The fundamental diagram of traffic flow and its role in modeling traffic dynamics

A Seminar Presentation at

Traffic Modeling and Management: Trends and Perspectives

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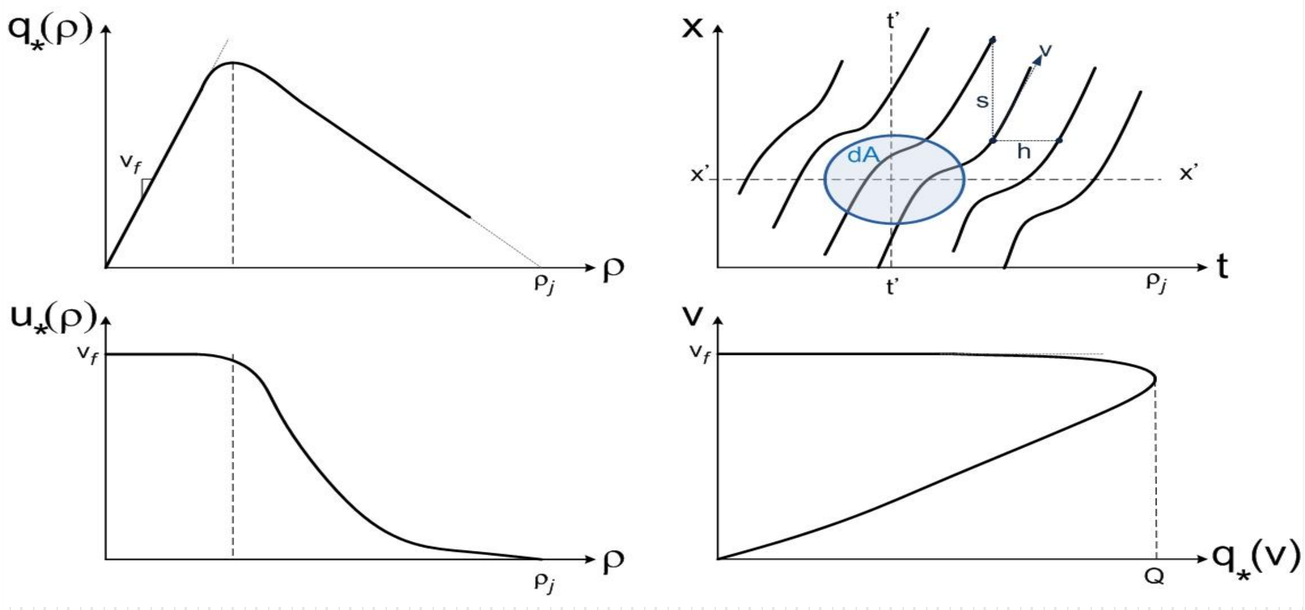
Outline of Presentation

- Concepts of the fundamental diagram (FD) and its role in mathematical traffic flow theory
- Overview of various forms of the FD and their empirical evidence
- The use of boundary conditions for screening FD forms
- The FD's influence on traffic dynamics
- Summary and outlook

The fundamental diagram of traffic flow

- First coined by Haight (1963),it refers to the flow-density curve either observed empirically or obtained from car-following models at that time
- In this talk we use the term to refer *any* of the pair-wise relations between flow (headway), density (spacing), and speed either at a fixed location, or for a moving platoon of vehicles



The Fundamental Diagram (FD)

- Embodies driver behavior that separates traffic flow from other material fluids
- Forms the foundation of some transportation applications (e.g., highway capacity and level of service analysis)
- Permeates in all levels of mathematical description of traffic flow
  - In microscopic, they are linked to steady-state behavior of car-following or CA models, or enter these models a priori
  - In macroscopic or mesoscopic, it enters into the relaxation process of the acceleration or "momentum" equation

The role of the FD in traffic models

- Microscopic
  - Modified Pipes' model
  - Newell' Model
  - Bando' model

$$\dot{x}_n = \min\left\{v_f, \left(s_n(t) - l\right)/\tau\right\}$$

$$\dot{x}_n(t+\tau) = v_f\left[1 - \exp\left\{-\lambda(s_n(t) - l)/v_f\right\}\right]$$

$$\ddot{x}_n(t) = a\left[\left(u_*(s_n) - \dot{x}_n\right)(t)\right], a = 1/\tau$$

$$\rho = 1/s, u_*(s) = v_*(\rho), q = \rho v, q_*(\rho) = \rho v_*(\rho)$$

- Macroscopic continuum
  - LWR model
  - Payne-Whitham model
  - Aw-Rascle, Zhang model

$$\rho_t + q_*(\rho)_x = 0$$

$$\rho_t + (\rho v)_x = 0, \quad v_t + (vv)_x + \frac{c_0^2}{\rho}\rho_x = \frac{v_*(\rho) - v}{\tau}$$

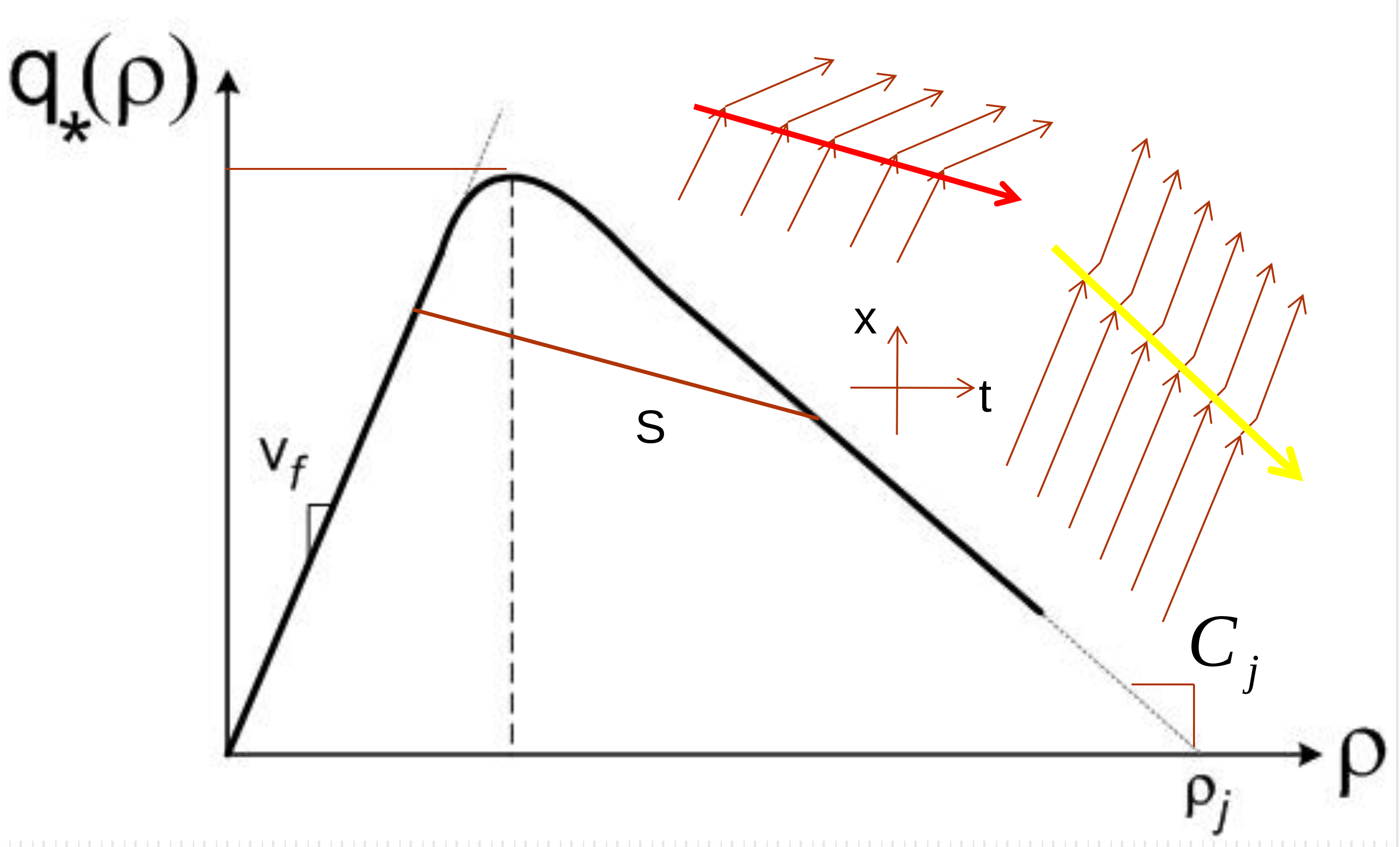
$$\rho_t + (\rho v)_x = 0, \quad v_t + \left(v - c(\rho)\right)v_x = \frac{v_*(\rho) - v}{\tau}$$

