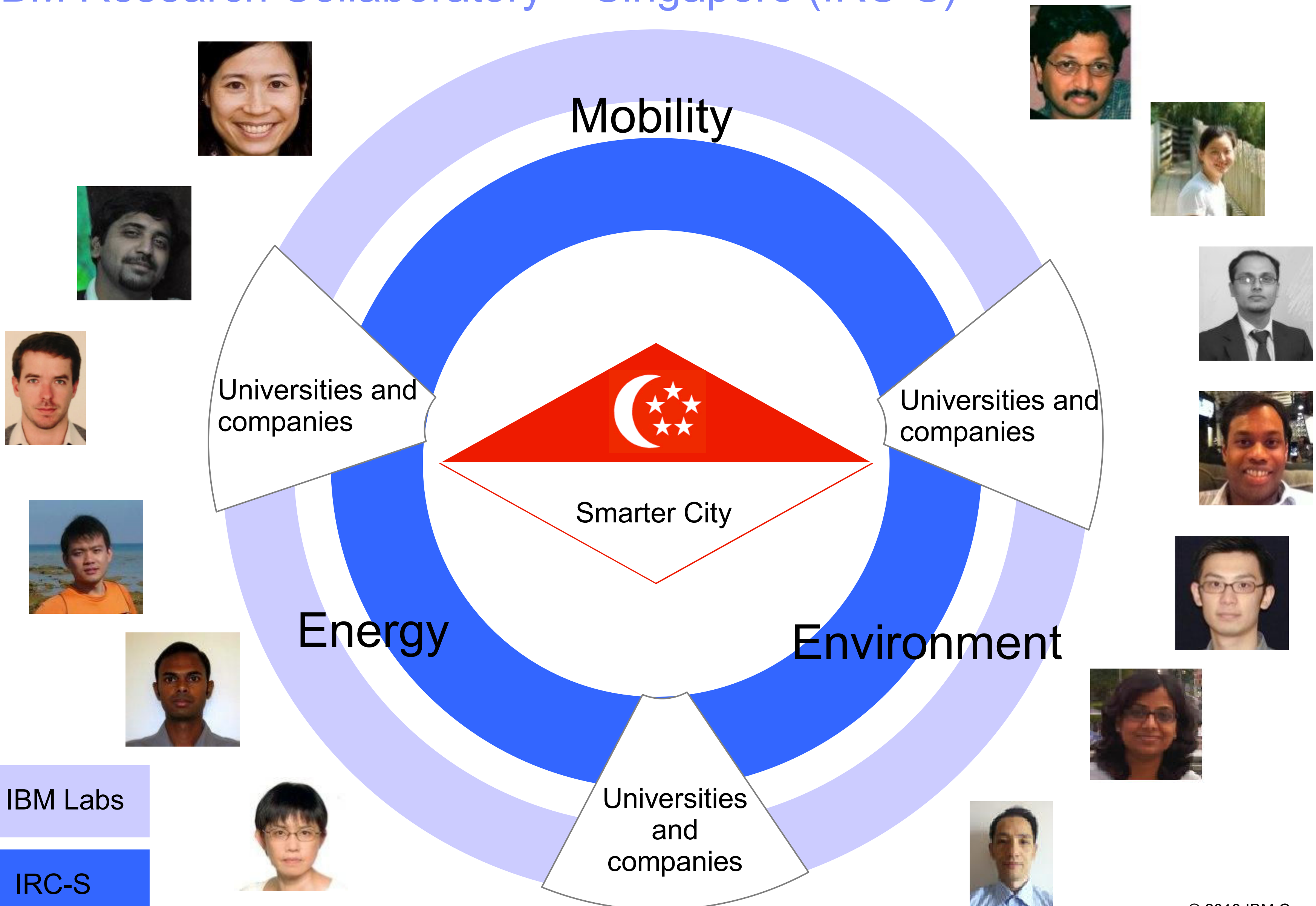
Sebastien Blandin, Research Scientist
IBM Research Collaboratory – Singapore

A general phase transition model for traffic flow on networks

Traffic Modeling and Management: trends and perspectives,
VI Workshop on Mathematical Foundations of Traffic (WMFT) ,
Sophia-Antipolis, March 20-22, 2013

IBM Research Collaboratory – Singapore (IRC-S)



IBM Labs

IRC-S

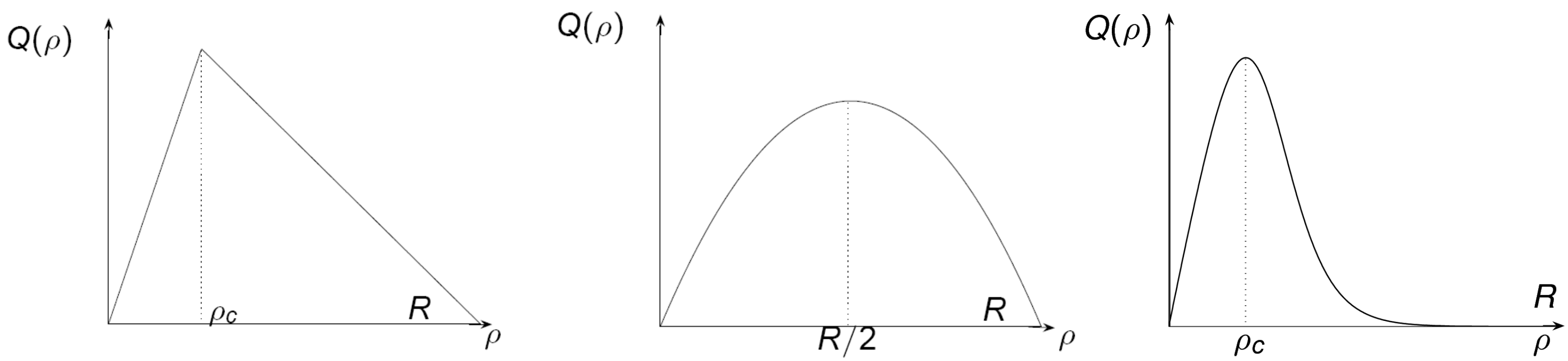
First order scalar conservation law models

- Traffic state: density $\rho(t,x)$ of vehicles at time t and location x
- Scalar one dimensional conservation law, transport equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial Q(\rho)}{\partial x} = 0$$

- Empirical flux function: the fundamental diagram

Newell-Daganzo Greenshields Kerner, Papageorgiou,Li



with R the maximal or *jam* density, and ρ_c the critical density

- Flux is increasing for $\rho \leq \rho_c$: free-flow phase
- Flux is decreasing for $\rho \geq \rho_c$: congestion phase

[Lighthill, Whitham, 1955], [Richards, 1956], [Greenshields, 1935],[Garavello, Piccoli, 2006]

Motivation for higher-order models

- Traffic satisfies "mass" conservation, what about other fundamental conservation principles from fluid dynamics; conservation of momentum, conservation of energy

[Lesort, Bourrel, Henn, 2003]

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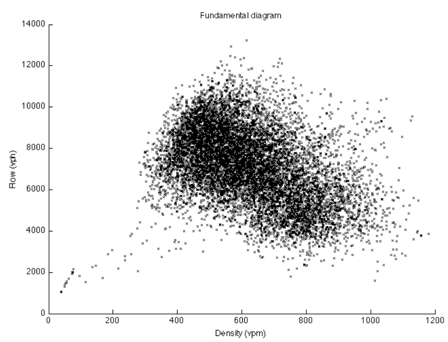
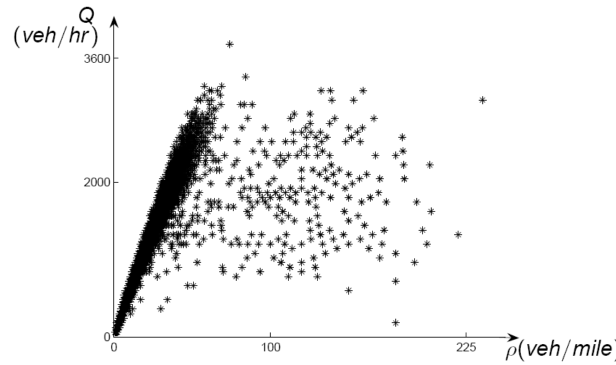
• Experimental observations of fundamental diagrams are more complex than postulated by first order traffic models

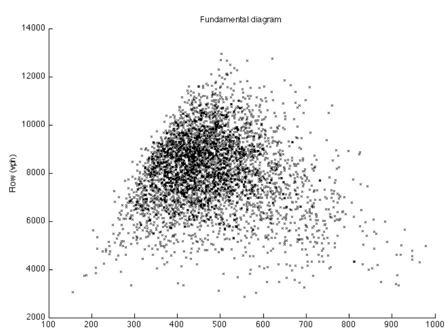
NGSIM : I-80

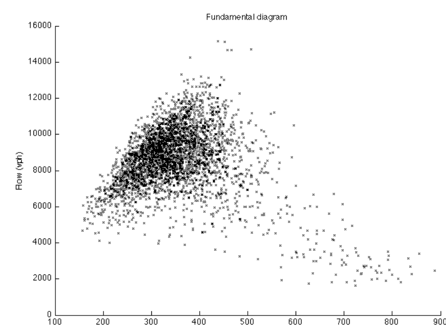




Roma: muro torto

NGSIM : I-80





NGSIM: US-101

[Blandin, Bretti, Cutolo, Piccoli, AMC, 2009]

Motivation for higher-order models

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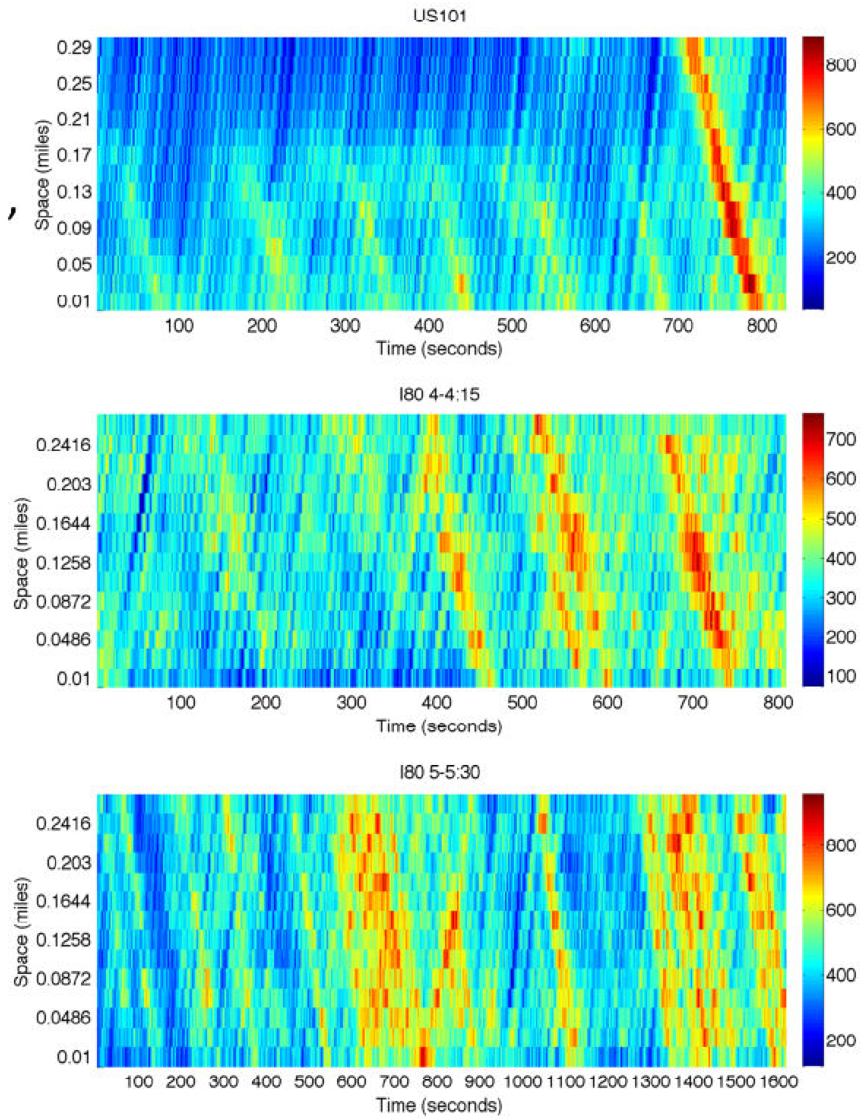
- Experimental observations of real-time dynamics show phenomena that are not accounted for by first-order models



