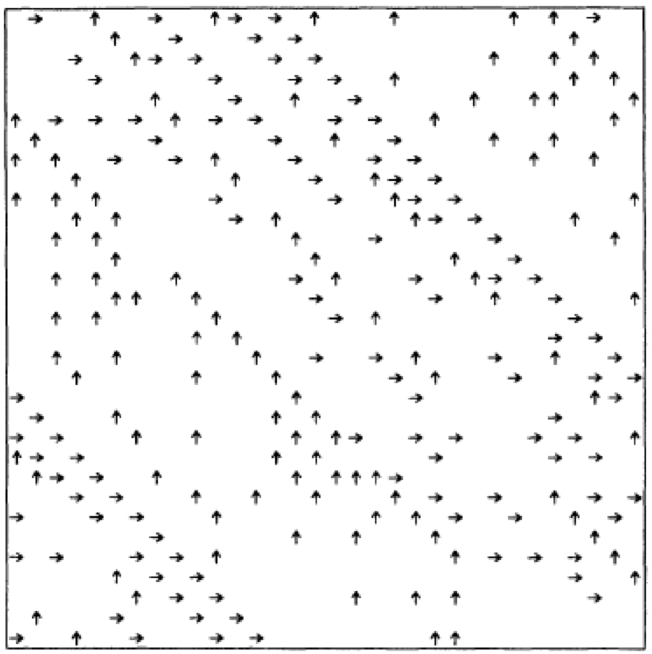
# Pattern formation in 2D traffic flows

J. Cividini, C. Appert-Rolland and H.J.Hilhorst

Laboratoire de Physique Théorique (Orsay)

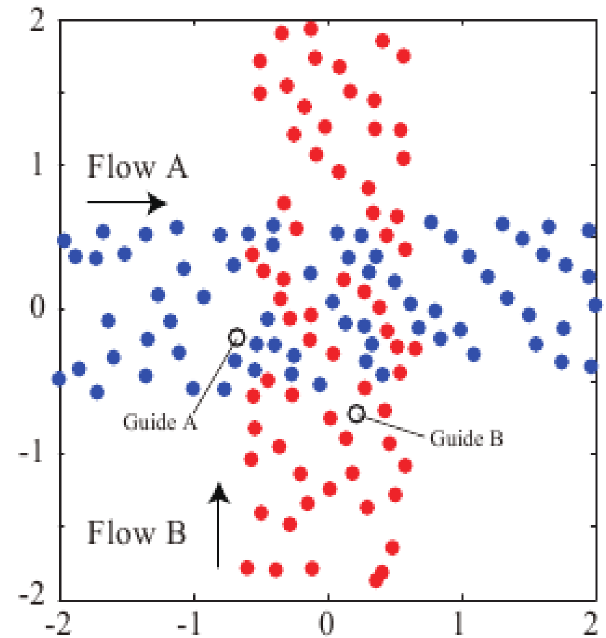
# Traffic flow in two dimensions

Observation of diagonal patterns



Road traffic

(Biham Middleton Levine, Phys. Rev. A, 46 (1992) R6124-R6127)




Model of pedestrians

(Yamamoto Okada, 2011 IEEE International conference on robotics and automation)

Simple explanation for this? Influence of the boundaries?

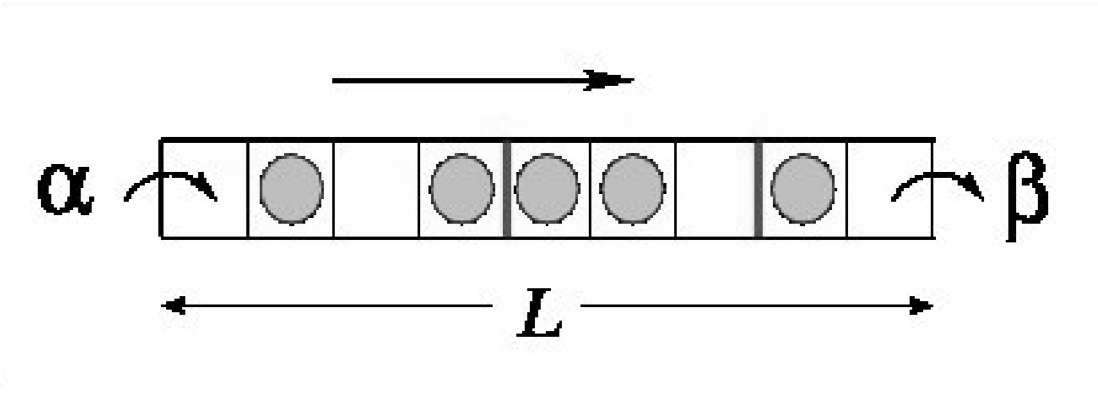
# The TASEP

Totally Asymmetric Simple Exclusion Process:

