An easy-to-use numerical approach for simulating traffic flow on networks

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Outline of the talk

1. Easy-to-use numerical algorithm based on a multi-path model.

2. Characterization into the known framework (Garavello-Piccoli 2006)
Known theory of traffic flow on networks

Roads network: a network where each edge and each vertex represents respectively an unidirectional road and a junction.

Main references

- M. Garavello, B. Piccoli, Traffic flow on networks, AIMS, 2006
At any time $t$, the evolution of the car density on the network is computed by a two-step procedure:

1) a classical conservation laws is solved at any internal point of the arcs;
2) the densities at endpoints, which correspond to a junction, are computed.
Known Theory

1) On each arc, the density $\rho_r(x, t)$ of all vehicles it is simply given by the entropic solution of

\[
\frac{\partial}{\partial t} \rho_r + \frac{\partial}{\partial x} f(\rho_r) = 0 \quad x \in I_r.
\]

with the flux $f \in C^1([0, \rho_{max}])$ for some maximal density $\rho_{max}$, and

\[
f(0) = f(\rho_{max}) = 0, \quad f \text{ is concave, } \quad f(\sigma) = \max_{\omega \in [0, \rho_{max}]} f(\omega)
\]

2) The computation of densities at endpoints has not in general a unique admissible solution, so that additional constraints must be added.

- Conservation of cars at junctions;
- Drivers behave in order to maximize the flux at junctions;
- Incoming roads are regulated by priorities (right of way).

This second step is performed by a linear programming method.
Multi-path model

Here we study from the numerical point of view a Multi-path model, following the idea that cars moving on the network are divided on the basis of their path i.e. on their origin-destination pair

▶ a modified version of the model proposed in M. Hilliges, W. Weidlich (1955);
Multi-path model

- \( N_P \) number of possible paths on the graph, \( P^1, \ldots, P^p, \ldots, P^{N_P} \) (paths can share some arcs of the networks);
- \( x^{(p)} \) is a generic point along the path \( P^p \) (a specific point on the network is characterized by both the path it belongs to and the distance from the origin of that path);
- \( \rho^p(x^{(q)}, t) \in [0, 1] \) is the density of the cars following the \( p \)-th path at point \( x^{(q)} \) along path \( P^q \) at time \( t > 0 \). (\( \rho^p(x^{(q)}, t) \) is, by definition, strictly positive if \( p = q \). Conversely, if \( p \neq q \), we have \( \rho^p(x^{(q)}, t) = 0 \) if \( x^{(q)} \notin P^p \) and \( \rho^p(x^{(q)}, t) > 0 \) if \( x^{(q)} \in P^p \))
- We define
  \[
  \omega^p(x^{(p)}, t) := \sum_{q=1}^{N_P} \rho^q(x^{(p)}, t),
  \]
  i.e. \( \omega^p(x^{(p)}, t) \) is the sum of all the densities living at \( x^{(p)} \) at time \( t \).
LWR-based model

System of $N_P$ conservation laws with space-dependent and discontinuous flux, for $p = 1, \ldots, N_P$,

$$
\frac{\partial}{\partial t} \rho^p(x^{(p)}, t) + \frac{\partial}{\partial x^{(p)}} \left( \rho^p(x^{(p)}, t) \ v(\omega^p(x^{(p)}, t)) \right) = 0, \quad x^{(p)} \in P^p, \ t > 0,
$$

or, equivalently, for $f(\omega) = \omega v(\omega)$ ($v$ is the velocity of cars)

$$
\frac{\partial}{\partial t} \rho^p(x^{(p)}, t) + \frac{\partial}{\partial x^{(p)}} \left( \frac{\rho^p(x^{(p)}, t)}{\omega^p(x^{(p)}, t)} \ f(\omega^p(x^{(p)}, t)) \right) = 0, \quad x^{(p)} \in P^p, \ t > 0,
$$

