Interaction of pedestrians and road-traffic

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pedestrians and cars

Outline

Cars

Single road - Lighthill-Witham

$$\partial_t \rho + \partial_x \left(v_{max} \rho \left(1 - \frac{\rho}{\rho_{max}} \right) \right) = 0$$

ρ density of cars, v_{max} maximal velocity

Network

For networks we use the coupling conditions of Coclite, Garavello and Piccoli.

Pedestrians

Hughes model

$$\partial_t \gamma + \nabla \left(\gamma \frac{\nabla \Phi}{\|\nabla \Phi\|} w(\gamma) \right) = 0 \ , \quad w(\gamma) = w_{max} \left(1 - \frac{\gamma}{\gamma_{max}} \right) \ ,$$

$$\|\nabla \Phi\| \ w(\gamma) = 1$$

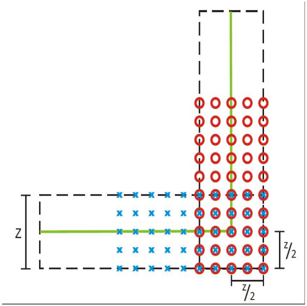
γ density of pedestrians, w_{max} maximal walking speed,

$\nabla \Phi(t, \cdot) : \mathbb{R}^2 \to \mathbb{R}^2$ direction of fastest path .

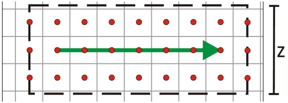
Projections

Map the respective densities on the ’other’ domains

recover the road as a $2D$ domain filled with cars



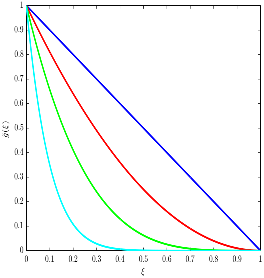
project the density of the pedestrians on the road



Coupling functions

$$v_{max} = v_{max}(\gamma) = v_{max}\left(1 - \frac{\gamma}{\gamma_{max}}\right)^{\alpha}$$

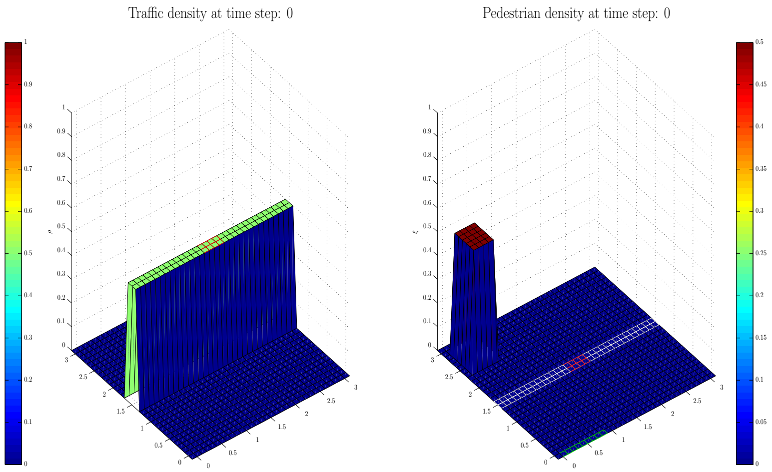
$$w_{max} = w_{max}(\rho) = w_{max}\left(1 - \frac{\rho}{\rho_{max}}\right)^{\beta}$$



Idea as in Lighthill-Witham:

the higher the density the slower the flow.

Crosswalk


Traffic density at time step: 0
Pedestrian density at time step: 0
ρ
ξ

Is this the only choice?

Cars see pedestrians

density: more pedestrians on the road $\rightarrow$ cars drive slower

Pedestrians see cars

- density (many cars $\rightarrow$ do not want to cross)

  $$w_{max}(\rho) = w_{max} \cdot \left(1 - \frac{\rho}{\rho_{max}}\right)^{\beta}$$

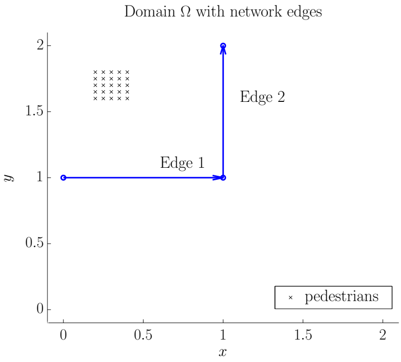
- velocity (fast cars $\rightarrow$ do not want to cross)

  $$w_{max}(\rho) = w_{max} \cdot \left(\frac{\rho}{\rho_{max}}\right)^{\beta}$$

- flux (attempt to cross, although there are either very few fast cars or there is a slow jam moving)

  $$w_{max}(\rho) = w_{max} \cdot \left(1 - \left(1 - \frac{\rho}{\rho_{max}}\right)\frac{\rho}{\rho_{max}}\right)^{\beta}$$

