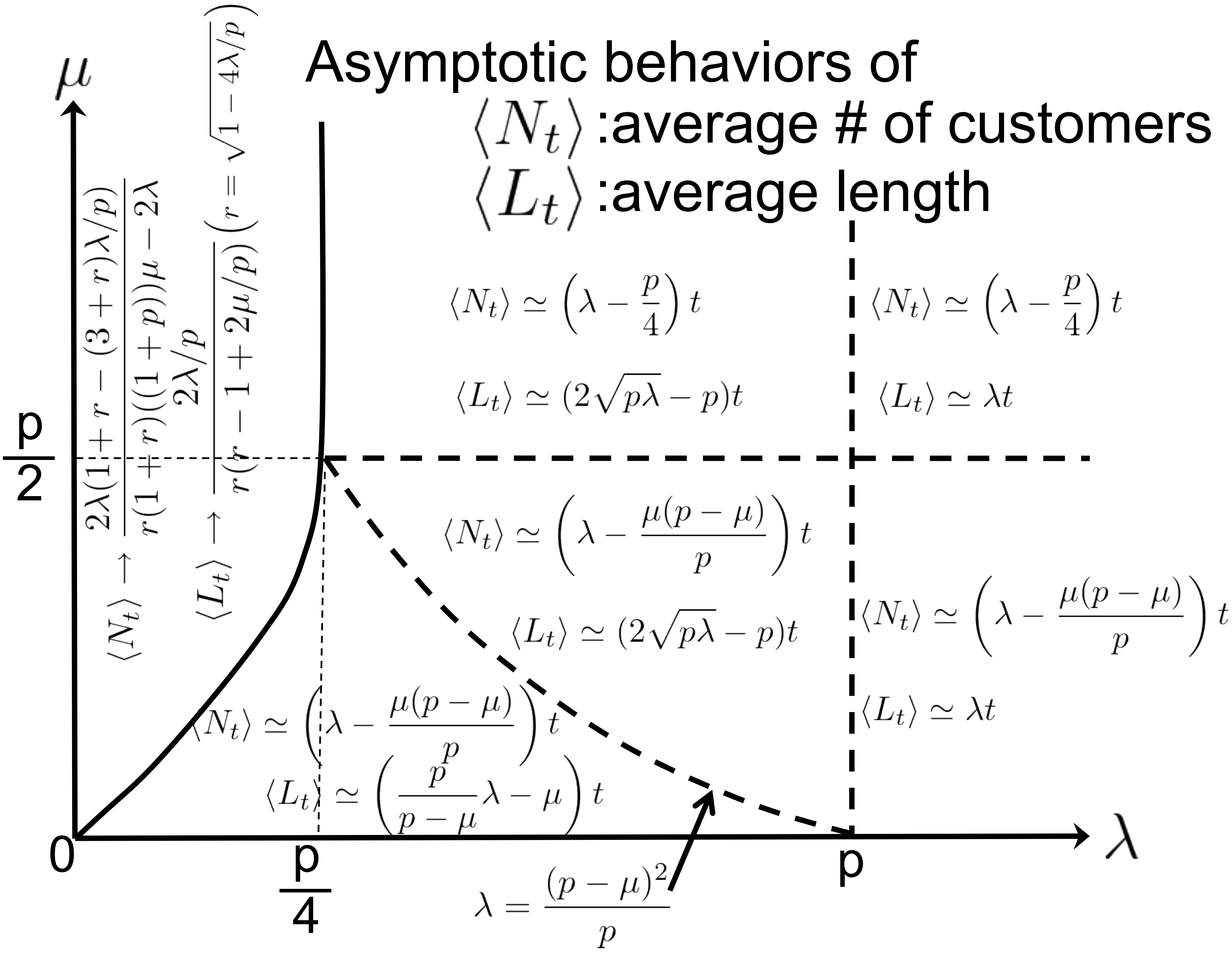
$$\lambda \begin{cases} \leq \frac{p}{4} & \mu > \frac{p}{2} \\ < \mu\left(1 - \frac{\mu}{p}\right) & \mu \leq \frac{p}{2} \end{cases}$$



