An easy-to-use numerical approach for simulating traffic flow on networks

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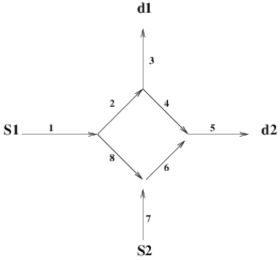
Outline of the talk

1. Easy-to-use numerical algorithm based on a multi-path model.
2. Characterization into the known framework (Garavello-Piccoli 2006)

Known theory of traffic flow on networks

Roads network: **a network where each edge and each vertex represents respectively an unidirectional road and a junction**



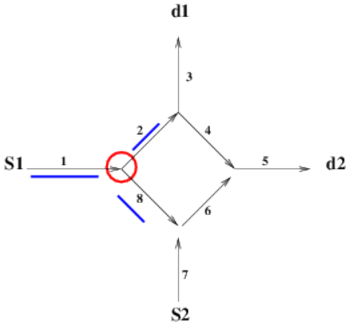
Main references

- H. Holden, N. H. Risebro, SIMA (1995)
- G. M. Coclite, M. Garavello, B. Piccoli, Traffic flow on a road network, SIAM J. Math. Anal. 36 (2005) 1862–1886
- M. Garavello, B. Piccoli, Traffic flow on networks, AIMS, 2006
- G. Bretti, R. Natalini, B. Piccoli, A fluid-dynamic traffic model on road networks, Arch. Comput. Methods Eng. 14 (2007) 139–172

Known Theory

At any time t, the evolution of the car density on the network is computed by a two-step procedure:

1) a classical conservation laws is solved at any internal point of the arcs;
2) the densities at endpoints, which correspond to a junction, are computed.



Known Theory

1) On each arc, the density $\rho_r(x,t)$ of all vehicles it is simply given by the entropic solution of

$$\frac{\partial}{\partial t}\rho_r + \frac{\partial}{\partial x} f(\rho_r) = 0 \qquad x \in I_r.$$

with the flux $f \in C^1([0,\rho_{max}])$ for some maximal density ρ_{max}, and

$$f(0) = f(\rho_{max}) = 0, \qquad f \text{ is concave}, \qquad f(\sigma) = \max_{\omega \in [0,\rho_{max}]} f(\omega)$$

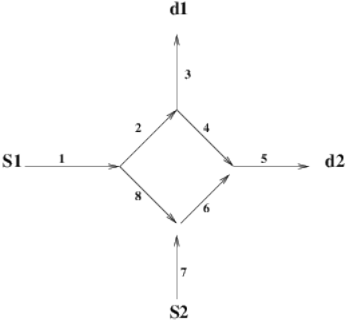
2) The computation of densities at endpoints has not in general a unique admissible solution, so that additional constraints must be added.

- Conservation of cars at junctions;
- Drivers behave in order to maximize the flux at junctions;
- Incoming roads are regulated by priorities (right of way).

This second step is performed by a linear programming method.

Multi-path model

Here we study from the numerical point of view a Multi-path model, following the idea that
cars moving on the network are divided on the basis of their path i.e. on their origin-destination pair



- a modified version of the model proposed in M. Hilliges, W. Weidlich (1955);

Multi-path model

- N_P number of possible paths on the graph, $P^1, \ldots, P^p, \ldots, P^{N_P}$ (paths can share some arcs of the networks);
- $x^{(p)}$ is a generic point along the path P^p (a specific point on the network is characterized by both the path it belongs to and the distance from the origin of that path);
- $\rho^p(x^{(q)}, t) \in [0, 1]$ is the density of the cars following the p-th path at point $x^{(q)}$ along path P^q at time $t > 0$. ($\rho^p(x^{(q)}, t)$ is, by definition, strictly positive if $p = q$. Conversely, if $p \neq q$, we have $\rho^p(x^{(q)}, t) = 0$ if $x^{(q)} \notin P^p$ and $\rho^p(x^{(q)}, t) > 0$ if $x^{(q)} \in P^p$)
- We define

$$\omega^p(x^{(p)}, t) := \sum_{q=1}^{N_P} \rho^q(x^{(p)}, t),$$

i.e. $\omega^p(x^{(p)}, t)$ is the sum of all the densities living at $x^{(p)}$ at time t.