[Blandin, Argote, Bayen, Work, 2011]

Motivation for higher-order models

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- Need for model able to integrate measurements of different traffic quantities (density/occupancy, flow, speed): data fusion

[Work, Blandin,Tossavainen, Piccoli, Bayen, 2010]

New data sources are available for understanding mobility patterns

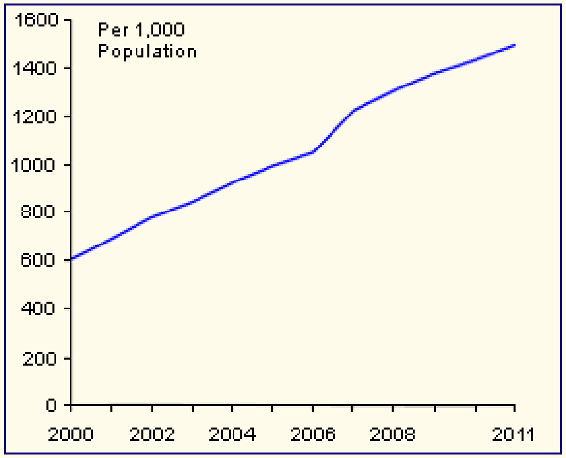
- Trip statistics [LTA, 2012]
  - 9.9 million trips per day (2010)
  - 5 millions public transport trips (2010)

- Citizen as Sensors
  – Mobile phones statistics [IDA, 2011]
    - Mobile phone penetration in Singapore: 149.6% (2011)
    - Penetration of 3G phones: 74.2% (2011)
    - Data volume: 2300 Millions SMS (Dec. 2011)

- Fused to form Real-time travel information
  – Integrate EZ-Link data with the telco data for a more complete picture of travel patterns

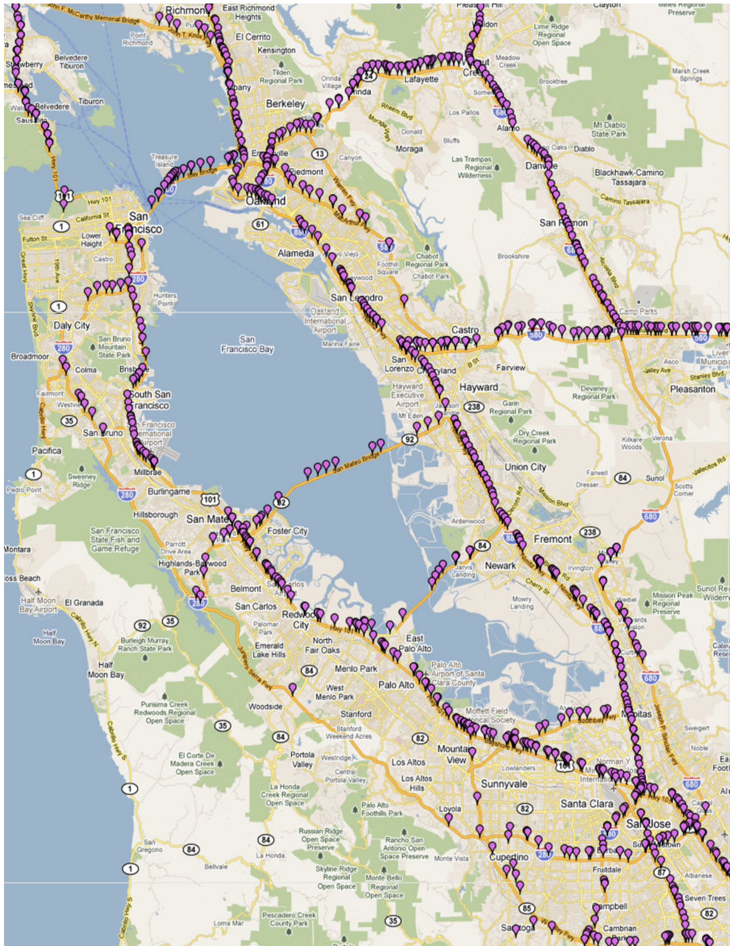
- Considerations: Citizen Privacy
  – Benefits from new insights should be balanced against risk of privacy infringement

Mobile Phones



Need for data fusion

PeMS loop detector stations

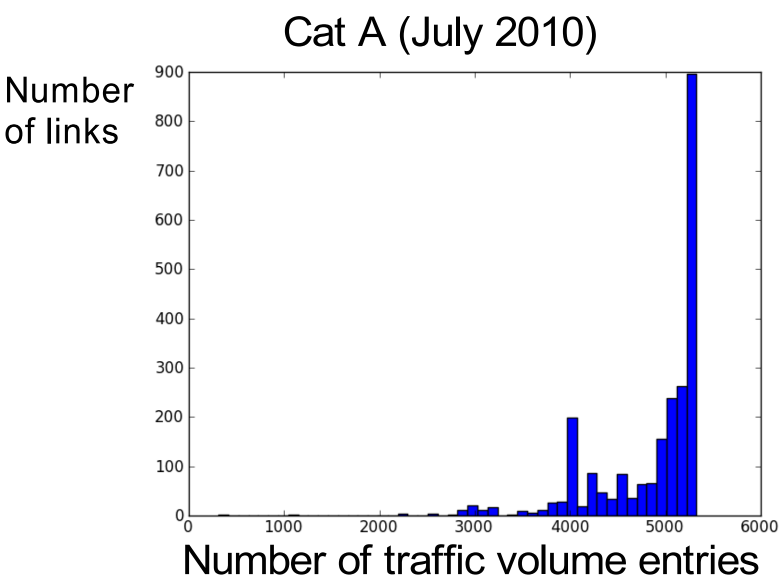


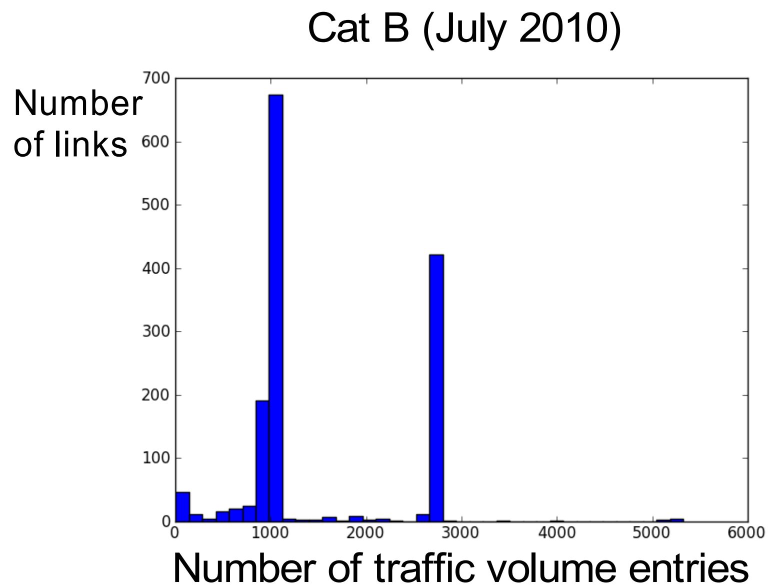
•Loop detectors
•Count and occupancy
•Localized in space

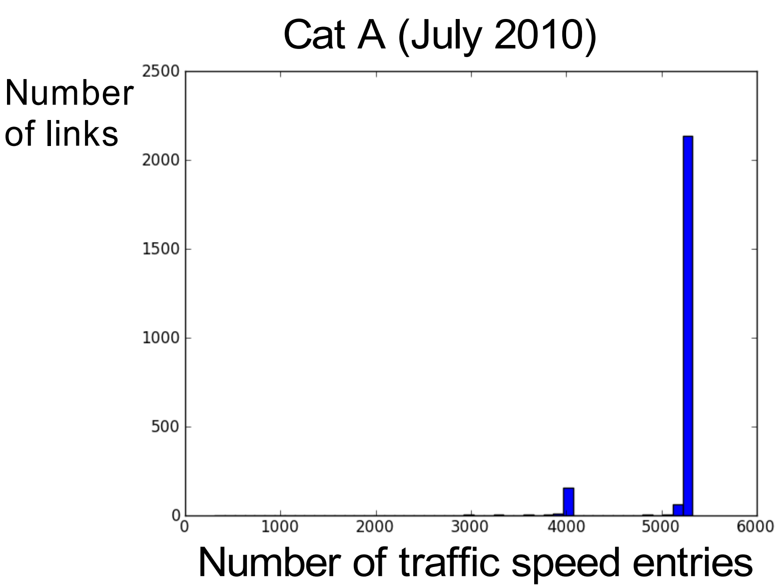
Mobile Millennium, GPS point speeds, July 29th, 2010

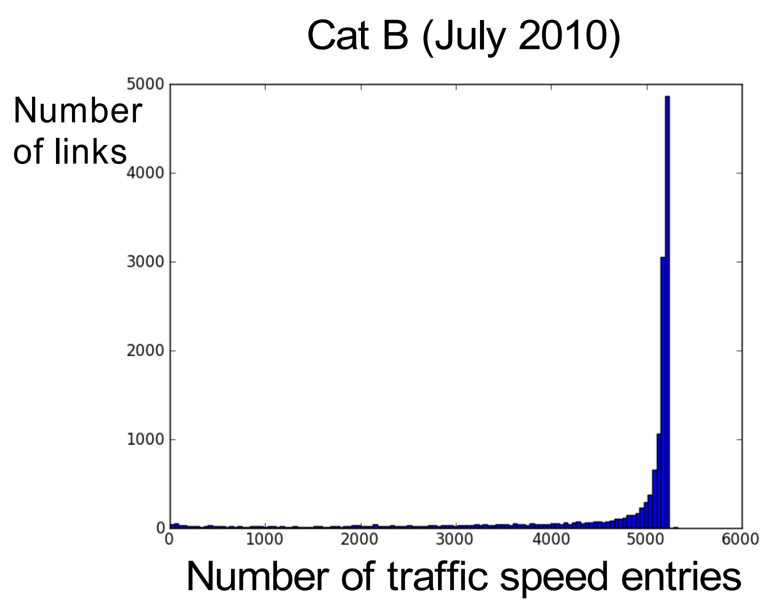
•Personal GPS
•Point speeds
•Distributed across the road network

Need for data fusion









Outline

- Motivation and history of macroscopic models of traffic flow
- Definition, properties and physical interpretation of phase transition model
- Junction problem formulation
- Numerical scheme for the phase transition model: the modified Godunov scheme
- Performance of phase transition model on NGSIM datasets

Macroscopic models of traffic flow

- First traffic measurements and traffic relationships (ancestor of the fundamental diagrams): [Greenshields, 1934], [Greenberg, 1938], [Edie, 1960]

- *"A 16 mm Simplex movie camera was set about 350 feet from the roadway so that each vehicle would appear at least in two frames"*.

- "*Frames of picture*" were "*superimposed upon a scale to show the distance traveled*".