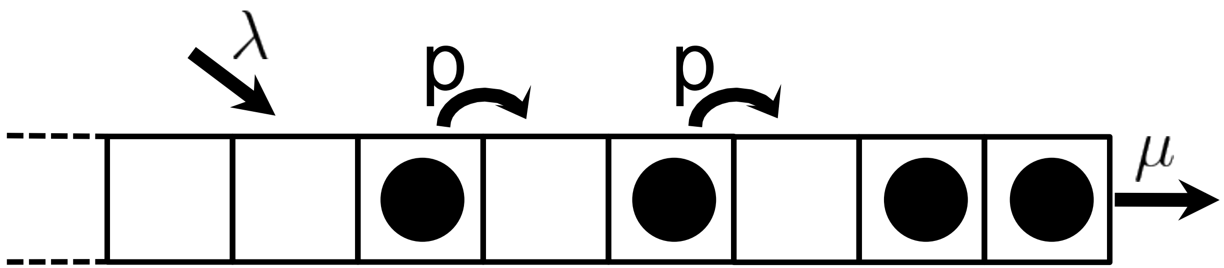

Recall condition for convergence in the MM1 case: $\lambda < \mu$

In the EQP, the queue itself is a bottleneck.

# Asymptotic behaviors of
$\langle N_t \rangle$ :average # of customers
$\langle L_t \rangle$ :average length



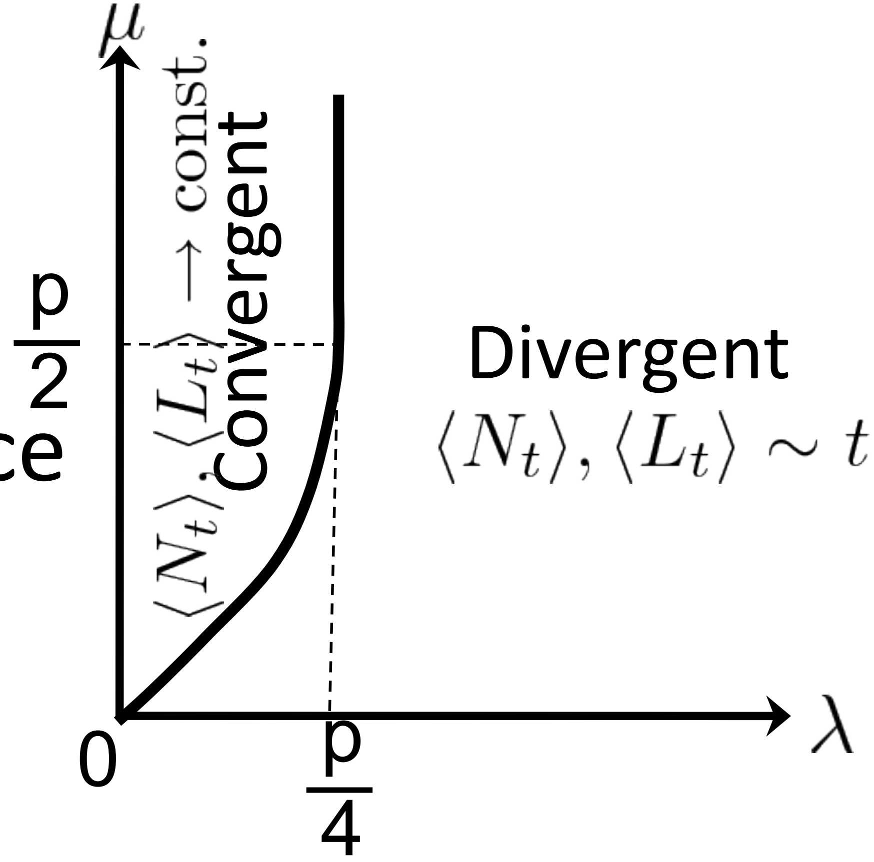
# Summary




- Matrix product stationary state

- Condition for convergence

$$\lambda < \begin{cases} \frac{p}{4} & (\mu > p/2) \\ \mu(1-\mu/p) & (\mu \le p/2) \end{cases}$$




THANK YOU