MOKALIEN Inria/McGill Associate Team - Paris Meeting - November 9-14, 2014

Schedule and Abstracts :

- ***Monday 9/11***

  10:00-12:00 Optimal Transportation and Fluid Models

  Yann Brenier CNRS, CMLS, Ecole Polytechnique, Palaiseau

  From free particles with sticky collisions to incompressible fluids
  Free particles with sticky collisions and incompressible fluids provide two almost opposite models of classical mechanics (discrete vs continuous, local vs non-local interactions, etc...) However, in the last 15 years, both have been approximately described by almost identical optimal transport models involving the Monge-Ampere equation. Very recently, we have related them to elementary linear models in high space dimensions (heat, free transport or Schroedinger equations), through suitable scaling limits and dimensional reductions.

  Jean-Marie Mirebeau U. Orsay

  TBA

  12:00-14:00 Lunch espace reception 7e etage

  14:00-16:30 Optimal Transportation and FreeForm Optics

  Brittany Froese NJIT

  Beam Shaping using the Monge-Ampere Equation.
  We consider the design of freeform optical surfaces to produce a beam with a desired intensity and shape, without any loss of energy. This can be accomplished through the solution of an optimal transportation problem. This approach is implemented using a convergent filtered finite difference method for the second boundary value problem for the Monge-Ampere equation. The method is used to design double freeform surfaces for converting a Gaussian beam into a triangular beam with large depth of field. Construction of the lenses and subsequent experiments validate this approach.

  Quentin Merigot

  Convergence of a Newton's algorithm for semi-discrete optimal transport.
  Many problems in geometric optics or convex geometry can be recast as optimal transport problems: this includes the far-field reflector problem, AlexandrovÕs curvature prescription problem, etc. A popular way to solve these problems numerically is to assume that the source probability measure is absolutely continuous while the target measure is finitely supported. We refer to this setting as semi-discrete optimal transport. Among the several algorithms proposed to solve semi-discrete optimal transport problems, one currently needs to choose between algorithms that are slow but come with a convergence speed analysis (and rely on coordinate-wise increments) or algorithms that are much faster in

practice but which come with no convergence guarantees (Newton/quasi-Newton). In this talk we will present a simple damped NewtonÕs algorithm with global linear convergence and which is also very efficient in practice, when the cost function satisfies the so-called Ma-Trudinger-Wang regularity condition. Joint work with Jun Kitagawa and Boris Thibert.

Kolja Brix Aachen U.

Designing Illumination.
Lenses and Mirrors by the Numerical Solution of Monge-Ampère Equations
The inverse refractor problem and the inverse reflector problem arise in geometrical non-imaging optics: given a point light source and a target, the question is how to design a refracting or reflecting optical free-form surface, respectively, such that a desired light density distribution is generated on the target, e.g. a projected image on a screen. Both problems can be modeled by strongly nonlinear second-order partial differential equations of Monge-Ampère type and constraints. We propose a collocation method based on spline ansatz functions in combination with nested iteration techniques for a fast and stable numerical solution. Special care has to be taken in the numerical method to handle the constraint, which enters the discrete problem formulation via a Picard-type iteration. The presentation concludes with numerical results of refracting and reflecting optical surfaces and their verification via ray tracing.

- ***Tuesday 10/11***

  10:00-12:30 OT linear prog. and extensions

  Adam Oberman U. Mc Gill

  An efficient Linear Programming method for Optimal Transportation.
  An efficient method for computing solutions to the Optimal Transportation (OT) problem with a wide class of cost functions is presented. The standard linear programming (LP) discretization of the continuous problem becomes intractible for moderate grid sizes. A grid refinement method results in a linear cost algorithm. Weak convergence of solutions is stablished. Barycentric projection of transference plans is used to improve the accuracy of solutions. The method is applied to more general problems, including partial optimal transportation, and barycenter problems. Computational examples validate the accuracy and efficiency of the method. Optimal maps between nonconvex domains, partial OT free boundaries, and high accuracy barycenters are presented.

