

# Two-Phase and Micro-Macro Descriptions of Traffic Flow

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# Two-Phase Description

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## The Classical LWR Model

$$\partial_t \rho + \partial_x (\rho V) = 0$$

$t$      $t \in [0, +\infty[$  time         $x$      $x \in \mathbb{R}$  space coordinate  
 $\rho$  traffic density                   $V$      $V = V(\rho)$  traffic speed

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$$\begin{cases} \partial_t \rho + \partial_x (\rho V) = 0 \\ V = w \psi(\rho) \end{cases} \quad \begin{array}{ll} \rho = \rho(t, x) & \text{traffic density} \\ V = V(w, \rho) & \text{traffic speed} \\ w = w(t, x) > 0 & \text{maximal traffic speed} \end{array}$$

$\psi \in C^2([0, R])$ ,  $\psi(0) = 1$ ,  $\psi(R) = 0$ ,  $\psi'(\rho) \leq 0$ , describes the attitude of drivers to choose their speed depending on the traffic density at their location

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$w$  is a specific feature of every single driver

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We assume that there exists an overall maximal speed  $V_{\max}$ :

$$\begin{cases} \partial_t \rho + \partial_x (\rho v(\rho, w)) = 0 \\ \partial_t w + v(\rho, w) \partial_x w = 0 \end{cases} \quad \text{with} \quad v = \min \{ V_{\max}, w \psi(\rho) \}$$

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Since  $\partial_t (\rho w) + \partial_x (\rho w v(\rho, w)) =$   
 $\rho (\partial_t w + v(\rho, w) \partial_x w) + w (\partial_t \rho + \partial_x (\rho v(\rho, w)))$  we obtain an equivalent form of the system:

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A  $2 \times 2$  system of conservation laws with a  $\mathbf{C}^{0,1}$  flow:

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S. Blandin, D. Work, P. Goatin, B. Piccoli, A. Bayen: SIAM J.Appl.Math. 2011  
R.M.Colombo, F.Marcellini, M.Rascle: SIAM J.Appl.Math. 2010

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With  $\eta = \rho w$ :

$$\begin{cases} \partial_t \rho + \partial_x (\rho v(\rho, w)) = 0 \\ \partial_t \eta + \partial_x (\eta v(\rho, \eta)) = 0 \end{cases} \quad \text{with} \quad v = \min \left\{ V_{\max}, \frac{\eta}{\rho} \psi(\rho) \right\}$$

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congested phase	$C = \{v = w \psi(\rho)\}$	low speed	high density

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low density

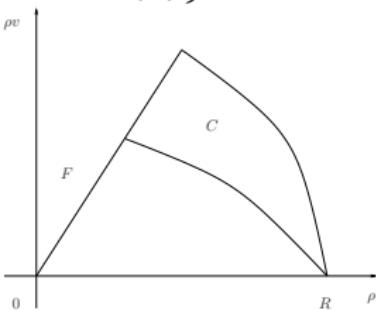
congested phase

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low speed

high density

The Fundamental Diagram



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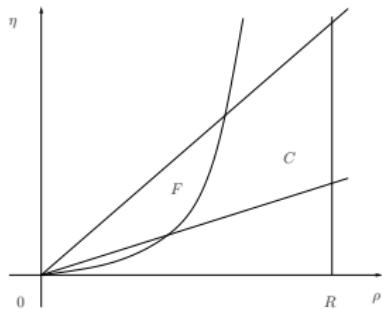
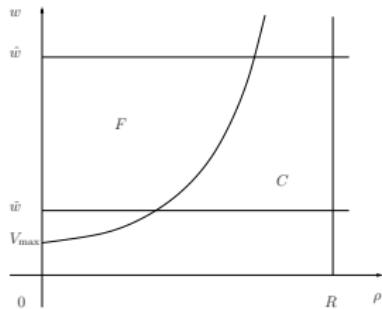
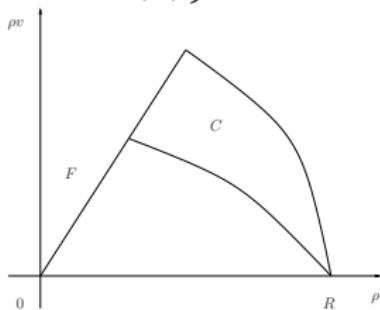
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The Fundamental Diagram