LWR-based model

System of N_P conservation laws with space-dependent and discontinuous flux, for $p = 1, \dots, N_P$,

$$\frac{\partial}{\partial t}\rho^p(x^{(p)},t) + \frac{\partial}{\partial x^{(p)}}\left(\rho^p(x^{(p)},t)\ v\big(\omega^p(x^{(p)},t)\big)\right) = 0, \quad x^{(p)} \in P^p,\ t > 0,$$

or, equivalently, for $f(\omega) = \omega v(\omega)$ (v is the velocity of cars)

$$\frac{\partial}{\partial t}\rho^p(x^{(p)},t) + \frac{\partial}{\partial x^{(p)}}\left(\frac{\rho^p(x^{(p)},t)}{\omega^p(x^{(p)},t)}\ f\big(\omega^p(x^{(p)},t)\big)\right) = 0, \quad x^{(p)} \in P^p,\ t > 0,$$

- the rate $\frac{\rho^p(x^{(p)},t)}{\omega^p(x^{(p)},t)}$ describes how the traffic distributes in percentage on the p-th path.

- If $\omega^p = 0$ we surely have $\rho^p = 0$ too, then we set $\frac{\rho^p}{\omega^p} = 0$ to avoid singularities.
- Equations are coupled by means of the velocity v, which depends on the total density ω and it is, in general, discontinuous at junctions.
- Paths do not have necessary arcs in common $\Rightarrow$ not all the equations are coupled with each other.

Numerical approximation by the Godunov scheme

$$\begin{cases} \frac{\partial}{\partial t}\rho + \frac{\partial}{\partial x} f(\rho) = 0, \ (x,t) \in [a,b] \times (0,T] \\ \rho(x,0) = \bar{\rho}(x), \ x \in [a,b]. \end{cases}$$

- initial condition of the problem approximated by:
  $\rho_j^0 = \frac{1}{\Delta x} \int_{x_{j-\frac{1}{2}}}^{x_{j+\frac{1}{2}}} \bar{\rho}(x) dx, \qquad \forall j$

$$\rho_j^{n+1} = \rho_j^n - \frac{\Delta t}{\Delta x}\Big(g(\rho_j^n, \rho_{j+1}^n) - g(\rho_{j-1}^n, \rho_j^n) \Big), \qquad \forall j$$

where the numerical flux g is defined as

$$g(\rho_-, \rho_+) = \begin{cases} \min\{f(\rho_-), f(\rho_+)\} & \text{if } \rho_- \leq \rho_+ \\ f(\rho_-) & \text{if } \rho_- > \rho_+ \text{ and } \rho_- < \sigma \\ f(\sigma) & \text{if } \rho_- > \rho_+ \text{ and } \rho_- \geq \sigma \geq \rho_+ \\ f(\rho_+) & \text{if } \rho_- > \rho_+ \text{ and } \rho_+ > \sigma \end{cases}$$

under the CFL condition

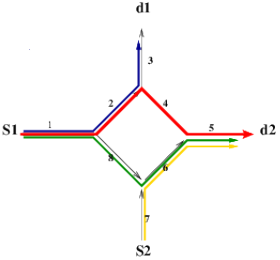
$$\Delta t \sup_\rho |f'(\rho)| \leq \Delta x.$$

Multi-path approach: Numerical approximation

Let us denote by $\rho^{n,p}_{j^q}$ the approximate density $\rho^p(x^{(q)}_j, t^n)$, where $j^{(q)}$ is the j-th node along the path P^q. We define for $n > 0$ and $p = 1, \ldots, N_P$

the sum of all the densities living at $x^{(p)}$ at time t

$$\omega^{n,p}_{j^{(p)}} := \sum_{q=1}^{N_P} \rho^{n,q}_{j^{(p)}}$$



computation of the discrete solutions at the internal nodes as

$$\rho^{n+1,p}_j = \rho^{n,p}_j - \frac{\Delta t}{\Delta x}\left(\frac{\rho^{n,p}_j}{\omega^{n,p}_j}\, g(\omega^{n,p}_j, \omega^{n,p}_{j+1}) - \frac{\rho^{n,p}_{j-1}}{\omega^{n,p}_{j-1}}\, g(\omega^{n,p}_{j-1}, \omega^{n,p}_j)\right)$$

- Junctions are hidden in the definition of ω_j functions.
- Note the intrinsic asymmetry of the scheme. The coefficients in front of the fluxes involve only the nodes j and $j-1$, and not $j+1$.

