# Parallel Tensor Train through Hierarchical Decomposition


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# Collaborators

This is joint work with ...

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# Overview

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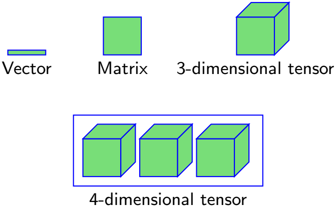
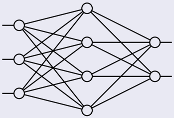
# Tensors: Multidimensional Arrays




- **Neuroscience**: Neuron × Time × Trial
- **Transportation**: Pickup × Dropoff × Time
- **Media**: User x Movie x Time
- **Ecommerce**: User x Product x Time
- **Cyber-Traffic**: IP x IP x Port x Time
- **Social-Network**: Person x Person x Time x Interaction-Type

## High Dimensional Tensors

- **Neural Network**:



- **Molecular Simulation**: To represent wave functions
- **Quantum Computing**: To represent qubit states

# Tensor computations

- Memory and computation requirements are exponential in the number of dimensions
  - A molecular simulation involving just 100 spatial orbitals manipulate a huge tensor with $4^{100}$ elements
- People work with low dimensional structure (decomposition) of the tensors
  - A tensor is represented with smaller objects
  - Improves memory and computation requirements
- Limited work on parallelization of tensor algorithms
- Most tensor decompositions rely on Singular Value Decomposition (SVD) of matrices

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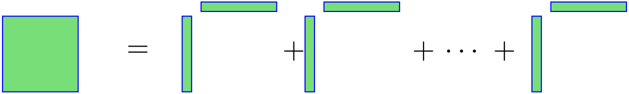
# Singular Value Decomposition (SVD) of Matrices

- It decomposes a matrix $A \in \mathbb{R}^{m \times n}$ to the form $U\Sigma V^T$
  - $U$ is an $m \times m$ orthogonal matrix
  - $V$ is an $n \times n$ orthogonal matrix
  - $\Sigma$ is an $m \times n$ rectangular diagonal matrix
- It represents a matrix as the sum of rank one matrices
  - $A = \sum_i \Sigma(i;i) U_i V_i^T$
  - Minimum number of rank one matrices required in the sum is called the rank of the original matrix



# Popular Tensor Decompositions

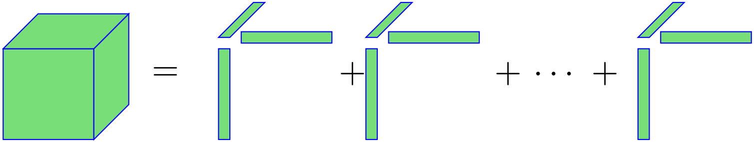
## Higher Order Generalization of SVD

- Canonical decomposition (equivalently known as Canonical Polyadic or CANDECOMP or PARAFAC)
- Tucker decomposition
- Tensor Train decomposition (equivalently known as Matrix Product States)

### Tensor Notations

- $\mathbf{A} \in \mathbb{R}^{n_1 \times \ldots \times n_d}$ is a $d$-dimensional tensor
- $\mathbf{A}(i_1, \cdots, i_d)$ represent elements of $\mathbf{A}$
- Use bold letters to denote tensors

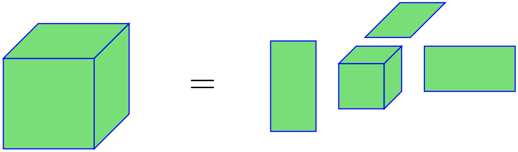
# Canonical Representation



- $\mathbf{A}(i_1, \cdots, i_d) = \sum_{\alpha=1}^{r} U_1(i_1, \alpha) U_2(i_2, \alpha) \cdots U_d(i_d, \alpha)$

- (+) For $n_1 = n_2 = \cdots n_d = n$, the number of entries $= \mathcal{O}(nrd)$
- (-) Determining the minimum value of $r$ is an NP-complete problem
- (-) No robust algorithms to compute this representation