- the rate $\frac{\rho^p(x^{(p)}, t)}{\omega^p(x^{(p)}, t)}$ describes how the traffic distributes in percentage on the $p$-th path.
- If $\omega^p = 0$ we surely have $\rho^p = 0$ too, then we set $\frac{\rho^p}{\omega^p} = 0$ to avoid singularities.
- Equations are coupled by means of the velocity $v$, which depends on the total density $\omega$ and it is, in general, discontinuous at junctions.
- Paths do not have necessary arcs in common $\Rightarrow$ not all the equations are coupled with each other.
Numerical approximation by the Godunov scheme

\[ \frac{\partial}{\partial t} \rho + \frac{\partial}{\partial x} f(\rho) = 0, \quad (x, t) \in [a, b] \times (0, T) \]
\[ \rho(x, 0) = \bar{\rho}(x), \quad x \in [a, b]. \]

initial condition of the problem approximated by:
\[ \rho_j^0 = \frac{1}{\Delta x} \int_{x_j-\frac{1}{2}}^{x_j+\frac{1}{2}} \bar{\rho}(x) dx, \quad \forall j \]

\[ \rho_j^{n+1} = \rho_j^n - \frac{\Delta t}{\Delta x} \left( g(\rho_j^n, \rho_{j+1}^n) - g(\rho_{j-1}^n, \rho_j^n) \right), \quad \forall j \]

where the numerical flux \( g \) is defined as

\[ g(\rho_-, \rho_+) = \begin{cases} 
\min\{f(\rho_-), f(\rho_+)\} & \text{if } \rho_- \leq \rho_+ \\
f(\rho_-) & \text{if } \rho_- > \rho_+ \text{ and } \rho_- < \sigma \\
f(\sigma) & \text{if } \rho_- > \rho_+ \text{ and } \rho_- \geq \sigma \geq \rho_+ \\
f(\rho_+) & \text{if } \rho_- > \rho_+ \text{ and } \rho_+ > \sigma 
\end{cases} \]

under the CFL condition
\[ \Delta t \sup_{\rho} |f'(\rho)| \leq \Delta x. \]
Multi-path approach: Numerical approximation

Let us denote by $\rho_{j^q}^{n,p}$ the approximate density $\rho^p(x_j^{(q)}, t^n)$, where $j^{(q)}$ is the $j$-th node along the path $P^q$. We define for $n > 0$ and $p = 1, \ldots, N_P$ the sum of all the densities living at $x^{(p)}$ at time $t$

$$\omega_{j^{(p)}}^{n,p} := \sum_{q=1}^{N_P} \rho_{j^{(p)}}^{n,q}$$

the sum of all the densities

computation of the discrete solutions at the internal nodes as

$$\rho_{j}^{n+1,p} = \rho_{j}^{n,p} - \frac{\Delta t}{\Delta x} \left( \frac{\rho_{j}^{n,p}}{\omega_j^{n,p}} g(\omega_j^{n,p}, \omega_{j+1}^{n,p}) - \frac{\rho_{j-1}^{n,p}}{\omega_{j-1}^{n,p}} g(\omega_{j-1}^{n,p}, \omega_j^{n,p}) \right)$$

▶ Junctions are hidden in the definition of $\omega_j$ functions.

▶ Note the intrinsic asymmetry of the scheme. The coefficients in front of the fluxes involve only the nodes $j$ and $j - 1$, and not $j + 1$. 
At each time step $n$

- Updates the values of $\omega_{j(p)}^{n,p}$, for each path $p$ and for each $j^{(p)}$ node of $p$-path;
- Compute the discrete solution, for each path $p$ and for each $j^{(p)}$ node of $p$-path:

$$\rho_{j}^{n+1,p} = \rho_{j}^{n,p} - \frac{\Delta t}{\Delta x} \left( \frac{\rho_{j}^{n,p}}{\omega_{j}^{n,p}} g(\omega_{j}^{n,p},\omega_{j+1}^{n,p}) - \frac{\rho_{j-1}^{n,p}}{\omega_{j-1}^{n,p}} g(\omega_{j-1}^{n,p},\omega_{j}^{n,p}) \right)$$

➢ The only challenging part: defining properly $\omega_{j}^{n,p}$ at every node
At each time step $n$

- Updates the values of $\omega_{j(p)}^{n,p}$, for each path $p$ and for each $j(p)$ node of p-path;
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$$\rho_{j}^{n+1,p} = \rho_{j}^{n,p} - \frac{\Delta t}{\Delta x} \left( \frac{\rho_{j}^{n,p}}{\omega_{j}^{n,p}} g(\omega_{j}^{n,p},\omega_{j+1}^{n,p}) - \frac{\rho_{j-1}^{n,p}}{\omega_{j-1}^{n,p}} g(\omega_{j-1}^{n,p},\omega_{j}^{n,p}) \right)$$