Easy-to-use!

At each time step n

- Updates the values of $\omega_{j^{(p)}}^{n,p}$, for each path p and for each $j^{(p)}$ node of p-path;
- Compute the discrete solution, for each path p and for each $j^{(p)}$ node of p-path:

$$\rho_j^{n+1,p} = \rho_j^{n,p} - \frac{\Delta t}{\Delta x}\left(\frac{\rho_j^{n,p}}{\omega_j^{n,p}}\, g(\omega_j^{n,p}, \omega_{j+1}^{n,p}) - \frac{\rho_{j-1}^{n,p}}{\omega_{j-1}^{n,p}}\, g(\omega_{j-1}^{n,p}, \omega_j^{n,p})\right)$$

▶ **The only challenging part: defining properly $\omega_j^{n,p}$ at every node**

THATS ALL!

Easy-to-use!

At each time step n

- Updates the values of $\omega_{j^{(p)}}^{n,p}$, for each path p and for each $j^{(p)}$ node of p-path;
- Compute the discrete solution, for each path p and for each $j^{(p)}$ node of p-path:

$$\rho_j^{n+1,p} = \rho_j^{n,p} - \frac{\Delta t}{\Delta x}\left(\frac{\rho_j^{n,p}}{\omega_j^{n,p}}\, g(\omega_j^{n,p}, \omega_{j+1}^{n,p}) - \frac{\rho_{j-1}^{n,p}}{\omega_{j-1}^{n,p}}\, g(\omega_{j-1}^{n,p}, \omega_j^{n,p})\right)$$

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▶ **The only challenging part: defining properly $\omega_j^{n,p}$ at every node**

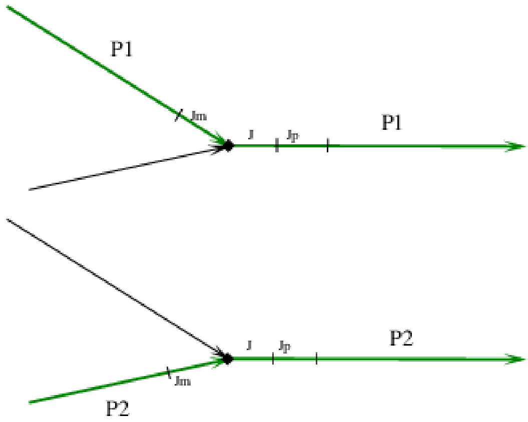
THATS ALL!

Outline of the talk

1. Easy-to-use numerical algorithm based on a multi-path model.
2. Characterization into the known framework (Garavello-Piccoli 2006)

Characterization into the classical framework

The case of two incoming roads and one outgoing road



Two incoming roads and one outgoing road

For simplicity,

- we assume that each arc has the same length, equal to $\frac{1}{2}$, then each path has the same length, equal to 1.
- We denote by J the node *just after* the junction.

We have

$$\omega_j^{n,1} = \begin{cases} \rho_j^{n,1} & j < J, \\ \rho_j^{n,1} + \rho_j^{n,2} & j \geq J. \end{cases}, \qquad \omega_j^{n,2} = \begin{cases} \rho_j^{n,2} & j < J, \\ \rho_j^{n,1} + \rho_j^{n,2} & j \geq J. \end{cases}$$

and the scheme becomes

$$\begin{cases} \rho_j^{n+1,1} = \rho_j^{n,1} - \frac{\Delta t}{\Delta x}\left(\frac{\rho_j^{n,1}}{\omega_j^{n,1}} g(\omega_j^{n,1}, \omega_{j+1}^{n,1}) - \frac{\rho_{j-1}^{n,1}}{\omega_{j-1}^{n,1}} g(\omega_{j-1}^{n,1}, \omega_j^{n,1})\right), \\ \\ \rho_j^{n+1,2} = \rho_j^{n,2} - \frac{\Delta t}{\Delta x}\left(\frac{\rho_j^{n,2}}{\omega_j^{n,2}} g(\omega_j^{n,2}, \omega_{j+1}^{n,2}) - \frac{\rho_{j-1}^{n,2}}{\omega_{j-1}^{n,2}} g(\omega_{j-1}^{n,2}, \omega_j^{n,2})\right). \end{cases}$$