- 1D lattice

- Particles can hop from their current site to its right neighbor (TA)

- Interaction between particles: maximum one particle per site (SE)




Freedom in the choice of the order in which the particles are enabled to hop:

Parallel update

Frozen shuffle update: particles are updated once per time step in a fixed randomly determined order (J. Stat. Mech. (2011) P10013)

# The model

A street = M neighboring TASEP lanes with free exit and deterministic hops

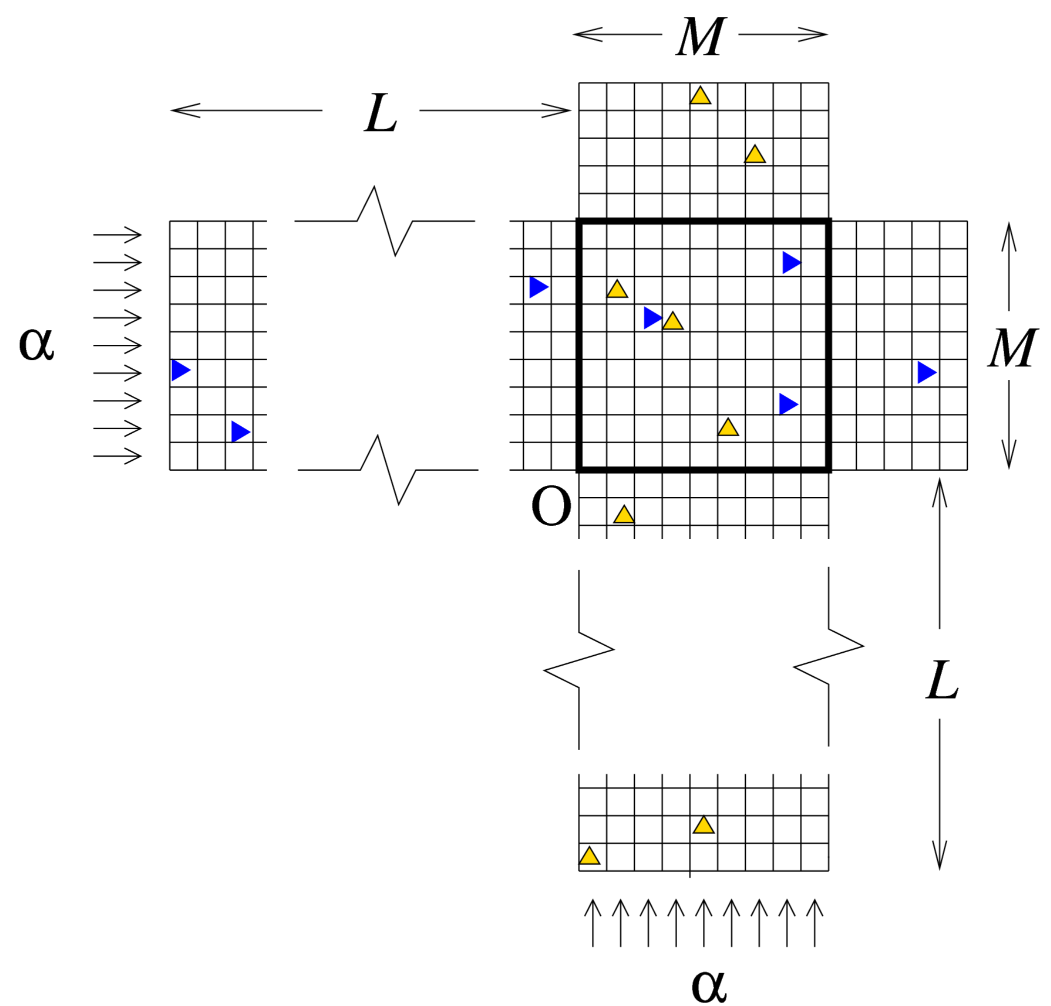
The intersection square = Intersection of two streets

Each particle of the intersection has a preferred direction of motion:  E (east) or N (north)

Simple exclusion

→ Simple 2-parameter out-of-equilibrium model (for infinite L)

We stay at low density (< jamming transition)




arXiv:1209.1529

# Mean field equations

In the exact equations a particle is blocked if there is an E or a N particle on its target site → The density is coupled to higher-order correlations.