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▶ The only challenging part: defining properly $\omega_{j}^{n,p}$ at every node

THATS ALL!
Outline of the talk

1. Easy-to-use numerical algorithm based on a multi-path model.

2. Characterization into the known framework (Garavello-Piccoli 2006)
Characterization into the classical framework

The case of two incoming roads and one outgoing road
Two incoming roads and one outgoing road

For simplicity,

- we assume that each arc has the same length, equal to $\frac{1}{2}$, then each path has the same length, equal to 1.
- We denote by $J$ the node just after the junction.

We have

$$\omega_{j}^{n,1} = \begin{cases} \rho_{j}^{n,1} & j < J, \\ \rho_{j}^{n,1} + \rho_{j}^{n,2} & j \geq J \end{cases}, \quad \omega_{j}^{n,2} = \begin{cases} \rho_{j}^{n,2} & j < J, \\ \rho_{j}^{n,1} + \rho_{j}^{n,2} & j \geq J. \end{cases}$$

and the scheme becomes

$$\begin{align*}
\rho_{j}^{n+1,1} &= \rho_{j}^{n,1} - \frac{\Delta t}{\Delta x} \left( \frac{\rho_{j}^{n,1}}{\omega_{j}^{n,1}} g(\omega_{j}^{n,1}, \omega_{j+1}^{n,1}) - \frac{\rho_{j-1}^{n,1}}{\omega_{j-1}^{n,1}} g(\omega_{j-1}^{n,1}, \omega_{j}^{n,1}) \right), \\
\rho_{j}^{n+1,2} &= \rho_{j}^{n,2} - \frac{\Delta t}{\Delta x} \left( \frac{\rho_{j}^{n,2}}{\omega_{j}^{n,2}} g(\omega_{j}^{n,2}, \omega_{j+1}^{n,2}) - \frac{\rho_{j-1}^{n,2}}{\omega_{j-1}^{n,2}} g(\omega_{j-1}^{n,2}, \omega_{j}^{n,2}) \right).
\end{align*}$$
Two incoming roads and one outgoing road

By the definition of \( \omega_{j}^{n,i} \), \( i = 1, 2 \) we have

\[
\text{for } j \geq J, \quad \omega_{j}^{n,1} = \omega_{j}^{n,2} = \omega_{j}^{n} = \rho_{j}^{n,3}
\]

With this notations \textbf{near the junction} we have

\[
\begin{align*}
\rho_{j}^{n+1,i} &= \rho_{j}^{n,i} - \frac{\Delta t}{\Delta x} \left( g(\rho_{j-1}^{n,i}, \omega_{j}^{n}) - g(\rho_{j-2}^{n,i}, \rho_{j-1}^{n,i}) \right), \quad i = 1, 2 \\
\omega_{j}^{n+1} &= \omega_{j}^{n} - \frac{\Delta t}{\Delta x} \left( g(\omega_{j}^{n}, \omega_{j+1}^{n}) - \left( g(\rho_{j-1}^{n,1}, \omega_{j}^{n}) + g(\rho_{j-1}^{n,2}, \omega_{j}^{n}) \right) \right) \\
\omega_{j+1}^{n+1} &= \omega_{j+1}^{n} - \frac{\Delta t}{\Delta x} \left( g(\omega_{j+1}^{n}, \omega_{j+2}^{n}) - g(\rho_{j+1}^{n,1}, \omega_{j+1}^{n}) \right).
\end{align*}
\]

\( \forall n, \) the value \( \omega_{j}^{n} \in [0, 1] \) under the CFL condition

\[
2\Delta t \sup_{\rho} |f'(\rho)| \leq \Delta x
\]
Similarities and differences with the classical theory