Two incoming roads and one outgoing road

By the definition of $\omega_j^{n,i}$, $i = 1, 2$ we have

$$\text{for } j \geq J,\ \omega_j^{n,1} = \omega_j^{n,2} = \omega_j^n = \rho_j^{n,3}$$

With this notations **near the junction** we have

$$\begin{cases} \rho_{J-1}^{n+1,i} = \rho_{J-1}^{n,i} - \frac{\Delta t}{\Delta x}\left(g(\rho_{J-1}^{n,i}, \omega_J^n) - g(\rho_{J-2}^{n,i}, \rho_{J-1}^{n,i})\right), \quad i = 1, 2 \\ \omega_J^{n+1} = \omega_J^n - \frac{\Delta t}{\Delta x}\left(g(\omega_J^n, \omega_{J+1}^n) - \left(g(\rho_{J-1}^{n,1}, \omega_J^n) + g(\rho_{J-1}^{n,2}, \omega_J^n)\right)\right) \\ \omega_{J+1}^{n+1} = \omega_{J+1}^n - \frac{\Delta t}{\Delta x}\left(g(\omega_{J+1}^n, \omega_{J+2}^n) - g(\rho_J^{n,1}, \omega_{J+1}^n)\right). \end{cases}$$

- $\forall n$, the value $\omega_J^n \in [0, 1]$ under the CFL condition

$$2\Delta t \sup_\rho |f'(\rho)| \leq \Delta x$$

Similarities and differences with the classical theory

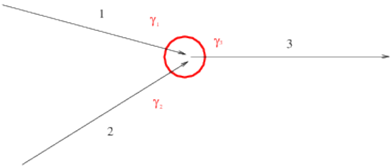
Going back to classical notations, we have

for $j \geq J$, $\rho_j^{n,3} = \omega_j^n$

and the classical algorithm near the junction, reads as

$$\begin{cases} \rho_{J-1}^{n+1,i} = \rho_{J-1}^{n,i} - \frac{\Delta t}{\Delta x} \left(\overbrace{\cancel{g(\rho_{J-1}^{n,i}, \omega_J^n)}}^{\gamma_i^*} - g(\rho_{J-2}^{n,i}, \rho_{J-1}^{n,i}) \right), \quad i = 1,2 \\ \omega_J^{n+1} = \omega_J^n - \frac{\Delta t}{\Delta x} \left(g(\omega_J^n, \omega_{J+1}^n) - \overbrace{\cancel{\left(g(\rho_{J-1}^{n,1}, \omega_J^n) + g(\rho_{J-1}^{n,2}, \omega_J^n) \right)}}^{\gamma_3^*} \right). \end{cases}$$

Classical approach: Two incoming roads and one outgoing road



The Riemann problem at junction:

$x < 0$	$x > 0$
$\begin{cases} \partial_t \rho^1 + \partial_x f(\rho^1) = 0, \\ \partial_t \rho^2 + \partial_x f(\rho^2) = 0, \end{cases}$	$\partial_t \rho^3 + \partial_x f(\rho^3) = 0,$
$\rho^1(x,0) = \rho^1_l$ $\rho^2(x,0) = \rho^2_l$	$\rho^3(x,0) = \rho^3_r$

Classical approach: Two incoming roads and one outgoing road

for $i = 1, 2$, the maximum flux for the incoming and outgoing roads

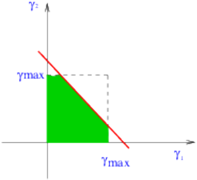
$$\gamma^i_{max} = \begin{cases} f(\rho^{n,i}_{J-1}) & \rho^{n,i}_{J-1} \in [0, \sigma], \\ f(\sigma) & \rho^{n,i}_{J-1} \in (\sigma, 1] \end{cases} \qquad \gamma^3_{max} = \begin{cases} f(\sigma) & \omega^n_J \in [0, \sigma], \\ f(\omega^n_J) & \omega^n_J \in (\sigma, 1] \end{cases}$$

The admissible solutions are given by the set

$$\Omega := \{(\gamma^1, \gamma^2) \in [0, \gamma^1_{max}] \times [0, \gamma^2_{max}] \mid (\gamma^1 + \gamma^2) \in [0, \gamma^3_{max}]\}$$

Red line: Drivers behave in order to maximize the flux at junctions

$$\{\gamma^1 + \gamma^2 = \gamma^3_{max}, \quad \gamma^i \leq \gamma^i_{max}, i = 1, 2\}$$

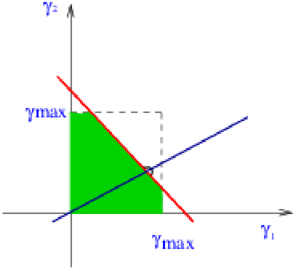


We do not have the uniqueness of the maximization problem ...