Greenshield, 1935

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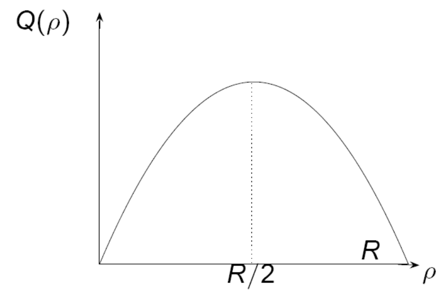

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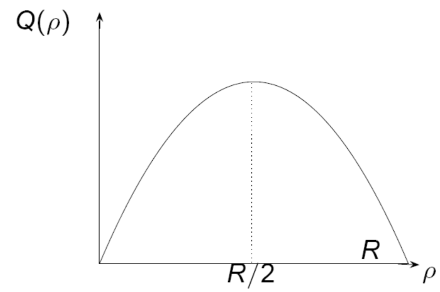

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- First-order models: [Lighthill and Whitham, 1956], [Richards, 1956]

- Applicability of Godunov scheme for discretization of macroscopic traffic models: [Lebacque, 1996]

$$\rho_i^{n+1} = \rho_i^n + \frac{\Delta t}{\Delta x}\left(q_G(\rho_{i-1}^n, \rho_i^n) - q_G(\rho_i^n, \rho_{i+1}^n)\right)$$

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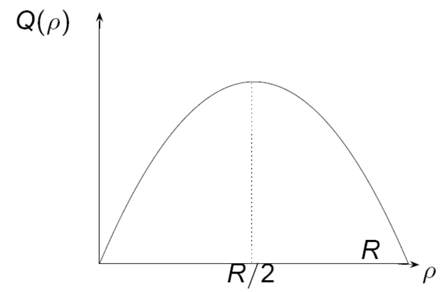

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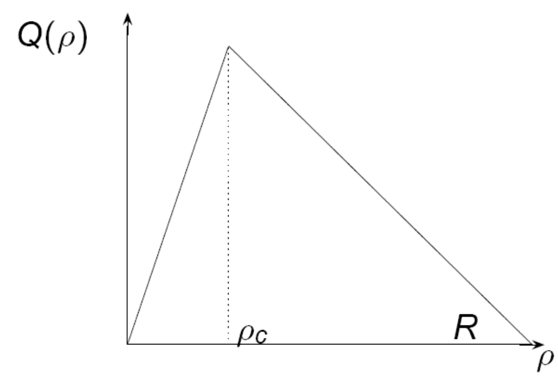
- Exhaustive analysis of case of triangular fundamental diagram: [Newell, Daganzo, 1990's]



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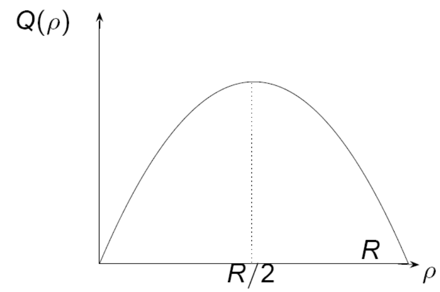

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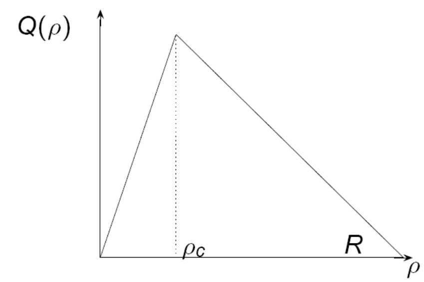

- Mathematical results for first-order traffic flow models on networks: [Garavello & Piccoli, 2003]

Macroscopic models of traffic flow

$$\begin{cases} \frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial x} = 0 \\ \frac{\partial v}{\partial t} + v\,\frac{\partial v}{\partial x} = -\frac{c_0^2}{\rho}\frac{\partial \rho}{\partial x} + \frac{v_*(\rho)-v}{\tau} \end{cases}$$

- First macroscopic models with two variables (2 X 2 sytems of PDEs): [Payne, 71], conservation of mass and momentum

- Modified discretized version for networks: [Papageorgiou et al, 90]

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• Critics of these 2 X 2 models: [Del Castillo et al, 94], [Daganzo, 95]

- Drivers should have only positive speed
- Drivers should have zero speed only at maximal density
- Anisotropy: drivers should react only to stimuli from the front

$$Jac_f(\rho, v) = \begin{pmatrix} v & \rho \\ -\frac{c_0^2}{\rho} & v \end{pmatrix}$$

$$\lambda_\pm(\rho, v) = v \pm c_0$$

with f the Payne-Whitham flux

Macroscopic models of traffic flow

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- [Aw, Rascle, 2000]: “Resurrection of second-order models of traffic flows”

- Non-equilibrium model: [Zhang, 2002]

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- Non-equilibrium model: [Zhang, 2002]

- Phase transition model: [Colombo, 2003], [Blandin et al, 2011]

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Phase transition model definition

- Definition of traffic state as
  - ρ in free-flow phase
  - (ρ, q) in congestion phase
- Definition of the standard speed function

$$v^{\mathrm{s}}(\rho) = \begin{cases} V & \text{in free-flow} \\ v_{\mathrm{c}}^{\mathrm{s}}(\rho) & \text{in congestion} \end{cases}$$

where $v_{\mathrm{c}}^{\mathrm{s}}(\cdot)$ is smooth with positive values.

- Definition of speed as a perturbation around the standard speed in congestion

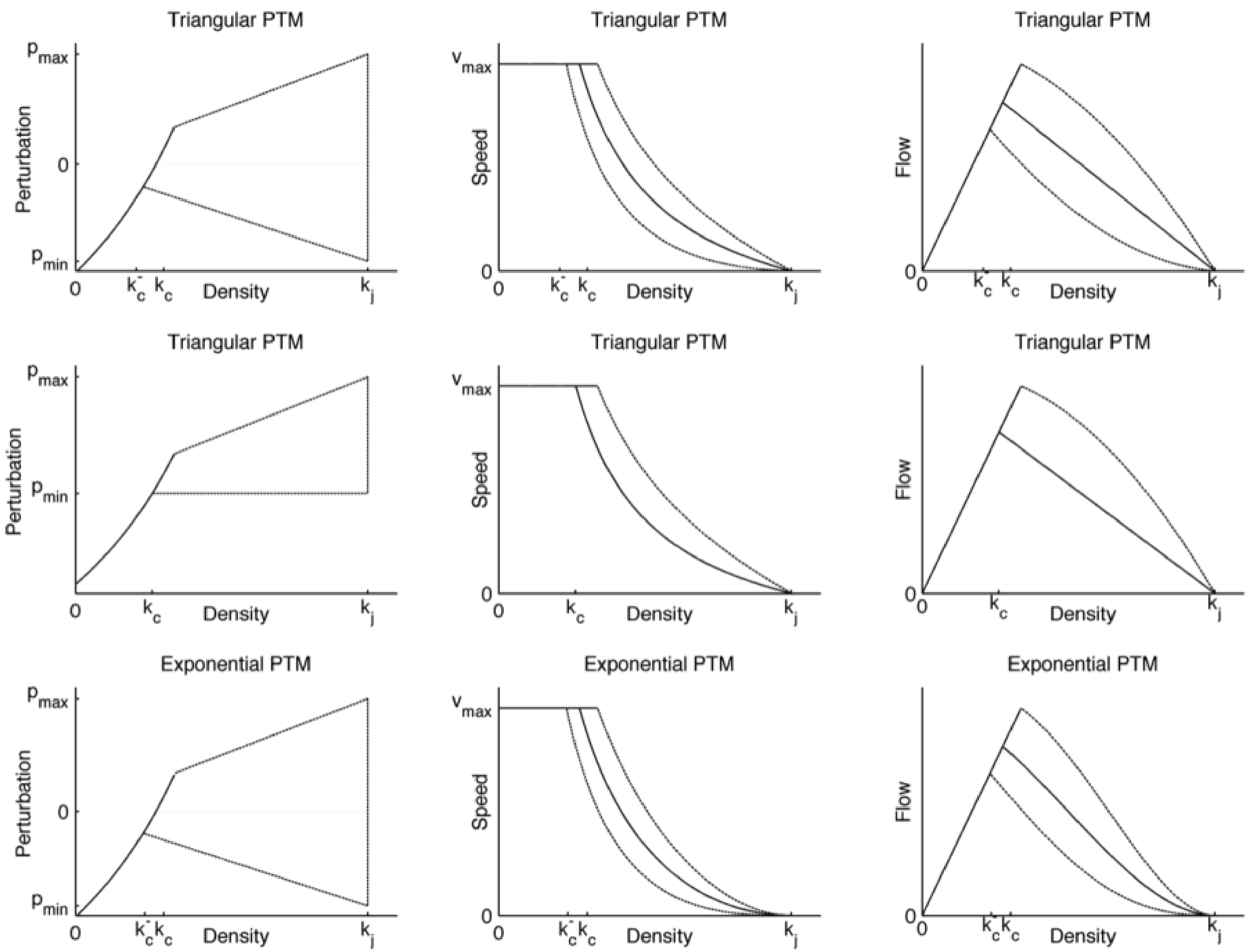
$$v = \begin{cases} V & \text{in free-flow} \\ v_{\mathrm{c}}(\rho, q) := v_{\mathrm{c}}^{\mathrm{s}}(\rho)(1+q) & \text{in congestion} \end{cases}$$

- Definition of two distinct dynamics

$$\begin{cases} \partial_t \rho + \partial_x(\rho\, v) = 0 & \text{in free-flow} \\ \begin{cases} \partial_t \rho + \partial_x(\rho\, v) = 0 \\ \partial_t q + \partial_x(q\, v) = 0 \end{cases} & \text{in congestion} \end{cases}$$

[Blandin, Work, Goatin, Piccoli, Bayen, SIAP, 2011]

Representations of phase transition model



[Blandin, Argote, Bayen, Work, TR-B, 2012]

Phase transition model definition

- Analysis of congestion phase
  - Eigenvalues and eigenvectors of Jacobian of flux
  - Lax-curve: integral curve of eigenvectors
  - Riemann-invariant: scalar quantity constant along Lax-curve

Eigenvalues	$\lambda_1(\rho, q) = \rho\,(1+q)\partial_\rho v_c^s(\rho) + v_c^s(\rho)\,(1+2\,q)$	$\lambda_2(\rho, q) = v_c^s(\rho)\,(1+q)$
Eigenvectors	$r_1 = \begin{pmatrix} \rho \\ q \end{pmatrix}$	$r_2 = \begin{pmatrix} v_c^s(\rho) \\ -(1+q)\,\partial_\rho v_c^s(\rho) \end{pmatrix}$
Nature of the Lax curves	$\nabla\lambda_1.r_1 = \rho^2\,(1+q)\partial^2_{\rho\rho} v_c^s(\rho) + 2\,\rho\,(1+2\,q)\,\partial_\rho v_c^s(\rho) + 2\,q\,v_c^s(\rho)$	$\nabla\lambda_2.r_2 = 0$
Riemann-invariants	q/ρ	$v_c^s(\rho)\,(1+q)$

- First family of Lax-curves is not genuinely-non-linear (in flux-density coordinates curves GNL equivalent to all Lax-curves have same concavity)
- Second family of Lax-curves is linearly degenerate (information propagates at a constant speed)