## Two-Phase Description – The Riemann Problem

For all states  $(\rho^l, \eta^l), (\rho^r, \eta^r) \in F \cup C$ , the Riemann problem of

$$\begin{cases} \partial_t \rho + \partial_x (\rho v(\rho, \eta)) = 0 \\ \partial_t \eta + \partial_x (\eta v(\rho, \eta)) = 0 \end{cases} \quad \text{with} \quad v(\rho, \eta) = \min \left\{ V_{\max}, \frac{\eta}{\rho} \psi(\rho) \right\}$$

with initial data

$$\rho(0, x) = \begin{cases} \rho^l & \text{if } x < 0 \\ \rho^r & \text{if } x > 0 \end{cases} \quad \eta(0, x) = \begin{cases} \eta^l & \text{if } x < 0 \\ \eta^r & \text{if } x > 0 \end{cases}$$

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Theorem (R.M.Colombo, F.Marcellini, M.Rascle: SIAM J.Appl.Math. 2010)

*The Riemann problem admits a unique self similar weak entropy solution  $(\rho, \eta)$*

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If  $(\rho^l, \eta^l), (\rho^r, \eta^r) \in F$  then  $\begin{cases} \partial_t \rho + \partial_x (\rho V_{\max}) = 0 \\ \partial_t \eta + \partial_x (\eta V_{\max}) = 0 \end{cases}$

(standard situation, single characteristic speed)

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If  $(\rho^l, \eta^l), (\rho^r, \eta^r) \in C$  then

$$\begin{cases} \partial_t \rho + \partial_x (\eta \psi(\rho)) = 0 \\ \partial_t \eta + \partial_x \left( \frac{\eta^2}{\rho} \psi(\rho) \right) = 0 \end{cases}$$

$$(\lambda_1(\rho, \eta) = \eta \psi'(\rho) + v(\rho, \eta) \text{ and } \lambda_2(\rho, \eta) = v(\rho, \eta))$$

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$$\left. \begin{array}{ll} \text{If } & (\rho^l, \eta^l) \in F \quad (\rho^r, \eta^r) \in C \\ & (\rho^l, \eta^l) \in C \quad (\rho^r, \eta^r) \in F \end{array} \right\} \Rightarrow \boxed{\text{Phase Transitions}}$$

## Follow-The-Leader Model

A single driver starting from  $p_o$  at  $t = 0$  follows the **particle path**  $p = p(t)$  that solves

$$\begin{cases} \dot{p} = v(\rho(t, p(t)), w((t, p(t))) \\ p(0) = p_o \end{cases} \quad v(\rho, w) = \min \{ V_{\max}, w \psi(\rho) \},$$

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$n$  drivers,  $l$  standard car length

$$\Rightarrow \rho(t, x) \simeq \sum_i \frac{l}{p_{i+1}(t) - p_i(t)} \chi_{[p_i(t), p_{i+1}(t)]}(x)$$

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the **follow-the-leader** model  $\begin{cases} \dot{p}_i = v\left(\frac{l}{p_{i+1} - p_i}, w_i\right) \\ \dot{p}_{n+1} = V_{\max} \\ p_i(0) = p_{o,i} \end{cases}$

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then  $\rho$  solves the macroscopic model.

# Follow-The-Leader Model

Scheme

$$(\tilde{\rho}, \tilde{w})$$



$$(\tilde{p}_i^n, \tilde{w}_i^n)$$

$$(\rho(t), w(t))$$



$$(p_i^n(t), w_i^n(t))$$

# Follow-The-Leader Model

Scheme

$$(\tilde{\rho}, \tilde{w})$$

$$(\rho(t), w(t))$$

discretize



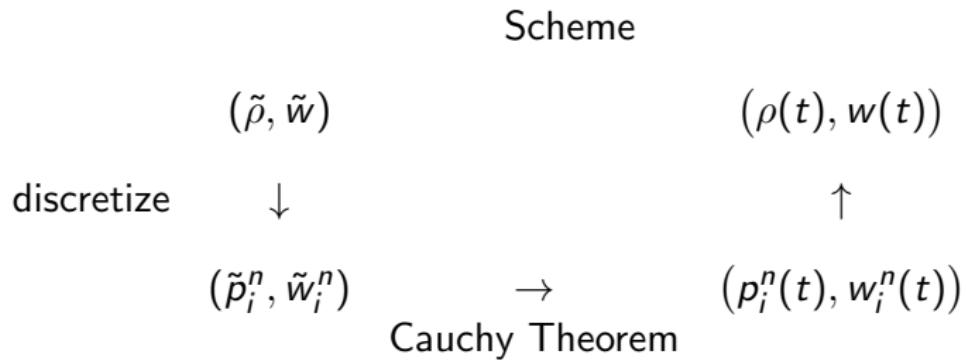
$\uparrow$

$$(\tilde{p}_i^n, \tilde{w}_i^n)$$

$\rightarrow$

$$(p_i^n(t), w_i^n(t))$$

# Follow-The-Leader Model



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↑

$n \rightarrow +\infty$

$$(\tilde{p}_i^n, \tilde{w}_i^n)$$



$$(p_i^n(t), w_i^n(t))$$

Cauchy Theorem

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$$(\tilde{\rho}, \tilde{w})$$

