Workshop TRAM2: Traffic Modeling and Management: Trends and Perspectives

Moving bottlenecks within the three representations of traffic flow: an historical review of numerical issues

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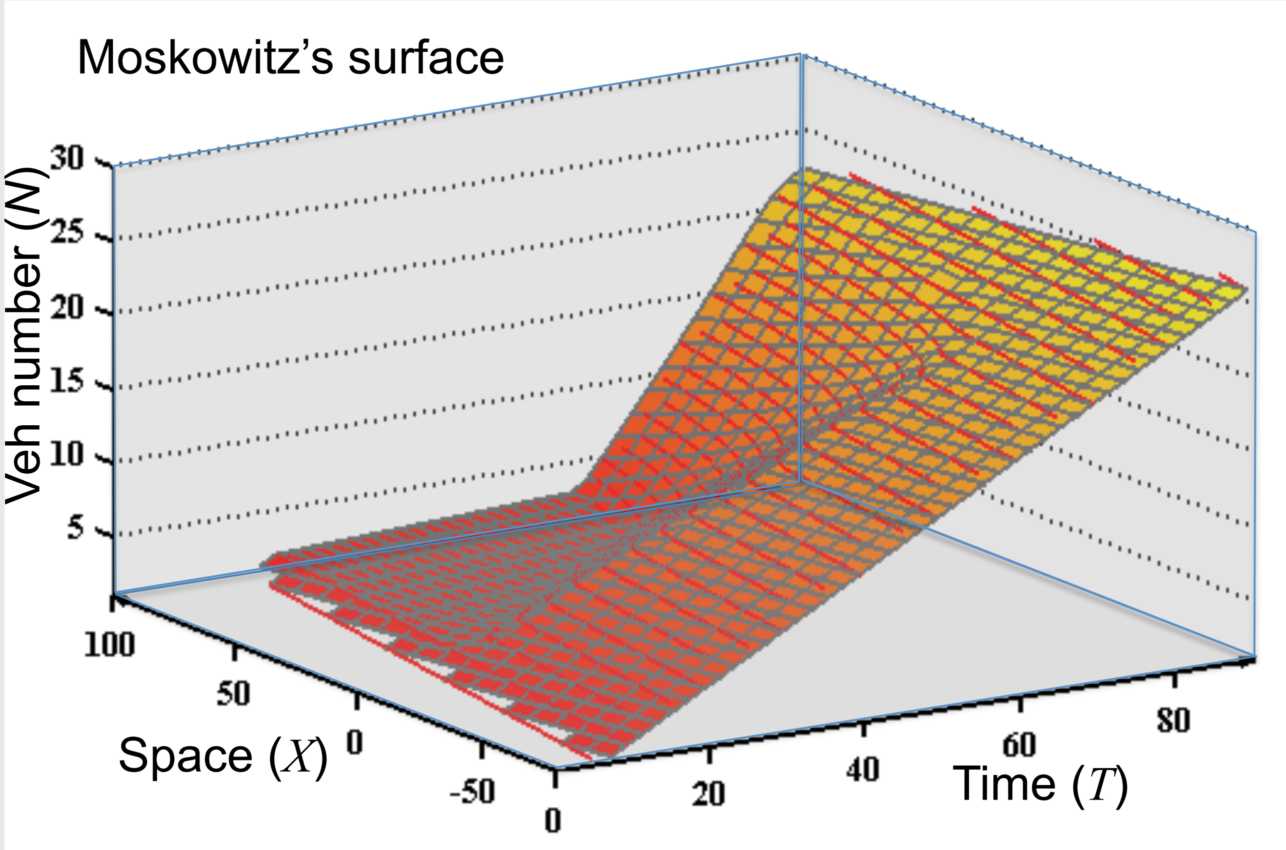
Outline

- The Three representation of traffic flow
- The moving bottleneck theory
- Numerical issues
  - Godunov based numerical schemes
    - Moving bottlenecks in Eulerian coordinates
    - Fixed bottlenecks in Lagrangian coordinates
  - Variational theory based numerical schemes
    - The shortcut principle
    - Applications to the three coordinate systems
    - Related approaches (viability theory)

The three representation of traffic flow (1)



Eulerian coordinates
$N(t,x)$

Lagrangian coordinates
$X(t,n)$

T coordinates
$T(t,n)$

The three representation of traffic flow (2)

	Eulerian		T coordinates		Lagrangian	
	$N(t,x)$		$T(n,x)$		$X(t,n)$	
partials	N_t	$-N_x$	$T_{[illegible]}$	T_x	X_t	$-X_n$
symbol	$q($[illegible]	[illegible]$)$	$h($[illegible]	[illegible]$, x)$	$v($[illegible]	[illegible]$)$
name	flow	[illegible]ensity	headwa[illegible]	[illegible]pace	speed	[illegible]pacing
	Macro		Meso		Micro	

$N(t,x)$ # of vehicles that have crossed location x by time t

$X(t,n)$ position of vehicle n at time t

$T(n,x)$ time vehicle n crosses location x

(Laval and Leclercq, 2013, part B)

First order macroscopic model (1)

- The PDE expression
  - in Eulerian coordinates

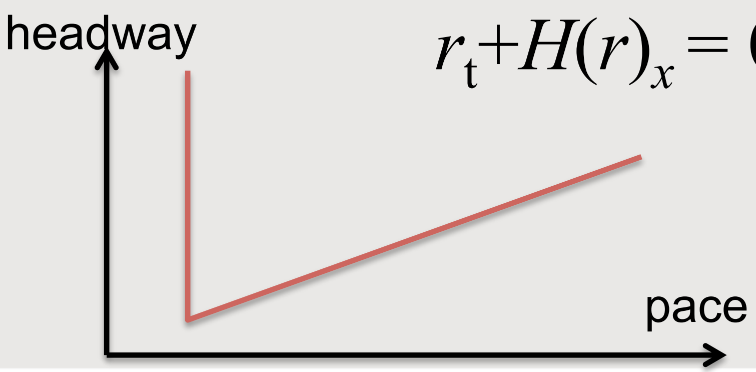
$$k_t + Q(k)_x = 0$$

  - in Lagrangian coordinates

$$s_t + V(s)_x = 0$$

  - in T coordinates

$$r_t + H(r)_x = 0$$



First order macroscopic model (2)