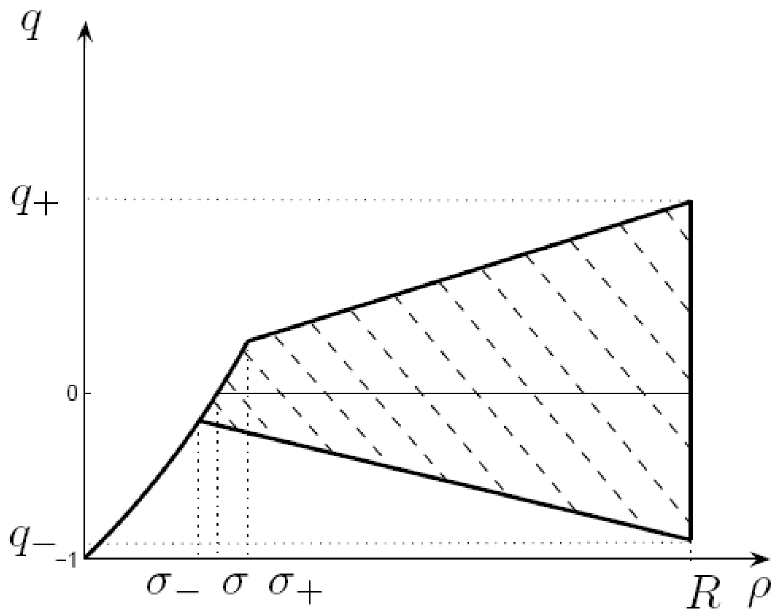
[Blandin, Work, Goatin, Piccoli, Bayen, SIAP, 2011]

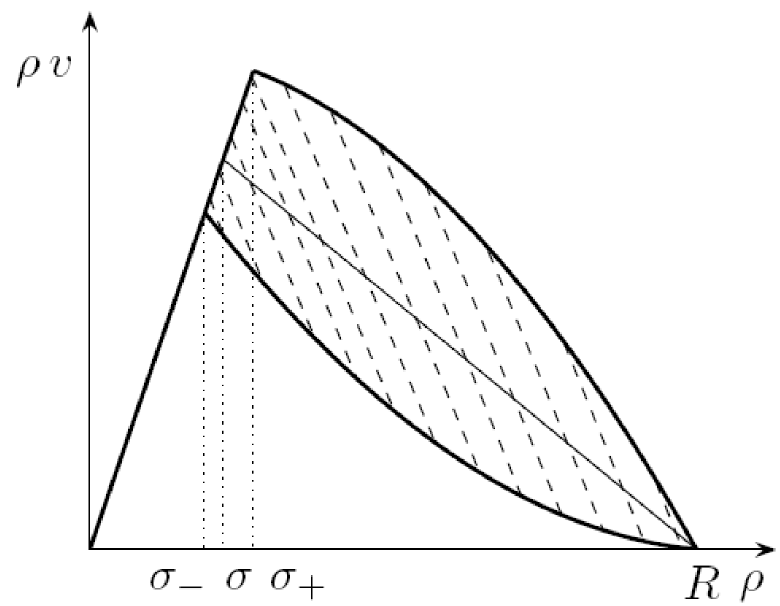
Phase transition model definition

- Definition of free-flow and congestion phase as invariant domains of dynamics

$$\begin{cases} \Omega_f = \{(\rho,q) \mid (\rho,q) \in [0,R] \times [0,+\infty[\ , v_c(\rho,q) = V \ , \ 0 \le \rho \le \sigma_+\} \\ \Omega_c = \left\{(\rho,q) \mid (\rho,q) \in [0,R] \times [0,+\infty[\ , v_c(\rho,q) < V \ , \ \frac{q_-}{R} \le \frac{q}{\rho} \le \frac{q_+}{R}\right\} \end{cases}$$

- Model parameters
  - Free-flow speed V, jam density R, critical density σ, upper and lower bound for perturbation, q_- and q_+.

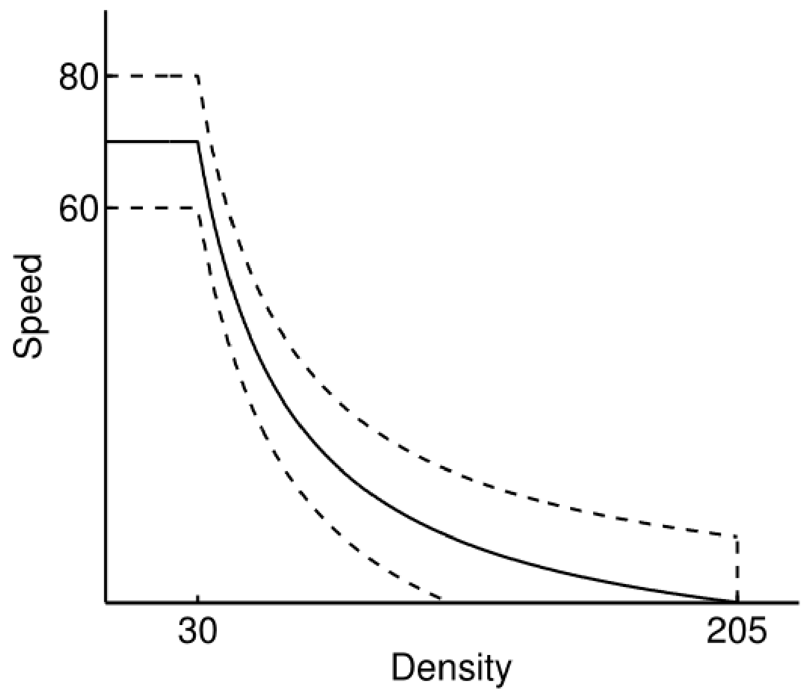


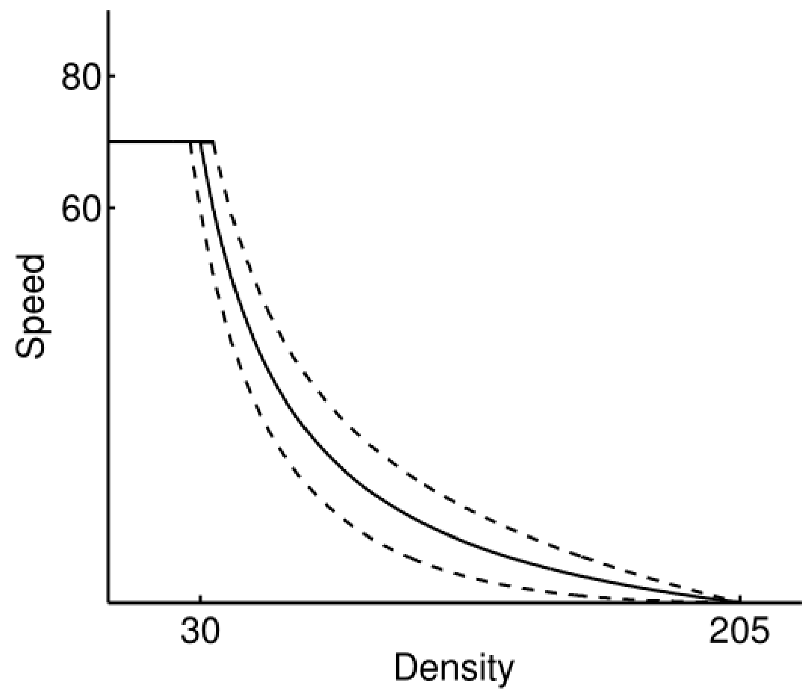


[Blandin, Work, Goatin, Piccoli, Bayen, SIAP, 2011]

Fundamental diagram for higher-order models

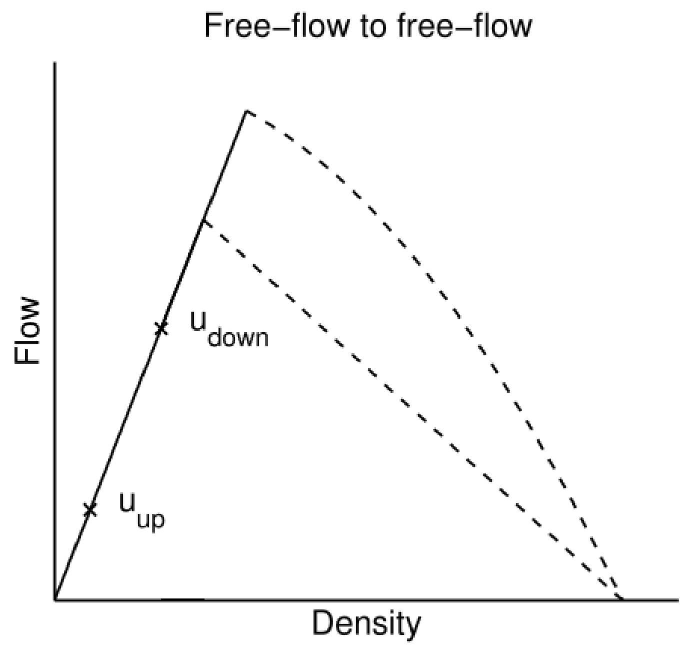
- Non-equilibrium model (left), phase transition model (right)

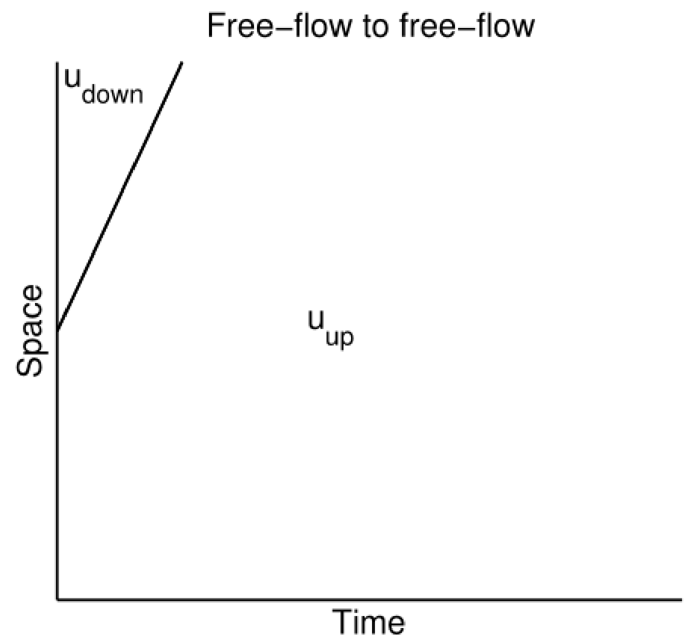




[Blandin, Argote, Bayen, Work, TR-B, 2012]

Phase transition model: Riemann problem





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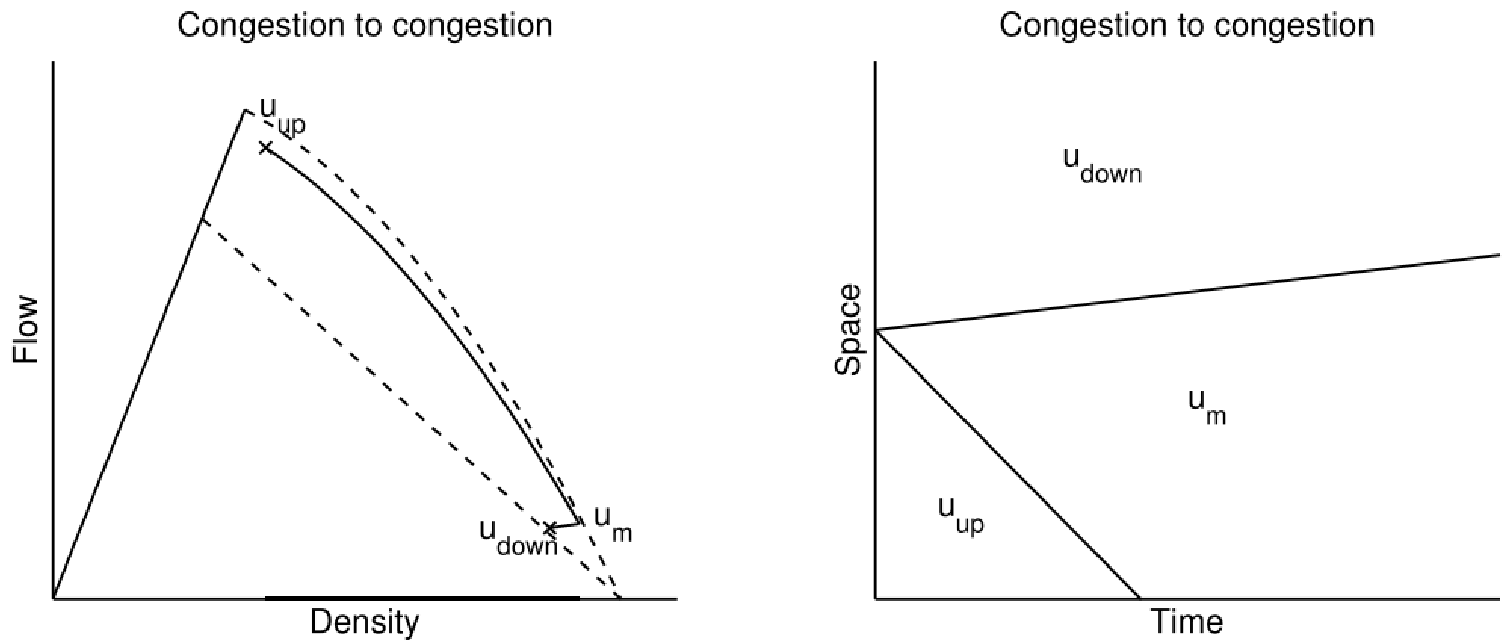