  Bernhart Schmitzer CEREMADE

  A Sparse Multi-Scale Algorithm for Dense Optimal Transport .
  Discrete optimal transport solvers do not scale well on dense large problems since they do not explicitly exploit the geometric structure of the cost function. In analogy to continuous optimal transport we provide a framework to verify global optimality of a discrete transport plan locally. This allows construction of an algorithm to solve large dense problems by considering a sequence of sparse problems instead. The algorithm lends itself to being combined with a hierarchical multi-scale scheme. Any existing discrete solver can be used as internal black-box. Several cost functions, including the noisy squared Euclidean distance, can be handled. We observe a significant reduction of run-time and memory requirements.

  Xavier Dupuis LUISS U.

  An iterated projection approach for b-convexity constraints.

I will present a numerical method for variational problems under b-convexity constraints, which generalize convexity constraints and are motivated by the principal-agent problem in economics. Convexity of such optimization problems requires conditions on b identified by Figalli, Kim and McCann. For a class of b which satisfy these conditions and are tractable numerically, we apply an iterated projection approach, namely Dykstra's algorithm, to projection problems. This is a joint work with Guillaume Carlier.

Lenaic Chizat CEREMADE

Unbalanced optimal transport and the Wasserstein-Fisher-Rao metric
In order to relax the equality of mass constraint which comes with the standard formulation of the optimal transport problem, we introduce a new formulation where the cost function is defined between couples of points in the product space (domain x mass). Informally, this accounts for the fact that, during the transport between the coupled measures, a particule of mass can both move in the domain and vary in mass. The resulting problem can be defined using two equivalent convex variational formulations: a static Kantorovich-like program and a dynamic Brenier-Benamou-like program. We also show that both problems enjoy dual formulations, and define a metric over positive Radon measures. In this line, the Wasserstein-Fisher-Rao metric plays a particular role as simple extension of 2-Wasserstein as a Riemannian-like metric over positive Radon measures.

12:30-14:00 Lunch Crous

Afternoon Free Discussions.

- ***Wenesday 11/11 Off***
- ***Thursday 12/11***

10:00-12:30 Wassertein Gradient Flows 1

Daniel Matthes T.U. Munich

Long-time asymptotics for discretized fourth order diffusion equations.
The long-time asymptotics of the Wasserstein gradient flow for a geodesically convex functional is essentially determined by the functional's modulus of convexity. A prominent application of that principle is Otto's proof for eventual self-similarity of solutions to porous medium equations. With a particular trick (mainly due to McCann), we could derive very similar long-time asymptotics for flows of certain non-convex functionals by exploiting a non-obvious relation to an associated convex flow. This trick applies, e.g., to the fourth order DLSS and the Hele Shaw/thin film equation. In this talk, I will discuss two different types of structure preserving discretization for these non-convex flows that fully inherit the original long-time asymptotics to their discrete solutions. The first discretization is a 1D Lagrangian scheme a la Gosse/Toscani (joint work with Osberger), the second is an nD finite volume scheme a la Maas/Mielke (joint work with Maas).

Virginie Ehrlacher CERMICS

Cross-diffusion equations in a one-dimensional moving domain (joint work with Athmane Bakhta)
The aim of this talk is to present preliminary existence results for a system of cross-diffusion equations defined on a one-dimensional domain with a moving boundary, which model the evolution of the concentrations of different chemical species in a solid during a Physical Vapor Deposition process. The system of equations, when the domain remains fixed over time, can be seen formally as a gradient flow system which can be analyzed using the boundedness by entropy method introduced by Burger and Jüngel.

Maxime Laborde CEREMADE

12:30-14:00 Lunch Crous

13:30-17:00 Wassertein Gradient Flows 2

Simone Di Marino U. Orsay

Crowd motion and Hele-Shaw model
We illustrate the recently found link between the crowd motion model (with strong congestion) by Maury et al and the Hele-Shaw equations. In particular the formulation in the Wasserstein space can be useful to prove uniqueness (and existence, too) for Hele-Shaw when the convection term is non-autonomous.