$$c(\rho) = -\rho v_*'(\rho)$$

FD and traffic waves

$$\rho_t + q_*(\rho)_x = 0$$

$$\frac{dx}{dt} = \frac{dq_*}{d\rho}, \quad \frac{d\rho}{dt} = 0$$

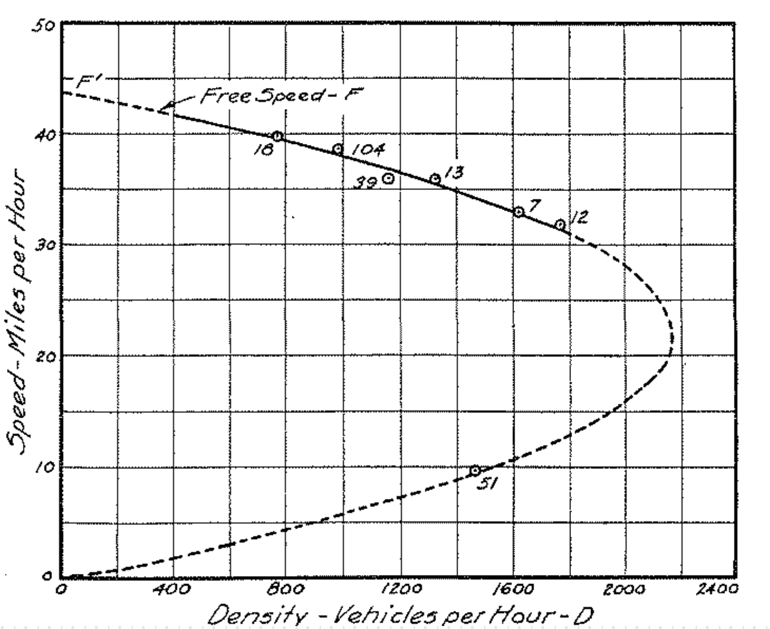


The FD comes in many forms

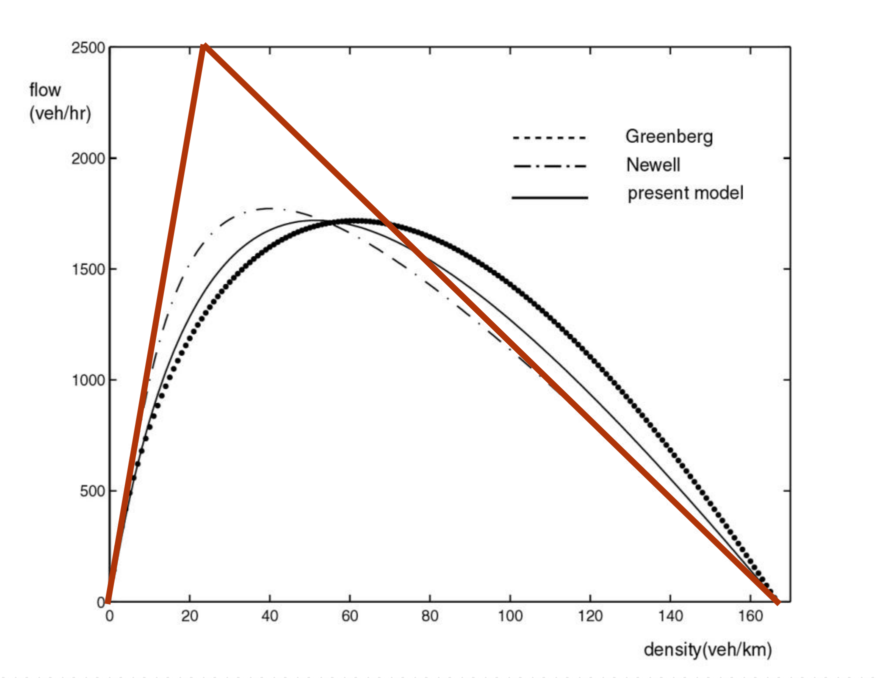
- Continuous/smooth and concave
- Continuous/smooth and concave-convex
- Discontinuous, piece-wise smooth (possibly multi-valued)
- Multi-phase, set-valued

Continuous/smooth, concave FD

$$u_*(s) = v_f\left[1 - \exp\left\{-\lambda(s-l)/v_f\right\}\right]$$

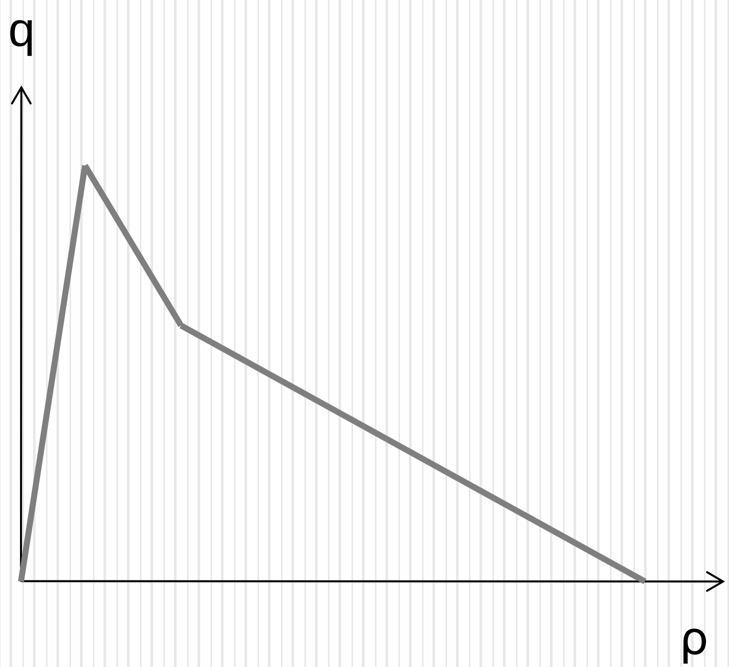


Greenshields (1935)

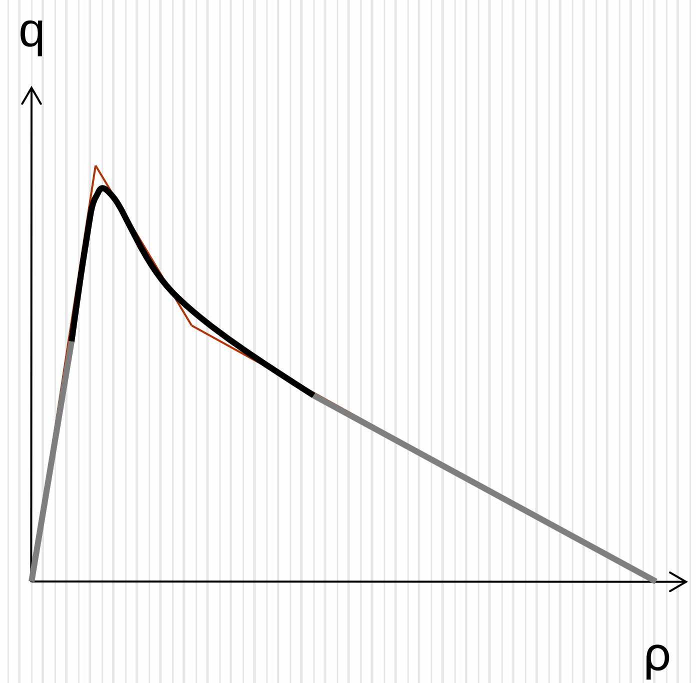


Other common concave FDs

Continuous/smooth, concave-convex

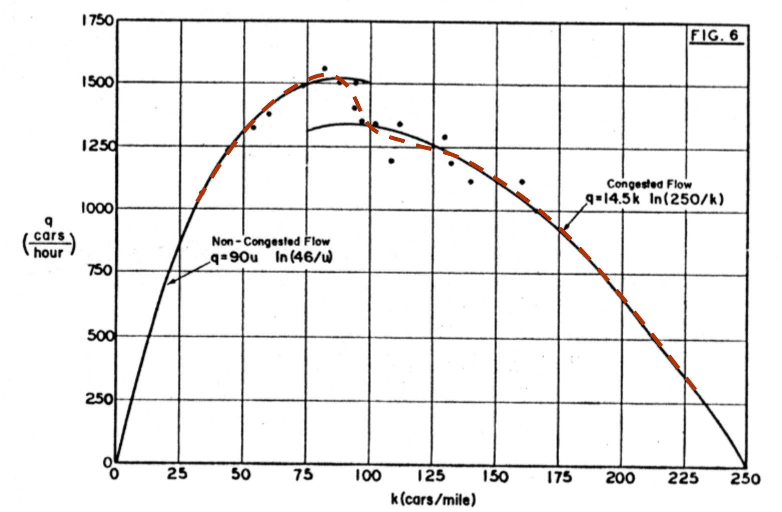


piece-wise linear, concave-convex FD

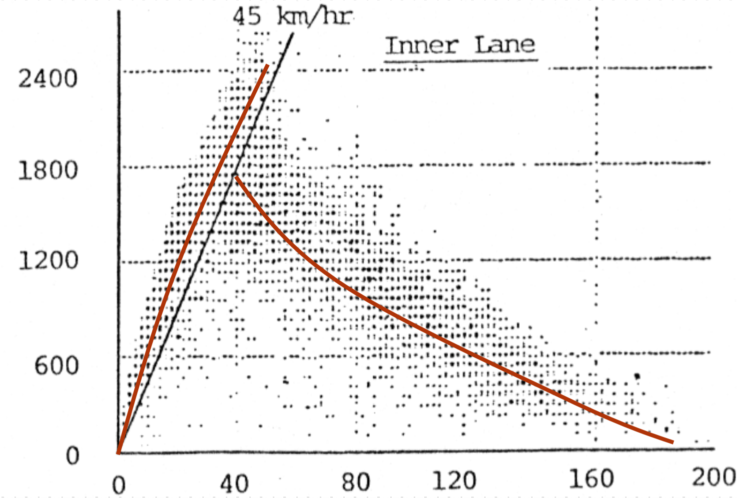


smooth, concave-convex FD
(e.g. Bando 1995)