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Cauchy Theorem

## Traffic Modeling – Other Macroscopic Models

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Aw, Rascle: SIAM J.Appl. Math., 2000  
Zhang: Transportation Research, 2002

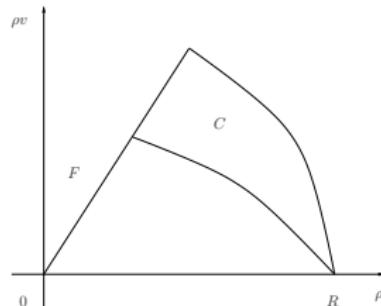
$$\begin{cases} \partial_t \rho + \partial_x [\rho v(\rho, y)] = 0 \\ \partial_t y + \partial_x [y v(\rho, y)] = 0 \end{cases}$$

$$v(\rho, y) = \frac{y}{\rho} - p(\rho)$$

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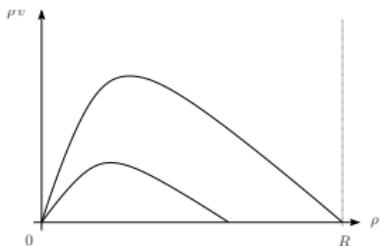
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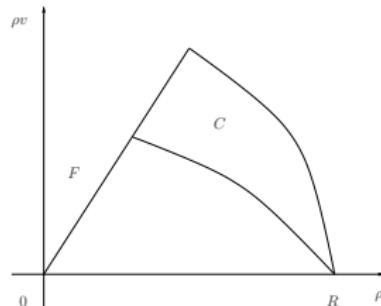
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Colombo: SIAM J.Appl. Math., 2002

Free flow:  $(\rho, q) \in F$ ,

$$\partial_t \rho + \partial_x [\rho \cdot v_F(\rho)] = 0,$$

$$v_F(\rho) = (1 - \frac{\rho}{R}) \cdot V$$

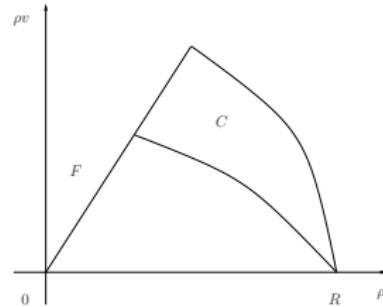
Congested flow:  $(\rho, q) \in C$ ,

$$\begin{cases} \partial_t \rho + \partial_x [\rho \cdot v_C(\rho, q)] = 0 \\ \partial_t q + \partial_x [(q - q_*) \cdot v_C(\rho, q)] = 0 \end{cases}$$
$$v_C(\rho, q) = (1 - \frac{\rho}{R}) \cdot \frac{q}{\rho}$$

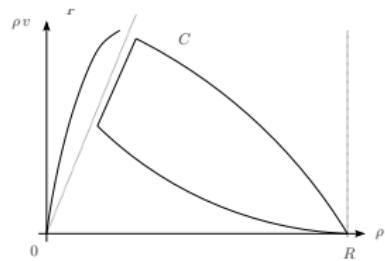
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Colombo: SIAM J.Appl. Math., 2002



# A Kinetic Model

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Benzoni-Gavage, Colombo, Gwiazda: Proc. Royal Soc. London, 2006

$$\partial_t r(t, x; w) + \partial_x \left[ w r(t, x; w) \psi \left( \int_{\check{w}}^{\hat{w}} r(t, x; w') dw' \right) \right] = 0.$$

The unknown  $r = r(t, x, w)$  is the probability density of vehicles having maximal speed  $w$  that at time  $t$  are at point  $x$

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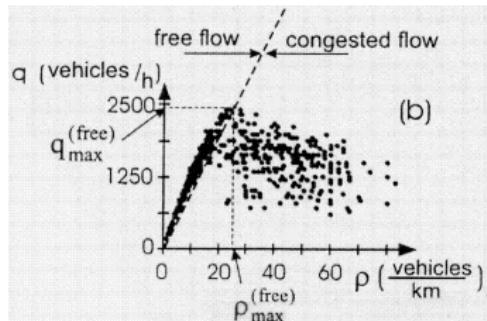
$$\partial_t r(t, x; w) + \partial_x \left[ w r(t, x; w) \psi \left( \int_{\check{w}}^{\hat{w}} r(t, x; w') dw' \right) \right] = 0.$$