Test case

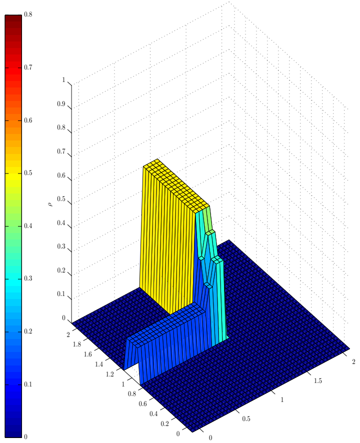


$$v_{max}^1 = 2$$
$$v_{max}^2 = 1$$

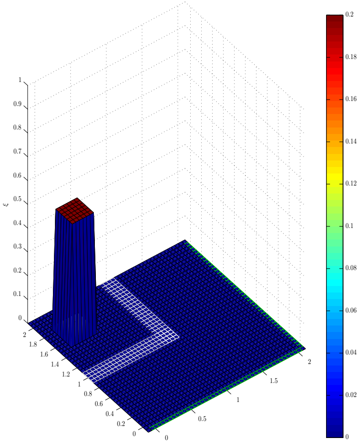
Coupling - density

$$w_{max}(\rho) = w_{max} \cdot \left(1 - \frac{\rho}{\rho_{max}}\right)^{\beta}$$

Traffic density at time step: 0

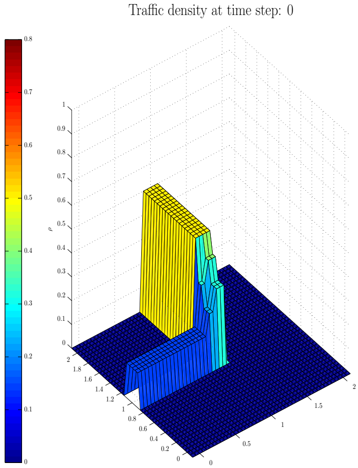


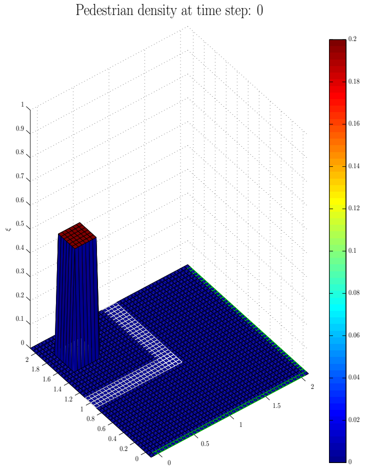
Pedestrian density at time step: 0



Coupling - velocity

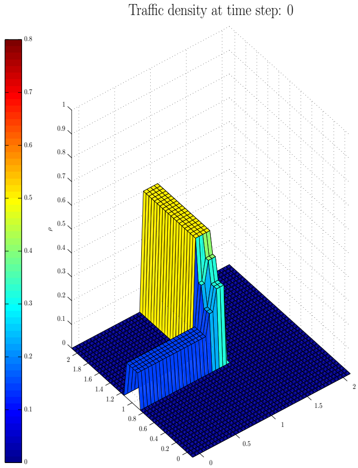
$$w_{max}(\rho) = w_{max} \cdot \left(\frac{\rho}{\rho_{max}}\right)^{\beta}$$

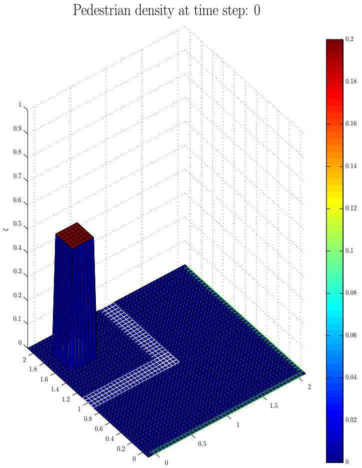




Coupling - flux

$$w_{max}(\rho) = w_{max} \cdot \left(1 - \left(1 - \frac{\rho}{\rho_{max}}\right) \frac{\rho}{\rho_{max}}\right)^{\beta}$$





Which one is the best?

Pedestrians see cars

- density (many cars $\rightarrow$ do not want to cross)
- velocity (fast cars $\rightarrow$ do not want to cross)
- flux (attempt to cross, although there are either very few fast cars or there is a slow jam moving)

Can we improve the situation?

- A second order traffic model, would allow to consider velocity and density simultaniuously.

Small Network