Discontinuous, piece-wise smooth (possibly multi-valued)

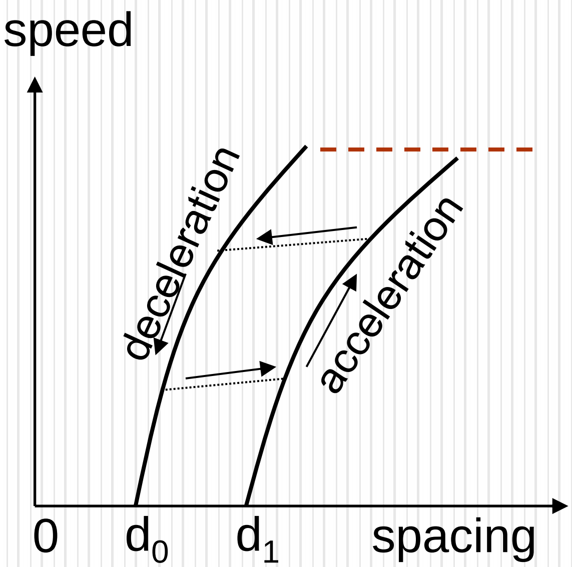


Eddie (1961)

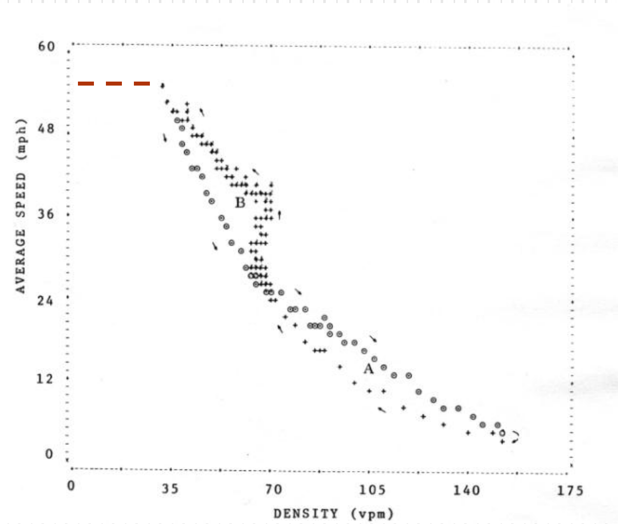


Koshi et al. (1983)

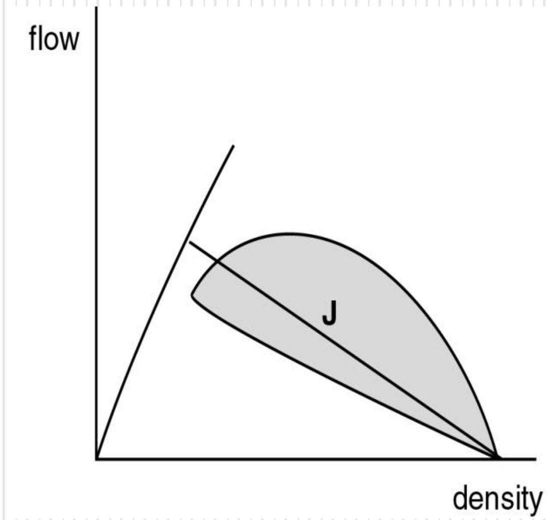
Multi-phase, set-valued



Newell(1962)



Treiterer and Myers(1974)



Kerner (1998), reproduced

Essential properties of FD

- No vehicle, no flow
- Physical limit on speed (vf)
- Physical storage limit (jam density)
- Jam packed, no velocity
- Physical bound on flow

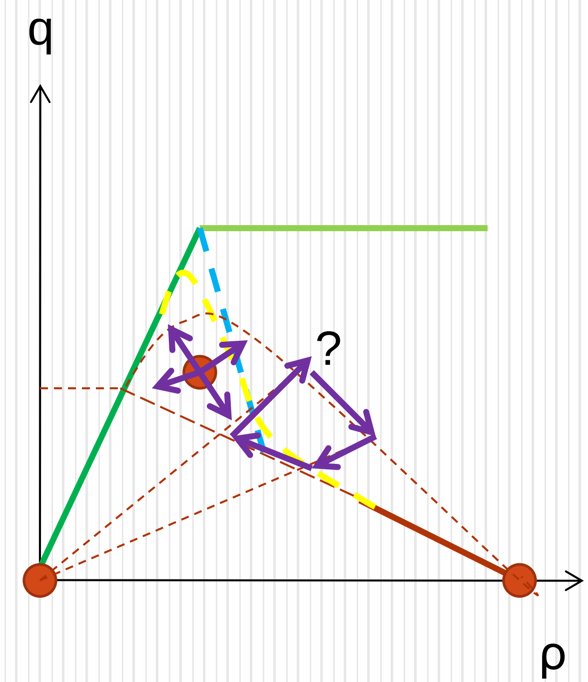
  vf / min safe spacing (=vf*reaction1+veh length)
- Jam dissipation
  - Wave speed

    Vehicle length / reaction time2

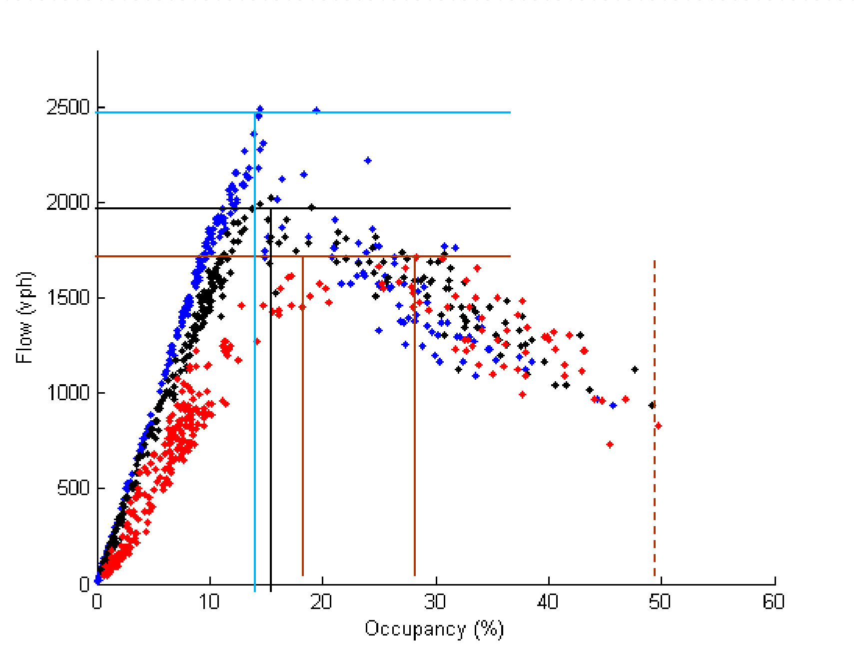
    (-) 6 meter / 1 sec = 21.6 km/hr = 13.4 mph

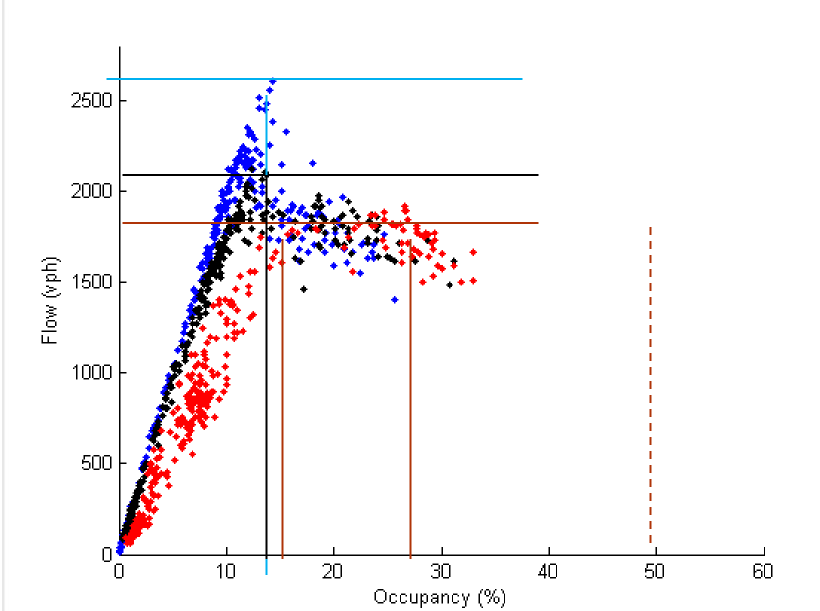
    (-) 6 meter / 1.8 sec =12 km/hr = 7.5 mph
  - Flow discharge rate