- The Hamilton-Jacobi (HJ) expression
  - In Eulerian coordinates

$$q=Q(k)$$

  - In Lagrangian coordinates

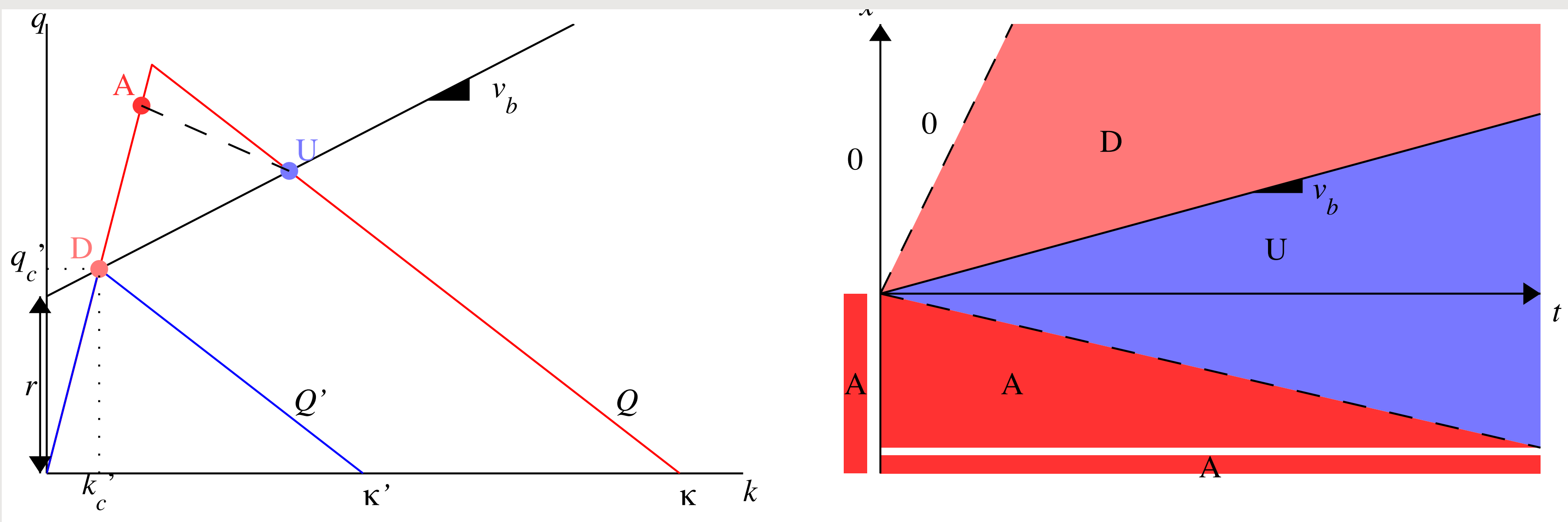
$$v=V(s)$$

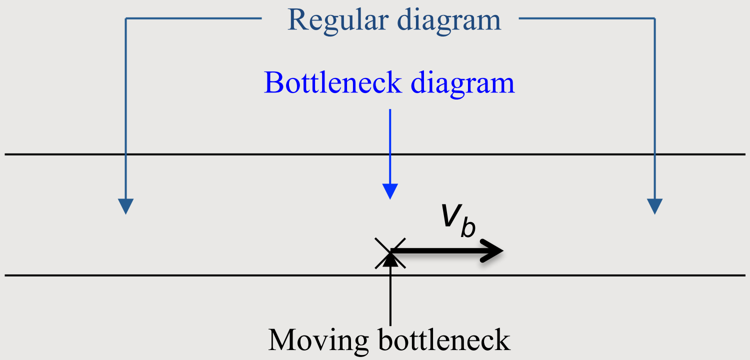
  - In T coordinates

$$h=H(r)$$

Appropriate expression of the fundamental diagram

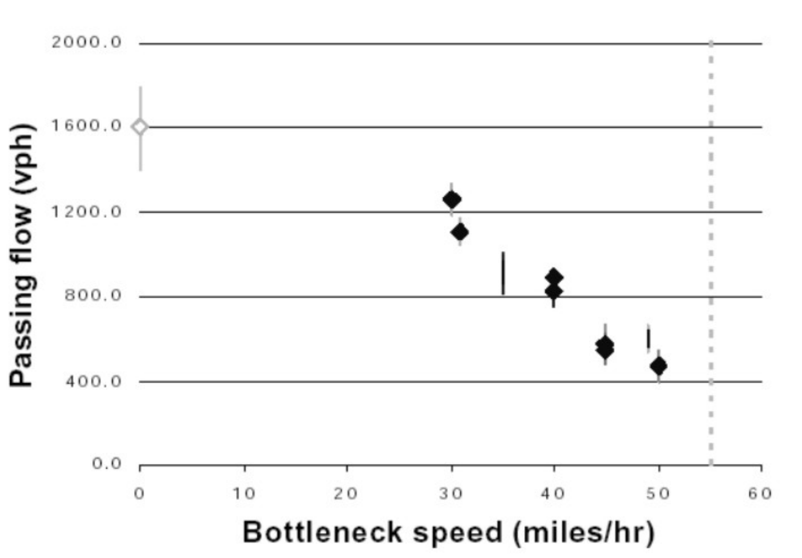
The moving bottleneck theory





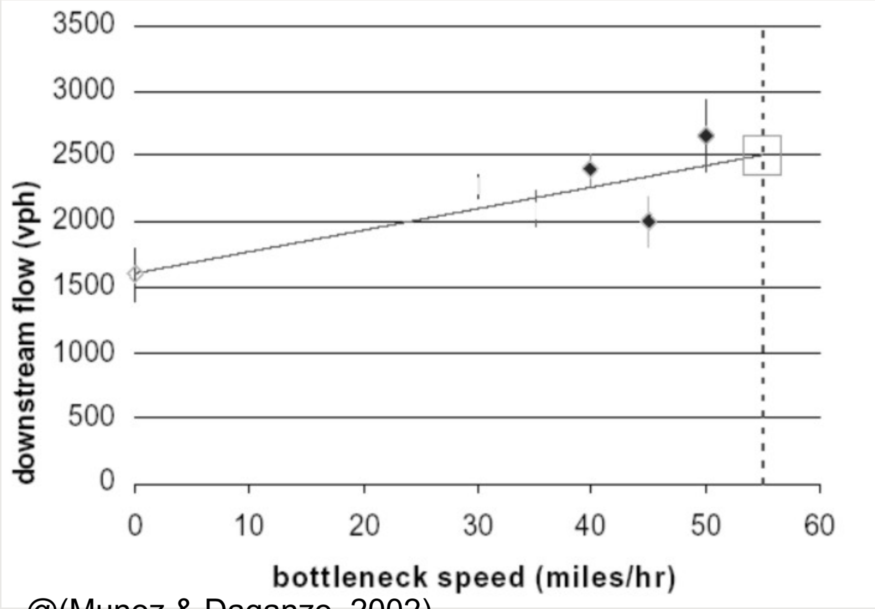
(Gazis and Herman, 1992, TS)
(Newell, 1998, part B)
(Lebacque et *al*, 1998, TRR)
(Leclercq et *al*, 2004, TRR)

The only dedicated experiment



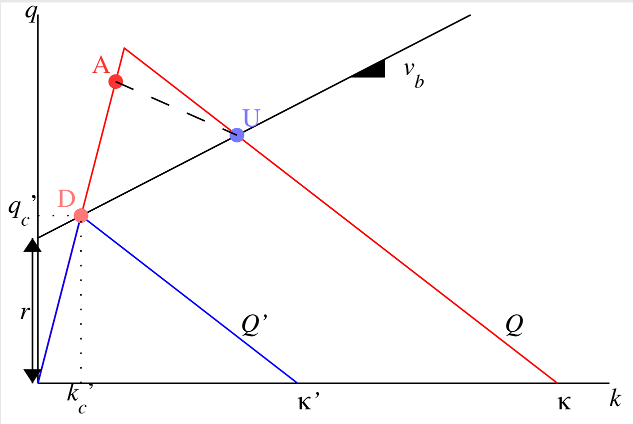
@(Munoz & Daganzo, 2002)

1 - (Munoz and Daganzo, 2002, ISTTT)



@(Munoz & Daganzo, 2002)

MB and FD


q
A
U
v_b
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q_c'
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Q'
Q
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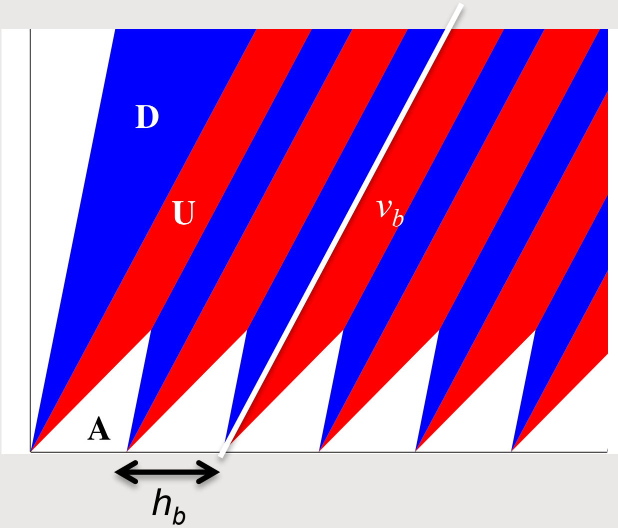
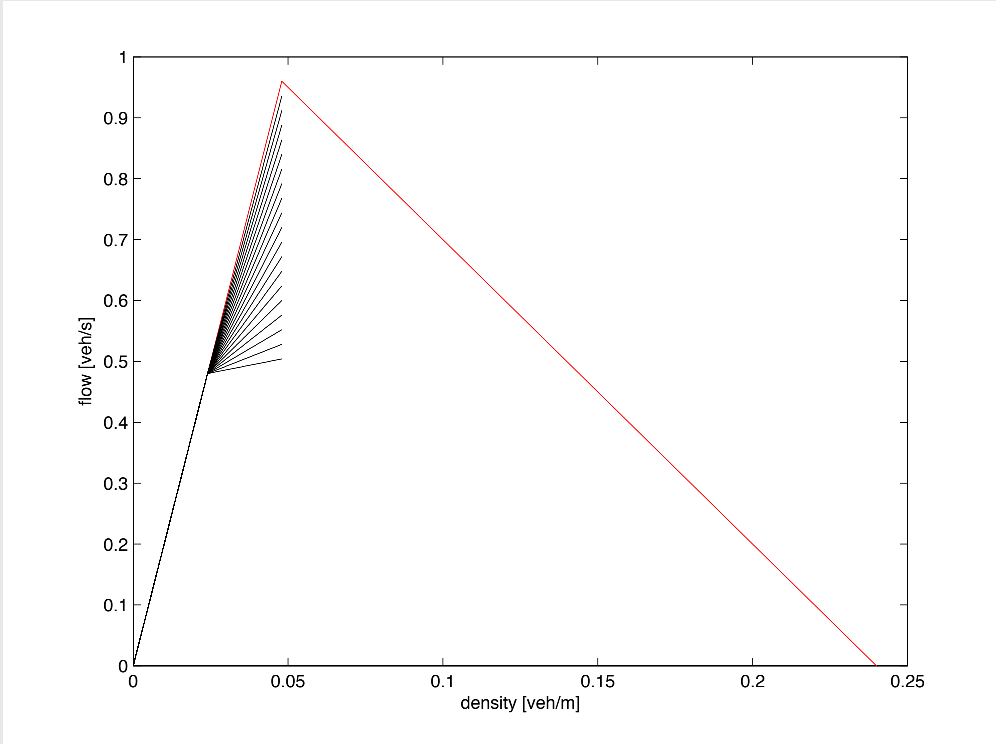
κ
k

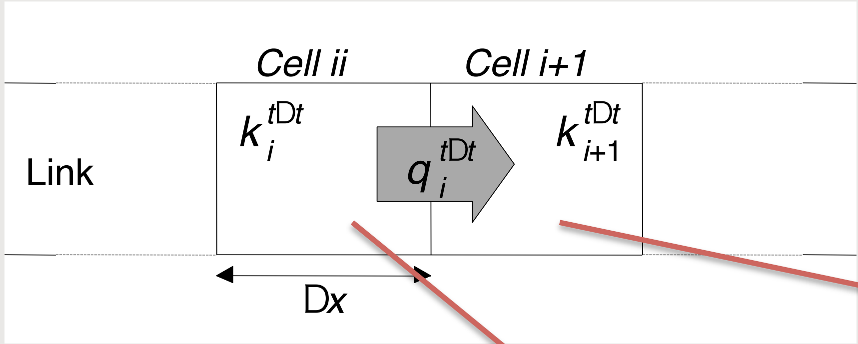

D
U
v_b
A
h_b


flow [veh/s]
density [veh/m]
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.05
0.1
0.15
0.2
0.25

Numerical issues

Approaches based on the Godunov scheme

Godunov scheme without MB

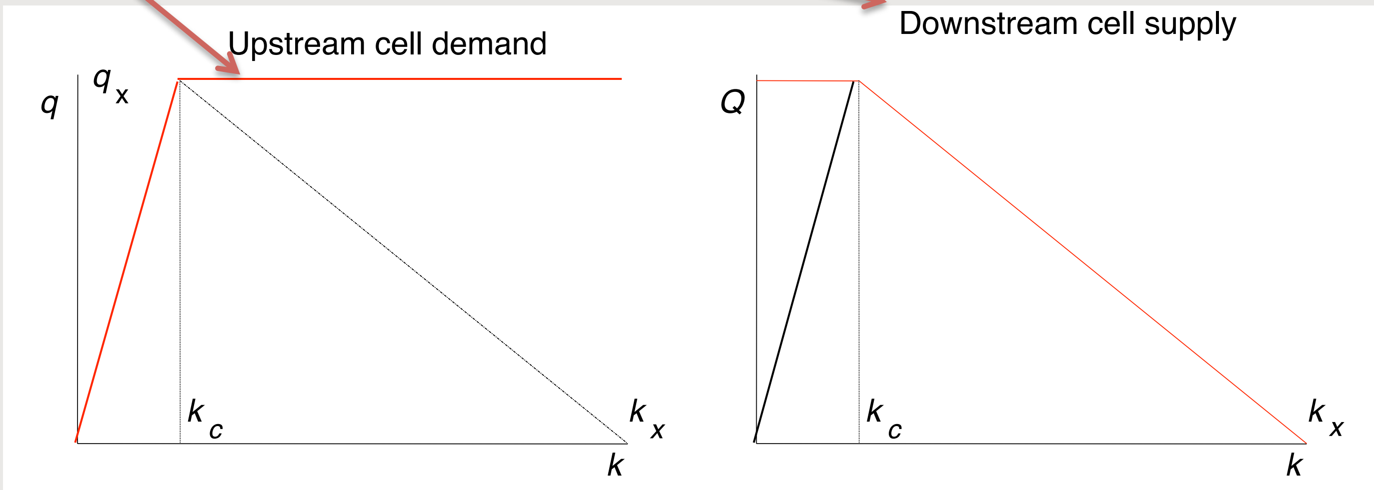


$$k_i^{t+\Delta t} = k_i^t + \left(q_{i-1}^{t\to t+\Delta t} - q_i^{t\to t+\Delta t}\right)\frac{\Delta t}{\Delta x}$$

$$q_i^{t\to t+\Delta t} = \min\left(\lambda\left(k_i^t\right), \mu\left(k_{i+1}^t\right)\right)$$

CFL condition:

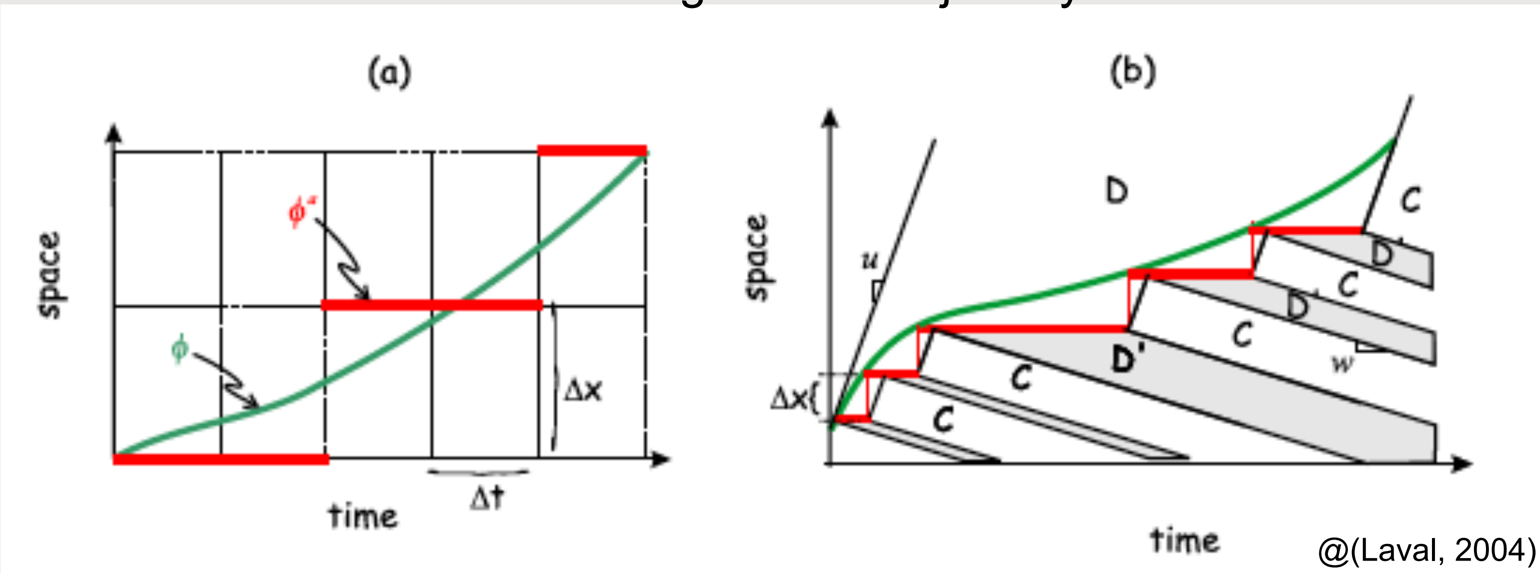
$$\Delta x \geq u\Delta t$$



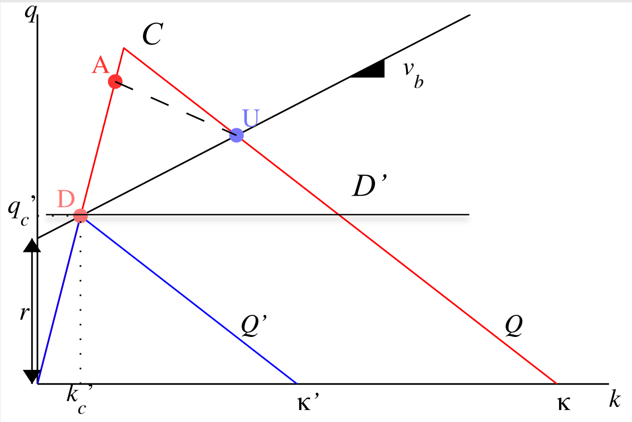
How to introduce moving discontinuities ?

First historical solution (1)

Discretizing the MB trajectory

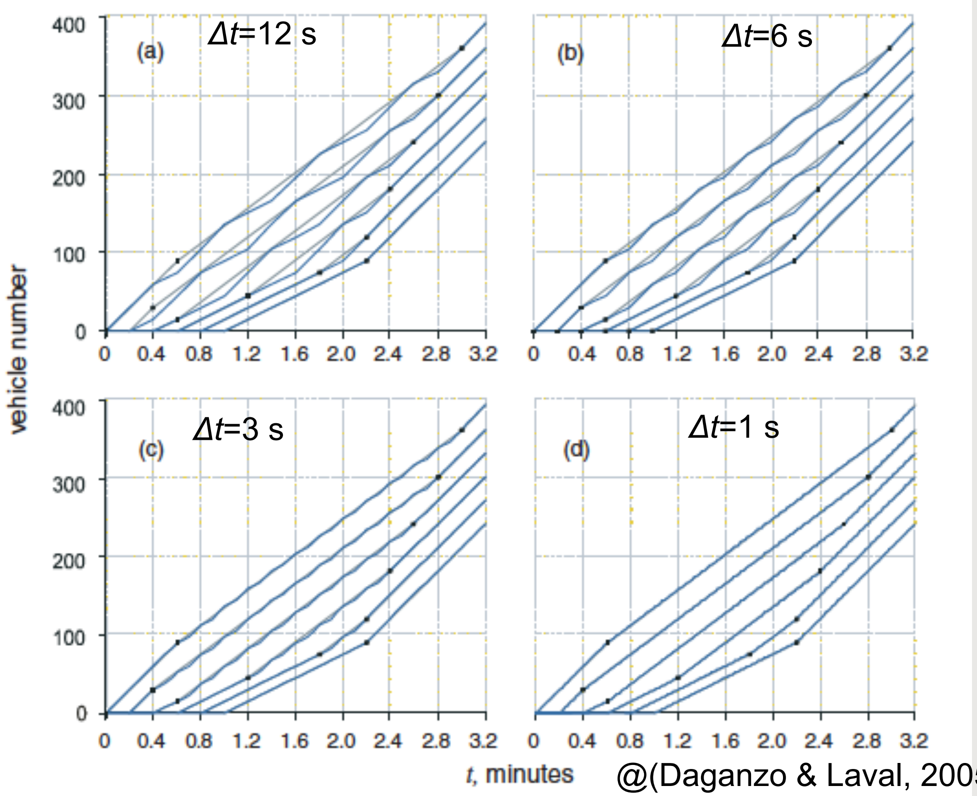


@(Laval, 2004)

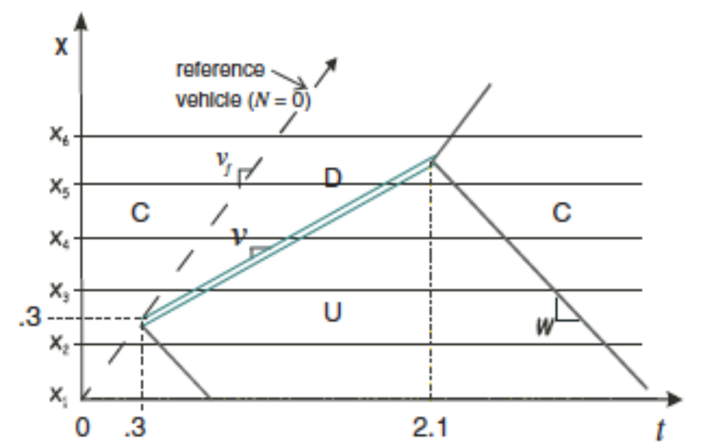


(Daganzo & Laval, 2005, part B)
(Laval, 2004, PhD thesis)

First Historical solution (2)



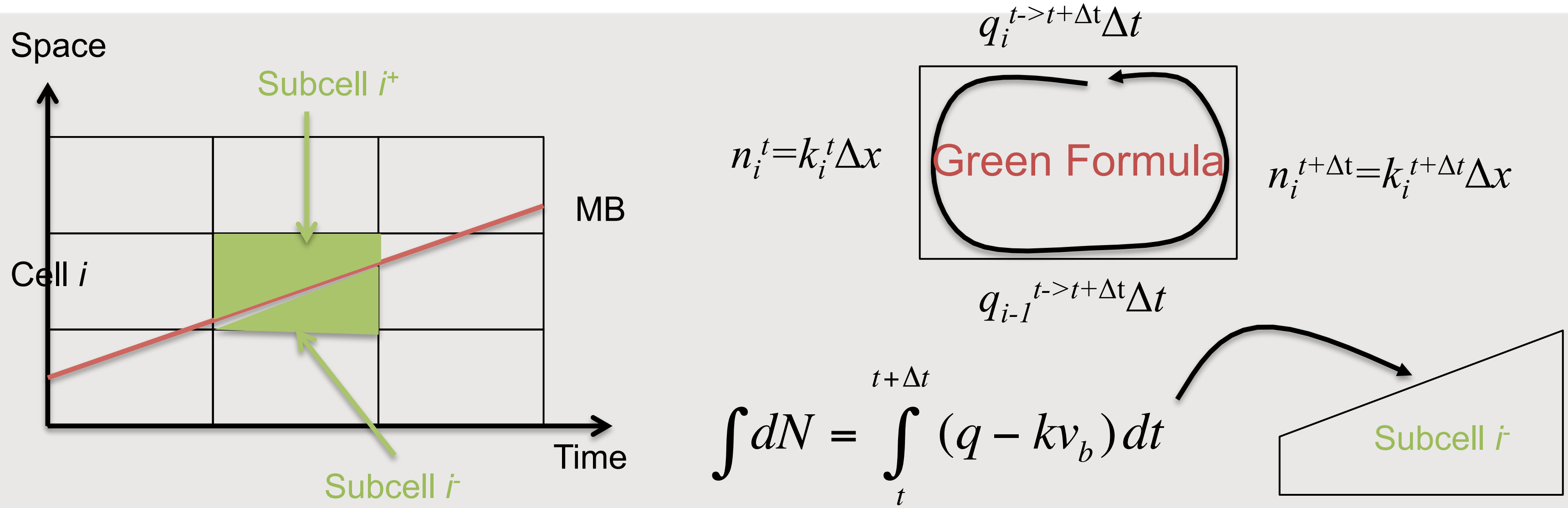
@(Daganzo & Laval, 2005)



@(Daganzo & Laval, 2005)

This method converges in *N* but not in flow

Splitting cells around MB (1)



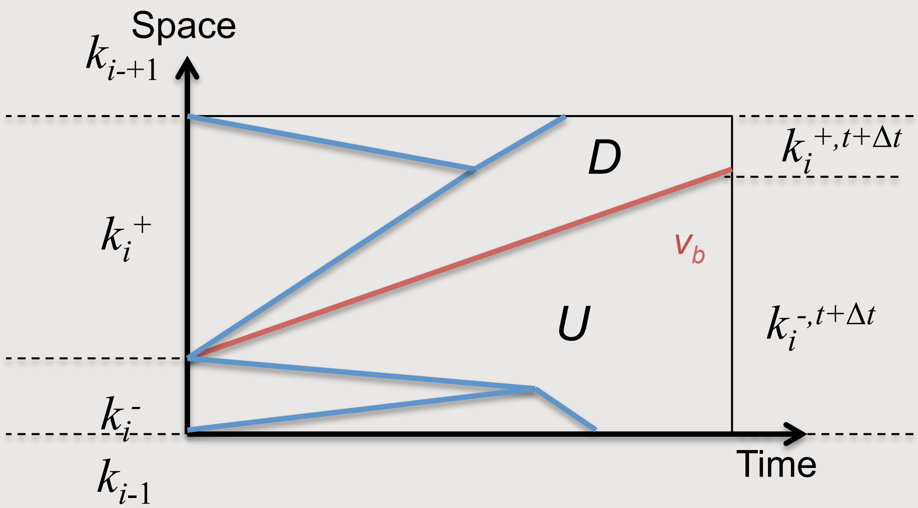
- Basic idea
  - Applying the original concept of the Godunov scheme (Green formula) when splitting cells that contain a MB
  - Properly calculating the increase of *N* over *t* on the cell border
  - The CFL condition no longer holds. Thus, Riemann problem on cell borders are no longer independent. We have to carefully track waves.

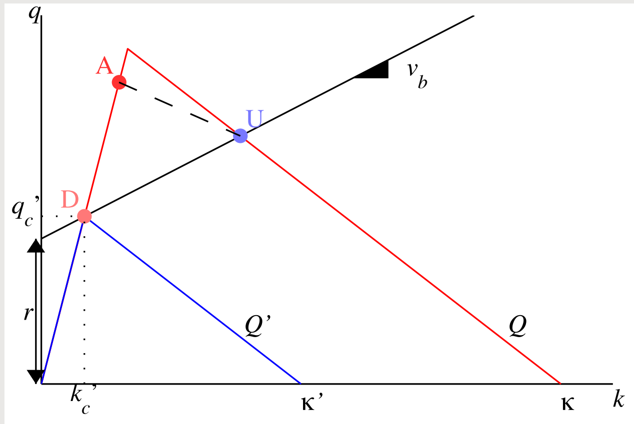
(Leclercq, 2005, unpublished)

Similar idea can be found in (Li & Zhang, 2012, TRB)

Splitting cells around MB (2)

Wave interactions between borders are easy to catch when the FD is piecewise linear because there is no rarefaction waves.