Mean-field approximation:

-EN and NE correlations are factorized

-EE and NN collisions are neglected (for low particle densities)

→ Nonlinear 2D equations for the densities in the intersection

$$\begin{aligned}
\rho_{t+1}^{\mathcal{E}}(\mathbf{r}) &= [1-\rho_t^{\mathcal{N}}(\mathbf{r})]\rho_t^{\mathcal{E}}(\mathbf{r}-\boldsymbol{e}_x) + \rho_t^{\mathcal{N}}(\mathbf{r}+\boldsymbol{e}_x)\rho_t^{\mathcal{E}}(\mathbf{r}) \\
\rho_{t+1}^{\mathcal{N}}(\mathbf{r}) &= [1-\rho_t^{\mathcal{E}}(\mathbf{r})]\rho_t^{\mathcal{N}}(\mathbf{r}-\boldsymbol{e}_y) + \rho_t^{\mathcal{E}}(\mathbf{r}+\boldsymbol{e}_y)\rho_t^{\mathcal{N}}(\mathbf{r})
\end{aligned}$$

Complemented with boundary conditions

A uniform density solves the equations in PBC

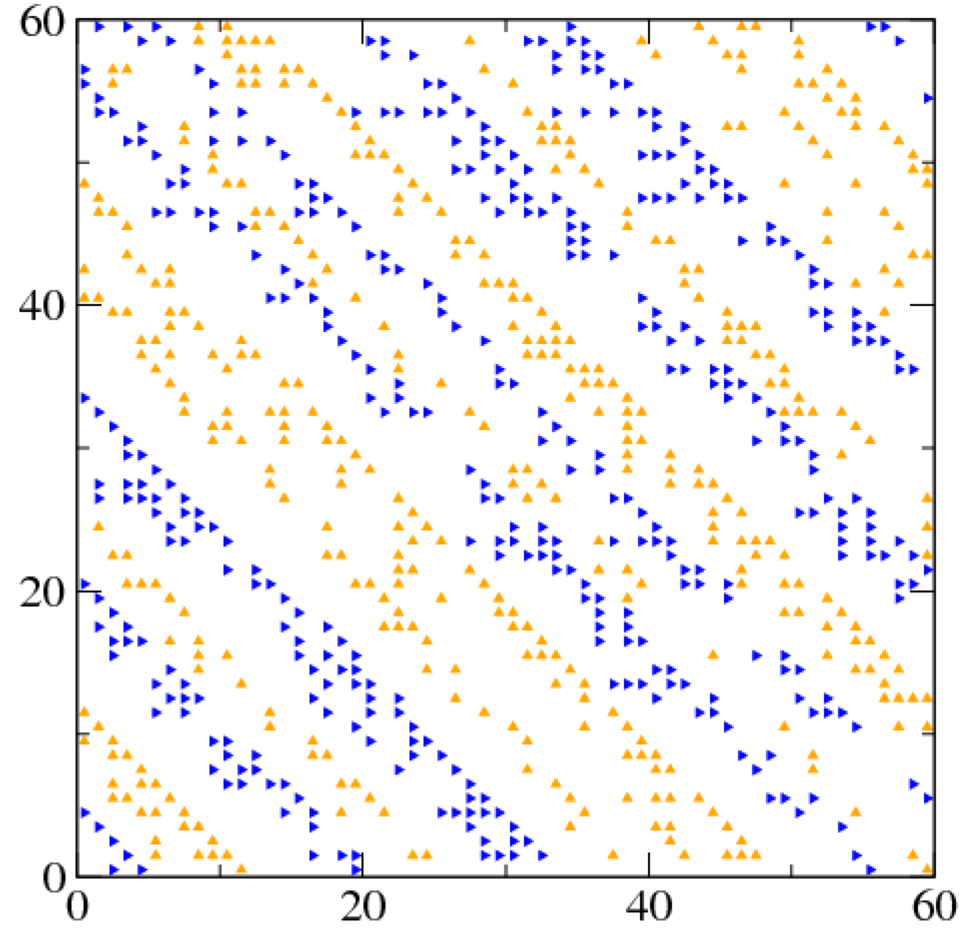
# Periodic boundary conditions

Easier problem (translational invariance)

Diagonal pattern : alignments of particles of the same type with an angle of 45° (direction(1,-1))

Already observed in BML-type models

Linear stability analysis → Maximum instability for $\lambda \approx 6$ in the (1,1) direction



