Inria International program
Associate Team proposal 2014-2016
Submission form

Title: Numerical Optimal Transportation in (Mathematical) Economics

Associate Team acronym: MOKALIEN

Principal investigator (Inria): Benamou Jean-David (Mokaplan - Rocquencourt)

Principal investigator (McGill): Oberman Adam (McGill U. - Montreal)

Other participants:

- Brittany Froese, University of Texas at Austin.
- Tiago Salvador McGill U. - Montreal.
- Guillaume Carlier, Mokaplan-Université Paris-Dauphine.
- Luca Nenna, Mokaplan.
- Xavier Dupuis, Mokaplan.
- Brendan Pass, University of Alberta.
- Martial Agueh, University of Victoria.
- Louis-Philippe Saumier, University of Victoria.
- Adrien Blanchet, Toulouse School of Economics and U. Toulouse.
- Quentin Mérigot, CNRS, Université de Grenoble, LJK and Mokaplan.
- Edouard Oudet, Universit de Grenoble, LJK, CNRS and Mokaplan.

1 Partnership

1.1 Detailed list of participants

Jean-David Benamou (INRIA DR - Mokaplan AEX) works on numerical methods for Optimal Transportation and related Monge-Ampère equations. He created the MOKAPLAN team in collaboration with G. Carlier.
`https://who.rocq.inria.fr/Jean-David.Benamou/`

Guillaume Carlier (U. Paris Dauphine Prof. - Mokaplan AEX) works in the fields of mathematical economics, optimal transport and calculus of variations and published more than 60 articles in these fields. He has been involved in the ANR projects OTARIE and EVAMEF and has been part time professor at Ecole Normale Supérieure de Paris (2008-2011).
`https://www.ceremade.dauphine.fr/~carlier/`

Luca Nenna (PhD Mokaplan) is starting a PhD on Multimarginal Optimal Transport.

Xavier Dupuis (Post Doc Mokaplan) is starting a PD on Infinite dimensional convexity constrained optimization and the Principal-Agent Problem.

Edouard Oudet (Prof. U. Grenoble, Mokaplan associate member) has been involved in several projects in the area of calculus of variations related to the approximation of optimal transport. Its main contributions are related to the study of generalized energies (crowd motion, incompresible flows) and optimization algorithms (Benamou-Brenier time-dependent setting, discrete dual approach, convex parametrization).
`http://www-ljk.imag.fr/membres/Edouard.Oudet/`

Quentin Mérigot (Junior CNRS researcher LJK, Grenoble, Mokaplan associate member) has been working on the numerical resolution of geometric instances of optimal transport problems using tools from computational geometry, and on the handling of convexity constraints in numerical calculus of variations.
`http://quentin.mrgt.fr/`

Adam Oberman (McGill U., Associate Prof. - Dept. of Math) works on numerical methods for fully nonlinear and degenerate elliptic PDEs. He is a winner of the 2011 Monroe H. Martin prize, and the 2011 CAIMS-PIMS Early Career Award in applied mathematics.
`http://wiki.math.mcgill.ca/doku.php/personal/staff/aoberman/home`

Brittany Froese (UT Austin Post-Doc - Department of Mathematics) works on numerical methods for nonlinear PDE and optimal transportation.
`http://www.ma.utexas.edu/users/bfroese/`.

Tiago Salvador (McGill U. Phd Student) is in his second year of a PhD program at McGill. He is working on filtered schemes for Non-Linear Elliptic equations. A. Oberman is his advisor.

Brendan Pass, Assistant Prof. (U. Alberta) is a specialist of multimarginal Optimal Transportation and is active in related problems in mathematical economics (including the principal-agent problem and matching for teams type problems) and mathematical physics (including Density Functional Theory). `http://www.ualberta.ca/~pass/`

Martial Agueh, Associate Prof. (U. Victoria) works in the fields of Nonlinear partial differential equations, kinetic theory and calculus of variations. He is interested in applications of optimal transport to geometric inequalities (e.g. Gagliardo-Nirenberg type inequalities), nonlinear PDEs and kinetic equations, as well as their numerical aspects. `http://www.math.uvic.ca/~agueh/`

Louis-Philippe Saumier, PhD (U. Victoria) is in his second year. Works on numerical methods for Monge-Ampère based Optimal Transportation solvers under M. Agueh supervision.