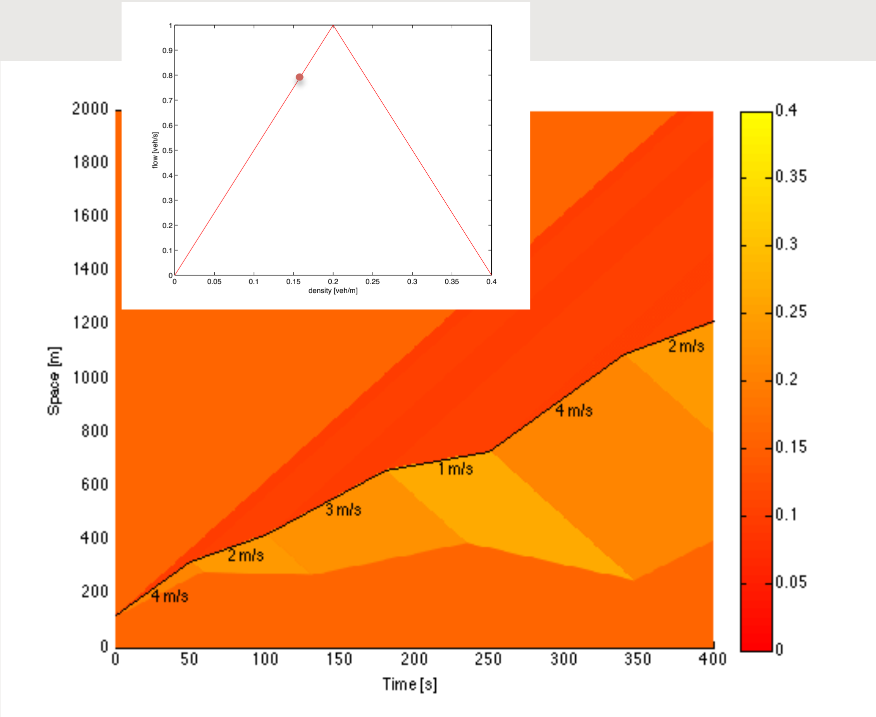
Intermediate time steps are introduced when MBs cross the border of the original rectangular cells

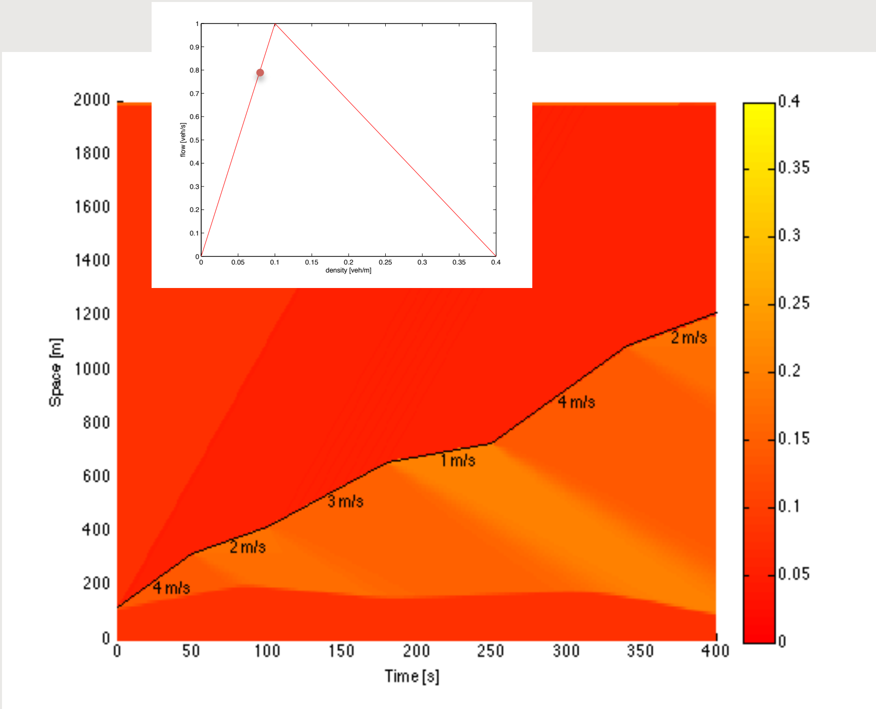
We have to determine if the MB is active or not at each time step. Inactive bottleneck may lead to change in v_b.

The simplest case is when the FD is triangular

Splitting cells around MB (3)

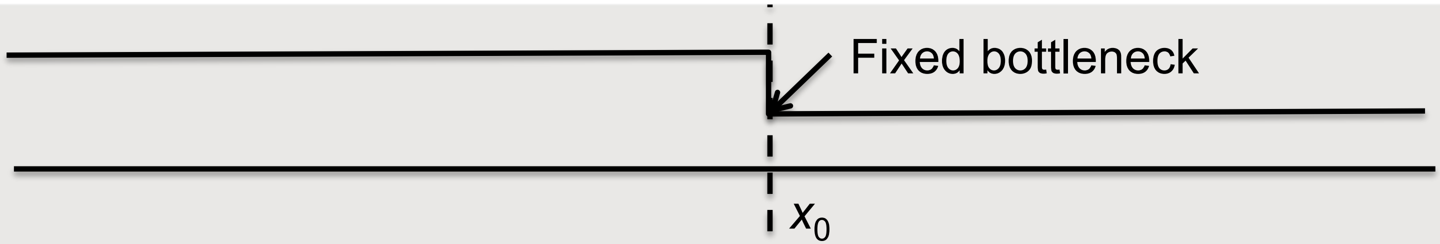
Numerical example (Δt=1s) ; q_0=0.8 veh/s

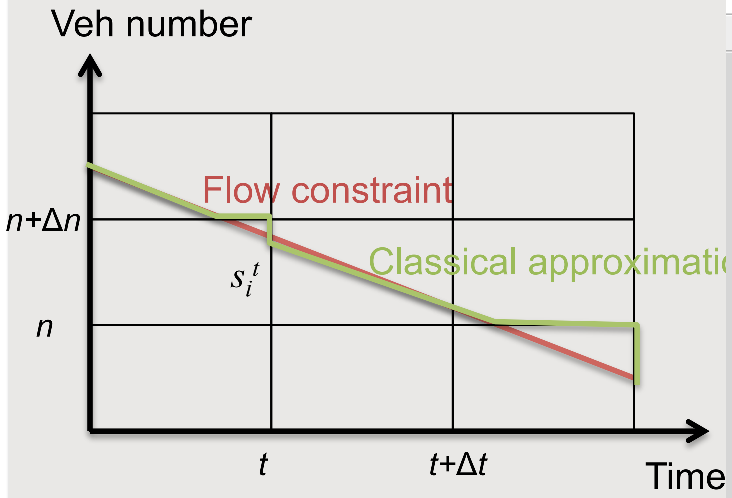


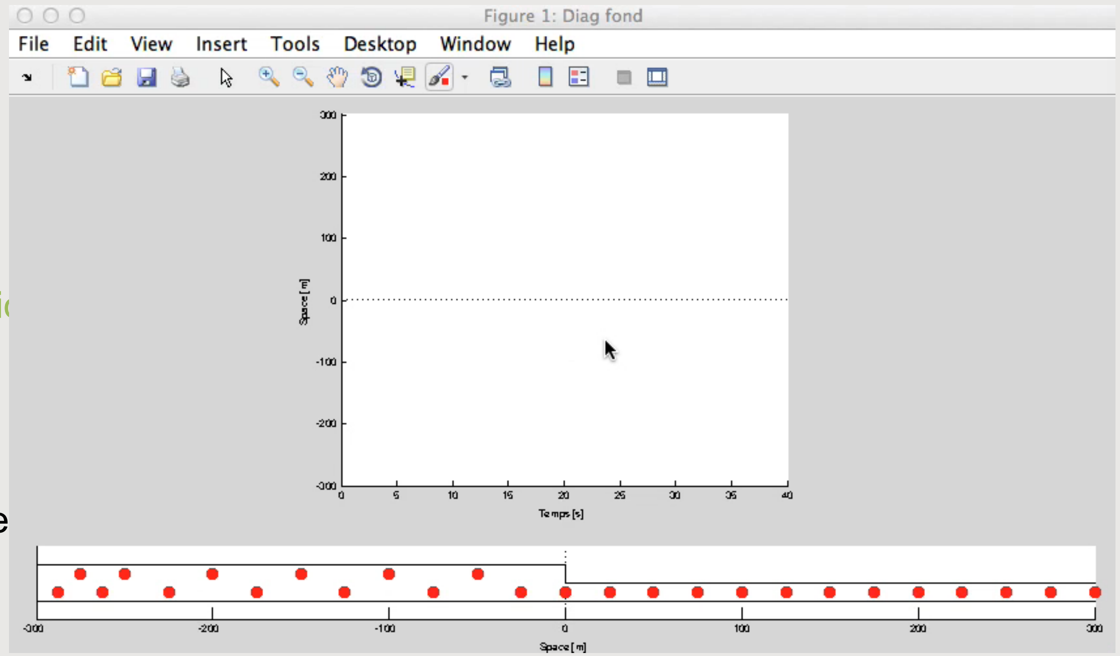


Solution remains exact
when the FD is isosceles

Applications to Lagrangian coordinates (1)





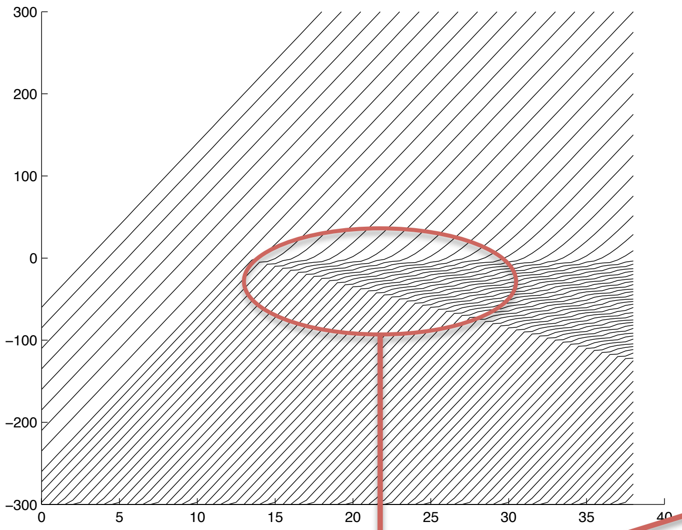


$$s_i^{t+\Delta t} = s_i^t + \frac{\Delta t}{\Delta n}\left(V^*\left(s_i^t\right) - V^*\left(s_{i-1}^t\right)\right)$$