Classical approach: Two incoming roads and one outgoing road

PRIORITIES (right of way)

$$q\gamma^2 = (1-q)\gamma^1$$



- *NO QUEUES*: $\gamma^* = \gamma_1 + \gamma_2$, then

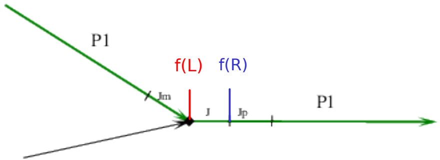
$$\gamma_1^* = \gamma_1, \quad \gamma_2^* = \gamma_2, \quad \gamma_3^* = \gamma^*$$

- *QUEUES*: $\gamma^* = \gamma_3$, then we need the priorities parameter q such that $\gamma_1 = (1-q)/q\gamma_2$ and

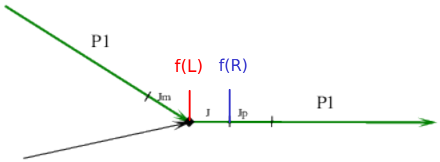
$$\gamma_1^* = q\gamma^*, \quad \gamma_2^* = (1-q)\gamma^*, \quad \gamma_3^* = \gamma^*.$$

Multi-path approach: Two incoming roads and one outgoing road

$$
\begin{cases}
\rho_{J-1}^{n+1,i} = \rho_{J-1}^{n,i} - \frac{\Delta t}{\Delta x}\left(g(\rho_{J-1}^{n,i}, \omega_J^n) - g(\rho_{J-2}^{n,i}, \rho_{J-1}^{n,i}) \right), \quad i = 1, 2 \\
\omega_J^{n+1} = \omega_J^n - \frac{\Delta t}{\Delta x}\left(g(\omega_J^n, \omega_{J+1}^n) - \left(g(\rho_{J-1}^{n,1}, \omega_J^n) + g(\rho_{J-1}^{n,2}, \omega_J^n) \right) \right) \\
\omega_{J+1}^{n+1} = \omega_{J+1}^n - \frac{\Delta t}{\Delta x}\left(g(\omega_{J+1}^n, \omega_{J+2}^n) - g(\rho_J^{n,1}, \omega_{J+1}^n) \right).
\end{cases}
$$



Multi-path approach: Two incoming roads and one outgoing road



- $F(L) = f_L^1 + f_L^2,$

$$f_L^1 = g(\rho_{J-1}^{n,1}, \omega_J^n)$$
$$f_L^2 = g(\rho_{J-1}^{n,2}, \omega_J^n)$$

- $F(R) = g(\omega_J^n, \omega_{J+1}^n)$

Godunov's function features

For f such that

$$f(0) = f(\rho_{max}) = 0, \qquad f \text{ is concave}, \qquad f(\sigma) = \max_{\omega \in [0, \rho_{max}]} f(\omega)$$

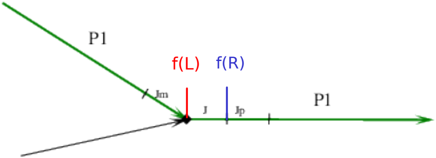
the Godunov's function verifies: for $\rho_-, \rho_+ \in [0, \rho_{max}]$,

$$g(\rho_-, \rho_+) = \min\left(g(\rho_-, \sigma), g(\sigma, \rho_+)\right).$$

$$g(\rho_-, \sigma) = \begin{cases} f(\rho_-), & \text{if } \rho_- \in [0, \sigma], \\ f(\sigma), & \text{if } \rho_- \in]\sigma, 1], \end{cases} = \gamma_{max}^{in}$$

$$g(\sigma, \rho_+) = \begin{cases} f(\sigma), & \text{if } \rho_+ \in [0, \sigma], \\ f(\rho_+), & \text{if } \rho_+ \in]\sigma, 1], \end{cases} = \gamma_{max}^{out}$$

