How Can Macroscopic Models Reveal Self-Organization in Traffic Flow?

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Traffic Modeling and Management: Trends and Perspectives
VI Workshop on Mathematical Foundations of Traffic
Sophia Antipolis, France

March 20–22, 2013

Notations

We are interested in modeling N agents moving in a domain Ω.

Their positions are denoted by

$$X = \big(X^1(t), \dots, X^N(t)\big) \in \Omega^N, \quad t > 0$$

and their average density by

$$\rho(x,t), \quad x \in \Omega, \; t > 0$$

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...a microscopic model...

$$\dot{X}^i = v_m[X], \; i = 1, \ldots, N, \qquad v_m[X](X^i) = v_{\text{des}} + \sum_j F(X^i; X^j)$$

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we can couple them by means of the velocity field

$$v_{mM}[X, \rho(t)](x) = \theta v_m[X](x) + (1 - \theta)\Lambda v_M[\rho(\cdot, t)](x), \quad \theta \in [0, 1]$$

Then, both models evolve driven by v_{mM}

Mathematical justification [CPT2011][1]

The system is described by one equation for a time-evolving measure μ_t,

$$\mu_t(E) = \text{mass of observed material in } E, \quad \forall E \in \mathbb{R}^d, \quad \forall t > 0$$

Equation for μ_t (conservation law for the mass)

$$\frac{\partial \mu_t}{\partial t} + \nabla \cdot (\mu_t \; v[\mu_t]) = 0, \qquad v[\mu_t](x) = v_{\text{des}} + \int_\Omega F(x;y) d\mu_t(y)$$

where

$$\mu_t = \sum_{i=1}^{N} \delta_{X^i(t)}$$

[1] E. Cristiani, B. Piccoli, A. Tosin, Multiscale modeling of granular flows with application to crowd dynamics, Multiscale Model. Simul., 9 (2011), 155–182.

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$$\mu_t = \qquad \Lambda \rho(\cdot, t)\mathcal{L}^d$$

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Both models are used

Both scales are observed (θ tunes the contributions of the scales)

Both discrete positions X^i and average density ρ are tracked

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Catch at large scale complex phenomena caused by granularity visible at small scale.
Take into account differences among individuals and random fluctuations in an averaged context.

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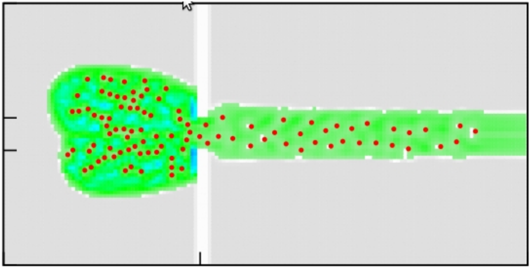
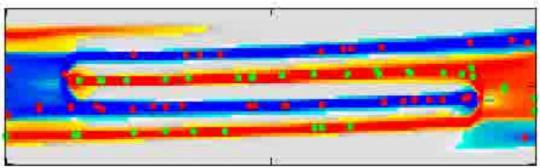
Drawbacks

A more complex algorithm and an higher CPU time.

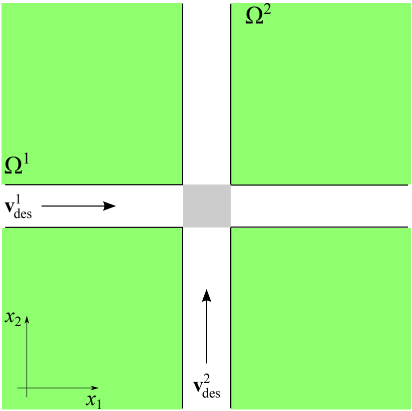
An application to pedestrian flow





Setting: 2 roads + 1 junction

No traffic light, no priorities, no rules (Roman style)



Toward vehicular traffic modeling

2D dynamics

$$v_{\text{des}} = \text{constant speed in the road direction}$$

$$F(x;y) = \frac{-C}{|y-x|^{\gamma}} \mathbf{1}_{\mathcal{S}(x)}(y) \frac{y-x}{|y-x|}$$

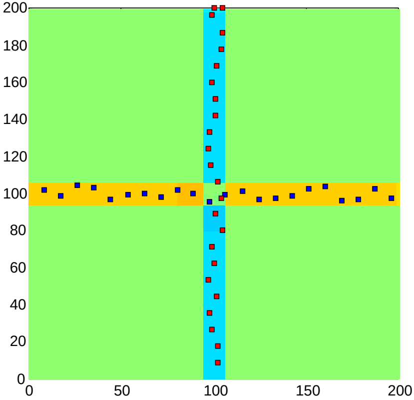
1D-2D coupling

If a 2D microscopic approach is possible, a 2D macroscopic approach is unfeasible. BCs at road sides are also numerically intractable.

After the evaluation of a fully 2D velocity field v_{mM}, the macroscopic scale is evolved by means of the projection of v_{mM} along the leading space dimension.

In this way we are able to couple a 2D microscopic model with a 1D macroscopic model.