# Tucker Representation

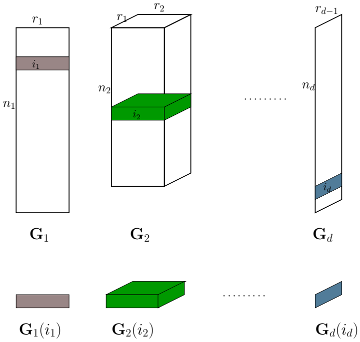


- Represents a tensor with $d$ matrices and a small core tensor
- $\mathbf{A}(i_1, \cdots, i_d) = \sum_{\alpha_1=1}^{r_1} \cdots \sum_{\alpha_d=1}^{r_d} \mathbf{g}_{\alpha_1 \cdots \alpha_d} U_1(i_1, \alpha_1) \cdots U_d(i_d, \alpha_d)$

- (+) SVD based stable algorithms to compute this representation
- (-) For $n_1 = n_2 = \cdots n_d = n$ and $r_1 = r_2 = \cdots = r_d = r$, the number of entries $= \mathcal{O}(ndr + r^d)$

# Tensor Train Representation: Product of Matrices View

- A $d$-dimensional tensor is represented with 2 matrices and $d$-2 3-dimensional tensors.




$$\mathbf{A}(i_1, i_2, \cdots, i_d) = \mathbf{G}_1(i_1)\mathbf{G}_2(i_2)\cdots\mathbf{G}_d(i_d)$$

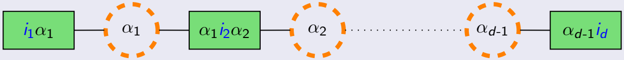
An entry of $\mathbf{A} \in \mathbb{R}^{n_1 \times \cdots \times n_d}$ is computed by multiplying corresponding matrix (or row/column) of each core.

# Tensor Train Representation

$\mathbf{A} \in \mathbb{R}^{n_1 \times \cdots \times n_d}$ is represented with cores $\mathbf{G}_k \in \mathbb{R}^{r_{k-1} \times n_k \times r_k}$, $k=1,2,\cdots d$, $r_0 = r_d = 1$ and its elements satisfy the following expression:

$$\mathbf{A}(i_1,\cdots,i_d) = \sum_{\alpha_0=1}^{r_0} \cdots \sum_{\alpha_d=1}^{r_d} \mathbf{G}_1(\alpha_0, i_1, \alpha_1) \cdots \mathbf{G}_d(\alpha_{d-1}, i_d, \alpha_d)$$

$$= \sum_{\alpha_1=1}^{r_1} \cdots \sum_{\alpha_{d-1}=1}^{r_{d-1}} \mathbf{G}_1(1, i_1, \alpha_1) \cdots \mathbf{G}_d(\alpha_{d-1}, i_d, 1)$$




- For $n_1 = n_2 = \cdots = n_d = n$ and $r_1 = r_2 = \cdots = r_{d-1} = r$, the number of entries $= \mathcal{O}(ndr^2)$

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# Unfolding Matrices of a Tensor & Notations

- Frobenius norm of a matrix $A$ is defined as, $||A||_F = \sqrt{\sum_{i,j} A(i;j)^2}$
- Frobenius norm of a $d$-dimensional tensor $\mathbf{A}$ is defined as, $||\mathbf{A}||_F = \sqrt{\sum_{i_1,i_2,\cdots,i_d} \mathbf{A}(i_1, i_2, \cdots, i_d)^2}$

## $k$-th unfolding matrix

$A_k$ denotes $k$-th unfolding matrix of tensor $\mathbf{A} \in \mathbb{R}^{n_1 \times \cdots \times n_d}$.

$$A_k = [A_k(i_1, i_2, \cdots, i_k; i_{k+1}, \cdots, i_d)]$$

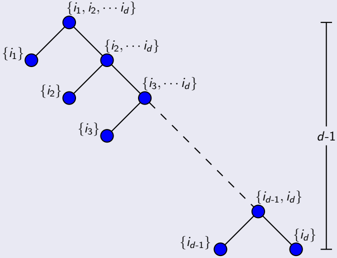
- Size of $A_k$ is $(\prod_{l=1}^{k} n_l) \times (\prod_{l=k+1}^{d} n_l)$
- $r_k$ denotes the rank of $A_k$.

- $(r_1, r_2, \cdots, r_{d-1})$ denotes the ranks of unfolding matrices of the tensor.