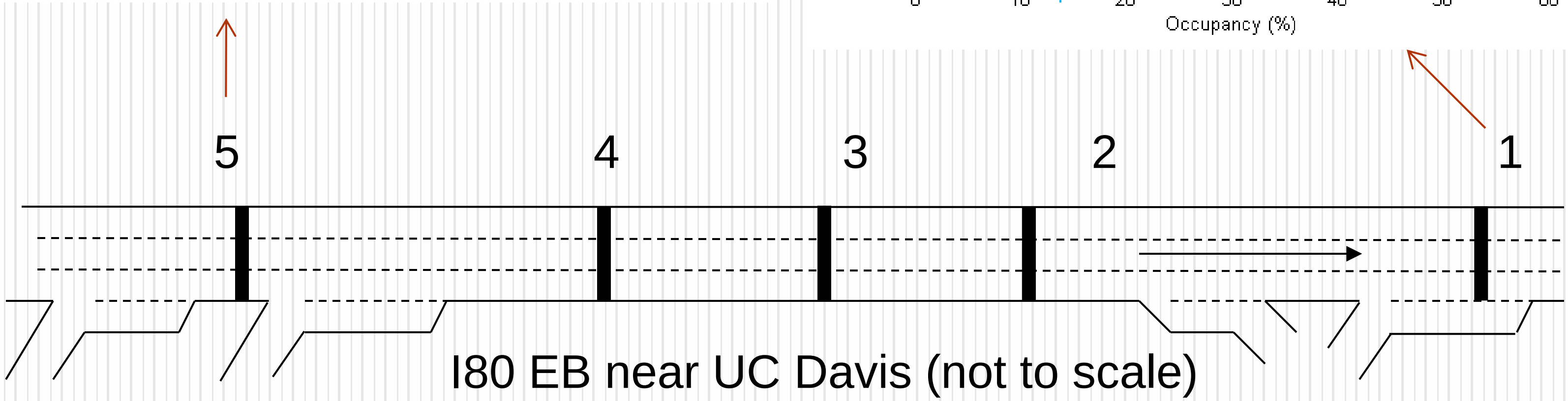
    2 seconds headway => 1800 vph



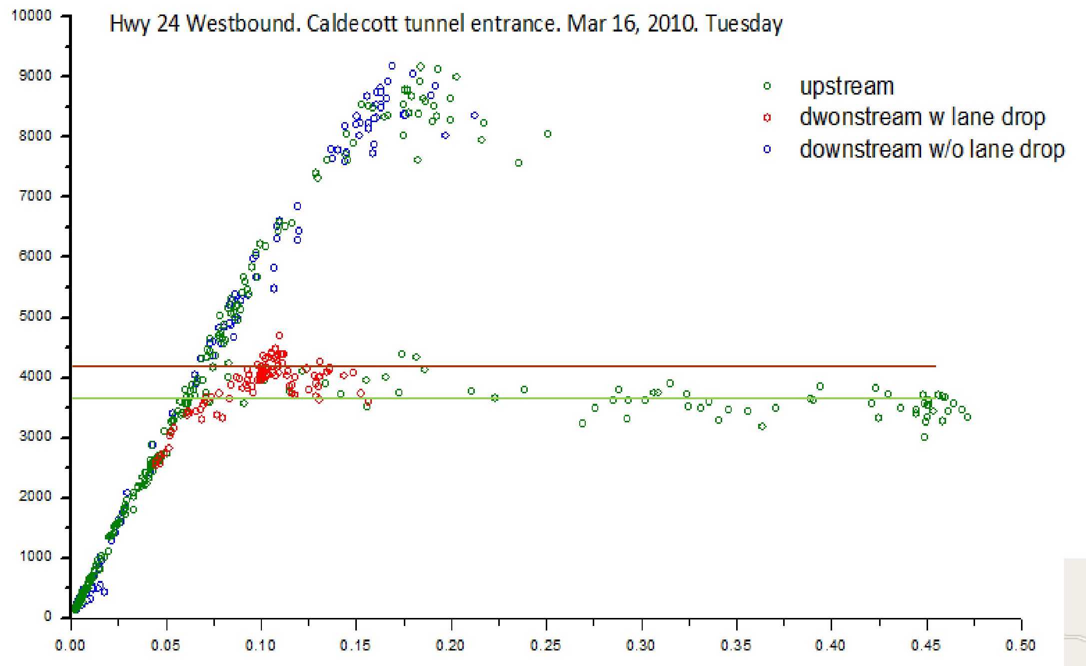
Empirical evidence: fixed locations

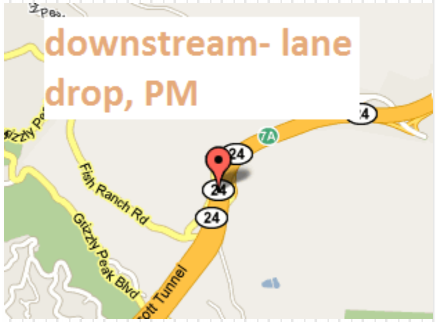


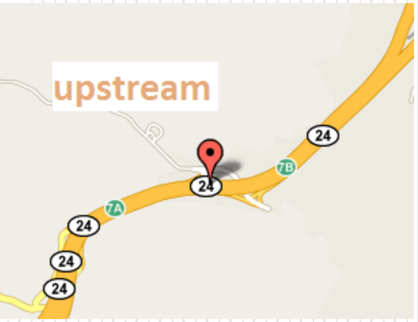




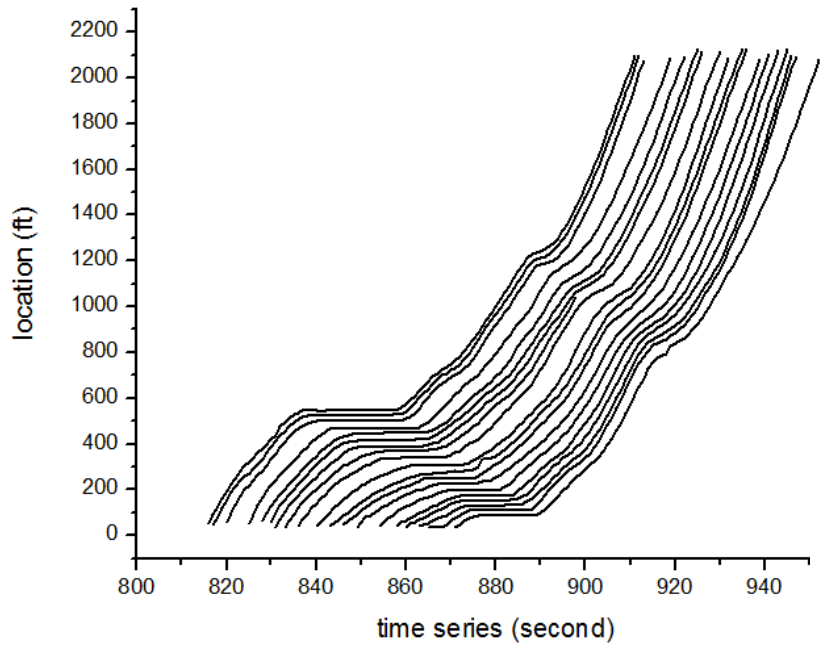
Empirical evidence: bottleneck effects



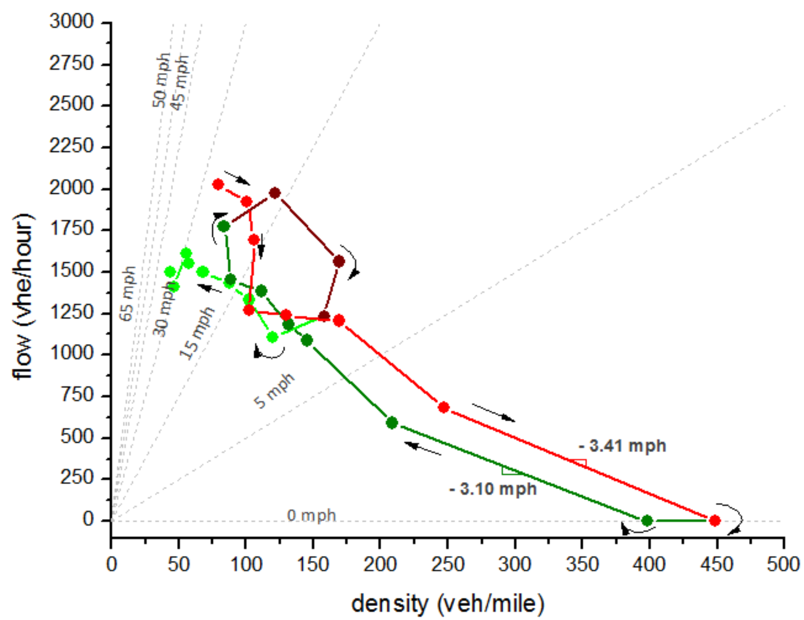




Empirical evidence: moving platoons (1)