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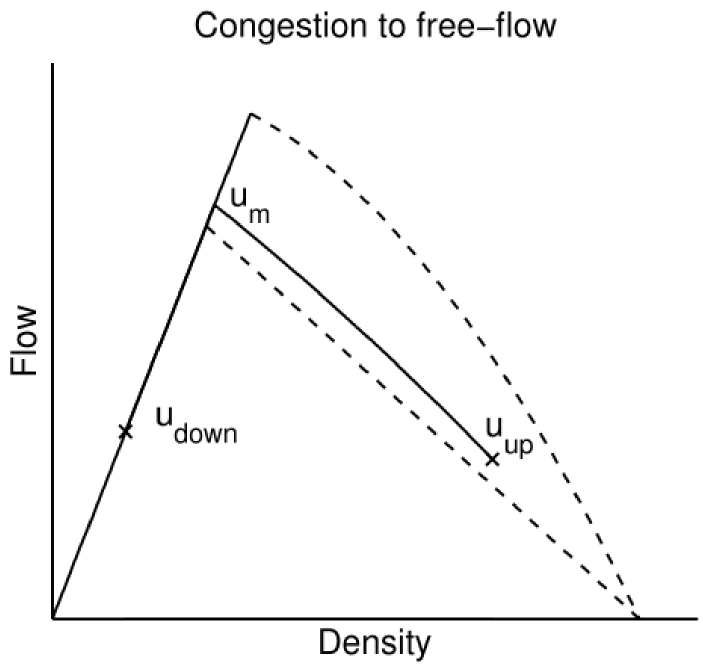

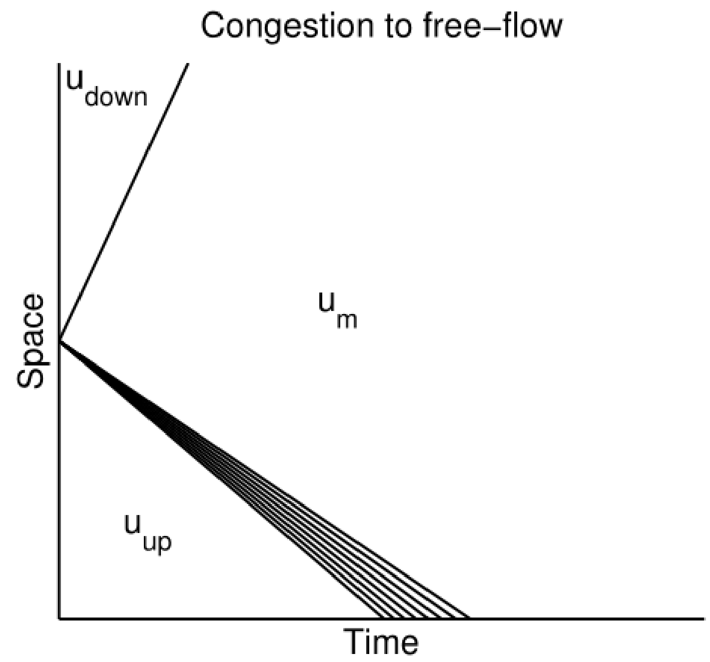

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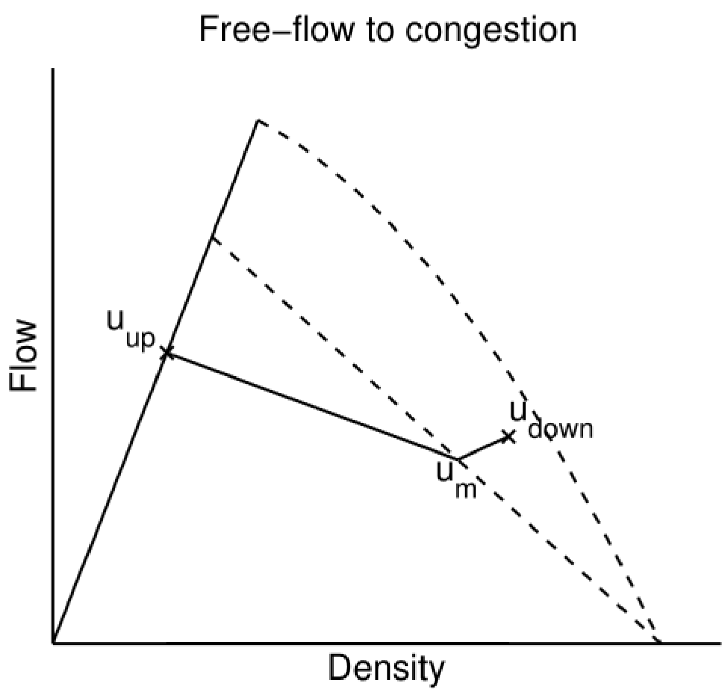
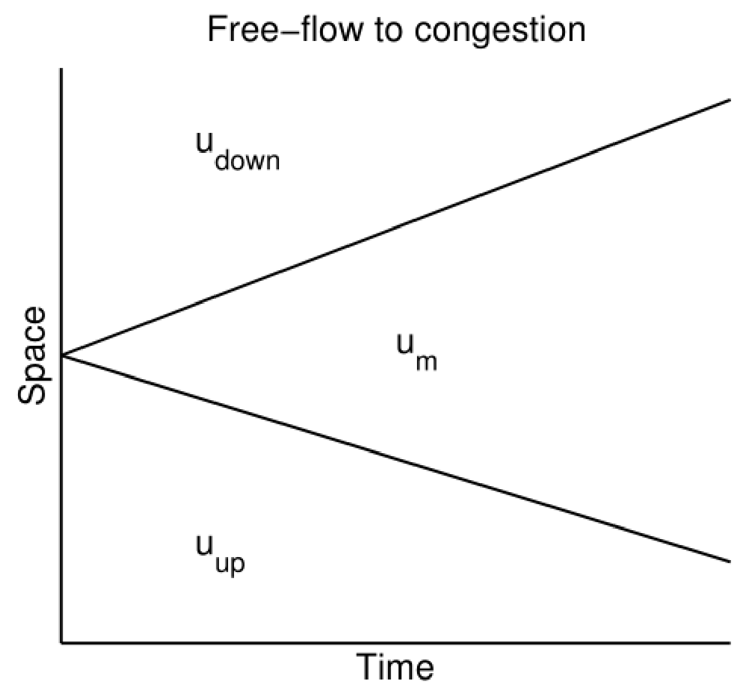


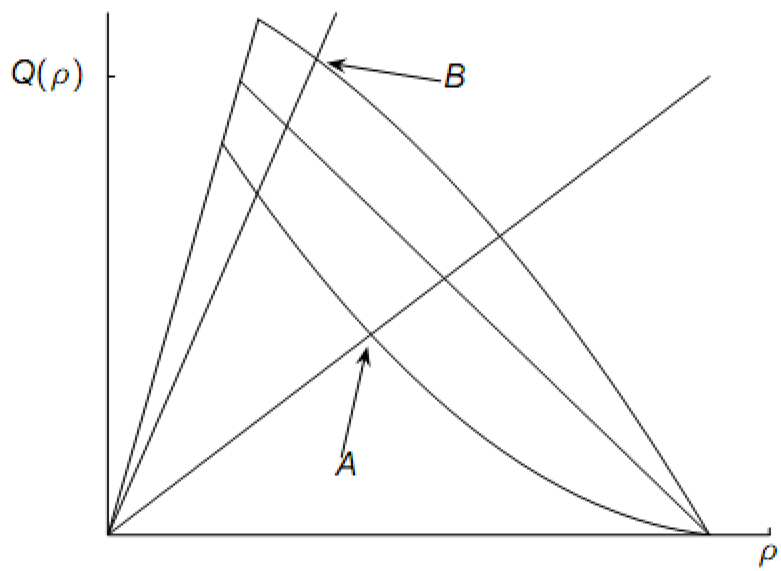


[Blandin, Argote, Bayen, Work, TR-B, 2012]

Phase transition model properties

- Riemann problem with initial data:

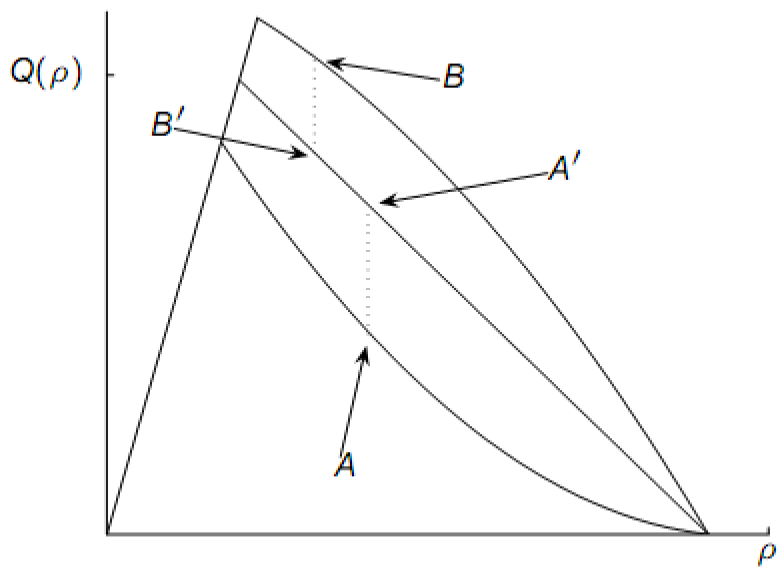
$$\begin{cases} u_{\text{left}} = (\rho_A, v_A) \\ u_{\text{right}} = (\rho_B, v_B) \end{cases}$$



[Blandin, Work, Goatin, Piccoli, Bayen, SIAP, 2011]

Phase transition model properties

- Scalar conservation law: solution given by a contact discontinuity between A’ and B’.