# Tensor Train algorithms and Separation of dimensions

- Sequential algorithms to compute Tensor Train decomposition and approximation exist [Oseledets, 2011]

## Sequential algorithm



# Tensor Train Decomposition [Oseledets, 2011]

**Algorithm 1** Tensor Train Decomposition

**Require:** $d$-dimensional tensor $\mathbf{A}$ and ranks $(r_1, r_2, \cdots r_{d-1})$

**Ensure:** Cores $\mathbf{G}_k(\alpha_{k-1}, n_k, \alpha_k)_{1 \leq k \leq d}$ of the Tensor Train representation with $\alpha_k \leq r_k$ and $\alpha_0 = \alpha_d = 1$

Temporary tensor: $\mathbf{C} = \mathbf{A}$, $\alpha_0 = 1$

**for** $k = 1 : d-1$ **do**

    $A_k = reshape(\mathbf{C}, \alpha_{k-1} n_k, \frac{numel(\mathbf{C})}{\alpha_{k-1} n_k})$

    Compute SVD: $A_k = U \Sigma V^T$

    Compute rank of $\Sigma$, $\alpha_k = \text{rank}(\Sigma)$

    New core: $\mathbf{G}_k := reshape(U(; 1 : \alpha_k), \alpha_{k-1}, n_k, \alpha_k)$

    $\mathbf{C} = \Sigma(1 : \alpha_k; 1 : \alpha_k) V^T(1 : \alpha_k; )$

**end for**

$\mathbf{G}_d = \mathbf{C}$, $\alpha_d = 1$

return $\mathbf{G}_1, \cdots, \mathbf{G}_d$

# Tensor Train Approximation [Oseledets, 2011]

**Algorithm 2** Tensor Train Approximation

**Require:** $d$-dimensional tensor $\mathbf{A}$ and expected accuracy $\epsilon$

**Ensure:** Cores $\mathbf{G}_k(\alpha_{k-1}, n_k, \alpha_k)_{1 \le k \le d}$ of the approximated tensor $\mathbf{B}$ in Tensor Train representation such that $||\mathbf{A} - \mathbf{B}||_F$ is not more than $\epsilon$

Temporary tensor: $\mathbf{C} = \mathbf{A}$, $\alpha_0 = 1$, $\delta = \frac{\epsilon}{\sqrt{d-1}}$

**for** $k = 1 : d-1$ **do**

  $A_k = reshape(\mathbf{C}, \alpha_{k-1} n_k, \frac{numel(\mathbf{C})}{\alpha_{k-1} n_k})$

  Compute SVD: $A_k = U \Sigma V^T$

  Compute $\alpha_k$ such that $A_k = U(; 1 : \alpha_k) \Sigma(1 : \alpha_k; 1 : \alpha_k) V^T(1 : \alpha_k; ) + E_k$ and $||E_K||_F \le \delta$

  New core: $\mathbf{G}_k := reshape(U(; 1 : \alpha_k), r_{k-1}, n_k, r_k)$

  $\mathbf{C} = \Sigma(1 : \alpha_k; 1 : \alpha_k) V^T(1 : \alpha_k; )$

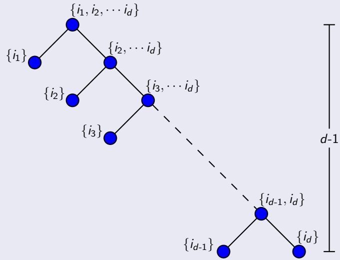
**end for**

$\mathbf{G}_d = \mathbf{C}$, $\alpha_d = 1$

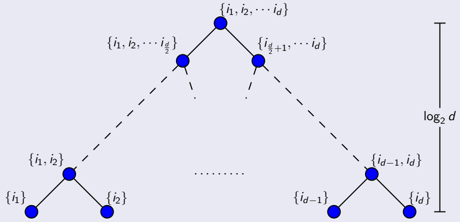
return $\mathbf{B}$ in Tensor Train representation with cores $\mathbf{G}_1, \cdots, \mathbf{G}_d$

# Tensor Train algorithms and Separation of dimensions

## Sequential algorithm




## For better parallelization




- Can obtain better parallelism by expressing the operation in a balanced binary tree shape
  - Proposed parallel algorithms based on this idea

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# Extra Definitions for Parallel Algorithms

- Original indices of a tensor are called external indices
- Indices obtained due to SVD are called internal indices
  - $\mathbf{A}(\alpha, i_1, i_2, i_3, \beta)$ has 3 external and 2 internal indices
- $nEI(\mathbf{A})$ denotes the number of external indices of $\mathbf{A}$

**$k$-th Unfolding Matrix**

$k$-th unfolding of a tensor with elements $\mathbf{A}(\alpha, i_1, i_2, \cdots, i_k, i_{k+1}, \cdots, \beta)$ is represented as, $A_k = [A_k(\alpha, i_1, i_2, \cdots, i_k; i_{k+1} \cdots, \beta)]$.