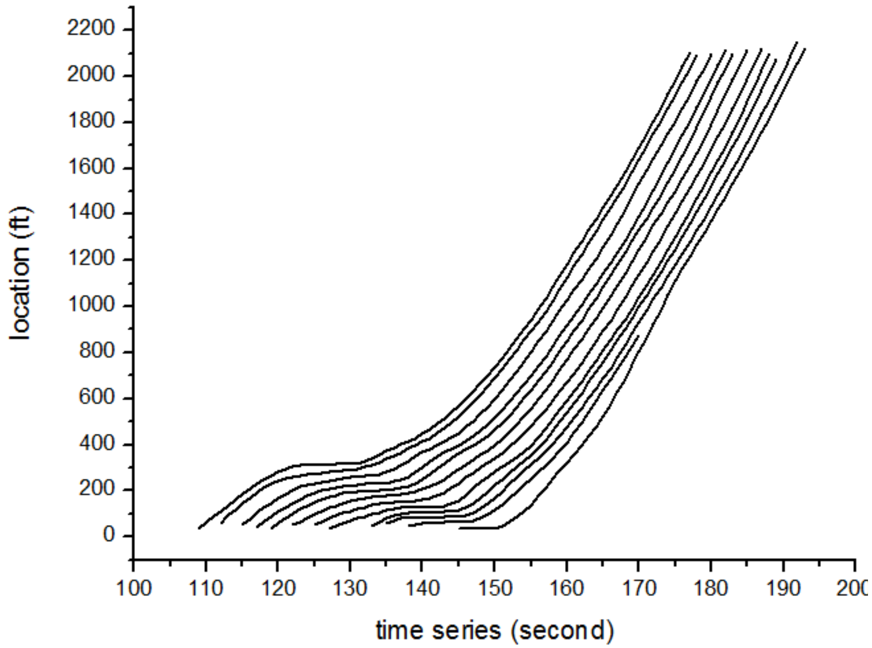


Vehicle platoon traveling through two shock waves

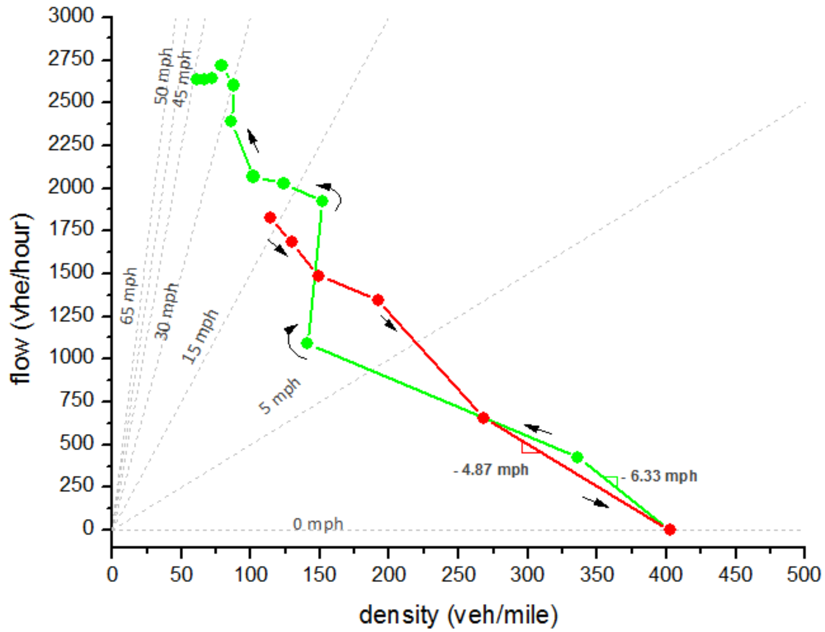


flow-density phase plot

Empirical evidence: moving platoons (2)