Multi-path approach: Two incoming roads and one outgoing road



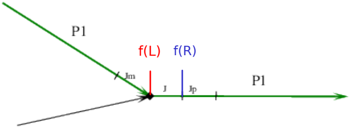
- $F(L) = f_L^1 + f_L^2,$

$$f_L^1 = g(\rho_{J-1}^{n,1}, \omega_J^n) = \min\left(g(\rho_{J-1}^{n,1}, \sigma), g(\sigma, \omega_J^n)\right) = \min\left(\gamma_{max}^1, \gamma_{max}^3\right)$$

$$f_L^2 = g(\rho_{J-1}^{n,2}, \omega_J^n) = \min\left(g(\rho_{J-1}^{n,2}, \sigma), g(\sigma, \omega_J^n)\right) = \min\left(\gamma_{max}^2, \gamma_{max}^3\right)$$

- $F(R) = g(\omega_J^n, \omega_{J+1}^n) = \min\left(g(\omega_J^n, \sigma), g(\sigma, \omega_{J+1}^n)\right)$

Left Flux



$$f_L^1 = \min\left(\gamma_{max}^1, \gamma_{max}^3\right), \quad f_L^2 = \min\left(\gamma_{max}^2, \gamma_{max}^3\right)$$

Drivers behave in order to maximize the flux of its own path

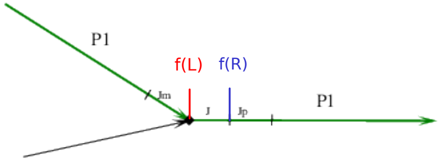
We have that,

$$(f_L^1, f_L^2) \in \{(\gamma^1, \gamma^2) \in [0, \gamma_{max}^1] \times [0, \gamma_{max}^2] \mid (\gamma^1 + \gamma^2) \in [0, 2\gamma_{max}^3]\}$$

so (f_L^1, f_L^2) is always an **admissible flux** for the problem with 2 incoming roads with flux $f(\rho)$ and 1 outgoing road with flux $2f(\rho)$.

- when $F(L) = 2g(\sigma, \omega_J^n)$, **automatically** the algorithm select for the two incoming road the **priority value** $q = 1/2$.

Right Flux



It is possible to prove that, for

$$F(R) = \min\left(g(\omega_J^n, \sigma), g(\sigma, \omega_{J+1}^n)\right)$$

we have

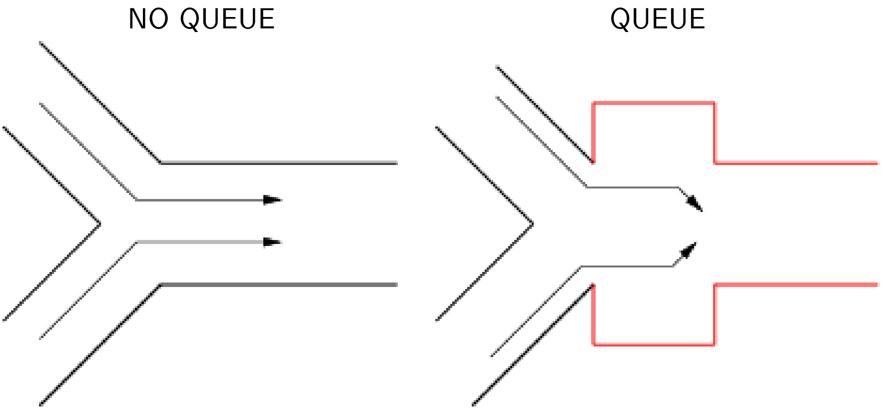
$$F(R) \leq \text{MIN}\Big(2g(\omega_J^n, \sigma), g(\sigma, \omega_{J+1}^n)\Big),$$

so $F(R)$ is always an **admissible flux** for the **Bottleneck problem** between left-flux $2f(\rho)$ and right-flux $f(\rho)$.