All indices from the beginning to $i_k$ denote the rows of $A_k$ and the remaining indices denote the columns of $A_k$.

- $\mathbf{Tensor}(A_l)$ converts an unfolding matrix $A_l$ to its tensor form

# Parallel Tensor Train decomposition I

Algorithm 3: PTT-decomposition (parallel Tensor Train Decomposition)

**Require:** $d$-dimensional tensor $\mathbf{A}$ and ranks $(r_1, r_2, \cdots r_{d-1})$

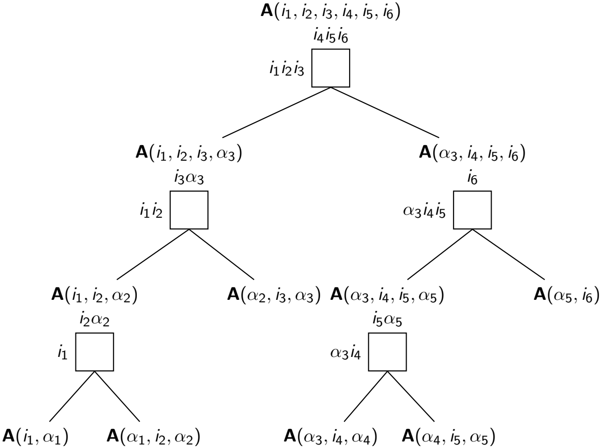
**Ensure:** Cores $\mathbf{G}_k(\alpha_{k-1}, n_k, \alpha_k)_{1 \le k \le d}$ of the Tensor Train representation with $\alpha_k \le r_k$ and $\alpha_0 = \alpha_d = 1$

```
1: if nEI(A) >1 then
2:     Find the middle external index k
3:     Compute unfolding matrix A_k
4:     Compute SVD: A_k = U Σ V^T
5:     Compute rank of Σ, α_k = rank(Σ)
6:     Select diagonal matrices X_k, S_k and Y_k such that
       X_k S_k Y_k = Σ(1 : α_k; 1 : α_k)
7:     A_left = Tensor(U(; 1 : α_k) X_k)
8:     list1 = PTT-decomposition(A_left, (r_1, ··· r_{k-1}, α_k) )
9:     A_right = Tensor(Y_k V^T(1 : α_k; ))
```

# Parallel Tensor Train decomposition II

list2= PTT-decomposition($\mathbf{A}_{right}$,($\alpha_k, r_{k+1}, \cdots r_{d-1}$))
  **return** {list1, list2}
**else**
  Find the external index $k$
  **if** $k$ is the last index of $\mathbf{A}$ **then**
    $\alpha_k = 1$
  **else if** $k$ is the first index of $\mathbf{A}$ **then**
    $\alpha_{k\text{-}1} = 1$
    $\mathbf{A}(i_k, \beta) = \sum_{\beta=1}^{\alpha_k} \mathbf{A}(i_k, \beta) S_k(\beta; \beta)$
  **else**
    $\mathbf{A}(\gamma, i_k, \beta) = \sum_{\beta=1}^{\alpha_k} \mathbf{A}(\gamma, i_k, \beta) S_k(\beta; \beta)$
  **end if**
  $\mathbf{G}_k = \mathbf{A}$
  return $\mathbf{G}_k$
**end if**

# Diagramatic Representation of the Algorithm



# Ranks of Tensor Train Representation (Algorithm 3)

## Theorem

*If for each unfolding $A_k$ of a $d$-dimensional tensor $\mathbf{A}$, $rank(A_k) = r_k$, then Algorithm 3 produces a Tensor Train representation with ranks not higher than $r_k$.*