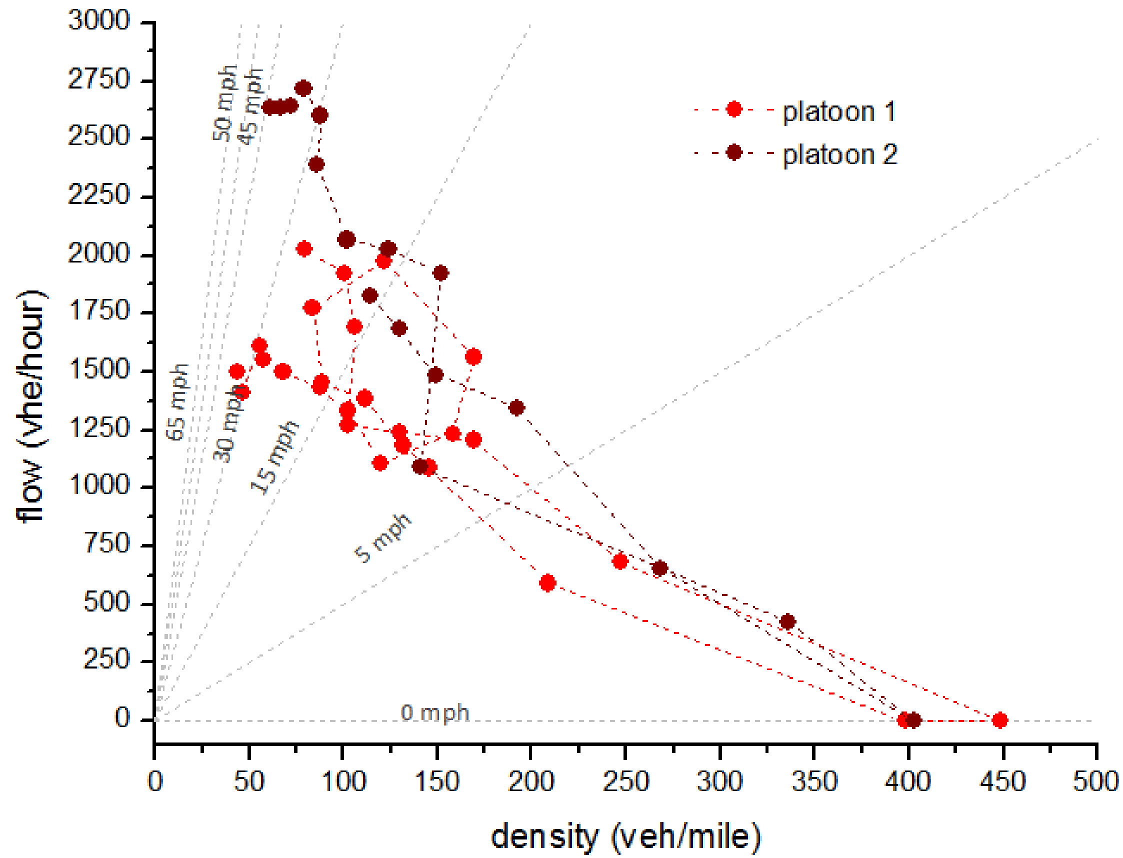


Vehicle platoon traveling through one shock wave

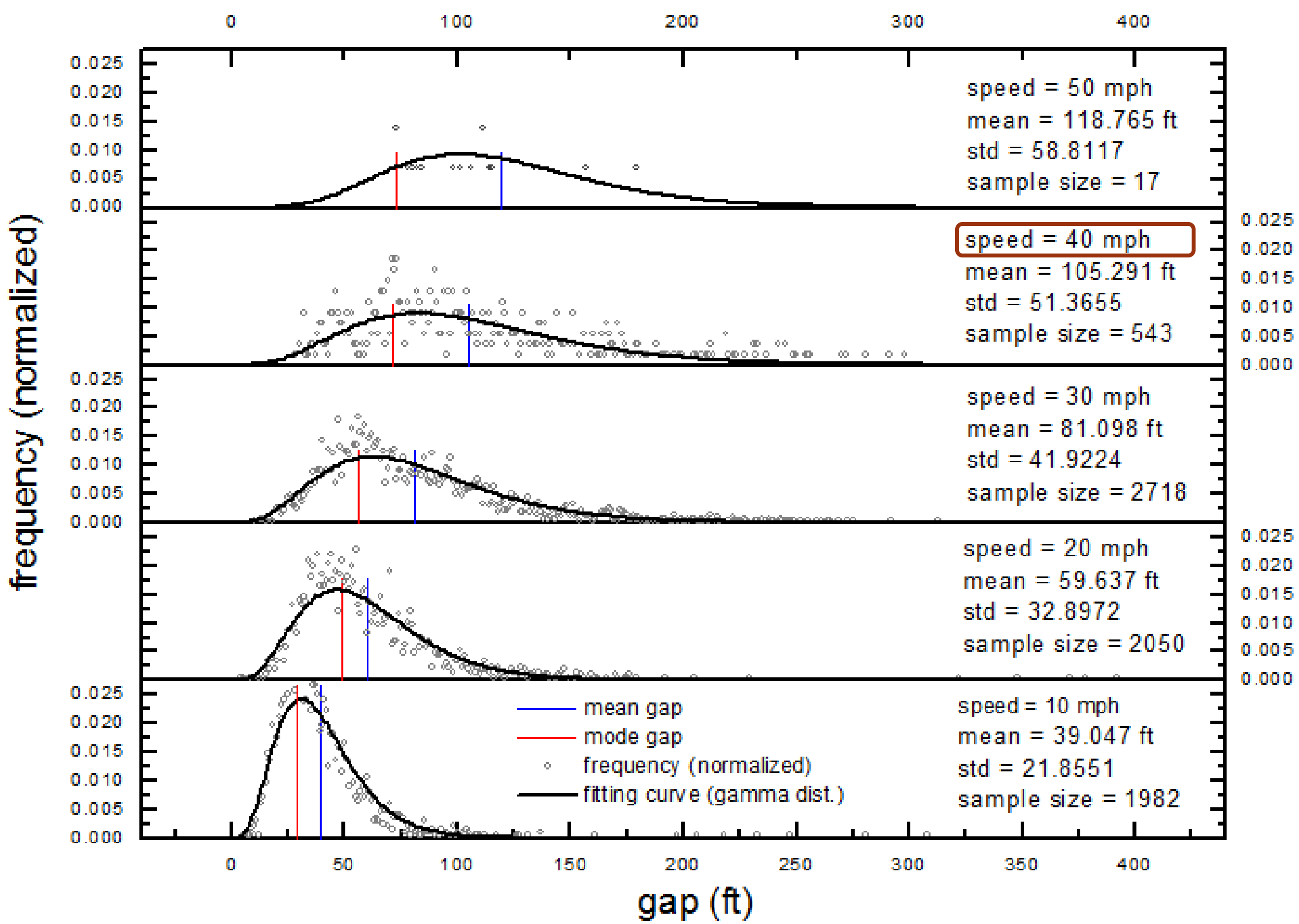


flow-density phase plot

Phase plots of both platoons

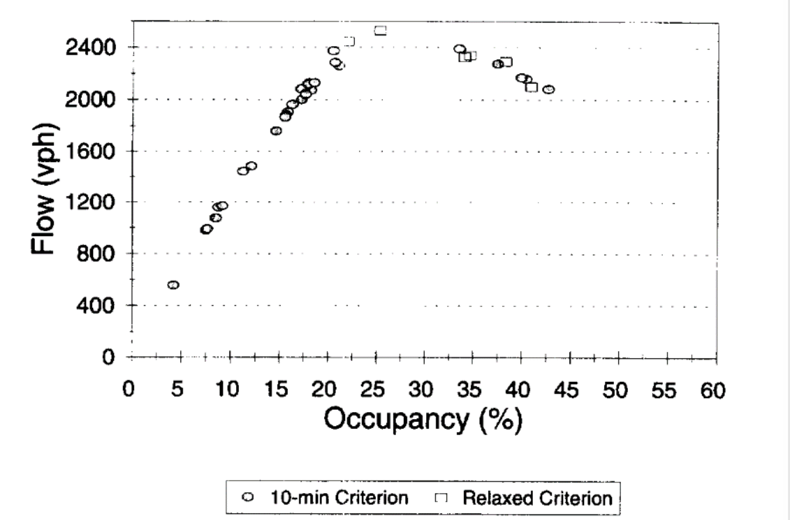


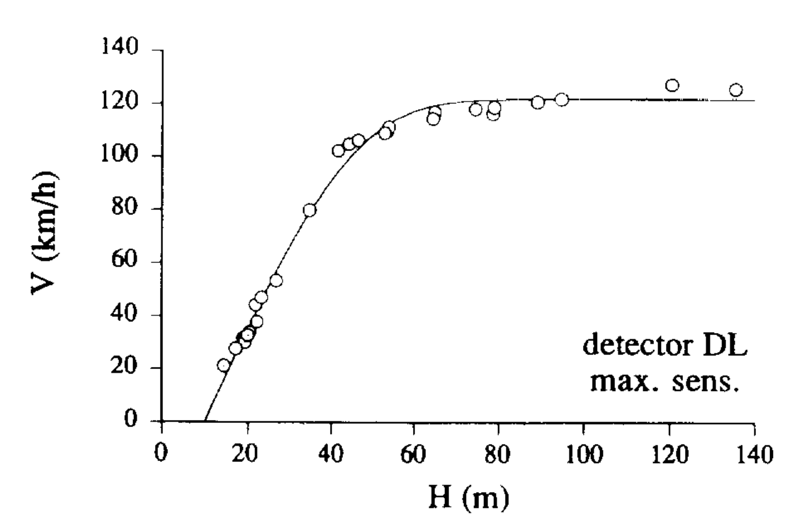
Empirical evidence: stochasticity in a large ensemble



Empirical evidence: stationary flow

$$V = V_f \left\{ 1 - \exp\left[1 - \exp\left(\frac{|C_j|}{V_f} \left(\frac{H}{H_j} - 1 \right) \right) \right] \right\}$$



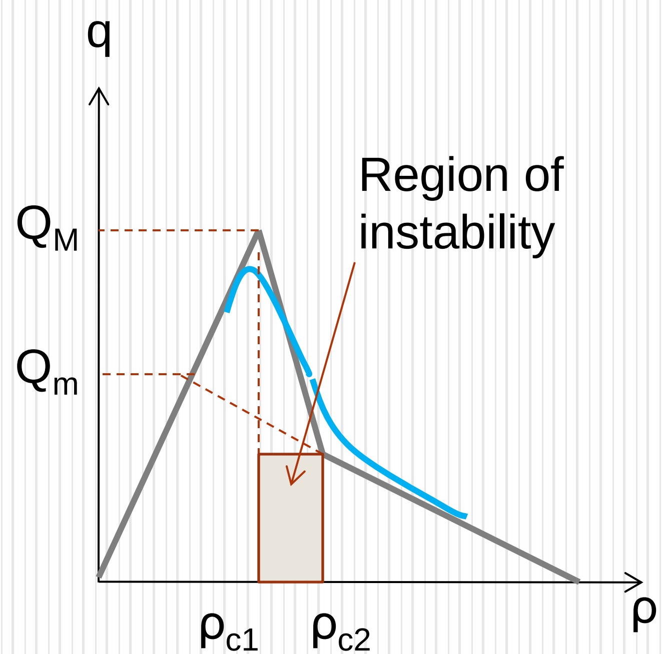


Cassidy (1998) [QEW data]

Del Castillo (1995) [Freeway A2 Amsterdam data]

Which form after all?

- The form we choose must
  - Respect the boundary conditions at zero and jam densities
  - Have physically meaningful and calibratable parameters
  - Produce reasonable capacity
  - Capture the flow drop at free-flow to congestion transition
  - Be simple, yet produce the essential dynamic features of traffic flow, such as accel/decel asymmetry, when being incorporated in traffic models
- The answer: single-valued concave-convex FD



Linear stability analysis

Newell $\dot{x}_n(t+\tau) = v_f\left[1 - \exp\left\{-\lambda(s_n(t) - l)/v_f\right\}\right]$

Bando $\ddot{x}_n(t) = a\left[\left(u_*(s_n) - \dot{x}_n\right)(t)\right],\ a = 1/\tau$

Payne-Whitham $\rho_t + (\rho v)_x = 0, \quad v_t + (vv)_x + \frac{c_0^2}{\rho}\rho_x = \frac{v_*(\rho) - v}{\tau}$

Obtained by perturbing a steady-state solution and study the growth/decay of the perturbation over vehicle numbers (micro) or time-space (macro) using Fourier transforms

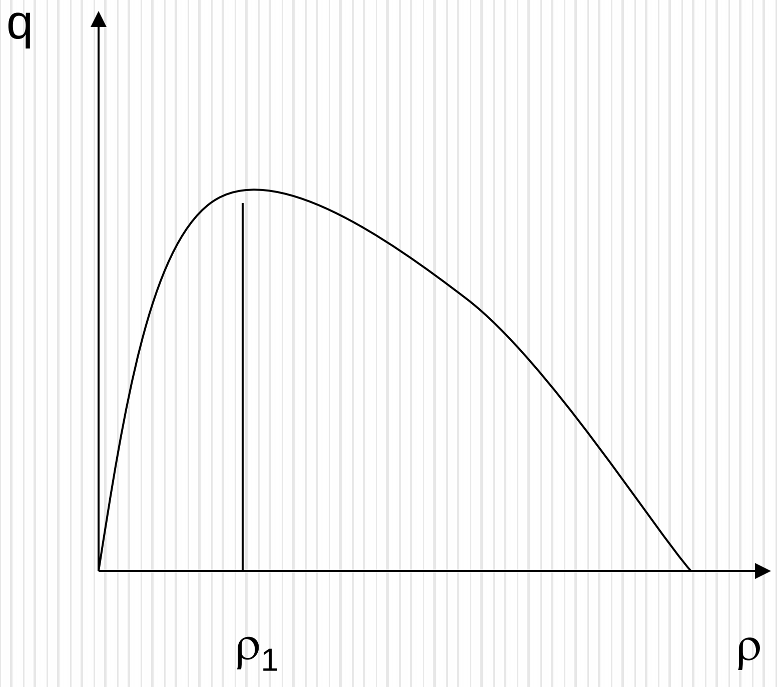
Instability and its relation to the shape of the fundamental diagram

$-\bar{\rho} v_*'(\bar{\rho}) > c_0$ $\quad -\bar{\rho}^2 v_*'(\bar{\rho}) = u_*'(\bar{s})$.