The *"modified"* Riemann problem at the junction

$x < 0$	$0 < x < \Delta x$	$x > \Delta x$
$\begin{cases} \partial_t u + \partial_x f(u) = 0, \\ \partial_t v + \partial_x f(v) = 0, \end{cases}$	$\partial_t z + \partial_x 2f(z) = 0,$	$\partial_t w + \partial_x f(w) = 0,$
$u(x,0) = u_l$ $v(x,0) = v_l$	$z(x,0) = z_c$	$w(x,0) = w_r$

Multi-path approach: Two incoming roads and one outgoing road



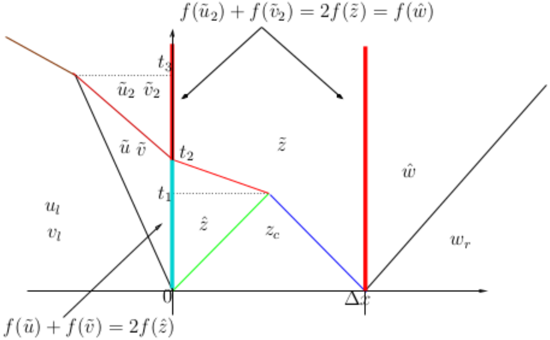
Similarities and differences with the classical theory

- The difference between the two algorithms is more negligible than we can expect from this example.
- Fixing the same Dirichlet boundary conditions, (assuming for the priority parameter $q = \frac{1}{2}$) after a **small transient during which the two solutions are different**, the two algorithms give the same solution, i.e. we get

$$F(L) = F(R)$$

The Riemann solver at the junction

For example in the case of two queues formed in the two incoming roads: $f(u_l) + f(v_l) > f(w_r)$:



1. in $x = 0$, $\tilde{u}$ and $\tilde{v}$ are the two attainable states in $N(u_s)$ and $N(v_s)$ respectively and $\hat{z}$ is the attainable state in $P(z_c)$.
2. in $x = \Delta x$, where $\tilde{z}$ is an attainable state in $N(z_c)$ and $\hat{w}$ is an attainable state in $P(w_r)$.

- $N(\cdot)$ = *right state* attainable by a wave of *negative* speed;
- $P(\cdot)$ = *left state* attainable by a wave of *positive* speed.

Features of the multi-pop alghoritm

- Similar arguments extend to the case of 1 incoming road and 2 outgoing roads, N incoming roads and 1 outgoing road, 1 incoming road and M outgoing roads, etc. ...
- All topics described apply to the case of roads with different flux functions.

Easy-to-use

$$\rho_j^{n+1,p} = \rho_j^{n,p} - \frac{\Delta t}{\Delta x}\left(\frac{\rho_j^{n,p}}{\omega_j^{n,p}}\, g(\omega_j^{n,p}, \omega_{j+1}^{n,p}) - \frac{\rho_{j-1}^{n,p}}{\omega_{j-1}^{n,p}}\, g(\omega_{j-1}^{n,p}, \omega_j^{n,p})\right)$$

1. The scheme selected automatically one solution at the junction, without the need of an additional separate procedure.
2. The solution chosen is *admissible in the sense of the classical theory*, assuming $\frac{\Delta t}{\Delta x}$ sufficiently small.

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Features of the multi-pop alghoritm

- **Drawback:** *The number of equations grows rapidly when the number of nodes of the graph increases*
- To keep the computational load within reasonable limits, **we propose a second version of the algorithm which splits the vehicles on the basis on their path only at junctions**.
  Drawback: *The global behavior of drivers is lost*

Conclusions: A real application

Rome: 6 two-lane roads and 7 junctions: 328.2km.

- Local version of the model.
- Four traffic lights coordinated in pairs.
- $\Delta x = 0.1$km, $\Delta t = 2.5$s. Final time $T = 1$h;
- The code is written in C++ (serial) and run on an Intel i3 2.27GHz processor.
- *The CPU time for the entire simulation was 0.5s.*

▶ This result suggests that the proposed technique can be actually used to forecast traffic flow in large networks, keeping to a minimum the implementing effort.

Thank you for your attention

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Thank you for your attention

multi-population or *multi-class* models

- S. Benzoni-Gavage, R. M. Colombo, An n-populations model for traffic flow, European J. Appl. Math. 14 (2003) 587–612
- G. C. K. Wong, S. C. Wong, A multi-class traffic flow model – an extension of LWR model with heterogeneous drivers, Transportation Research Part A 36 (2002) 827–841

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Bibliography

Several papers investigate from the theoretical point of view the (systems of) scalar conservation laws.

- R. Bürger, K. H. Karlsen, Conservation laws with discontinuous flux: a short introduction, J. Eng. Math. 60 (2008) 241–247;
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▶ Systems of scalar conservation laws with discontinuous flux are instead less studied.