# Open boundary conditions: observations

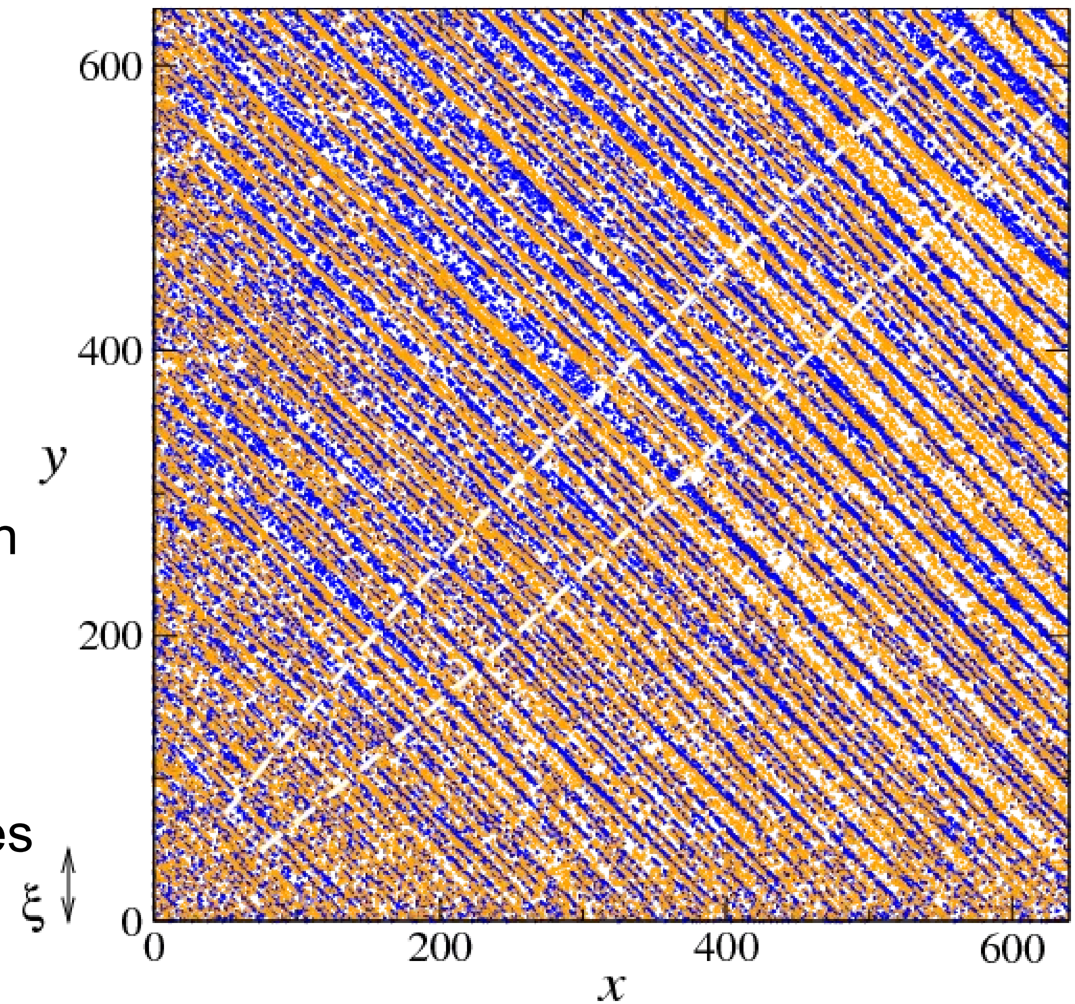
No more translational invariance

Alignments of particles still exist

Penetration length at the entrance boundaries

Asymmetry with respect to the main diagonal (difference of color between the upper and the lower triangle)

Division into two symmetric triangles



# Open boundary conditions: zoom

Upper triangle:

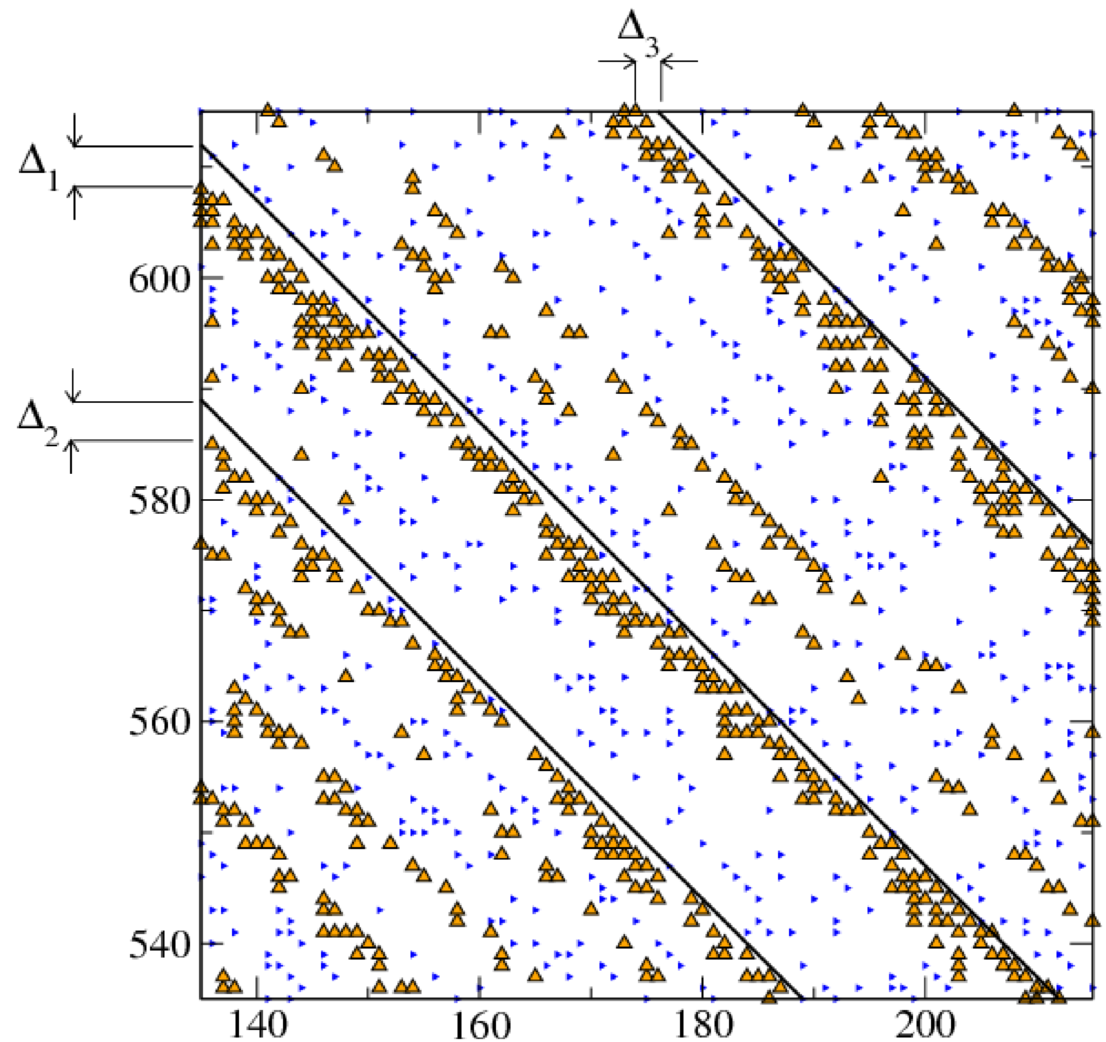
- Dense, well-organized alignments of N particles

- Sparse, apparently randomly distributed E particles

- Angle of the alignments = 45° + $\Delta\theta$

- $\Delta\theta \approx 1°$

Same phenomenon in the numerical resolution of the mean-field equations



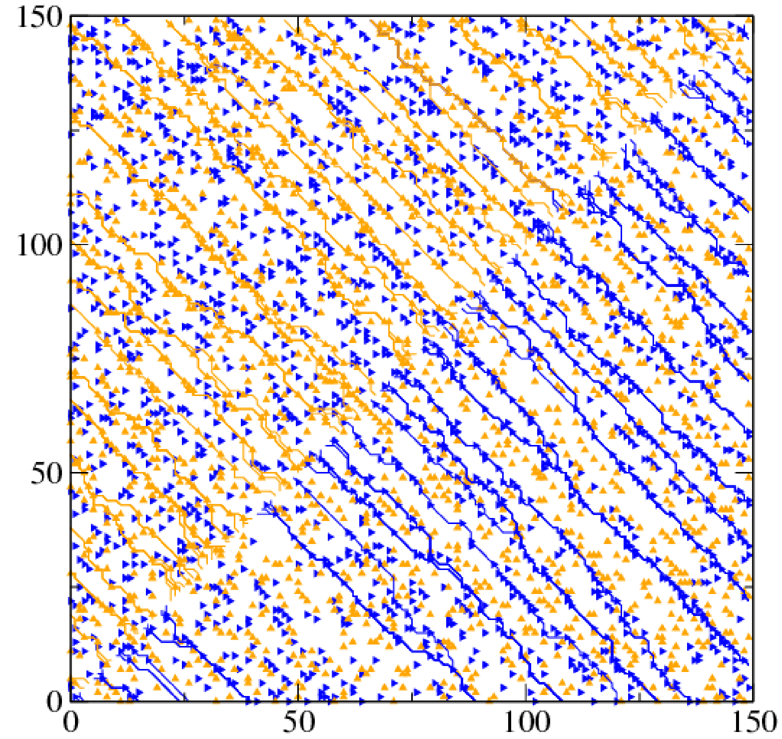
# How to measure $\Delta\theta$?