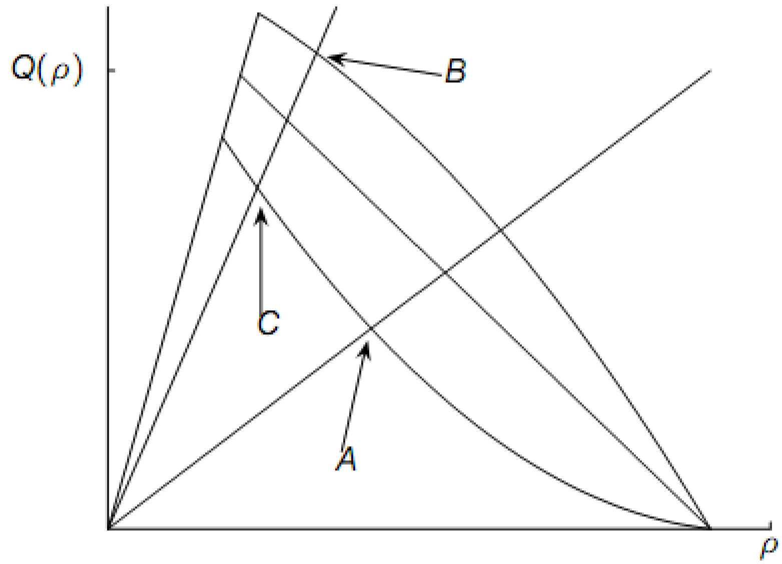


- Stationary state is A’

[Blandin, Work, Goatin, Piccoli, Bayen, SIAP, 2011]

Phase transition model properties

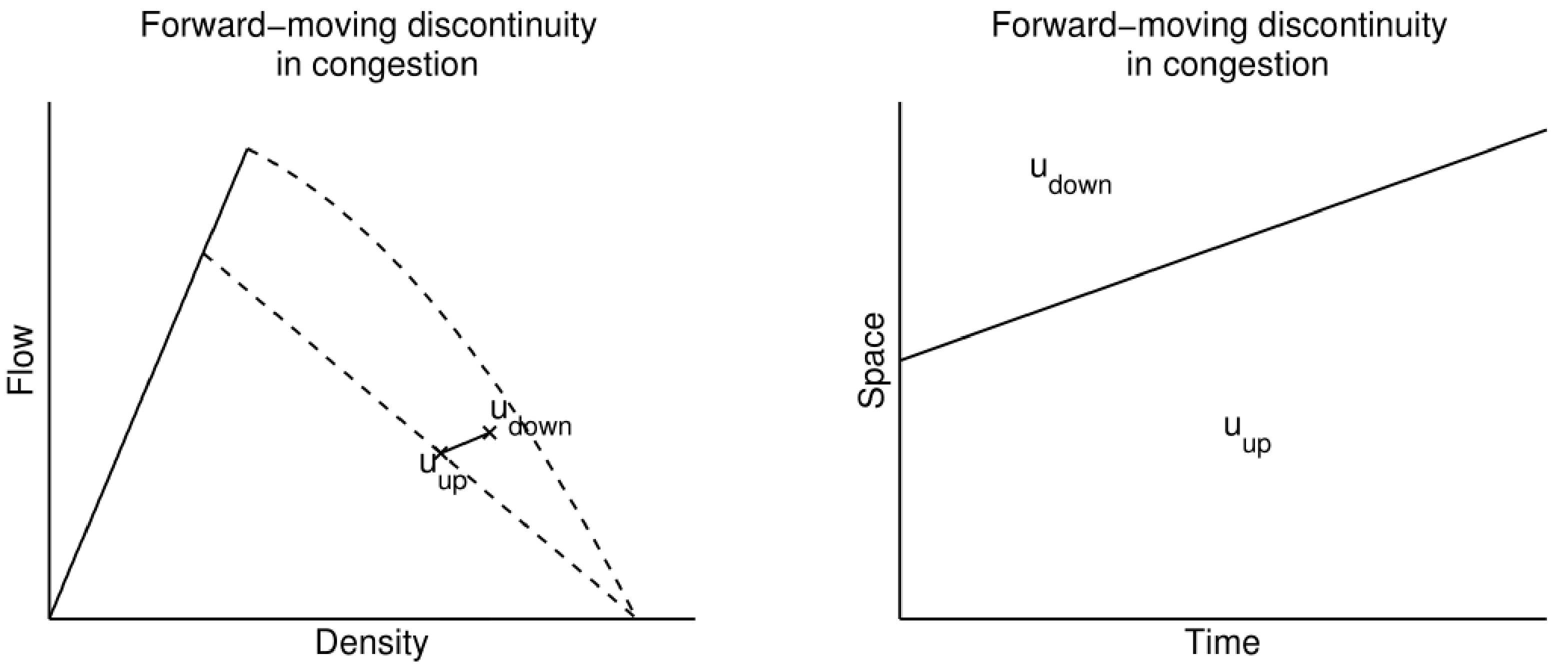
- Phase transition model: solution given by a backward moving shockwave between A and C and by a forward moving contact discontinuity between C and B



- Stationary state is C

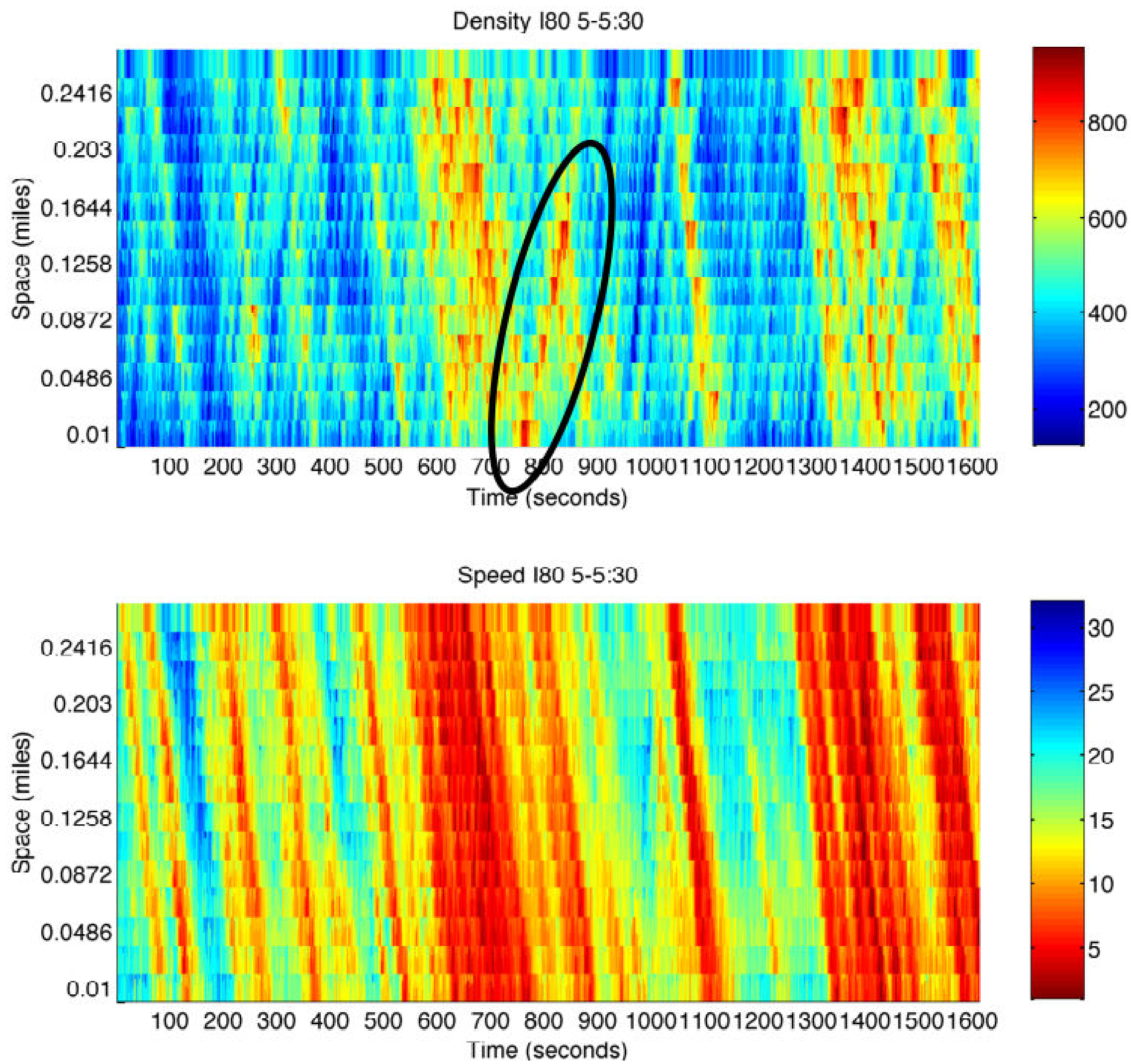
[Blandin, Work, Goatin, Piccoli, Bayen, SIAP, 2011]

Modeling forward moving discontinuity in congestion



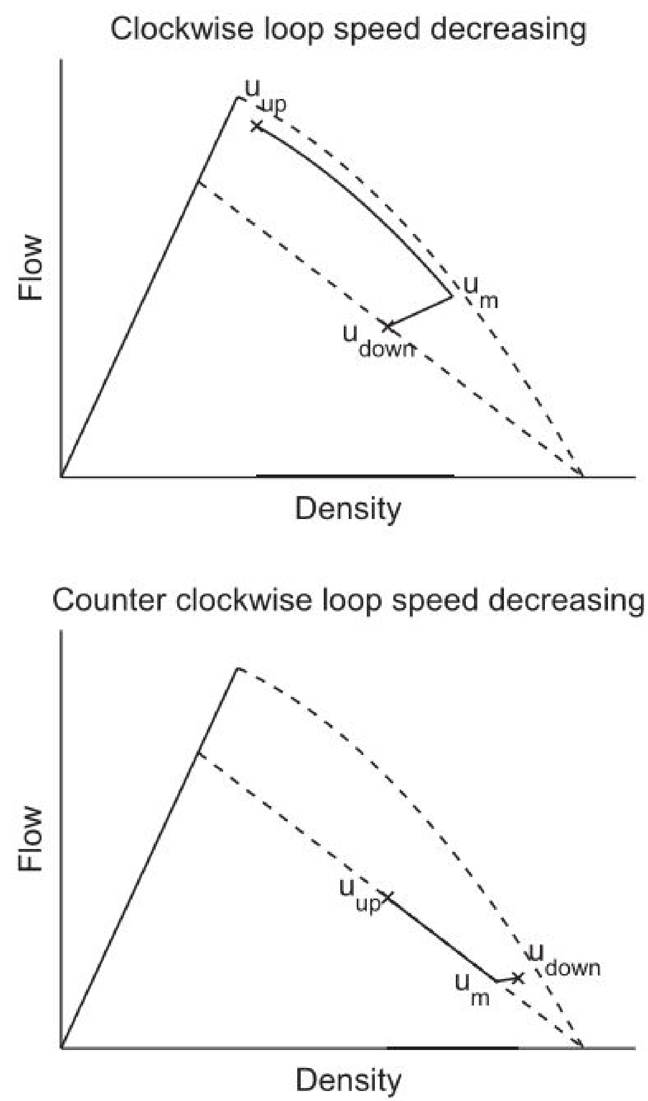
[Blandin, Argote, Bayen, Work, TR-B, 2012]

Forward moving discontinuity in congestion: NGSIM data



[Blandin, Argote, Bayen, Work, TR-B, 2012]

Hysteresis and disturbances



Outline

- Motivation and history of macroscopic models of traffic flow
- Definition, properties and physical interpretation of phase transition model
- Junction problem formulation
- Numerical scheme for the phase transition model: the modified Godunov scheme
- Performance of phase transition model on NGSIM datasets

Junction formulation

• Given an assignment matrix, an admissible solution to the network phase transition model should conserve the number of vehicles at the junction (conservation of the perturbation is not required)

• We require that the solution consists of
  - waves with negative speeds on incoming links
  - waves with positive speeds on outgoing links

• For uniqueness, we require that that flux at the junction is maximized under the constraints above