The rank of the $k$th-unfolding matrix is $r_k$; hence it can be written as:

$$\begin{aligned} A_k(i_1, \cdots, i_k; i_{k+1}, \cdots, i_d) &= \sum_{\alpha=1}^{r_k} U(i_1, \cdots, i_k; \alpha)\Sigma(\alpha; \alpha)V^T(\alpha; i_{k+1}, \cdots, i_d) \\ &= \sum_{\alpha=1}^{r_k} U(i_1, \cdots, i_k; \alpha)X(\alpha; \alpha)S(\alpha; \alpha)Y(\alpha; \alpha)V^T(\alpha; i_{k+1}, \cdots, i_d) \\ &= \sum_{\alpha=1}^{r_k} B(i_1, \cdots, i_k; \alpha)S(\alpha; \alpha)C(\alpha; i_{k+1}, \cdots, i_d). \end{aligned}$$

In matrix form we obtain, $A_k = BSC, B = A_k C^{-1} S^{-1} = A_k Z, C = S^{-1} B^{-1} A_k = W A_k$.
or in the index form, $B(i_1, \cdots, i_k; \alpha) = \sum_{i_{k+1}=1}^{n_{k+1}} \cdots \sum_{i_d=1}^{n_d} \mathbf{A}(i_1, \cdots, i_d) Z(i_{k+1}, \cdots, i_d; \alpha)$,

$C(\alpha; i_{k+1}, \cdots, i_d) = \sum_{i_1=1}^{n_1} \cdots \sum_{i_k=k}^{n_k} \mathbf{A}(i_1, \cdots, i_d) W(\alpha; i_1, i_2, \cdots, i_k)$.

$B$ and $C$ can be treated as $k+1$ and $d$-$k+1$ dimensional tensors $\mathbf{B}$ and $\mathbf{C}$ respectively. We prove that rank($B_{k'}$)$\leq r_{k'}$ $_{1 \leq k' \leq k-1}$ and rank($C_{k'}$)$\leq r_{k'}$ $_{k+1 \leq k' \leq d-1}$.

# Parallel Tensor Train Approximation I

**Algorithm 4:** PTT-approx (Parallel Tensor Train Approximation)

**Require:** $d$-dimensional tensor $\mathbf{A}$ and expected accuracy $\epsilon$

**Ensure:** Cores $\mathbf{G}_k(\alpha_{k-1}, n_k, \alpha_k)_{1 \le k \le d}$ of the approximated tensor $\mathbf{B}$ in TT-representation such that $||\mathbf{A} - \mathbf{B}||_F$ is close to or less than $\epsilon$

```
if nEl(A) > 1 then
    Find the middle external index k
    Compute unfolding matrix A_k
    Compute SVD: A_k = U Σ V^T
    Compute truncation accuracy Δ
    Compute α_k such that A_k = U(; 1 : α_k) Σ(1 : α_k; 1 : α_k) V^T(1 : α_k; ) + E_k
    and ||E_K||_F ≤ Δ
    Select diagonal matrices X_k, S_k and Y_k such that
    X_k S_k Y_k = Σ(1 : α_k; 1 : α_k)
    A_left = Tensor(U(; 1 : α_k) X_k)
    list1 = PTT-approx(A_left, ε_1)
    A_right = Tensor(Y_k V^T(1 : α_k; ))
```

# Parallel Tensor Train Approximation II

list2 = PTT-approx($\mathbf{A}_{right}$, $\epsilon_2$)<br>
**return** {list1, list2}<br>
**else**<br>
Find the external index $k$<br>
**if** $k$ is the last index of **A then**<br>
$\alpha_k = 1$<br>
**else if** $k$ is the first index of **A then**<br>
$\alpha_{k\text{-}1} = 1$<br>
$\mathbf{A}(i_k, \beta) = \sum_{\beta=1}^{\alpha_k} \mathbf{A}(i_k, \beta) S_k(\beta; \beta)$<br>
**else**<br>
$\mathbf{A}(\gamma, i_k, \beta) = \sum_{\beta=1}^{\alpha_k} \mathbf{A}(\gamma, i_k, \beta) S_k(\beta; \beta)$<br>
**end if**<br>
$\mathbf{G}_k = \mathbf{A}$<br>
return $\mathbf{G}_k$<br>
**end if**

# Frobenius Error with Product of Approximated Matrices

The SVD of a real matrix $A$ can be written as,

$$A = (U_1 U_2) \begin{pmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{pmatrix} (V_1 V_2)^T = U_1 \Sigma_1 V_1^T + U_2 \Sigma_2 V_2^T$$
$$= U_1 \Sigma_1 V_1^T + E_A = BSC + E_A.$$