Crests algorithm:

Follow the crest of the organized type of particles.

Measure the angle between the ends of the crest.

Measure of $\Delta\theta$ based on short-range correlations

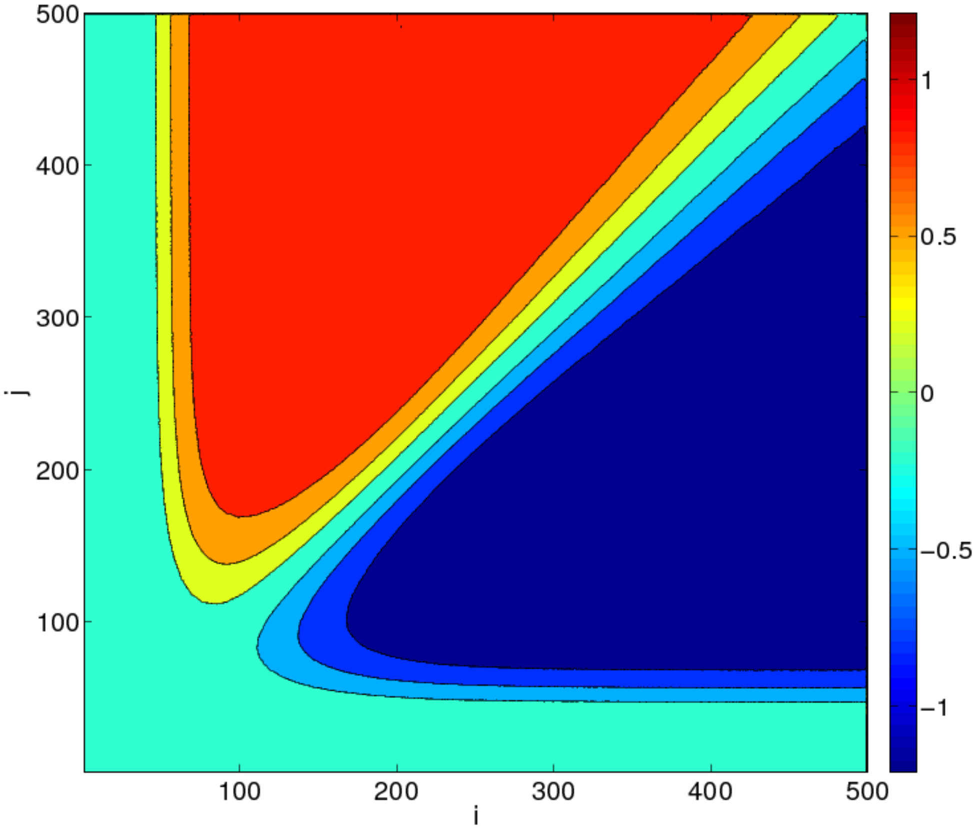


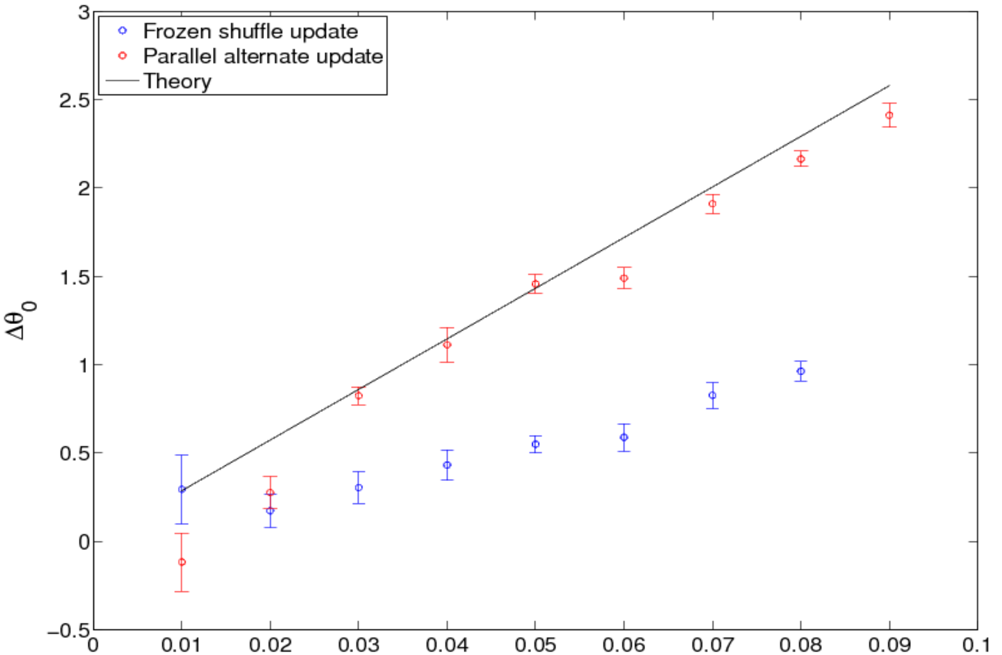

Simple hypothesis: impenetrability of the alignments

$$\tan\theta(\mathbf{r}) = v^{\mathcal{E}}(\mathbf{r})/v^{\mathcal{N}}(\mathbf{r})$$

Good agreement between the two methods.