Nicolas Vauchelet LJLL U. P6

Mathematical study of a cell model for tumor growth : travelling front and incompressible limit.
We consider mathematical models at macroscopic scale to describe tumor growth. In this view, tumor cells are considered as an elastic material subjected to mechanical pressure. Two main classes of model can be encountered: those describing the dynamics of tumor cells density and those describing the dynamic of the tumor thanks to the motion of its domain. These latter models are free boundary problem. We will show that such free boundary problem of Hele-Shaw type can be derived thanks to an incompressible limit from models describing the dynamics of cells density. Moreover, for this model we study the existence of travelling waves, allowing to describe the spread of the tumor.

Thomas Gallouet Ecole Polytechnique

Finite time Blow-up for a particle system modeling a 1D Keller-Segel equation with non linear diffusion
We discuss a dichotomy theorem for a particle gradient flow of homogenous reaction-diffusion functional in $\mathbb{R}^N$. This problem can be seen as a discrete and deterministic approximation of the gradient flow, in Wasserstein space, of the energy

$$\mathcal{F}_m[\rho] = \frac{1}{m-1}\int_{\mathbb{R}^d} \rho^m(x)\,dx - \chi\frac{1}{d(m-1)}\iint_{\mathbb{R}^d\times\mathbb{R}^d} |x-y|^{d(1-m)}\rho(x)\rho(y)\,dxdy\,.$$

It is an extension, with a non linear diffusion, of the classical Keller-Segel problem for which it is easy to prove a dichotomy theorem. We focus on the super critical case and we show that, unless for very peculiar situations, the blow-up occurs in finite time.

Athmane Bakhta Ecole des Ponts

Cross diffusion equations in a moving domain.
We show global-in-time existence of bounded weak solutions to non linear degenerate cross diffusion equations in a one dimensional moving domain. These equations stem from the modelisation of the evolution of the concentration of chemical species composing a crystalline solid during a physical vapor deposition process. To do so we use the so called boundness-by-entropy technique developed in [1], [2] and [3] based on the gradient flow formalism of the system. Moreover, we are interested in controlling the fluxes of the different atomic species during the process in order to reach a certain desired final profile of concentrations. This problem is formulated as an optimal control problem to which we prove the existence of a solution. Finally, some numerical results and comparaison with true experiments are presented.

- ***Friday 13/11***

10:00-12:30 Optimal Transportation and Applications

B. Pass U. Alberta

Multi - to one - dimensional transportation.
I will discuss joint work with Pierre-Andre Chiappori and Robert McCann on the Monge-Kantorovich problem of transporting a probability measure on R^n to another on the real line. We introduce a nestededness criterion relating the cost to the marginals, under which it is possible to solve this problem uniquely (and essentially explicitly), by constructing an optimal map one level set at a time. I plan to discuss examples for which the nestedness condition holds, as well as some for which it fails; some of these examples arise from a matching problem in economics which originally motivated our work. If time permits, I will also briefly discuss how level set dynamics can be used to develop a local regularity theory in the nested case.

M. Lewin Ceremade

Density Functional Theory and the Lieb-Oxford inequality: recent results and conjectures.
Density functional theory is a very effective method used in quantum chemistry and condensed matter physics. Yet, its mathematical properties are not fully understood and any new insight could influence the models used by practitioners. In this talk I will focus on the classical Coulomb interaction, which can be recast in the form of a problem in optimal transport. In particular, I will discuss a lower bound due to Lieb-Oxford and the conjectures on its best constants. Finally, I will present a new inequality with gradient corrections, obtained in a joint work with Elliott H. Lieb (Princeton).

Y.-H. Kim UBC

Optimal martingale transport in general dimensions
We discuss the optimal solutions to a transport problem where mass has to move under martingale constraint; this constraint forces the transport to split the mass. There have been intensive studies on the one-dimensional case, but, rarely in higher dimensions. We present structural results in general dimensions. This is a joint work with Nassif Ghoussoub and Tongseok Lim,

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