If the measure  $r$  solves the kinetic model and is such that

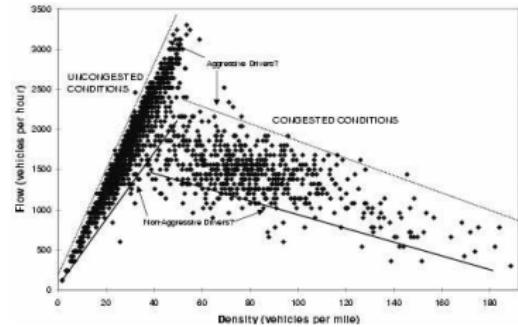
$$r(t, x; \cdot) = \rho(t, x) \delta_{w(t, x)}$$

then  $(\rho, w)$  solves the 2-phase model

# Experimental Fundamental Diagrams

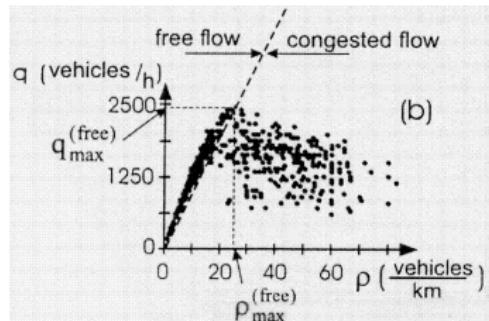


Kerner  
Traffic and Granular Flow  
Springer Verlag, 2000

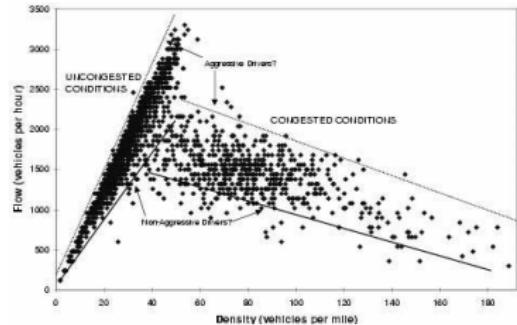


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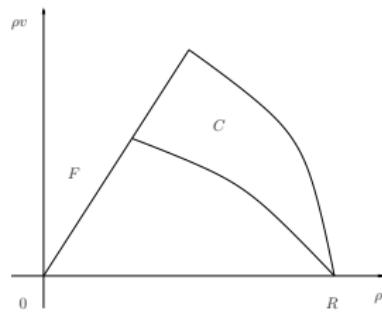
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Fundamental Diagram by  
R.M. Colombo, F. Marcellini, M.Rascle  
SIAM J.Appl. Math. 2010

# Micro-Macro Description

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The Macroscopic Part: LWR Model

$$\partial_t \rho + \partial_x (\rho V) = 0$$

$t$   $t \in [0, +\infty[$  time       $x$   $x \in \mathbb{R}$  space coordinate  
 $\rho$  traffic density       $V$   $V = V(\rho)$  traffic speed

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$$\begin{array}{ll} t & t \in [0, +\infty[ \text{ time} \\ \rho & \text{traffic density} \end{array} \quad \begin{array}{ll} x & x \in \mathbb{R} \text{ space coordinate} \\ V & V = V(\rho) \text{ traffic speed} \end{array}$$

The Microscopic Part: Follow-the-Leader Model

$$\dot{p}_i = v \left( \frac{l}{p_{i+1} - p_i} \right)$$

$p_i = p_i(t)$  is the position of the  $i$ -th driver, for  $i = 1, \dots, n$   
 $l$  is the vehicles' lenght and

$$p_{i+1} - p_i \geq l$$

# Micro-Macro Description

## The Case LWR-FtL

$$\begin{cases} \partial_t \rho + \partial_x (\rho v(\rho)) = 0 & t \in \mathbb{R}^+ \text{ and } x < p_1(t) \\ \dot{p}_i = v \left( \frac{\ell}{p_{i+1} - p_i} \right) & t \in \mathbb{R}^+ \text{ and } i = 1, \dots, n-1 \\ \dot{p}_n = w(t) & t \in \mathbb{R}^+ \\ \rho(0, x) = \bar{\rho}(x) & x \leq \bar{p}_1 \\ p(0) = \bar{p} \end{cases}$$

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Theorem (R.M. Colombo, F. Marcellini: Preprint 2013)

*The Problem admits a unique solution*

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- ▶ C. Lattanzio, B. Piccoli: Math. Models Methods Appl. Sci., 2010

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## The Case FtL-LWR

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LWR and FtL alternated along the real line

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Well Posedness

# Micro-Macro Description

## The Case LWR-FtL