(PW)

$$u_*'(\bar{s}) > \frac{1}{2\tau}$$ (Payne, Newell)

$$u_*'(\bar{s}) > \frac{a}{2}$$ (Bando)



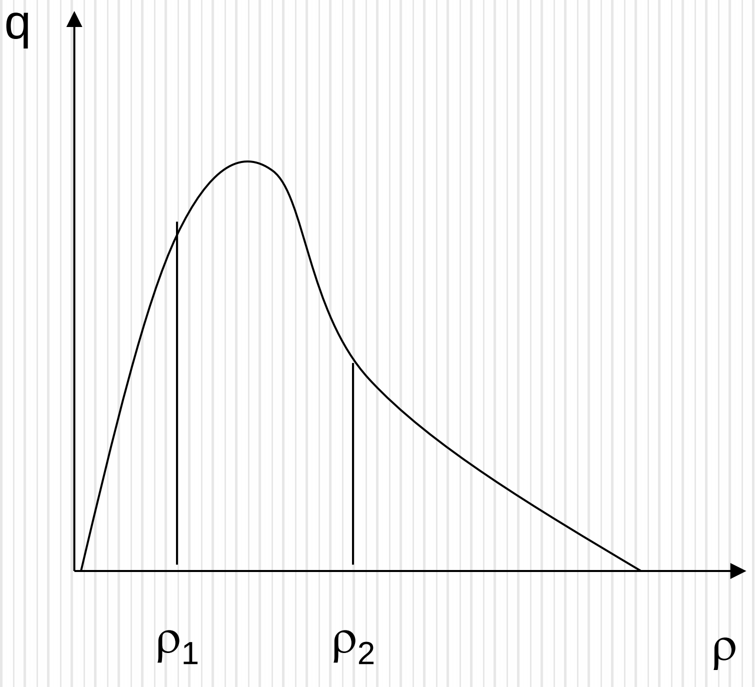
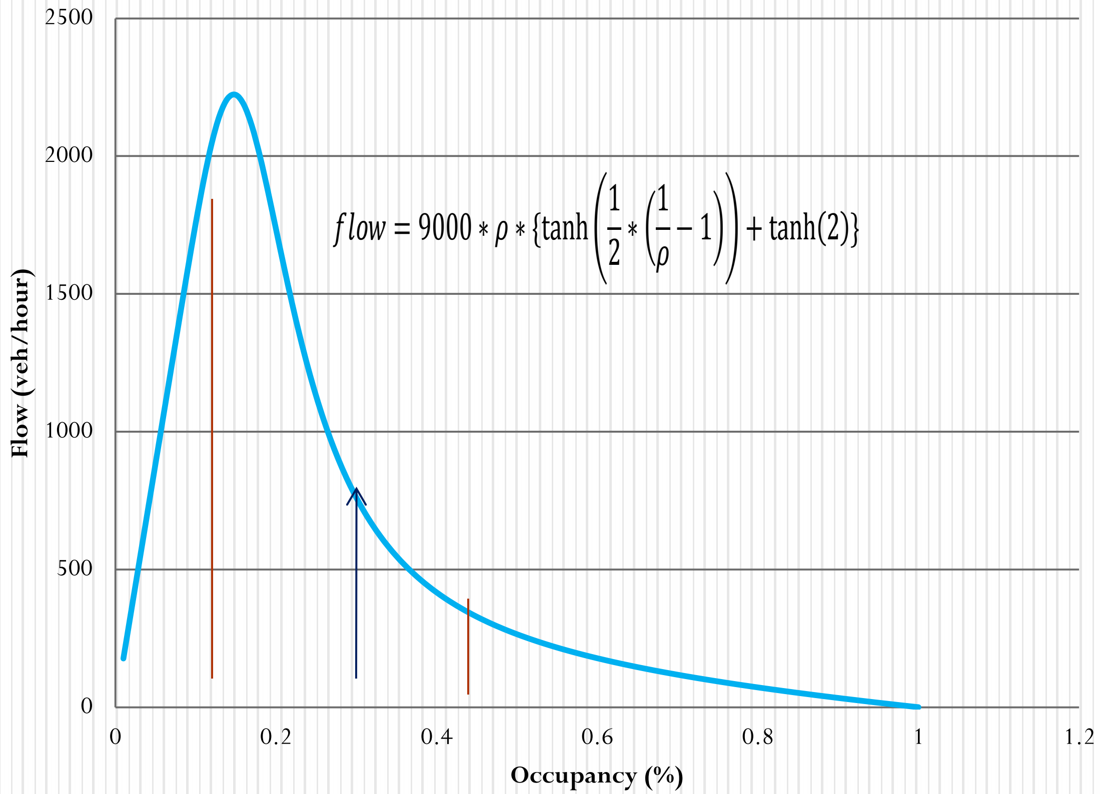


Illustration: cluster solutions in the Bando model with a non-concave FD



L=6,000 m, l=6m, T=600s, dt=0.1s, ρj=167 veh/km, N=300 veh, average gap=14 m, Avg. occ is 0.3 . Vehicles randomly placed on circular road with 0 speed

Illustration: cluster solutions in the Bando model with a non-concave FD

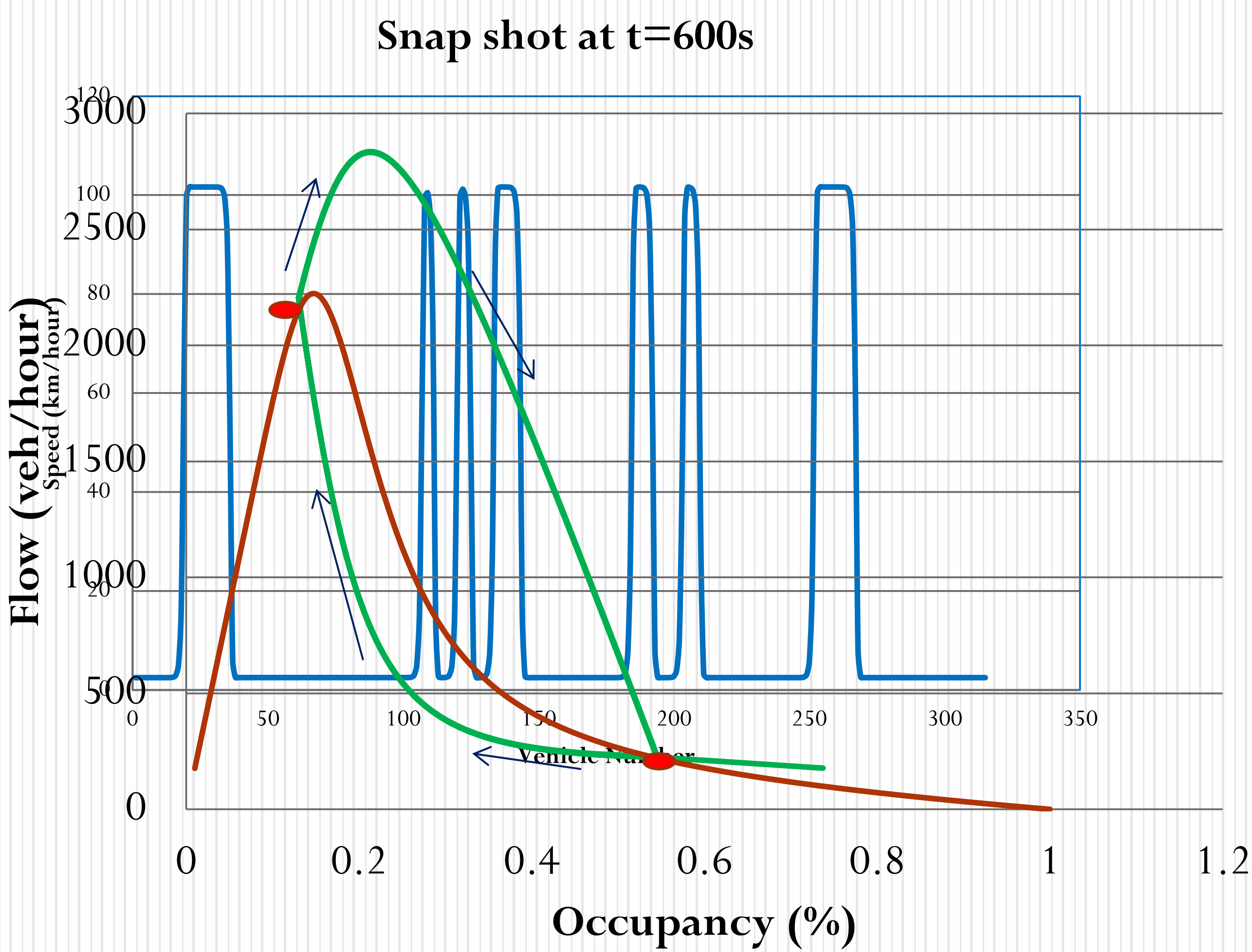


Illustration: cluster solutions in PW model with a non-concave FD

$$\begin{aligned} \rho(x,0) &= \rho_h + \Delta\rho_0 \cos(2\pi x/L), \quad x \in [0,L], \\ v(x,0) &= v_*(\rho_h) + \Delta v_0 \cos(2\pi x/L), \quad x \in [0,L]; \end{aligned}$$