Going back to classical notations, we have

\[ \text{for } j \geq J, \rho_{j}^{n,3} = \omega_{j}^{n} \]

and the classical algorithm near the junction, reads as

\[
\begin{align*}
\rho_{J-1}^{n+1,i} &= \rho_{J-1}^{n,i} - \frac{\Delta t}{\Delta x} \left( \frac{\gamma_{i}^{*}}{g(\rho_{J-1}^{n,i}, \omega_{J}^{n}) - g(\rho_{J-2}^{n,i}, \rho_{J-1}^{n,i})} \right), & i &= 1, 2 \\
\omega_{J}^{n+1} &= \omega_{J}^{n} - \frac{\Delta t}{\Delta x} \left( \frac{\gamma_{3}^{*}}{g(\omega_{J}^{n}, \omega_{J+1}^{n}) - \left( g(\rho_{J-1}^{n,1}, \omega_{J}^{n}) + g(\rho_{J-1}^{n,2}, \omega_{J}^{n}) \right)} \right).
\end{align*}
\]
Classical approach: Two incoming roads and one outgoing road

The Riemann problem at junction:

\[
\begin{align*}
\begin{cases}
\partial_t \rho^1 + \partial_x f(\rho^1) &= 0, \\
\partial_t \rho^2 + \partial_x f(\rho^2) &= 0,
\end{cases}
& \quad x < 0 \\
\partial_t \rho^3 + \partial_x f(\rho^3) &= 0, \\
\rho^1(x, 0) &= \rho^1_l \\
\rho^2(x, 0) &= \rho^2_l \\
\rho^3(x, 0) &= \rho^3_r
& \quad x > 0
\end{align*}
\]
Classical approach: Two incoming roads and one outgoing road

for $i = 1, 2$, the maximum flux for the incoming and outgoing roads

$$
\gamma_{i}^{\text{max}} = \begin{cases} 
    f(\rho_{J-1}^{n,i}) & \rho_{J-1}^{n,i} \in [0, \sigma], \\
    f(\sigma) & \rho_{J-1}^{n,i} \in (\sigma, 1]
\end{cases}
$$

$$
\gamma_{3}^{\text{max}} = \begin{cases} 
    f(\sigma) & \omega_{J}^{n} \in [0, \sigma], \\
    f(\omega_{J}^{n}) & \omega_{J}^{n} \in (\sigma, 1]
\end{cases}
$$

The admissible solutions are given by the set

$$
\Omega := \{(\gamma^{1}, \gamma^{2}) \in [0, \gamma_{\text{max}}^{1}] \times [0, \gamma_{\text{max}}^{2}] | (\gamma^{1} + \gamma^{2}) \in [0, \gamma_{\text{max}}^{3}]\}
$$

**Red line:** Drivers behave in order to maximize the flux at junctions

$$
\{\gamma^{1} + \gamma^{2} = \gamma_{\text{max}}^{3}, \quad \gamma^{i} \leq \gamma_{\text{max}}^{i}, i = 1, 2\}
$$

We do not have the uniqueness of the maximization problem ...
Classical approach: Two incoming roads and one outgoing road

PRIORITIES (right of way)

\[ q\gamma^2 = (1 - q)\gamma^1 \]

► NO QUEUE: \( \gamma^* = \gamma_1 + \gamma_2 \), then

\[ \gamma_1^* = \gamma_1, \quad \gamma_2^* = \gamma_2, \quad \gamma_3^* = \gamma^* \]

► QUEUE: \( \gamma^* = \gamma_3 \), then we need the priorities parameter \( q \) such that \( \gamma_1 = (1 - q)/q \gamma_2 \) and

\[ \gamma_1^* = q\gamma^*, \quad \gamma_2^* = (1 - q)\gamma^*, \quad \gamma_3^* = \gamma^*. \]
Multi-path approach: Two incoming roads and one outgoing road

\[
\begin{cases}
    \rho_{J-1}^{n+1,i} = \rho_{J-1}^{n,i} - \frac{\Delta t}{\Delta x} \left( g(\rho_{J-1}^{n,i}, \omega_{J}^{n}) - g(\rho_{J-2}^{n,i}, \rho_{J-1}^{n,i}) \right), & i = 1, 2 \\
    \omega_{J}^{n+1} = \omega_{J}^{n} - \frac{\Delta t}{\Delta x} \left( g(\omega_{J}^{n}, \omega_{J+1}^{n}) - \left( g(\rho_{J-1}^{n,1}, \omega_{J}^{n}) + g(\rho_{J-1}^{n,2}, \omega_{J}^{n}) \right) \right) \\
    \omega_{J+1}^{n+1} = \omega_{J+1}^{n} - \frac{\Delta t}{\Delta x} \left( g(\omega_{J+1}^{n}, \omega_{J+2}^{n}) - g(\rho_{J}^{n,1}, \omega_{J+1}^{n}) \right).
\end{cases}
\]
Multi-path approach: Two incoming roads and one outgoing road