Numerical results



$\theta = 0$ (pure macro)

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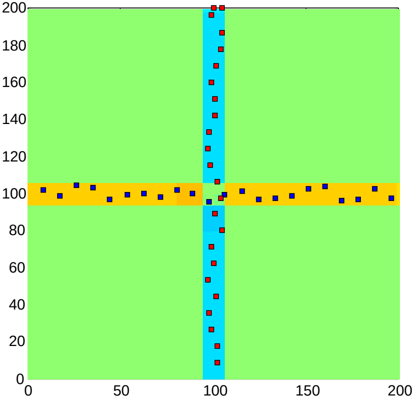

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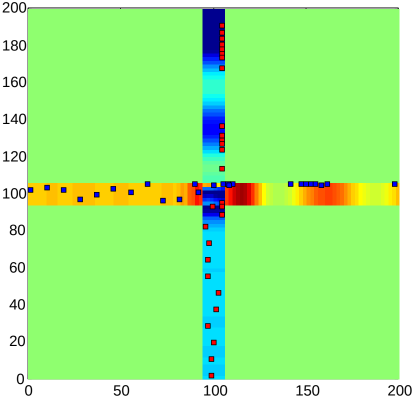

$\theta = 0.7$

Micro dynamics induces a self-organized oscillatory pattern at junction

Space-dependent θ

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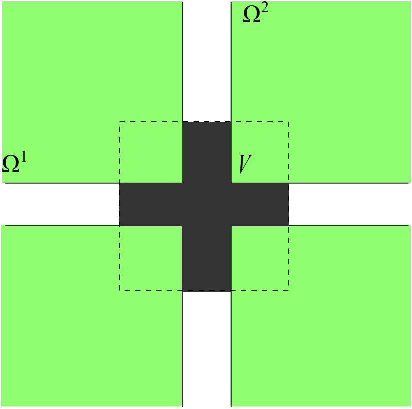
The multiscale parameter θ is assumed to be space-dependent, i.e. $\theta=\theta(x)$.
This way we account for the proper scale in the proper portion of the domain.

NATURAL CHOICE: micro at junctions, macro elsewhere.

Numerical results for space-dependent θ

$$\theta(x) = \begin{cases} 1 & x \in V \\ 0 & x \in (\Omega^1 \cup \Omega^2) \setminus V \end{cases}$$



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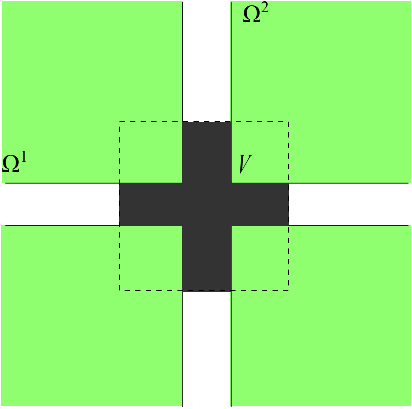

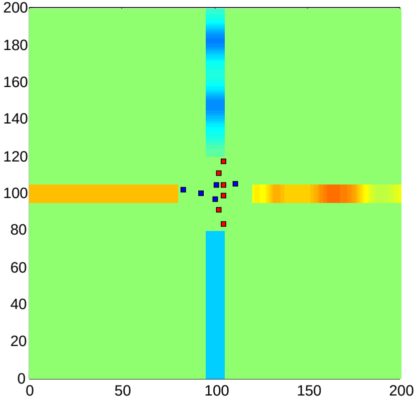

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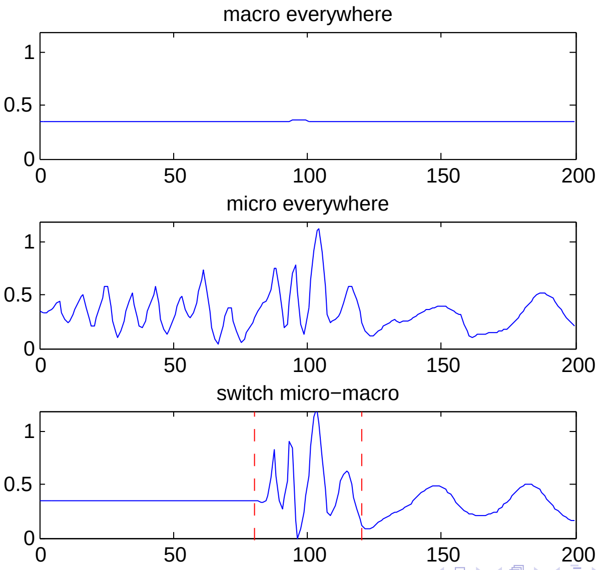

Conclusions and future work

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We got macroscopic simulations where granular effects are not lost.

Future work

- Explore the potential of this technique investigating other fields of applications.
- Control of traffic flow

References

1. E. Cristiani, B. Piccoli, A. Tosin, *Multiscale modeling of granular flows with application to crowd dynamics*, Multiscale Model. Simul., 9 (2011), pp. 155–182.
2. E. Cristiani, B. Piccoli, A. Tosin, *How can macroscopic models reveal self-organization in traffic flow?*, Proc. IEEE CDC 2012 Conference, Maui, HI, December 10-13, 2012.