L=22.4km, T=0.7 hr
τ=5s,

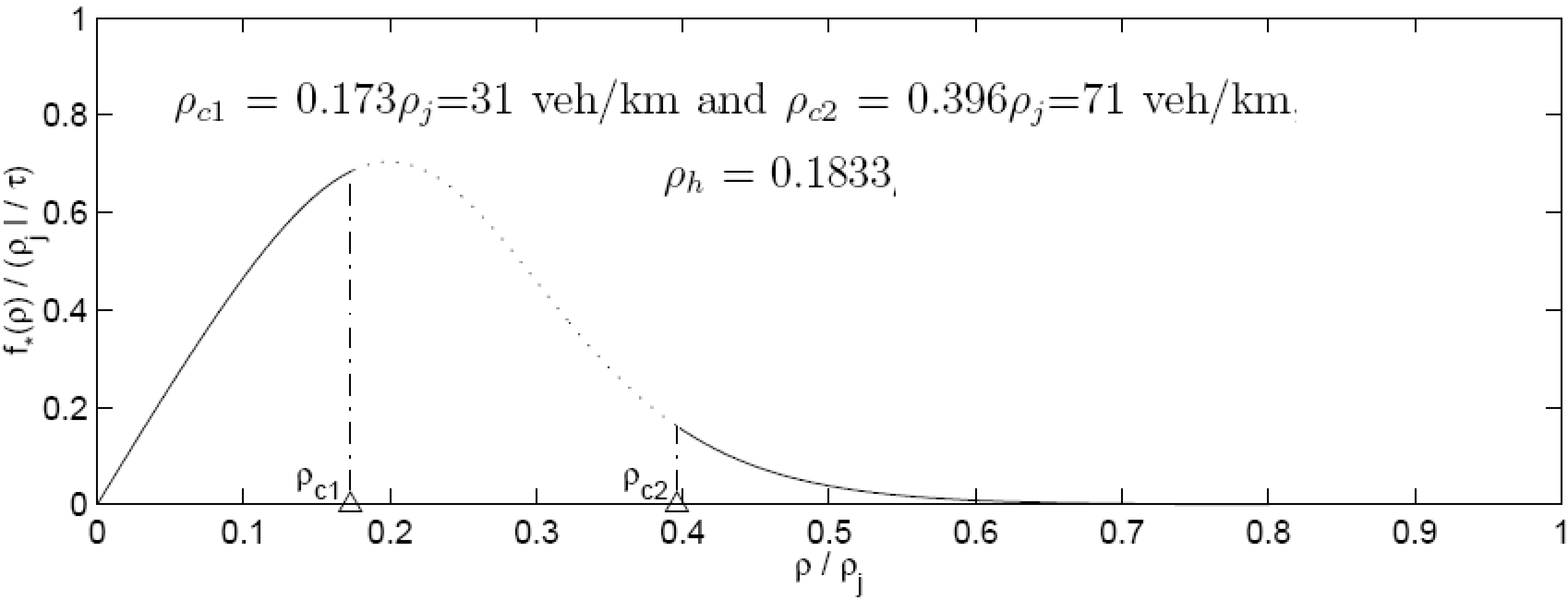
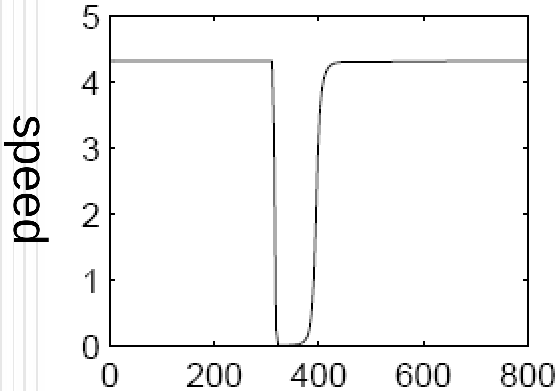
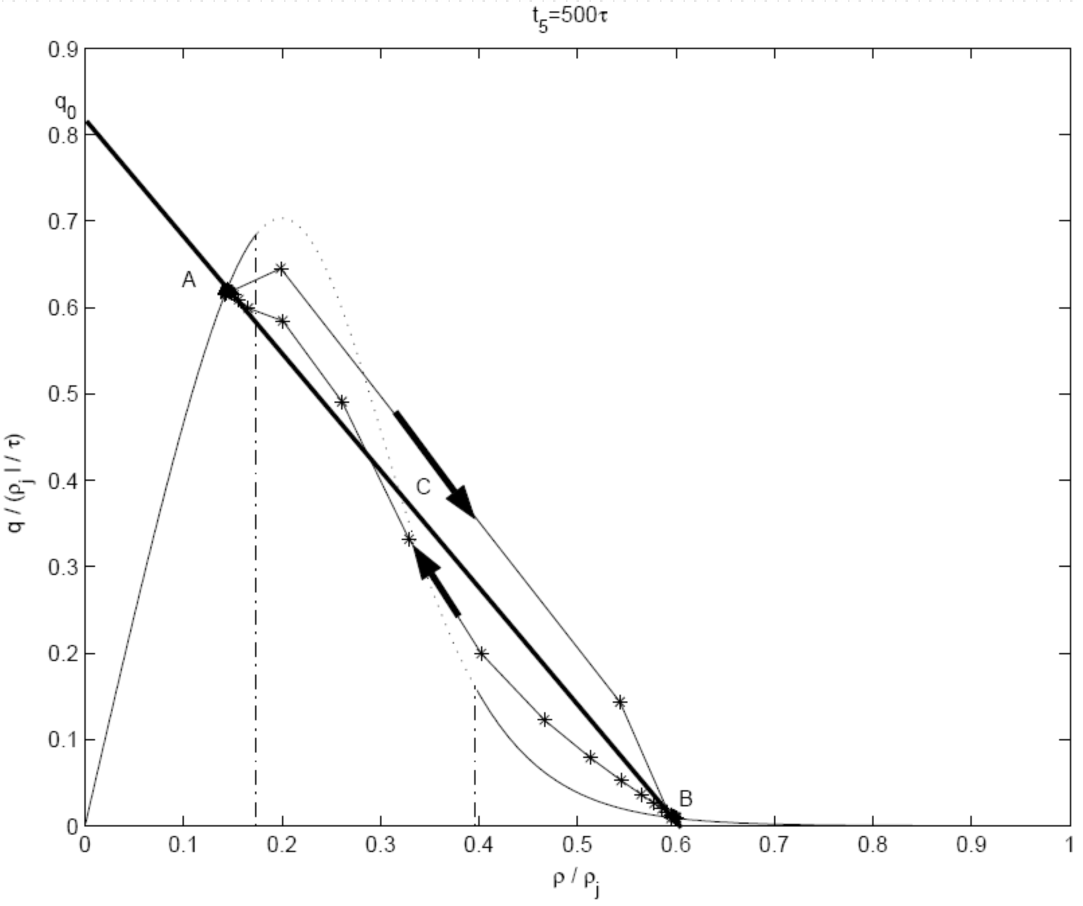


Illustration: cluster solutions in PW model with a non-concave FD

Time=500 τ



location



Concluding remarks

- The FD plays a pivotal role in traffic flow models and applications
- Diverse forms have been suggested in response to observed complexity in traffic flow
- Showed that the single-valued, continuous, non-concave FD, exhibits complexity with simplicity, and can be adopted to model quasi steady state flow (transitions between equilibrium states)

Concluding remarks

- The FD plays a pivotal role in traffic flow models and applications
- Diverse forms have been suggested in response to observed complexity in traffic flow
- Provided a set of criteria for assessing FD forms
- Showed that the single-valued, continuous, non-concave FD, exhibits complexity with simplicity, and can be adopted to model quasi steady state flow (transitions between equilibrium states)

Concluding remarks-continued

- One can extend the modeling capabilities of the simpler FDs by
  - ascribing each lane a FD to reflect lane differences in macro models
  - ascribing each group of drivers/vehicles a unique FD to reflect diversity in driver/vehicle units in micro models
- It is possible to incorporate more complex FDs, such as FDs with hysteresis, into mico or macro models, or in the extreme ascribe to each vehicle a unique complex FD.
- Realism, tractability, numerics

Discovery of a New Hysteresis