Here $B = U_1 X$, $C = Y V_1^T$ and $XSY = \Sigma_1$. Matrices $B$ and $C$ are approximated by $\hat{B}$ and $\hat{C}$, i.e., $B = \hat{B} + E_B$ and $C = \hat{C} + E_C$. $X$, $Y$ and $S$ are diagonal matrices. $E_A$, $E_B$ and $E_C$ represent error matrices corresponding to low-rank approximations of $A$, $B$ and $C$.

$$||A - \hat{B} S \hat{C}||_F^2 \approx ||E_A||_F^2 + ||BSE_C||_F^2 + ||E_B SC||_F^2$$

# Our Approximation Approaches

We propose 3 approaches based on how leading singular values of the unfolding matrix are passed to the left and right subtensors in Algorithm 4.

- *Leading Singular values to Right subtensor* (*LSR*)
- *Square root of Leading Singular values to Both subtensors* (*SLSB*)
- *Leading Singular values to Both subtensors* (*LSB*)

| Approach | Description | $\Delta$ | $\epsilon_1$ | $\epsilon_2$ |
|---|---|---|---|---|
| *LSR* | $X = I,\ Y = \Sigma_\alpha,\ S = I$ | $\frac{\epsilon}{\sqrt{d-1}}$ | $\epsilon\sqrt{\frac{(d-2)(d_1-1)}{(d-1)(d_2-1+(d_1-1)tr(\Sigma_\alpha^2))}}$ | $\epsilon\sqrt{\frac{(d-2)(d_2-1)}{(d-1)(d_2-1+(d_1-1)tr(\Sigma_\alpha^2))}}$ |
| *SLSB* | $X = Y = \Sigma_\alpha^{\frac{1}{2}},\ S = I$ | $\frac{\epsilon}{\sqrt{d-1}}$ | $\epsilon\sqrt{\frac{d_1-1}{(d-1)tr(\Sigma_\alpha)}}$ | $\epsilon\sqrt{\frac{d_2-1}{(d-1)tr(\Sigma_\alpha)}}$ |
| *LSB* | $X = Y = \Sigma_\alpha,\ S = \Sigma_\alpha^{-1}$ | $\frac{\epsilon}{\sqrt{d-1}}$ | $\epsilon\sqrt{\frac{d_1-1}{d-1}}$ | $\epsilon\sqrt{\frac{d_2-1}{d-1}}$ |
| *STTA* | $X = I,\ Y = \Sigma_\alpha,\ S = I$ | $\frac{\epsilon}{\sqrt{d-1}}$ | 0 | $\epsilon\sqrt{\frac{d_2-1}{d-1}}$ |

- *STTA* represents *Sequential Tensor Train Approximation*

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# Low Rank Functions

| | |
|---|---|
| *Log* | $\log(\sum_{j=1}^{d} j i_j)$ |
| *Sin* | $\sin(\sum_{j=1}^{d} i_j)$ |
| Inverse-Square-Root (*ISR*) | $\frac{1}{\sqrt{\sum_{j=1}^{d} i_j^2}}$ |
| Inverse-Cube-Root (*ICR*) | $\frac{1}{\sqrt[3]{\sum_{j=1}^{d} i_j^3}}$ |
| Inverse-Penta-Root (*IPR*) | $\frac{1}{\sqrt[5]{\sum_{j=1}^{d} i_j^5}}$ |

We consider $d = 12$ and $i_j \in \{1, 2, 3, 4\}_{1 \leq j \leq d}$. This setting produces a 12-dimensional tensor with $4^{12}$ elements for each low rank function.

# Comparison of All Approaches for 12-dimensional Tensors

- Prescribed accuracy $= 10^{-6}$
- compr: compression ratio, ne: number of elements in aprroximation, OA: approximation accuracy

