Inria International program
MOKALIEN Associate Team Report 2015 (2nd year)

Title: Numerical Optimal Transportation

Associate Team acronym: MOKALIEN

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1 Abstract of the scientific program

The overall scientific goals is to develop numerical methods for large scale optimal transport and models based on optimal transport tools.
see `https://team.inria.fr/mokaplan/files/2014/09/MOKALIEN_Proposal_2013.pdf`, section 2.

A few additional applications were suggested at our annual workshop in october 2014
`https://team.inria.fr/mokaplan/first-meeting-in-montreal-at-u-mcgill-october-20-24-2014/`

2 Scientific progress

- **Linear Programing approach to OT and applications**

  The Entropic regularisation of the linear programming approach to the Monge Kantorovich problem [11] has been applied to the Density Functional Theory problem in Quantum Chemistry (see [19]) [13] and also to Euler generalised solutions [11]. see also [18] for an alternative approach based on semi-dis

  Multi-scale and adaptative support acceleration of linear programming for OT has been proposed in [4] and [3].

  A dual approach with application to matching for teams in economy can be found in [15].

- **Monge Ampère solvers, convexity constraints and applications**

  A multi grid approach is proposed in [8].

  The semi-discrete approach to Monge-Ampère solver has been applied in FreeForm optics [9] [10], see also the MOKABAJOUR ADT project `https://project.inria.fr/mokabajour/` and also to Euler generalised solutions [18].

An Aleksandrov Monge-Ampère solver based on a mixed Aleksandrov/Viscosity formulation see [2].

- **JKO gradient flows**

  A JKO approach to kinetic granular flows [6] [7].

3 Next year's work program

In 2016, the Mokalien Workshop will take place at Mc Gill (July 18-22) and will be part of the CRM Thematic Semester on Computational Mathematics in Emerging Applications (January-July 2016).

JD Benamou and Guillaume Carlier will also give a mini-course on Optimal Transportation at Mc Gill July 14-15.

- **Linear Programing approach to OT**

  We will continue to work on the Entropic regularisation method coupled with Bregman Iterate [11]. In order to attack very large problems we are trying to couple the approach with a Monte-Carlo evaluation of marginals.

- **Monge Ampère solvers and convexity constraints**

  Extension of the BV2 boundary conditions to the MA-LBR Monotone and consistent scheme [1]. We plan in particular to use it on Optics FreeForming see `https://project.inria.fr/mokabajour/`. We will also test the theoretical result on the convexity of the support of the Wasserstein interpolate in [17].

  For the principal agent problem and in the realistic case where line of products is fixed, finite and discrete, the problem becomes a non convex variant of the semi-discrete Optimal Transportation. Customer and product may live in high dimensional space of characteristics, the unknowns have the finite size of the prices/products. The difficulties are reduced to High Dimensionnal Laguerre cells computations for which tools are available and a non convex polynomial cost function to minimise. We are trying various heuristics and stochastic methods.

- **JKO gradient flows**

  An interesting and open extension are CFD models with non-linear constraints linking density of agents and speed of advection. They do not fit the classical JKO gradient flows framework and some analysis is needed. One motivation is the numerical simulation of congested motion macroscopic models like in [16].

  We propose a semi-implicit convex formulation for each step of the JKO scheme for Wasserstein gradient flows which can be attacked by an augmented Lagrangian method. We will test the algorithm on the porous medium equation. We will also consider a semi implicit variant which enables us to treat nonlocal interactions as well as systems of interacting species.

4 Record of activities

- Guillaume Carlier spent 6 month in U. Victoria to work with M. Agueh.
- The 2015 annual workshop will be held at Paris Dauphine November 9-13. `https://team.inria.fr/mokaplan/second-meeting-in-paris-at-u-paris-dauphine-november-9-`

References

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[16] A. R. Mészáros - F. Santambrogio (Preprint) A diffusive model for macroscopic crowd motion with density constraints (2015)

[17] F. Santambrogio - X. J. Wang (Preprint) Convexity of the support of the displacement interpolation: counterexamples (2015)

[18] Mérigot, Quentin and Mirebeau, Jean-Marie (Preprint) Minimal geodesics along volume preserving maps, through semi-discrete optimal transport (2015)

[19] C. Cotar, G. Friesecke and B. Pass. Infinite-body optimal transport with Coulomb Cost. Calc. Var. Partial Differential Equations. 54 (2015) 717-742.

5 Non-Public information

6 Changes on the team

MOKAPLAN is now a joint CNRS U. Paris Dauphine EPC. See `https://team.inria.fr/mokaplan/team-members/` for the full list.

Xavier Dupuis now a post-doc at Luiss U. Roma, is in the project.

7 Budget requested for the coming year

As requested in the initial proposal : 20K Euro to organise a one week workshop gathering the Canadians and French researchers. Half of the funding goes directly to the canadian partner from the FRQNT.