\[ F(L) = f_1^L + f_2^L, \]
\[ f_1^L = g(\rho_{J-1}^{n,1}, \omega_J^n) \]
\[ f_2^L = g(\rho_{J-1}^{n,2}, \omega_J^n) \]

\[ F(R) = g(\omega_J^n, \omega_{J+1}^n) \]
Godunov’s function features

For $f$ such that

$$f(0) = f(\rho_{\text{max}}) = 0, \quad f \text{ is concave}, \quad f(\sigma) = \max_{\omega \in [0, \rho_{\text{max}}]} f(\omega)$$

the Godunov’s function verifies: for $\rho_-, \rho_+ \in [0, \rho_{\text{max}}]$,

$$g(\rho_-, \rho_+) = \min (g(\rho_-, \sigma), g(\sigma, \rho_+)).$$

$$g(\rho_-, \sigma) = \begin{cases} f(\rho_-), & \text{if } \rho_- \in [0, \sigma], \\ f(\sigma), & \text{if } \rho_- \in ]\sigma, 1], \end{cases} = \gamma_{\text{in}}^{\text{max}}$$

$$g(\sigma, \rho_+) = \begin{cases} f(\sigma), & \text{if } \rho_+ \in [0, \sigma], \\ f(\rho_+), & \text{if } \rho_+ \in ]\sigma, 1], \end{cases} = \gamma_{\text{out}}^{\text{max}}$$
Multi-path approach: Two incoming roads and one outgoing road

\[
F(L) = f^1_L + f^2_L,
\]

\[
f^1_L = g(\rho^{n,1}_{J-1}, \omega^n_J) = \min \left( g(\rho^{n,1}_{J-1}, \sigma), g(\sigma, \omega^n_J) \right) = \min \left( \gamma^{1\text{max}}, \gamma^{3\text{max}} \right)
\]

\[
f^2_L = g(\rho^{n,2}_{J-1}, \omega^n_J) = \min \left( g(\rho^{n,2}_{J-1}, \sigma), g(\sigma, \omega^n_J) \right) = \min \left( \gamma^{2\text{max}}, \gamma^{3\text{max}} \right)
\]

\[
F(R) = g(\omega^n_J, \omega^n_{J+1}) = \min \left( g(\omega^n_J, \sigma), g(\sigma, \omega^n_{J+1}) \right)
\]
Drivers behave in order to maximize the flux of its own path

We have that,

\[(f_1^L, f_2^L) \in \{ (\gamma^1, \gamma^2) \in [0, \gamma_{max}^1] \times [0, \gamma_{max}^2] \mid (\gamma^1 + \gamma^2) \in [0, 2\gamma_{max}^3] \}\]

so \((f_1^L, f_2^L)\) is always an admissible flux for the problem with 2 incoming roads with flux \(f(\rho)\) and 1 outgoing road with flux \(2f(\rho)\).

\(\blacktriangleright\) when \(F(L) = 2g(\sigma, \omega^n_J)\), automatically the algorithm select for the two incoming road the priority value \(q = 1/2\).
It is possible to prove that, for

\[ F(R) = \min \left( g(\omega_j^n, \sigma), g(\sigma, \omega_{j+1}^n) \right) \]

we have

\[ F(R) \leq \text{MIN}\left( 2g(\omega_j^n, \sigma), g(\sigma, \omega_{j+1}^n) \right), \]

so \( F(R) \) is always an **admissible flux** for the **Bottleneck problem** between left-flux \( 2f(\rho) \) and right-flux \( f(\rho) \).
The "modified" Riemann problem at the junction

\[
\begin{align*}
\begin{cases}
    x < 0 & \quad 0 < x < \Delta x & \quad x > \Delta x \\
    \partial_t u + \partial_x f(u) &= 0, \\
    \partial_t v + \partial_x f(v) &= 0, \\
    u(x, 0) &= u_l \\
    v(x, 0) &= v_l \\
    \partial_t z + \partial_x 2f(z) &= 0, \\
    \partial_t w + \partial_x f(w) &= 0, \\
    z(x, 0) &= z_c \\
    w(x, 0) &= w_r
\end{cases}
\end{align*}
\]
Multi-path approach: Two incoming roads and one outgoing road

NO QUEUE

QUEUE
Similarities and differences with the classical theory

- The difference between the two algorithms is more negligible than we can expect from this example.