# Measurements of $\Delta\theta$




Existence of two opposite plateau values $\pm\,\Delta\theta_0(\alpha)$

$\Delta\theta_0(\alpha) \approx c\alpha$ ,c depends on the particular features of the system (updating scheme, cellular automaton/equations...)

Theoretical prediction?

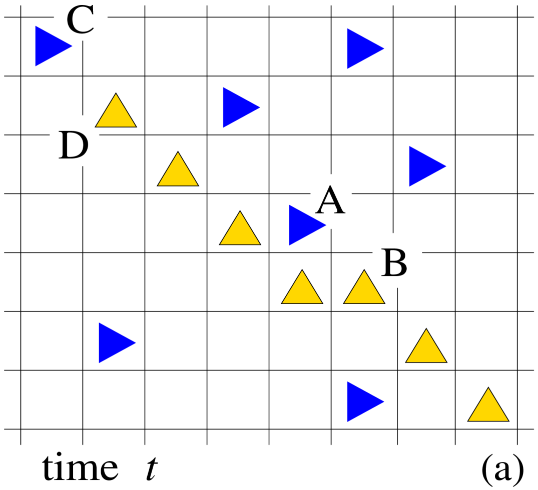
# Chevron mechanism

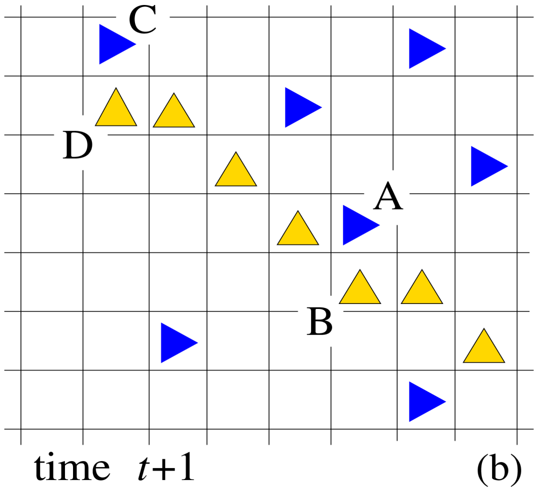
Limiting case:

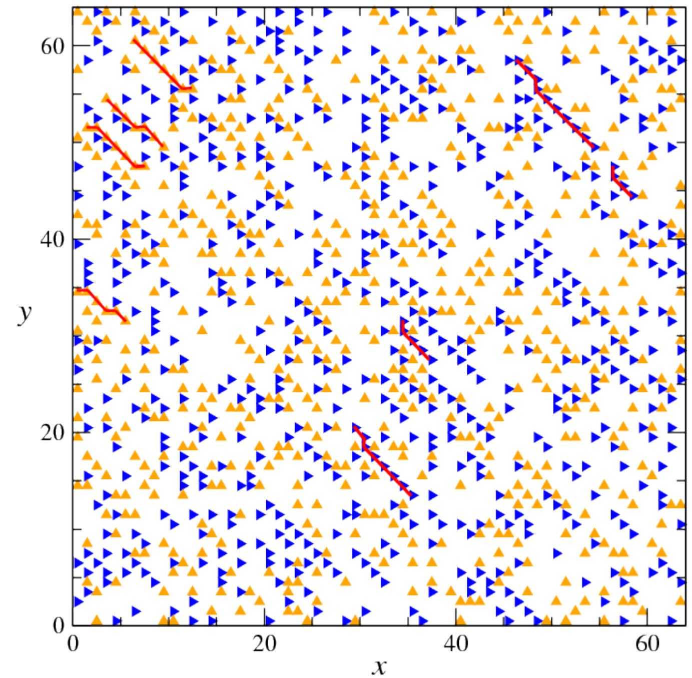
- N particles all organized in a strip composed of diagonal sections and kinks

- Kinks are linked to the presence of an E particle blocking an N particle at each time step