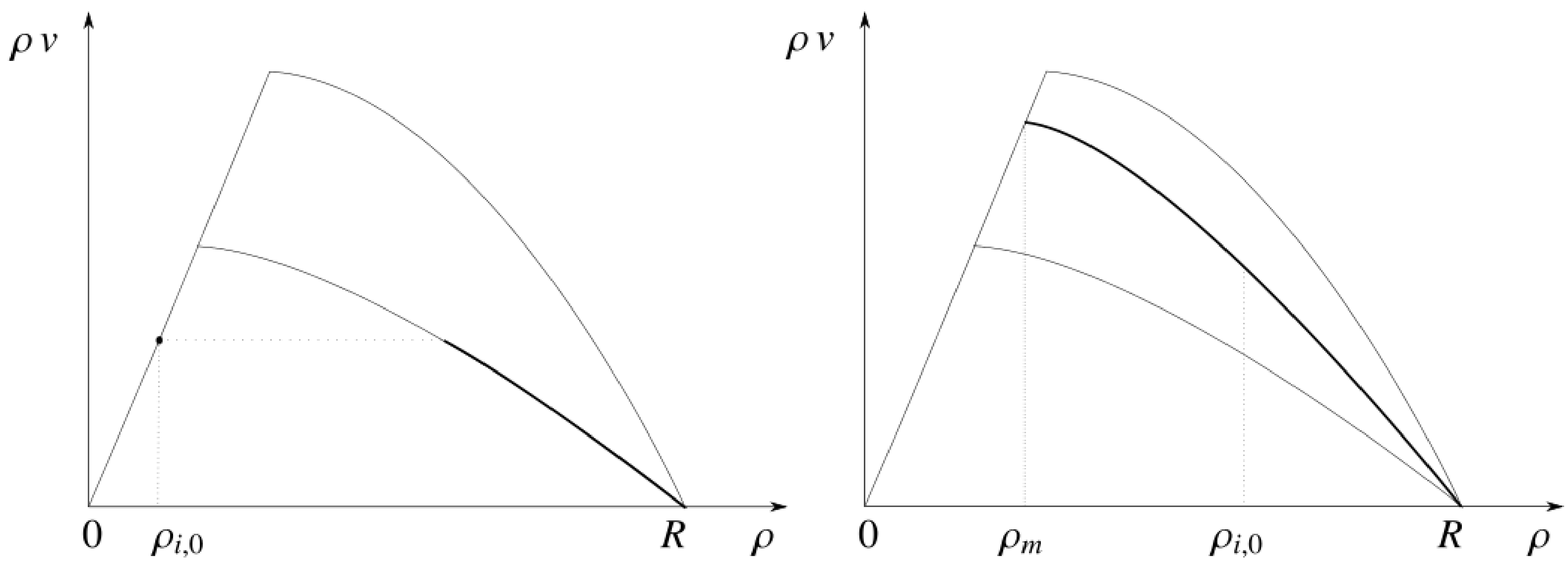
• The junction problem is formulated as
  - Computation of allowable states on incoming links and outgoing links
  - Computation of maximum allowable flux for each incoming and outgoing link
  - Maximization of flux across the junction

[Garavello, Piccoli, 2006], [Blandin, Goatin, Piccoli, Bayen, Work, 2012]

Admissible flux on incoming roads

$$\begin{cases} \dfrac{p_m}{\rho_m} = \dfrac{p_{i,0}}{\rho_{i,0}}, \\ v_c(\mathbf{u}_m) = V, \end{cases}$$

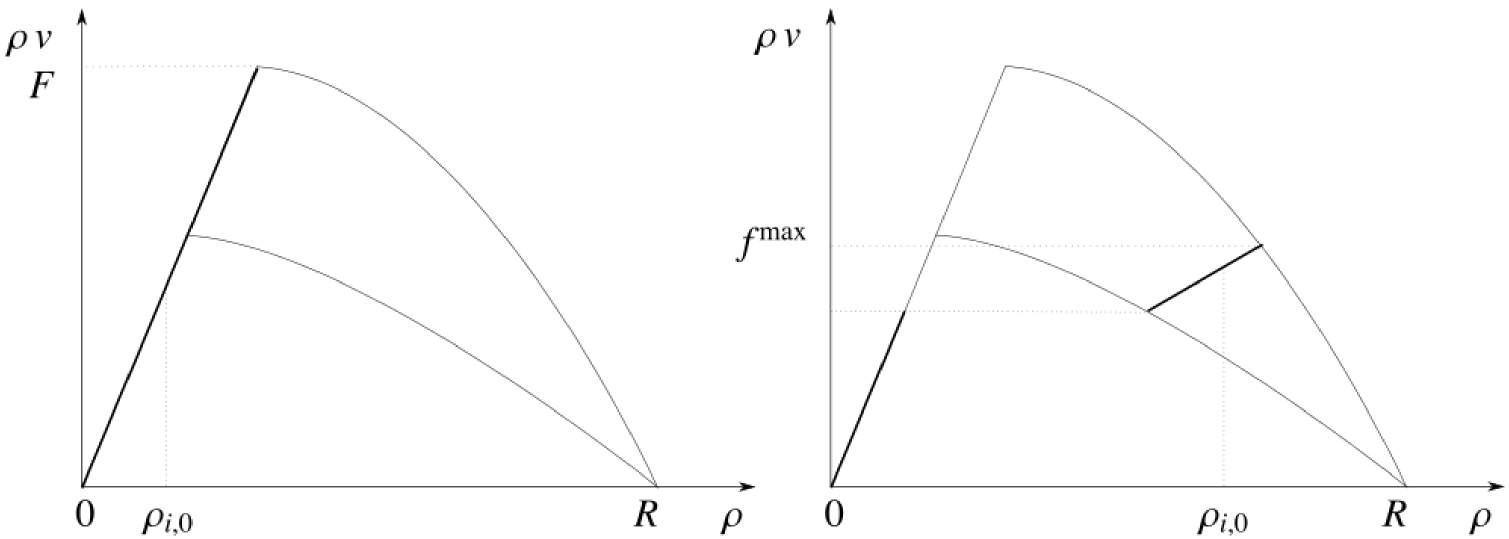
$$O_i(\mathbf{u}_{i,0}) = \begin{cases} [0, \rho_{i,0}\, V] & \text{if } \mathbf{u}_{i,0} \in \Omega_f, \\ [0, \rho_m\, V] & \text{if } \mathbf{u}_{i,0} \in \Omega_c, \end{cases}$$



[Blandin, Goatin, Piccoli, Bayen, Work, 2012]

Admissible flux on outgoing roads

$$\begin{cases} \dfrac{p^{\max}}{\rho^{\max}} = \dfrac{p_+}{R}, \\ v_c(\mathbf{u}^{\max}) = v_c(\mathbf{u}_{j,0}). \end{cases} \qquad O_j(\mathbf{u}_{j,0}) = \begin{cases} [0, F] & \text{if } \mathbf{u}_{j,0} \in \Omega_f\,, \\ [0, f^{\max}(\mathbf{u}_{j,0})] & \text{if } \mathbf{u}_{j,0} \in \Omega_c\,, \end{cases}$$



[Blandin, Goatin, Piccoli, Bayen, Work, 2012]

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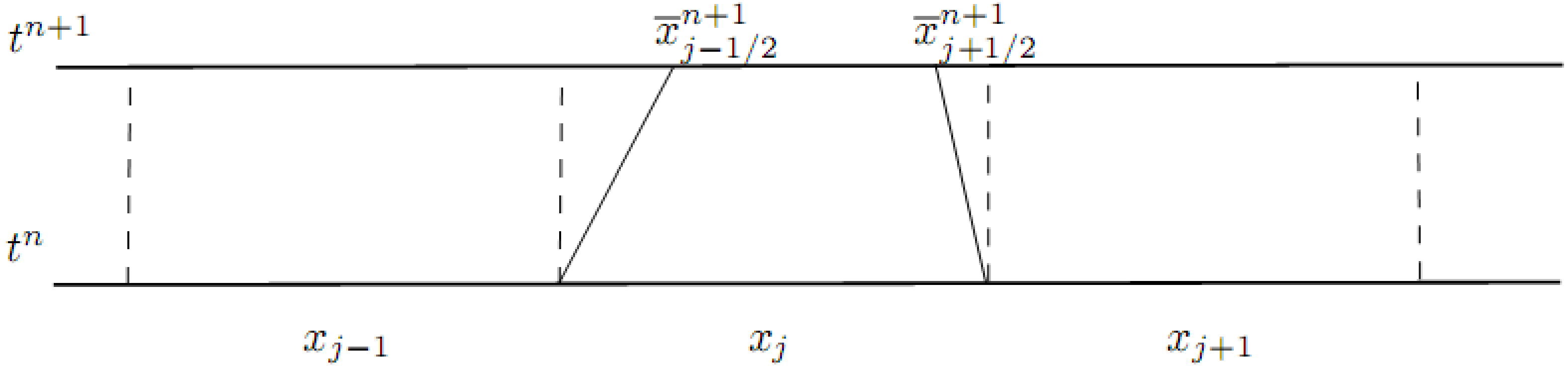
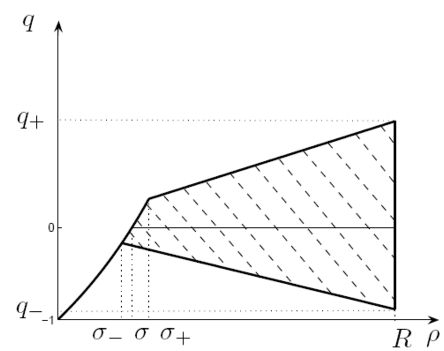
Solution method: Godunov scheme

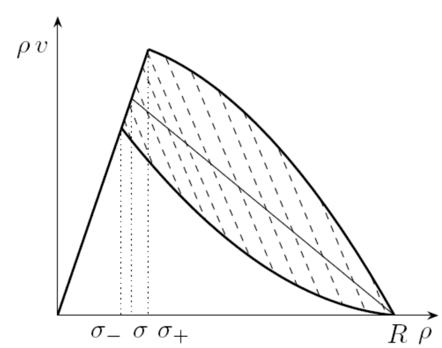
- Notations
  - space-time discretization with cells of size Δt , Δx
  - u_j^n denotes value of solution at time $n\,\Delta t$ at location $(j-1/2)\,\Delta x$



- Numerical scheme:
  - Solve the Riemann problem between two neighboring cells
  - Average solutions on each cell at the next time step
  - Iterate

- State space is not convex!

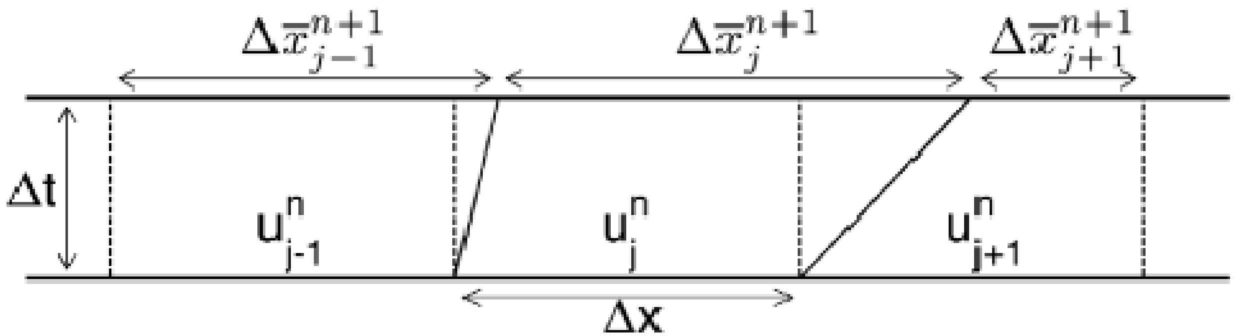




[Godunov, 1959]

Solution method: modified Godunov scheme

- Modified Godunov scheme
  - Riemann problems solved between neighboring cells
  - Solution averaged over single phases
  - Projection step in the case of congestion
  - Sampling method added to compute solution on original cells (Van der Corput sequence)



$$\Delta\overline{x}^n_j\,\overline{u}^{n+1}_j = \Delta x\,u^n_j - \Delta t\left(g\left(\nu^{n,-}_{j+1/2}, u^n_j, u^n_{j+1}\right) - \nu^n_{j+1/2}\,u_R\left(\nu^{n,-}_{j+1/2}, u^n_j, u^n_{j+1}\right)\right)$$
$$+\Delta t\left(g\left(\nu^{n,+}_{j-1/2}, u^n_{j-1}, u^n_j\right) - \nu^n_{j-1/2}\,u_R\left(\nu^{n,+}_{j-1/2}, u^n_{j-1}, u^n_j\right)\right).$$