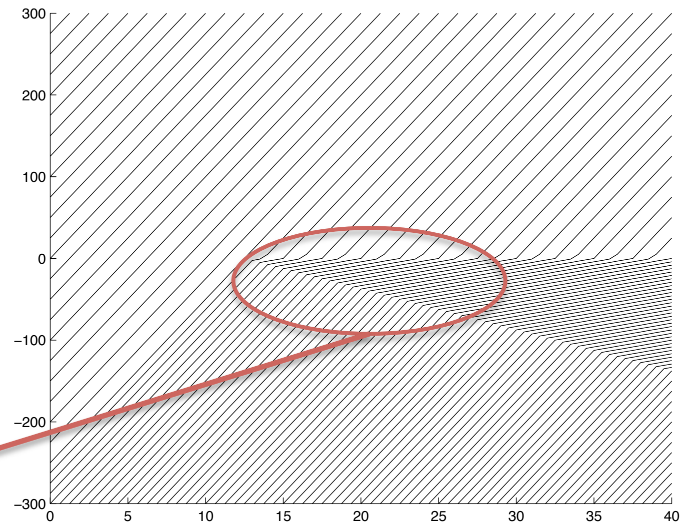
Godunov scheme

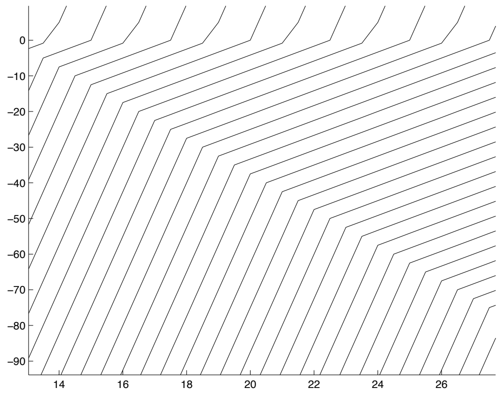
Applications to Lagrangian coordinates (2)

Classical approximation



Cell splitting





Numerical issues

Approaches based on the variational theory

VT : General principles (1)

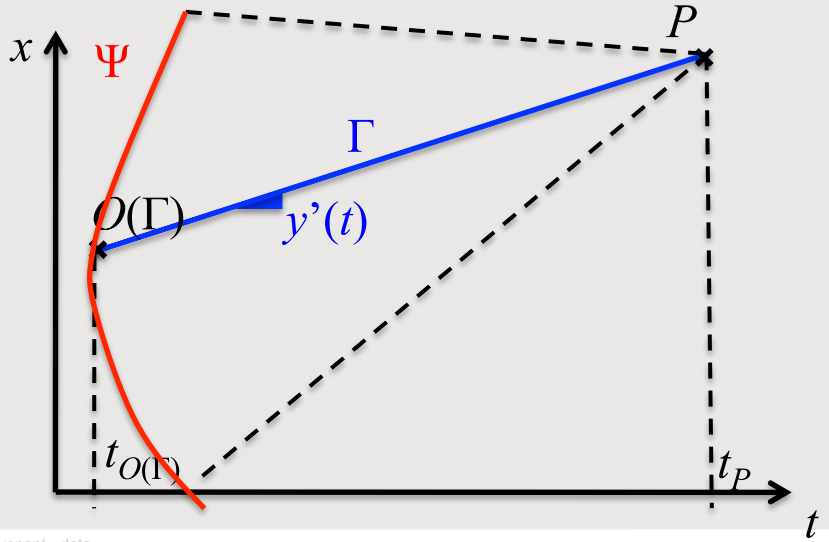
HJ Equation:

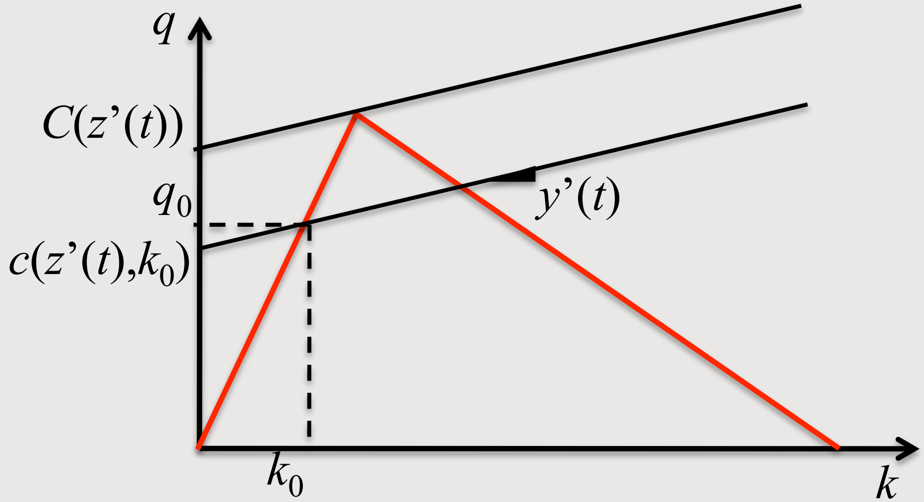
$$q = Q(k) \quad \Leftrightarrow \quad \partial_t k = Q(-\partial_x k)$$

General expression for the solutions

$$N_P = \min_{\Gamma \in D_P} \left(N_{O(\Gamma)} + \Delta(\Gamma) \right)$$

$$\Delta(\Gamma) = \int_{t_{O(\Gamma)}}^{t_P} r(y'(t), k)\, dt$$





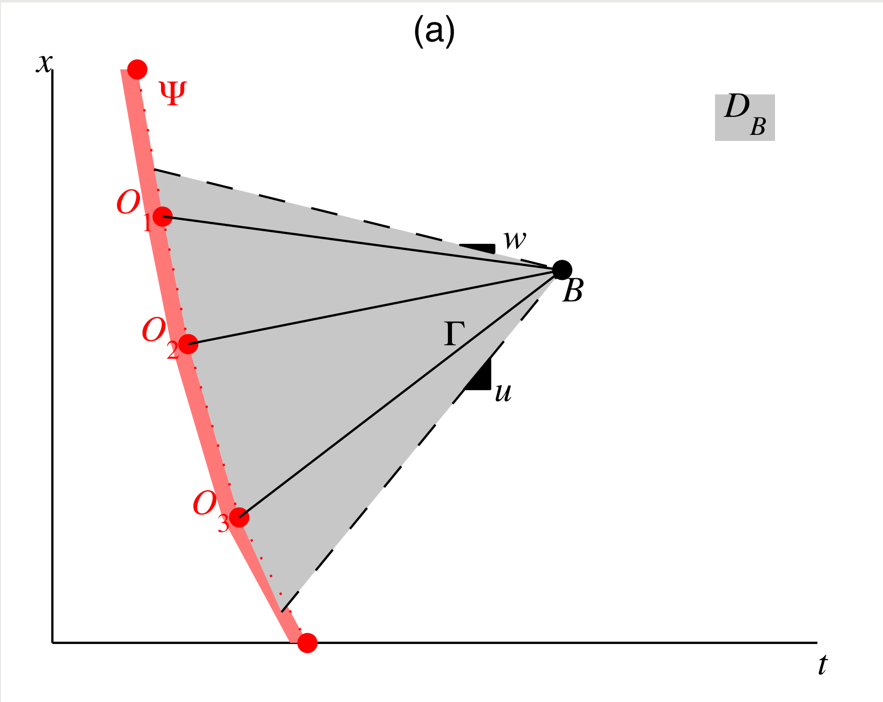
$$r(y'(t), k) = d_t N = \partial_t N + y'(t)\partial_n N$$

$$r(y'(t), k) = Q(k(t)) - y'(t)k(t)$$

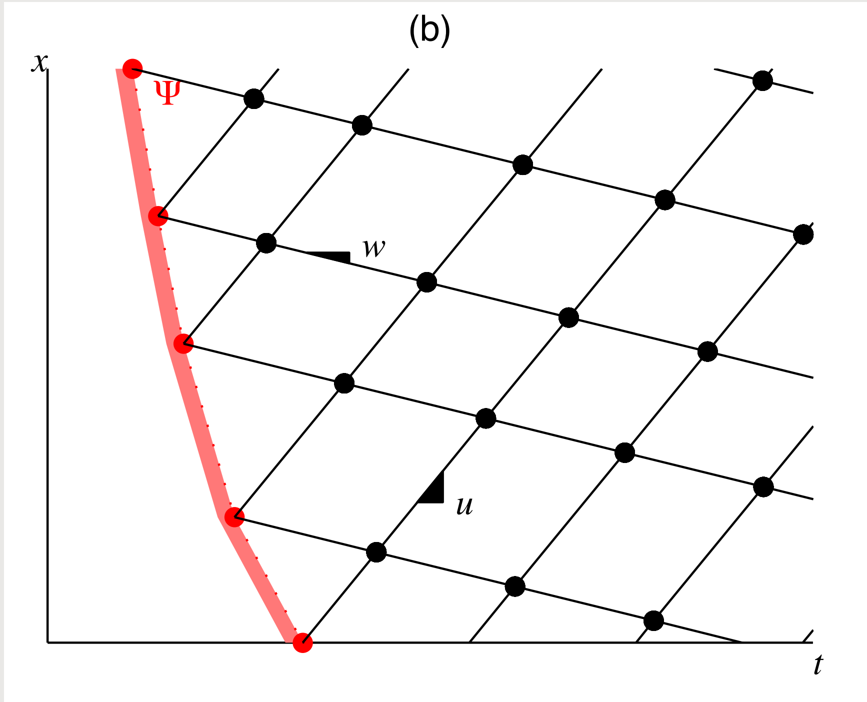
$$R(y'(t)) = \sup_s \left(c(y'(t)), k \right)$$

Legendre transformation

VT : General principles (2)