- Fixing the same Dirichlet boundary conditions, (assuming for the priority parameter $q = \frac{1}{2}$) after a small transient during which the two solutions are different, the two algorithms give the same solution, i.e. we get

  $$F(L) = F(R)$$
The Riemann solver at the junction

For example in the case of two queues formed in the two incoming roads:

\[ f(u_l) + f(v_l) > f(w_r) \]

1. In \( x = 0 \), \( \tilde{u} \) and \( \tilde{v} \) are the two attainable states in \( N(u_s) \) and \( N(v_s) \) respectively and \( \hat{z} \) is the attainable state in \( P(z_c) \).

2. In \( x = \Delta x \), where \( \hat{z} \) is an attainable state in \( N(z_c) \) and \( \hat{w} \) is an attainable state in \( P(w_r) \).

- \( N(\cdot) = \) right state attainable by a wave of negative speed;
- \( P(\cdot) = \) left state attainable by a wave of positive speed.
Features of the multi-pop algorithm

- Similar arguments extend to the case of 1 incoming road and 2 outgoing roads, \( N \) incoming roads and 1 outgoing road, 1 incoming road and \( M \) outgoing roads, etc. ...

- All topics described apply to the case of roads with different flux functions.

Easy-to-use

\[
\rho_{j}^{n+1,p} = \rho_{j}^{n,p} - \frac{\Delta t}{\Delta x} \left( \frac{\rho_{j}^{n,p}}{\omega_{j}^{n,p}} g(\omega_{j}^{n,p}, \omega_{j+1}^{n,p}) - \frac{\rho_{j-1}^{n,p}}{\omega_{j-1}^{n,p}} g(\omega_{j-1}^{n,p}, \omega_{j}^{n,p}) \right)
\]

1. The scheme selected automatically one solution at the junction, without the need of an additional separate procedure.

2. The solution chosen is *admissible in the sense of the classical theory*, assuming \( \frac{\Delta t}{\Delta x} \) sufficiently small.
Features of the multi-pop algorithm

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\]

1. The scheme selected automatically one solution at the junction, without the need of an additional separate procedure.

2. The solution chosen is \textit{admissible in the sense of the classical theory}, assuming \( \frac{\Delta t}{\Delta x} \) sufficiently small.
Features of the multi-pop algorithm

- **Drawback:** *The number of equations grows rapidly when the number of nodes of the graph increases*

- To keep the computational load within reasonable limits, we propose a second version of the algorithm which splits the vehicles on the basis on their path only at junctions.

  **Drawback:** *The global behavior of drivers is lost*
Conclusions: A real application

Rome: 6 two-lane roads and 7 junctions: 328.2km.

- Local version of the model.
- Four traffic lights coordinated in pairs.
- $\Delta x = 0.1\text{km}, \Delta t = 2.5\text{s}$. Final time $T = 1\text{h}$;

The code is written in C++ (serial) and run on an Intel i3 2.27GHz processor.

$\Delta t = 2.5\text{s}$. Final time $T = 1\text{h}$;

This result suggests that the proposed technique can be actually used to forecast traffic flow in large networks, keeping to a minimum the implementing effort.

Thank you for your attention
Conclusions: A real application

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- Local version of the model.
- Four traffic lights coordinated in pairs.
- $\Delta x = 0.1\text{km}$, $\Delta t = 2.5\text{s}$. Final time $T = 1\text{h}$;
- The code is written in C++ (serial) and run on an Intel i3 2.27GHz processor.
- The CPU time for the entire simulation was 0.5s.

▶ This result suggests that the proposed technique can be actually used to forecast traffic flow in large networks, keeping to a minimum the implementing effort.

Thank you for your attention
**Bibliography**

*multi-population or multi-class models*

Several papers investigate from the theoretical point of view the (systems of) scalar conservation laws.


Systems of scalar conservation laws with discontinuous flux are instead less studied.