- Alternating parallel update



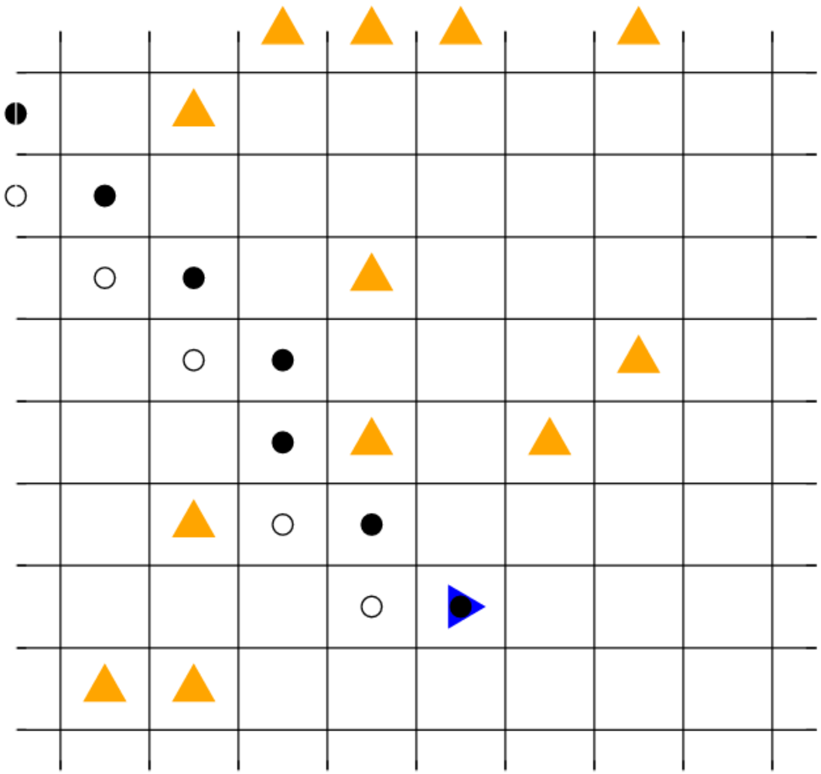



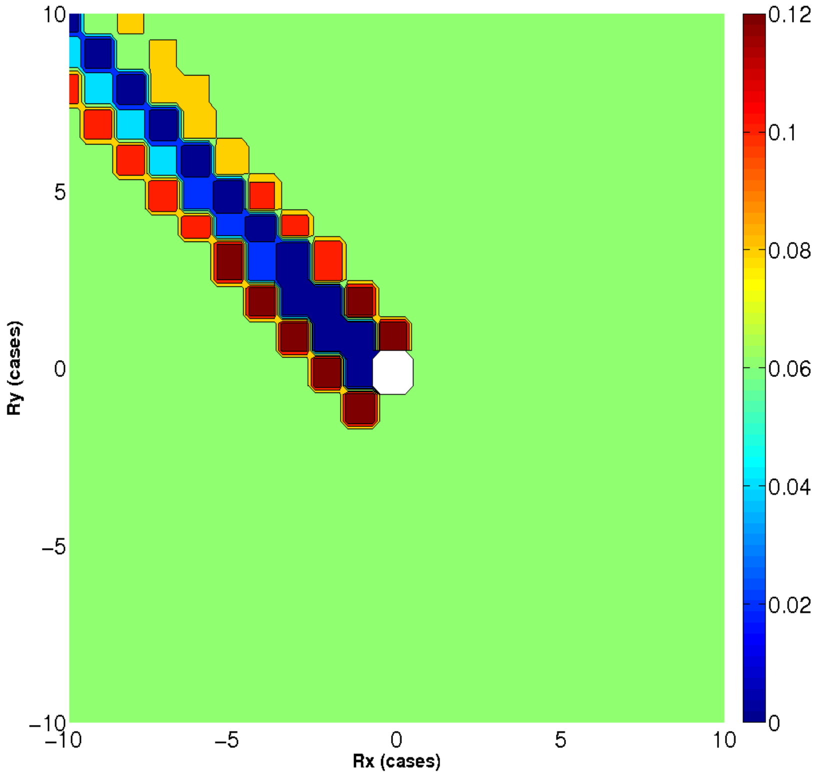

The impenetrability condition gives: $\tan\theta = (1 - \rho^{\mathcal{E}})^{-1}$

# Wake of a particle

Perturbation of the density of N particles created by a single E particle



For a given realization
Dots = empty sites




Ensemble averaged
density perturbation

→ The wake stabilizes a second E particle

# Conclusion and perspectives

-Diagonal pattern formation and chevron effect observed for various models (different updates, mean field equations) → Robustness of the effect

-Inclination of the stripes because of random open boundaries

-Theory for the plateau values in a simple but relevant limiting case

-What happens in the infinite M limit?

-Lane changes? Turning cars?

Thank you