| Appr. | Metric | *Log* | *Sin* | *ISR* | *ICR* | *IPR* |
|---|---|---|---|---|---|---|
| *STTA* | compr | 99.993 | 99.999 | 99.987 | 99.981 | 99.971 |
| | ne | 1212 | 176 | 2240 | 3184 | 4864 |
| | OA | 2.271e-07 | 2.615e-09 | 1.834e-07 | 4.884e-07 | 4.836e-07 |
| *LSR* | compr | 99.817 | 99.998 | 99.915 | 99.874 | 99.824 |
| | ne | 30632 | 344 | 14196 | 21176 | 29524 |
| | OA | 3.629e-08 | 1.412e-11 | 1.118e-07 | 8.520e-08 | 5.811e-08 |
| *SLSB* | compr | 99.799 | 99.999 | 99.952 | 99.912 | 99.870 |
| | ne | 33772 | 176 | 8068 | 14824 | 21792 |
| | OA | 2.820e-08 | 6.144e-12 | 1.118e-07 | 8.518e-08 | 5.664e-08 |
| *LSB* | compr | 99.993 | 99.999 | 99.987 | 99.981 | 99.970 |
| | ne | 1212 | 176 | 2240 | 3184 | 4964 |
| | OA | 2.265e-07 | 1.252e-11 | 1.834e-07 | 4.884e-07 | 3.999e-07 |

# Alternatives to SVD

- SVD is expensive
- Good alternatives to SVD: QR factorization with column pivoting (QRCP), randomized SVD (RSVD)

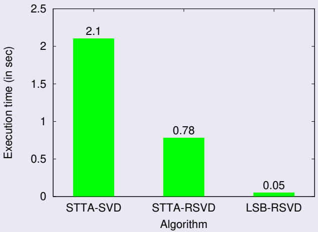
## SVD vs QRCP+SVD vs RSVD for *Log* tensor

| Approach | Rank | compr | ne | *LSB*-OA | *STTA*-OA |
|---|---|---|---|---|---|
| SVD | 5 | 99.994 | 992 | 6.079e-06 | 6.079e-06 |
| QRCP+SVD | | | | 1.016e-05 | 1.384e-05 |
| RSVD | | | | 6.079e-06 | 6.079e-06 |
| SVD | 6 | 99.992 | 1376 | 1.323e-07 | 1.340e-07 |
| QRCP+SVD | | | | 3.555e-07 | 5.737e-07 |
| RSVD | | | | 1.322e-07 | 1.322e-07 |
| SVD | 7 | 99.989 | 1824 | 2.753e-09 | 2.279e-08 |
| QRCP+SVD | | | | 6.620e-09 | 1.167e-08 |
| RSVD | | | | 2.760e-09 | 2.774e-09 |

# Performance Comparison

## Sequential performance with *Log* tensor

- Number of computations for both RSVD algorithms = $\mathcal{O}(n^d)$
- LSB-SVD is very slow
- LSB-RSVD is much faster




## RSVD Algorithm of LSB-RSVD

- Input matrix is $A$ and the desired rank is $r$
- Multiply with a random sketch matrix (depends on $r$), Y = A *RS
- Perform QR factorization, [Q, $\sim$] = QR(Y)
- Compute SVD decomposition, [U S V] = SVD($Q^T$ * A)
- Update U, U = Q*U

# Parallel performance counts on $P$ processors

## Communication cost analysis along the critical path

- To perform A*RS, #data transfers $= \mathcal{O}(\frac{n^{\frac{d}{2}}}{\sqrt{P}} \log P)$
- To perform $Q^T * A$, #data transfers $= \mathcal{O}(\frac{n^{\frac{d}{2}}}{\sqrt{P}} \log P)$
- To perform reshape operation, #data transfers $= \mathcal{O}(\frac{n^{\frac{d}{2}}}{\sqrt{P}} \log P)$
- At each step, #messages $= \mathcal{O}(\log P)$

| Algorithm | # Computations | Communications[1] | # Messages |
|---|---|---|---|
| *LSB*-RSVD | $\mathcal{O}(\frac{n^d}{P})$ | $\mathcal{O}(\frac{n^{\frac{d}{2}}}{\sqrt{P}} \log P)$ | $\mathcal{O}(\log d \log P)$ |
| *STTA*-RSVD | $\mathcal{O}(\frac{n^d}{P})$ | $\mathcal{O}(\frac{n^{d-1}}{P}(1 + \frac{\log P}{d}))$ | $\mathcal{O}(d \log P)$ |

[1] Assuming $n$ is large.

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# Conclusion & Ongoing Work

## Conclusion

- Proposed parallel algorithms to compute tensor train decomposition and approximation of a tensor
- LSB approach achieves similar compression to the sequential algorithm
- Accuracies of all approaches are within prescribed limit

## Ongoing Work

- Proving quasi optimality for parallel approximation algorithms
- Implementation of parallel algorithms for distributed memory systems

Thank You!