Adrien Blanchet (TSE, Assistant Prof.) works in the field of nonlinear PDEs with a special emphasis on optimal transport methods and problems arising in economics, he was the coordinator of the ANR resarch project EVAMEF. `http://idei.fr/vitae.php?i=1854&site=TSE&data=TSE&lang=fr`

1.2 Nature and history of the collaboration

Oberman, Froese and Benamou collaborate on the application of Monge-Ampère solvers in Optimal transportation since 2008.

Oberman was the advisor of Froese PhD at SFU (2009-2012) on the application of the Wide-Stencil technique for non-linear elliptic problems in particular the Monge-Ampère problem.

Carlier and Benamou have recently started the MOKAPLAN INRIA "exploratory action", they coadvise Nenna PhD on multi-marginal transport and Dupuis Post-Doc on the convexity constraint/Principal Agent problem.

Carlier and Agueh collaborated in 2009 on a calculus of variation problem related to sharp L^1 Gagliardo-Nirenberg type inequalities and their applications to partial differential equations involving the 1-Laplacian. They also collaborated on the notion of barycenters in the Wasserstein space which is related to optimal transportation.

Carlier and Blanchet started working together in 2010 within the scope of the ANR project EVAMEF devoted to variational methods for mathematical finance and economics. They proposed a new Optimal Transportation based modelisation of Cournot-Nash equilibria in Economics.

Oudet and Mérigot have worked together on the numerical implementation of convexity constraints in problems of calculus of variations such as the principal-agent problem, and on discrete optimal transport.

Agueh and Blanchet worked on the long time asymptotics of the doubly nonlinear equation using entropy methods in the case when the associated free energy functional in not displacement convex.

In July 2013, Oudet, Carlier, Agueh, Pass, Oberman, Froese and Benamou gathered in Banff for a "focussed research group" week : `http://www.birs.ca/events/2013/focussed-research-groups/13frg167`. The meeting was very productive and several new collaborations were started on the occasion which are listed in the objectives of this proposal. The proposed group is an natural extension of this French-Canadian gathering. It brings together a combination of Optimal Transportation analysts (Agueh, Pass) , Numericians (Oberman, Froese, Oudet, Mérigot, Benamou), Specialists of mathematical economics (Carlier, Blanchet, Ekeland) and students

2 Scientific program

2.1 Context

Optimal Transportation is a mathematical research topic which began two centuries ago with Monge's work on "des remblais et déblais". This engineering problem consists in minimizing the transport cost between two given mass densities. In the 40's, Kantorovitch [Kan42] solved the dual problem and interpreted it as an economic equilibrium. The *Monge-Kantorovitch* problem became a specialized research topic in optimization and Kantorovitch obtained the 1975 Nobel prize in economics for his contributions to resource allocations problems. Following the seminal discoveries of Brenier in the 90's, Optimal Transportation has received renewed attention from mathematical analysts and the Fields Medal awarded in 2010 to C. Villani, who gave important contributions to Optimal Transportation[Vil09], arrived at a culminating moment for this theory. *Optimal Transportation* is today a mature area of mathematical analysis connected with : regularity theory for nonlinear elliptic equations and in particular Monge-Ampère [CC95] ; gradient flow formulation (a.k.a. JKO) of nonlinear diffusion equations [Ott01] [JKO98] ; image warping [CWVB09] ; frontongenesis models in meteorology [CP84] ; mesh adaptation in fluid models [CDF11] [CS11b] ; cosmology [FMMjS02] ; reflector design [CO08] [CKO99] ; finance [GHLT11] and mathematical economics (see below).

The modern Optimal Transportation problem between two densities : μ defined on $X \in R^d$ and ν defined on $Y \in R^d$, can be mathematically sketched as

$$\min_{T:x\in X\rightarrow y\in Y, T\#\mu=\nu} \int_X c(x,T(x))\mu(x)dx$$

where $c(x,y)$ is the ground cost measuring a "traveling" distance (the Euclidean distance squared is the classical Monge-Kantorovitch problem) and $T\#\mu = \nu$ is a notation ("push forward") expressing a conservation of mass property for the transportation map T.

Several important extension to the classical L^2 Monge Kantorovich have recently received attention : more general ground cost (c-convex functions, concave costs, non-linear mobilities), multi-marginal Optimal Transportation, [GŚ98] [Pas12] [Pas13], partial optimal transport [CM10] [Fig10].