Piece-wise linear initial condition

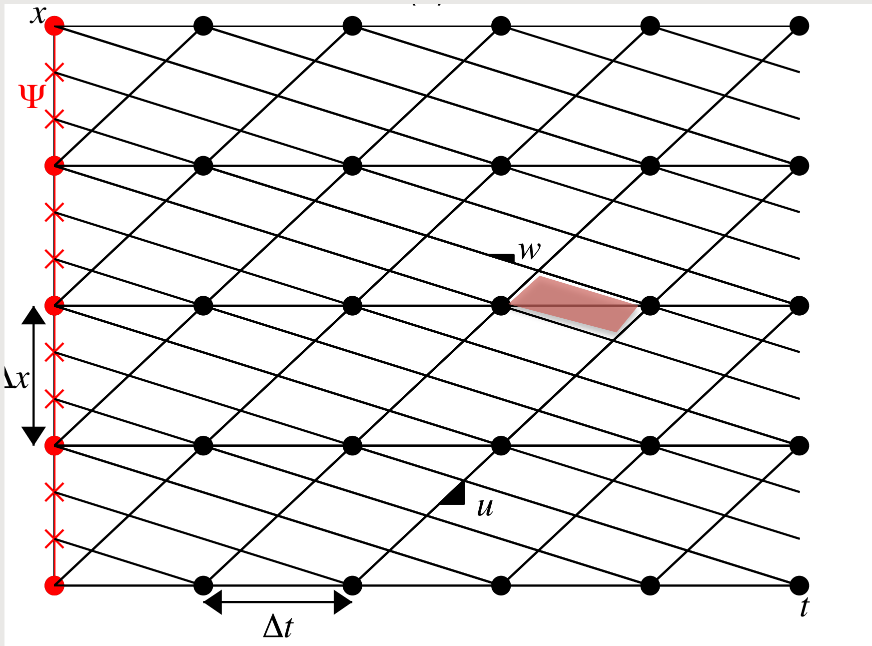


Sufficient grid for triangular FD

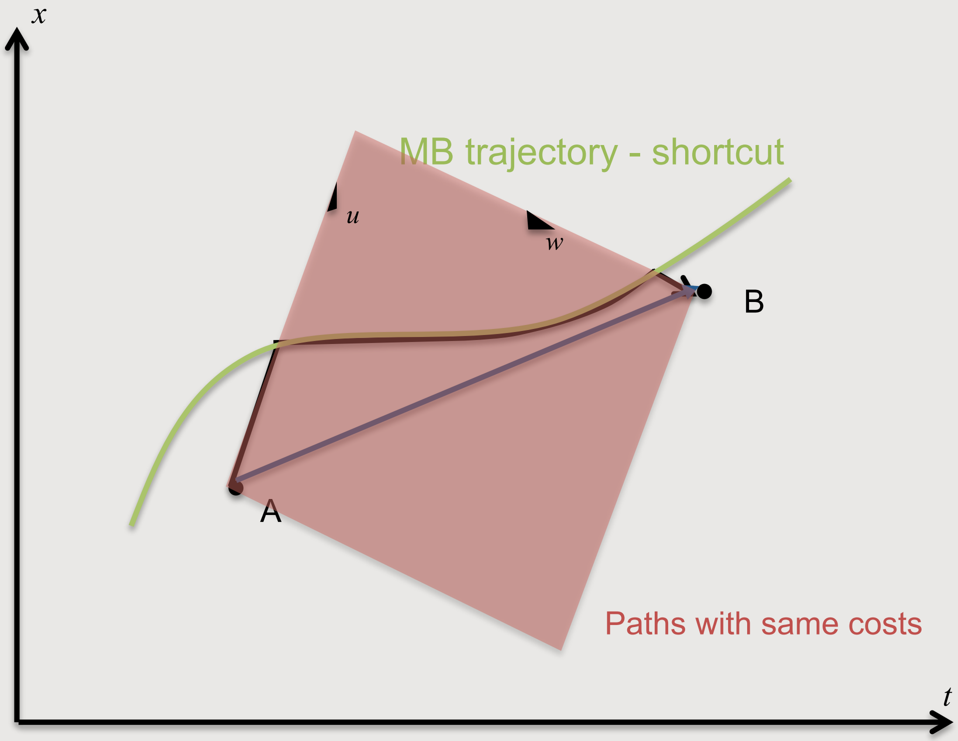
VT is really relevant for PWL FD
and especially triangular FD

VT : General principles (3)

Defining a sufficient rectangular grid



VT: The Shortcut Theorem (1)

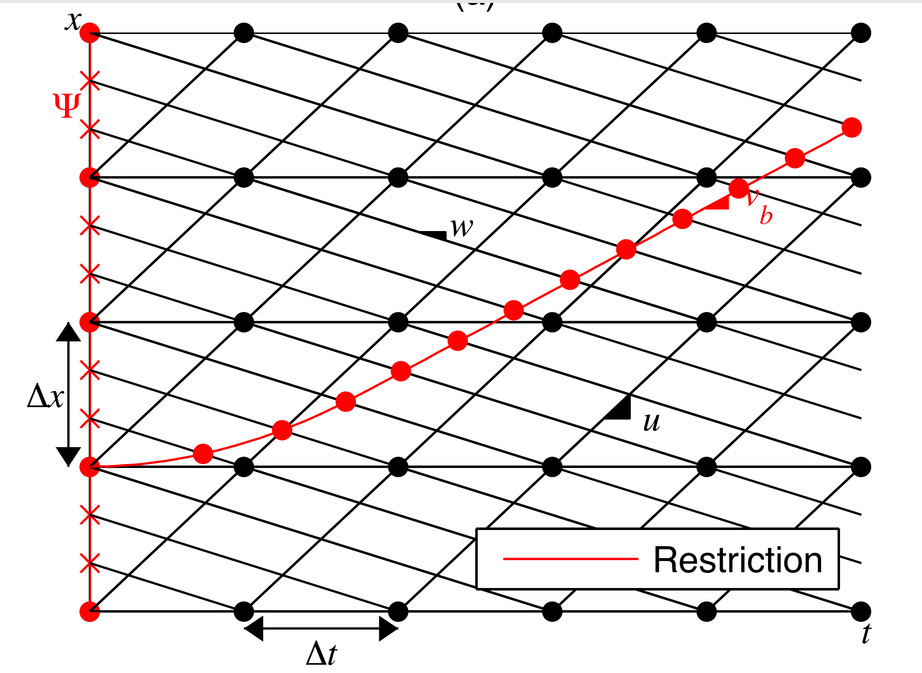


(Daganzo & Menendez, 2005, ISTTT)

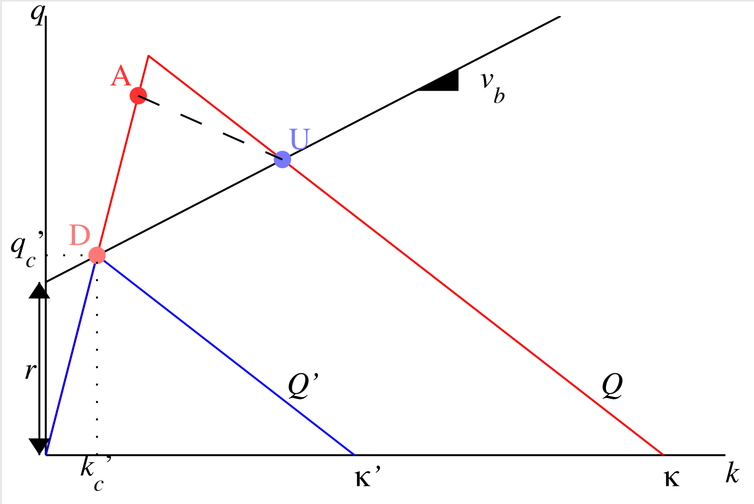
Optimal paths with shortcuts are composed with:

- Continuous sections of bottlenecks
- Least duration access and egress sub-paths
- Inter-bottleneck sub-paths with speed *u*

VT: The Shortcut Theorem (2)



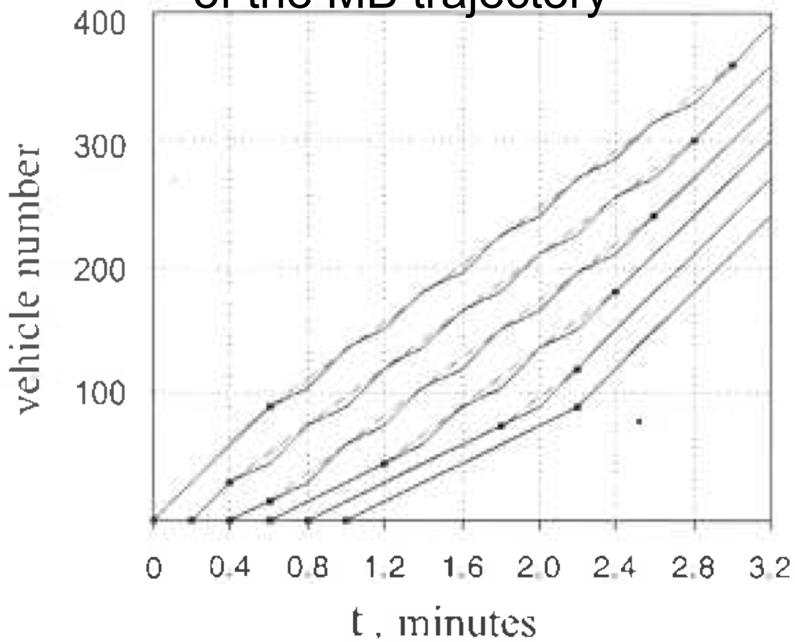
Modification of the numerical grid
when MBs are present



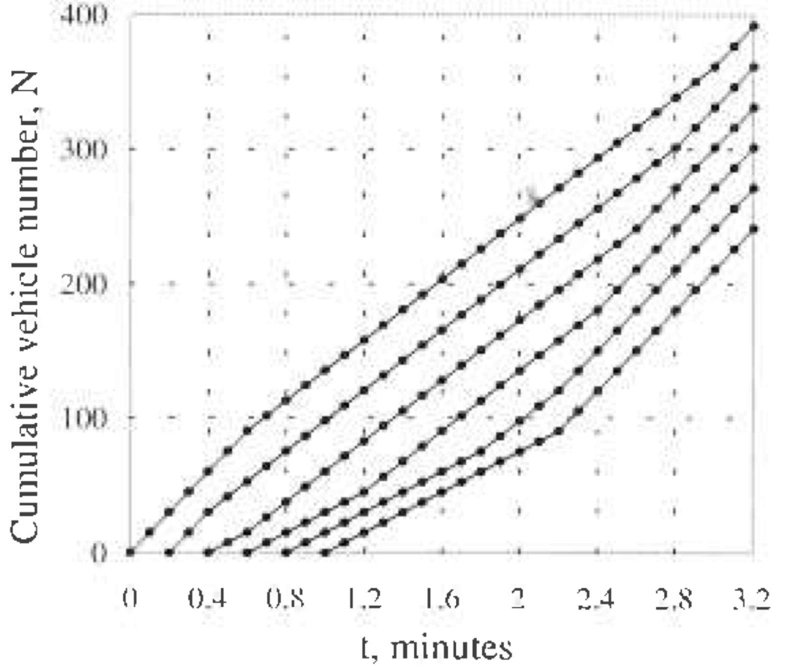
The main difficulty is to account for feedback
(downstream traffic conditions => MB trajectory)

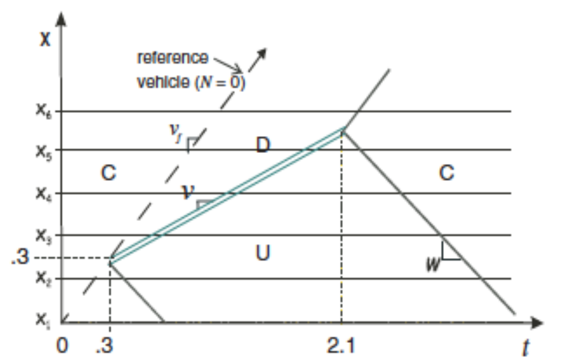
Historical example

Godunov scheme & discretization of the MB trajectory



VT scheme

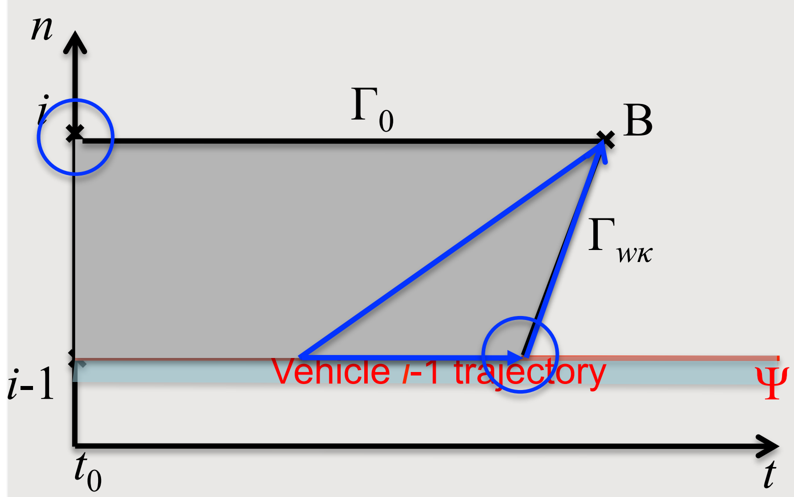


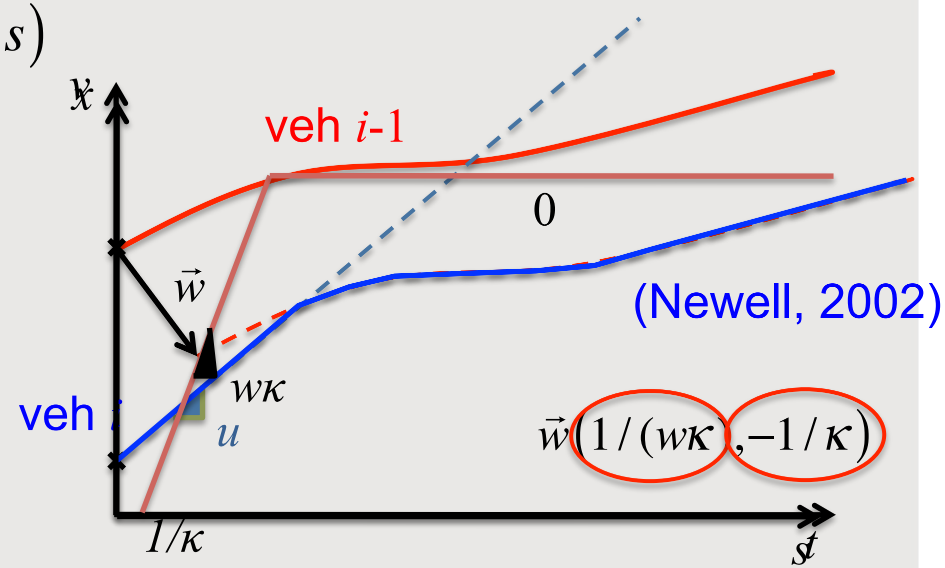


@(Daganzo & Menendez, 2005, ISTTT)

