How to solve boundary value problems in unbounded domains?

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Although there exists no infinite volume of material in reality - even the universe is supposed to be finite - boundary value problems in unbounded domains are used as models. As for fluids one may simply think of a submarine in the ocean or a flow through a pipe, where the cross section is small in comparison with the length. Then again numerical computations can only be performed in bounded domains.

To approximate solutions of boundary value problems in unbounded domains there exists mainly two options: either boundary integral methods or the construction of so called artificial boundary conditions. While the first method is often used for exterior boundary value problems, one is stuck with the second option if the boundary of the starting domain Ω is non compact. The basic idea there is to consider the boundary value problem on a bounded domain $\Omega_R \subset \Omega$. It seems reasonable to keep the original partiol differential equation (or a system of them) in the domain Ω_R, but on the truncation surface $\partial\Omega_R\setminus \partial \Omega$ an additional boundary condition has to be imposed - a so called artificial boundary condition. For the numerical calculations
this means one has to deal with two kind of errors: the truncation error and the error due to the discretization of the problem.
Here we deal with the former: How to choose artificial boundary conditions that lead to a well posed problem with a small truncation error?