[Chalons, Goatin, 2008], [Blandin, Work, Goatin, Piccoli, Bayen, SIAP, 2011]

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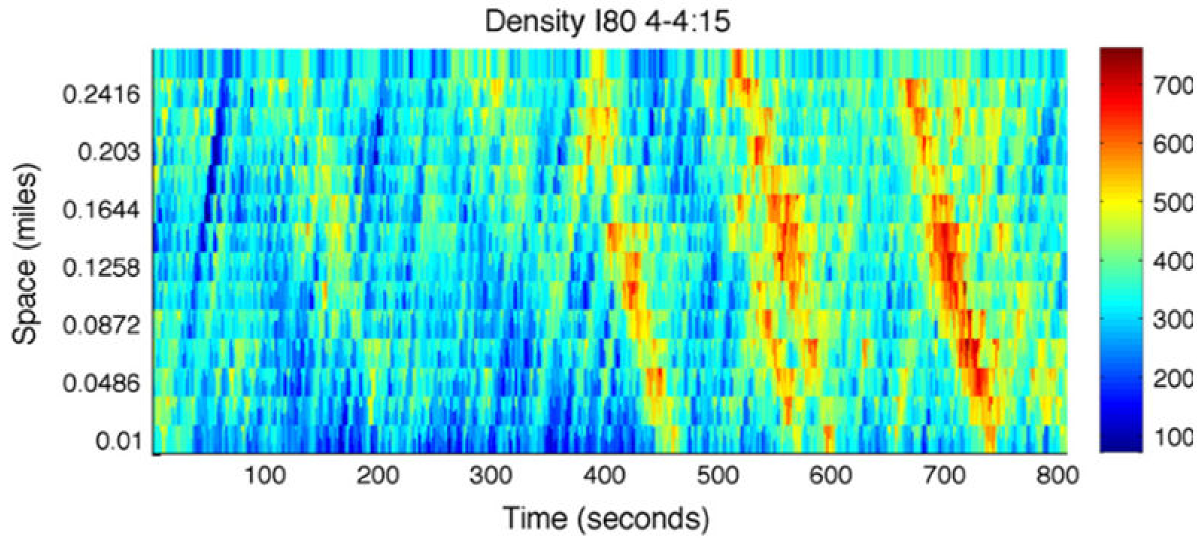
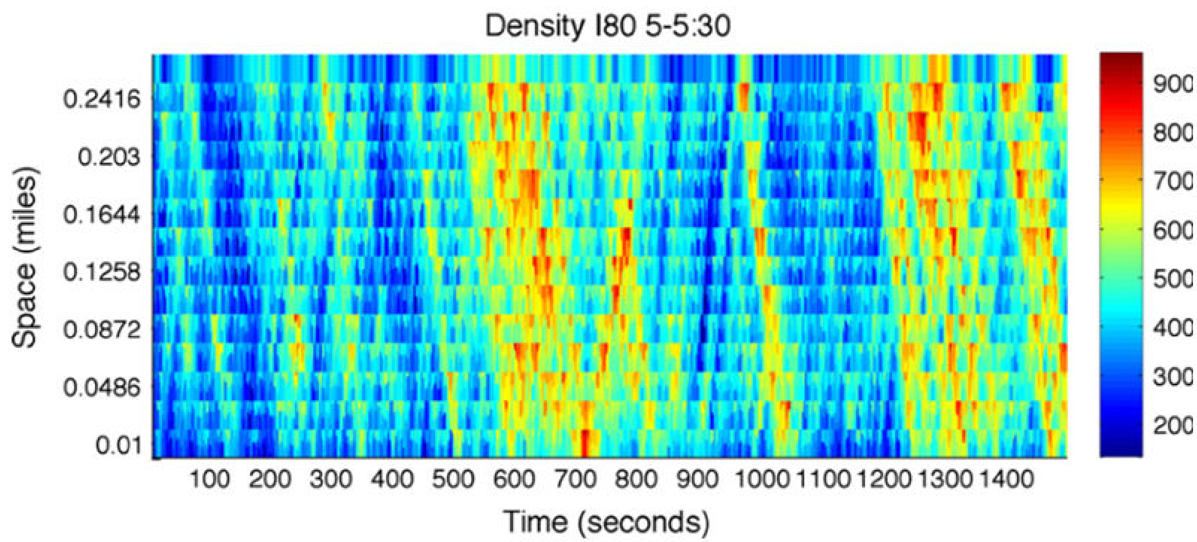
NGSIM dataset

- Vehicle trajectories recorded at a 0.1 sec resolution on a 0.34 miles stretch of freeway I-80 Northbound, with six lanes including a HOV lane, from 4 pm to 4:15 pm, and from 5 pm to 5:30 pm

- Dataset is discretized using 1s time steps and 100 feet cells





Parameter calibration

- Minimization of L1 error metric for density, flow and speed

- I-80 4-4:15

Error variable	Optimal parameters					L^1 Error		
	k_j^*	$v_{\max}^*$	w^*	$p_{\min}^*$	$p_{\max}^*$	k	q	v
Density (k)	160	50	14	−0.01	0.94	**0.152**	0.185	0.164
Flow (q)	150	40	15.5	−0.21	0.77	0.164	**0.176**	0.157
Speed (v)	120	40	13.5	−0.99	0.99	0.160	0.190	**0.137**

- I-80 5-5:30

Error variable	Optimal parameters					L^1 error		
	k_j^*	$v_{\max}^*$	w^*	$p_{\min}^*$	$p_{\max}^*$	k		v
Density (k)	190	55	13	−0.25	0.95	**0.130**	0.170	0.165
Flow (q)	180	55	13.5	−0.75	0.45	0.141	**0.164**	0.170
Speed (v)	150	55	13	−0.05	0.35	0.145	0.172	**0.161**

Parameter calibration

- Model performance

	Density	Flow	Speed
(a) Optimal density parameters			
PTM	0.139	0.167	0.165
CTM	0.146	0.242	0.227
(b) Optimal flow parameters			
PTM	0.141	0.173	0.163
CTM	0.146	0.195	0.191
(c) Optimal speed parameters			
PTM	0.142	0.171	0.165
CTM	0.147	0.190	0.189

Parameter calibration

- Model performance

	Density	Flow	Speed
(a) Optimal density parameters			
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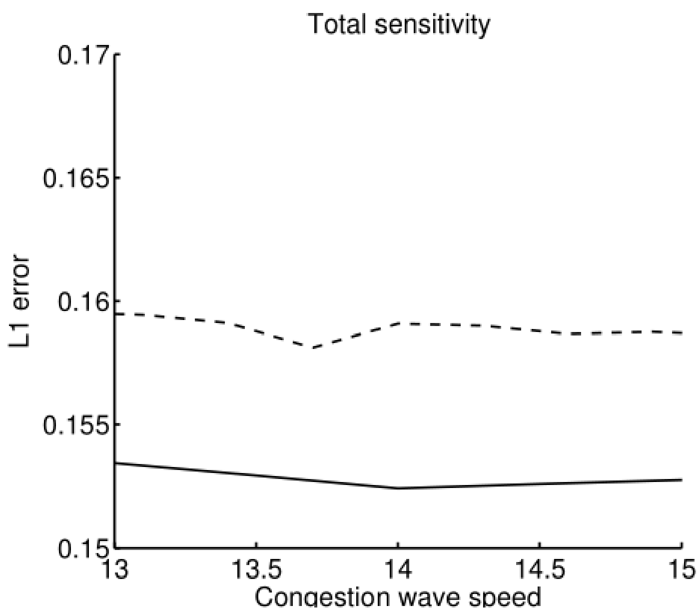
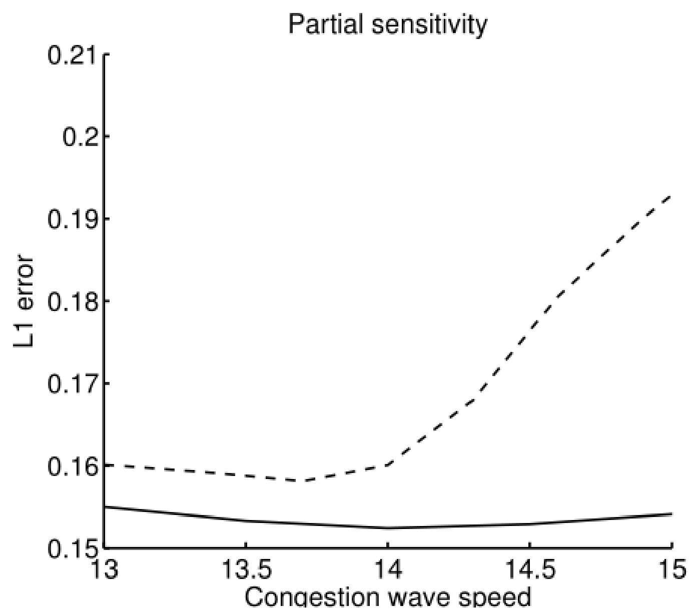
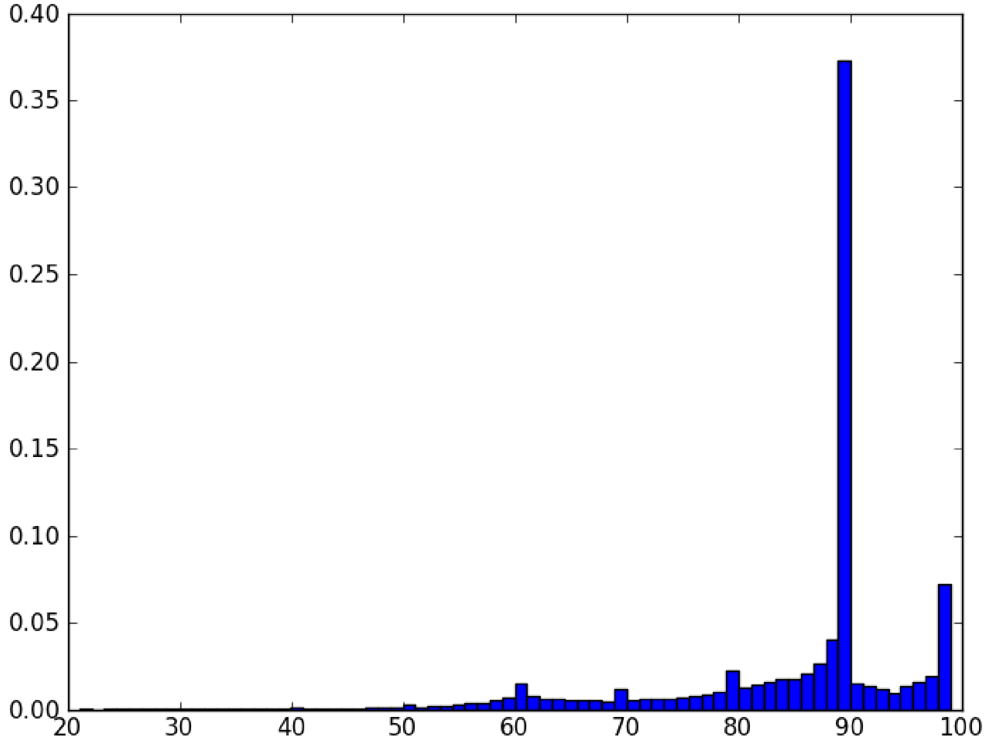
- Sensitivity to model parameters





Speed distribution on a link

Probability



Speed (kph)

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IBM Research Collaboratory – Singapore

A general phase transition model for traffic flow on networks

Traffic Modeling and Management: trends and perspectives,
VI Workshop on Mathematical Foundations of Traffic (WMFT) ,
Sophia-Antipolis, March 20-22, 2013