VT in Lagrangian coordinates (1)

HJ Equation: $v = V(s) \quad \Leftrightarrow \quad \partial_t s = V(-\partial_n s)$



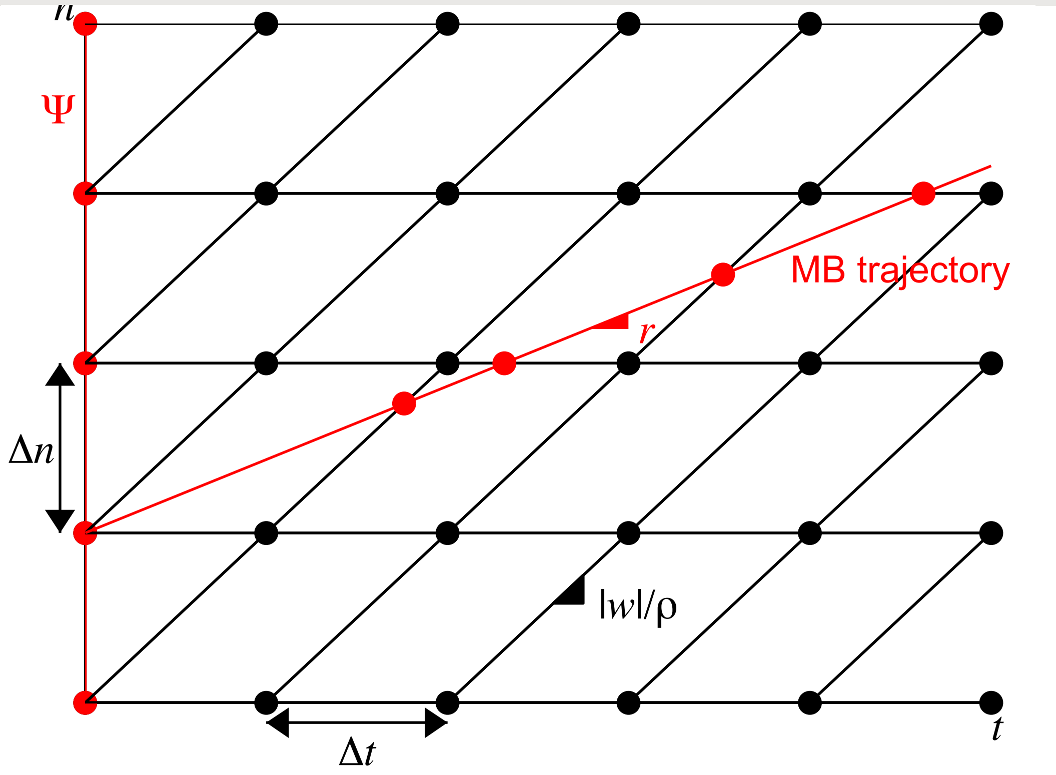


$$X_B = \min\left(X_{O(\Gamma_0)} + \Delta(\Gamma_0), X_{O(\Gamma_{w\kappa})} + \Delta(\Gamma_{w\kappa})\right)$$

$$X_B = X(t,i) = \min\left(\underbrace{X(t_0,i) + u(t-t_0)}_{free-flow}, \underbrace{X(t - 1/(w\kappa), i-1) - 1/\kappa}_{congestion}\right)$$

VT in Lagrangian coordinates (2)

Numerical grid in Lagrangian

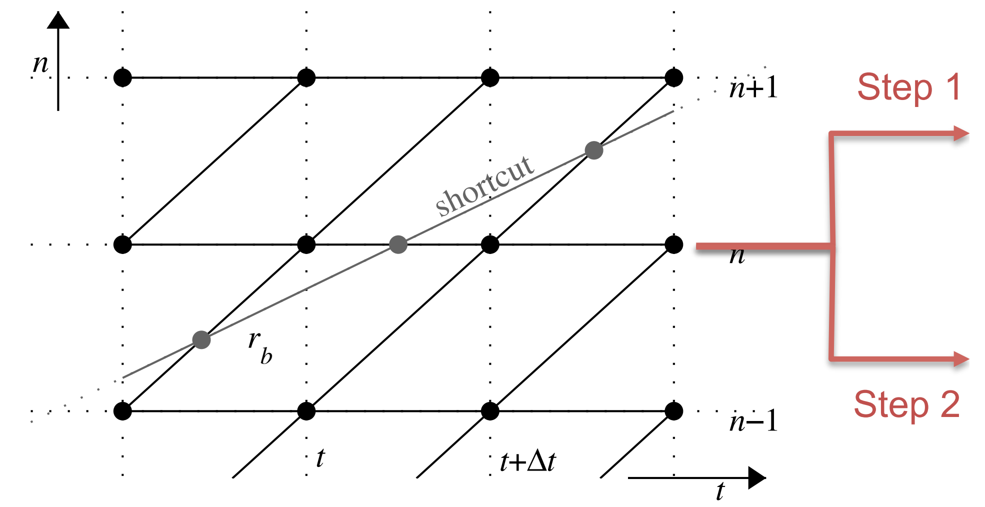


In Lagrangian, MB can be slower vehicles but also fixed Eulerian discontinuities

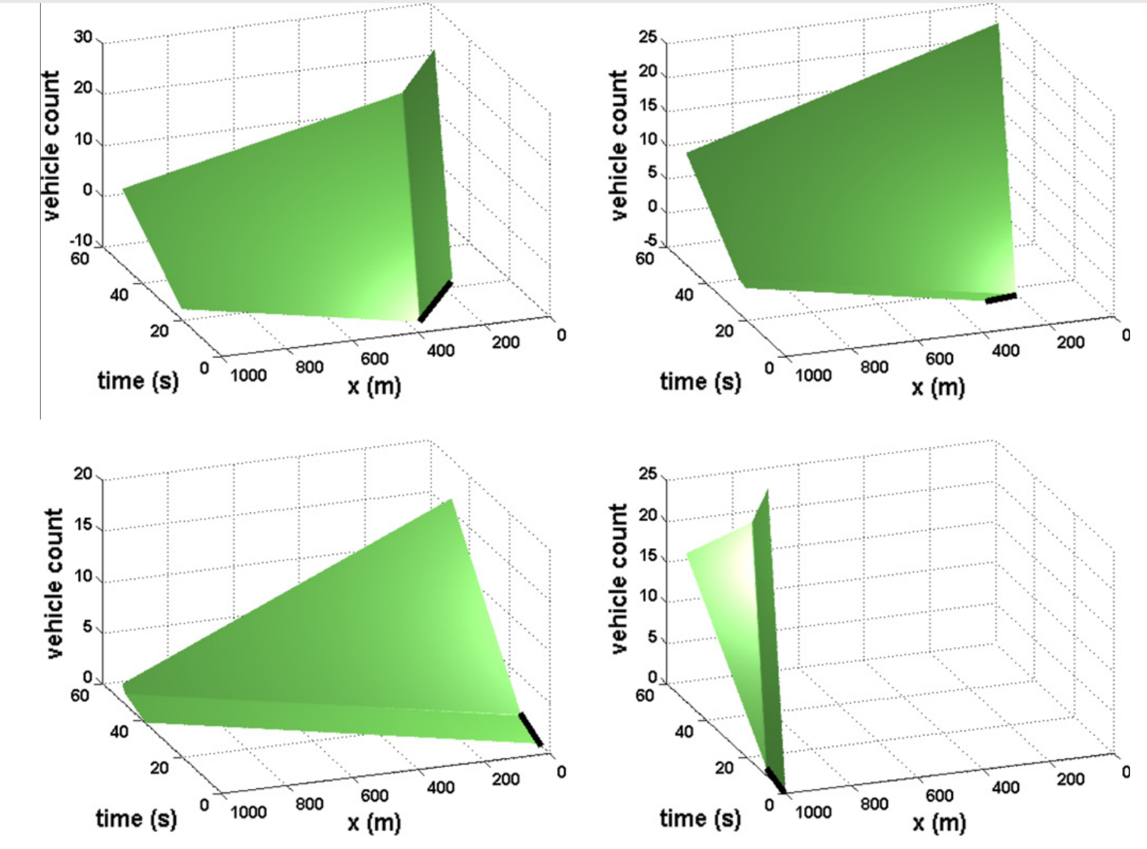
VT in Lagrangian coordinates (3)

A careful attention should be paid to determine if the bottleneck is active or not

In some cases, inactivity changes the bottleneck path (feedback)



Related numerical approach - Viability Theory



@(Mazaré, Dehwah, Claudel, Bayen, 2011, part B)

Viability theory is the sister of the variationnal theory

Search for solutions from the initial and boundary (linear) conditions

Grid free numerical scheme that are exact even if the FD is PWQ

As for VT, MB are straightforward to account for in such a framework, except for the feedback component

Conclusion

- Solving the LWR model with internal boundary conditions is now a well-addressed problem
- The choice of the numerical method depends on the initial assumptions
- Piecewise linearity (for initial conditions and/or FD) is really convenient properties because it permits to use exact numerical scheme
- The three representations of traffic flow with MB provide a versatile and consistent framework to represent lots of traffic flow problems

Thank you for your attention

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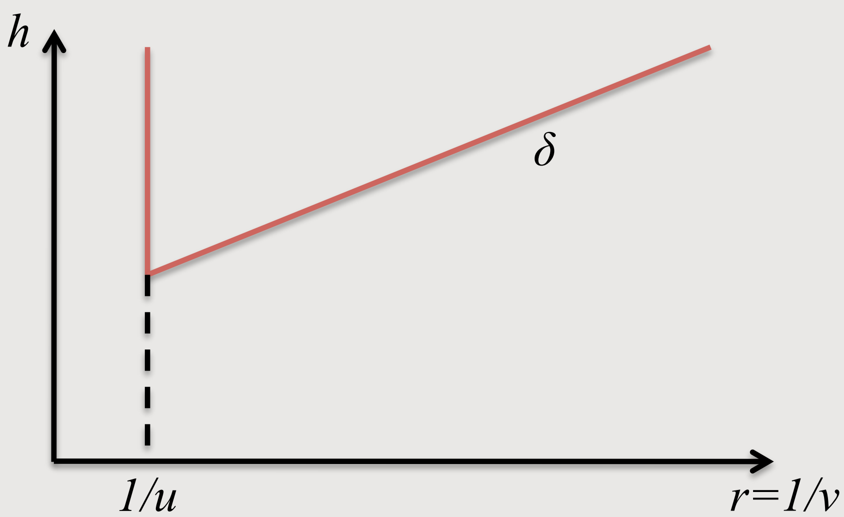
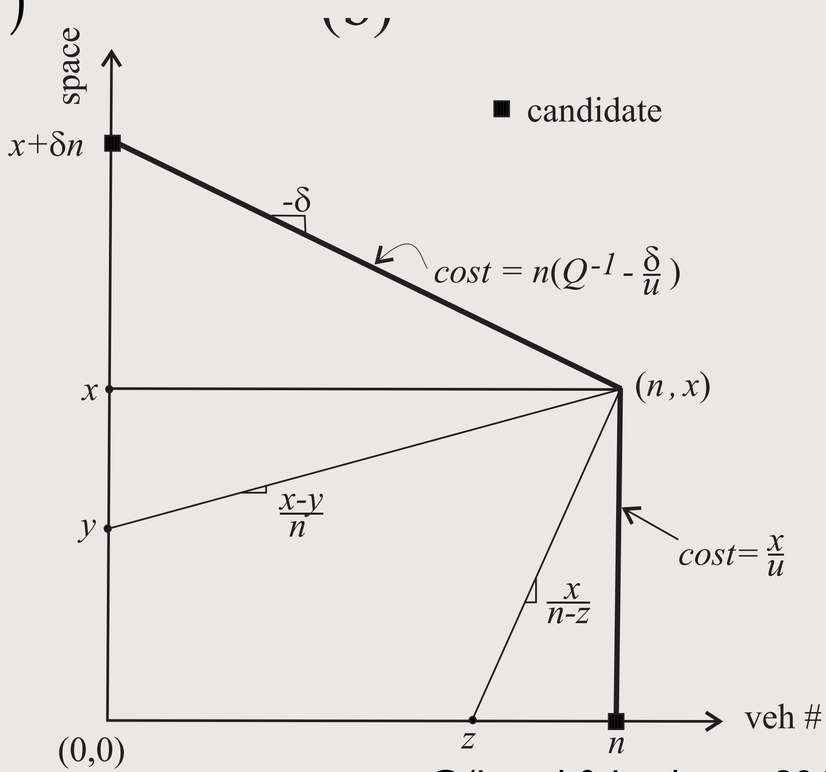
VT in T coordinates (1)

HJ Equation: $h = H(r) \quad \Leftrightarrow \quad \partial_n t = H(-\partial_x t)$

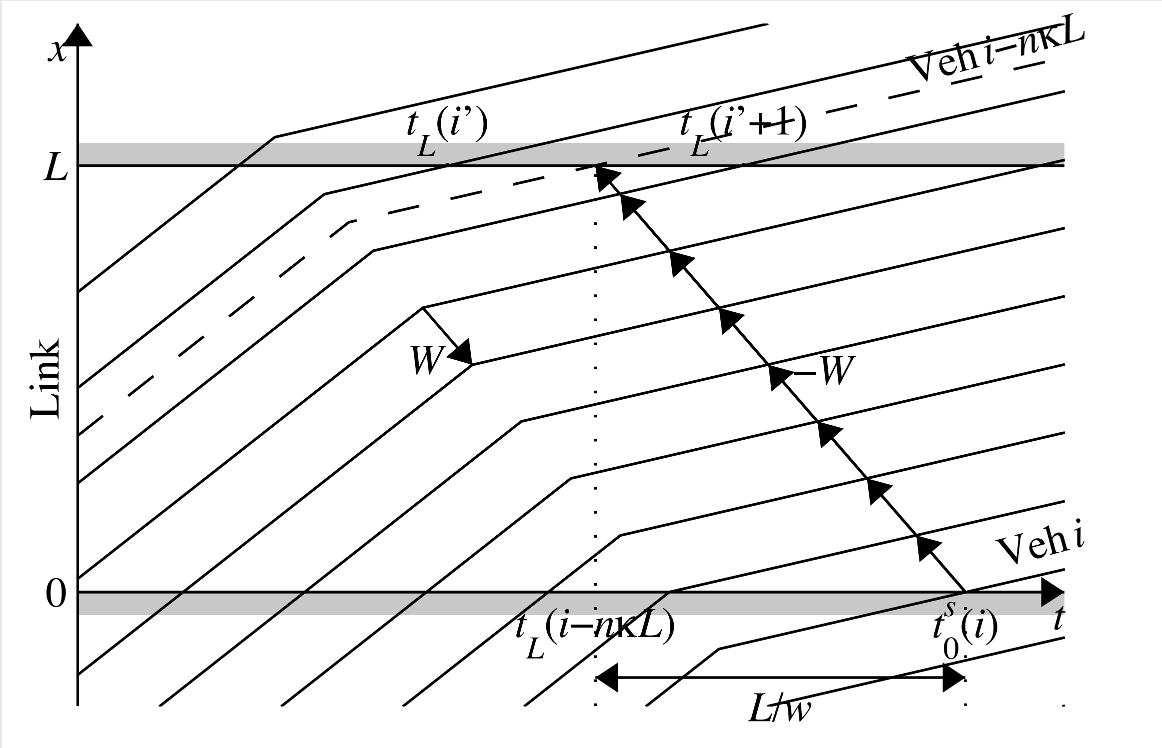




@(Laval & Leclercq, 2013)

VT in T coordinates (2)

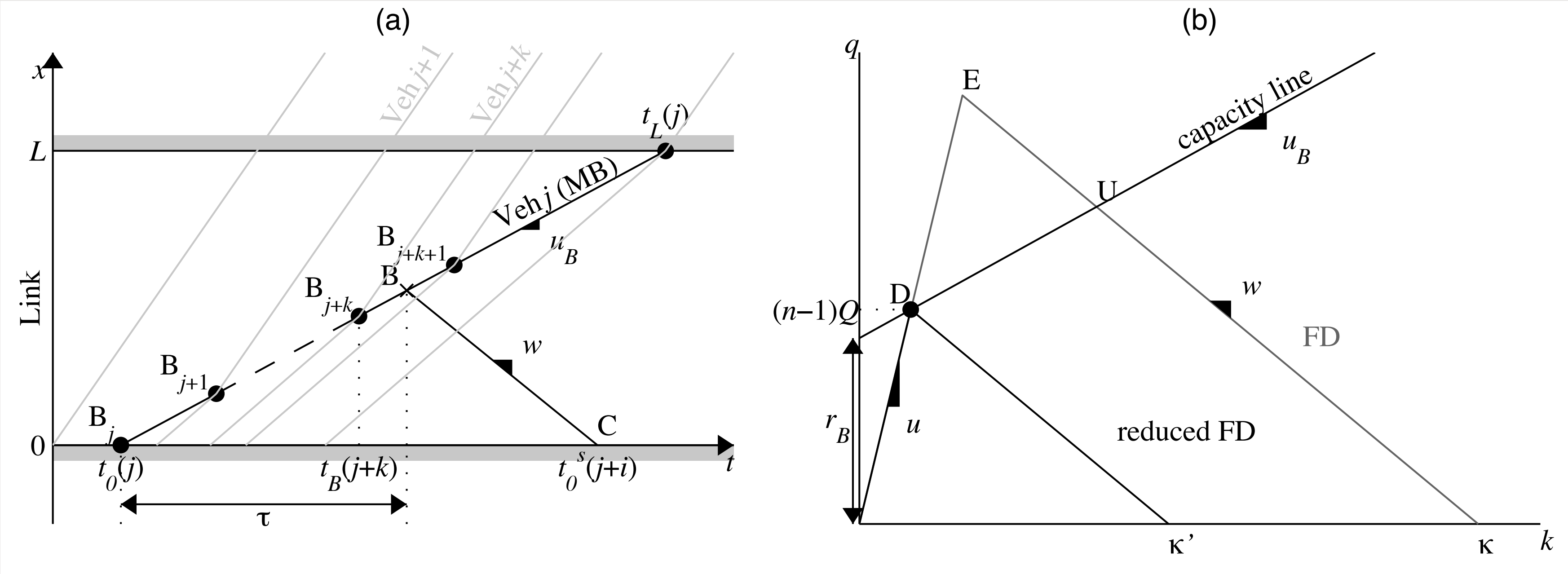
Practical numerical scheme (mesoscopic scale)



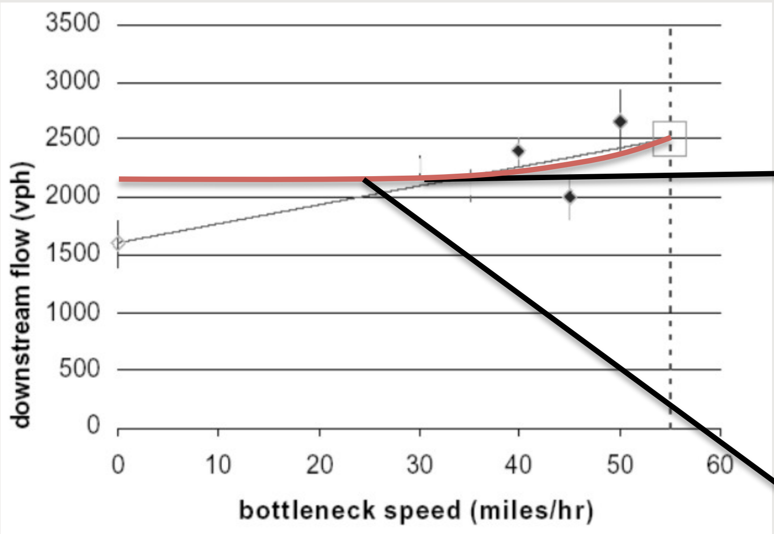
Supply time for next entering vehicle:

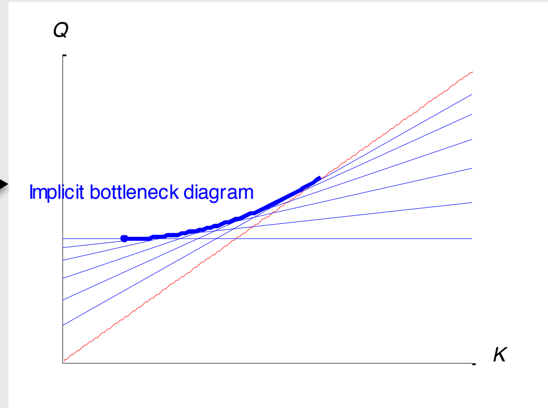
$$t_0^s(i) = t_L\left(i - n\kappa L\right) + L/w$$

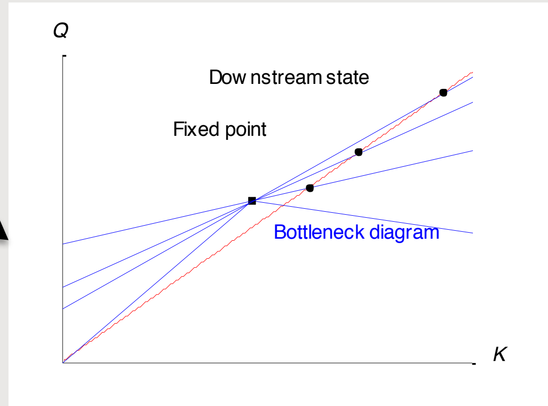
Introducing MB in T coordinates



The only dedicated experiment (2)







@(Munoz & Daganzo, 2002)