These extension appear naturally in several academic economic models : the Principal agent (aka Monopolist) problem [Car01] [Sal97] [FMCK11],, Equilibrium models [BC] [CS11a] matching for teams [CE10], more matching [GS10] [GS11] ...

2.2 Objectives (for the three years)

A - Improve and extend existing numerical method for "classical" Optimal Transportation, we mean here the L^2 Monge-Kantorovitch problem. The main existing tool is the Benamou-Froese-Oberman Monge-Ampere/Optimal Transportation solver [BFO]. Open problems are the extension to 3D, the capture of weaker than viscosity (Aleksandrov) solutions, treatment of data of unequal mass (aka partial Optimal Transportation).

B - Design or generalized numerical solution methods for more general ground cost $c(x, y)$ measuring the "distance" between x and y appearing in Economic models [FMCK11] A considerable theoretical effort has been devoted to understand the conditions on c to ensure that well posenedness results obtained for the "classical" Optimal Transportation (where c can be simplified as $-x \cdot y$) still hold see [Vil09]), the most recent result being known as the Ma-Trudinger-Wang conditions [MTW05]. This leads to new Monge-Ampère equations where the convexity of the solution is replaced by the possibly non-local notion of c-convexity. Numerics are non existent.

C - A natural generalization of Optimal Transportation and of its relaxed Kantorovitch formulation is the multi-marginal Optimal Transportation. This is when the data consists in more than 2 (possibly an infinite number of) densities and the ground cost is now of the form $c(x, y, z, ...)$, each of these variables corresponding to each density space. Multi-marginal Optimal Transportation appears naturally in the matching for teams paradigm. The simplest ground cost has been studied in [GŚ98] and correspond to the generalization of the notion of barycenters in the space of densities functions. An degraded numerical methods for this problem is used in image processing [RPDB11]. Multi-marginal type problems also have have applications in physics (Density Functional Theory [CFK13] [BDPGG12] , and finance (model independent derivative pricing) [BHLP13] [GHLT]. A serious numerical effort in needed on this problem.[1].

D - An important by-product of Optimal Transportation is the observation that the Optimal Transportation cost is a distance, called Wasserstein distance, in the space of density functions and that in this metric a large number of non-linear diffusion equation (the prototype being the Focker Planck equation) can be approximated by a time discrete gradient flow in the induced metric. This is known as JKO gradient flows after their inventors [JKO98]. This idea is now a popular analysis tool and some numerical studies have recently been published. Several of our target equilibrium models can be formulated as one JKO step. The Wasserstein distance and its gradient are unfortunately difficult and expensive to compute. We also plan to address numerically this problem.

E - Because of the fundamental Brenier characterization of the Optimal Transportation map as the gradient of a convex potential, the field is intimately linked with "convexity constraint" problem. By this we mean infinite dimensional optimization where the variable is constrained to live in cone of convex functions.. A simplified version of the principal agent problem falls in this category. Several member of the group already contributed (including very recently) numerical method in this field [CLRM01] [EM10] [MO13] [Obe13]. A study/comparison of these methods is needed and will possibly lead to new methods [2].

[1]This is the PhD subject of Luca Nenna

[2]This is the Post-Doc subject of Xavier Dupuis

2.3 Work-program (for the first year)

A1 (Benamou, Froese, Oberman) - When the Optimal Transportation data is not balanced, i.e. the densities do not have equal mass. A natural extension of the optimal transport has been proposed by McCann and Caffareli [CM10] and revisited by Figalli [Fig10]. It is formulated as an obstacle problem which automatically select the portion of mass corresponding to Optimal Transportation. The numerical resolution of this problem is open and we believe ideas linked the state constraint reformulation contained in paper [BFO] may be applied to obtain a tractable reformulation.

A2 (Benamou, Froese) Design a scheme for Aleksandrov solution of Optimal Transportation between atomic measure and continuous densities. The idea is to couple the notion of viscosity solution with an adapted sub gradient discretization at dirac points where the notion of Aleksandrov solution is relevant. This would offer a "PDE" alternative to the classical gradient methods based on costly computational geometry tools [MÍ3].McCann

C1 (Carlier, Oudet, Oberman) A new numerical method for the Multi-Marginal Barycenter problem [GŚ98]. The method uses linear programming, in an implementation that was more efficient than expected: the cost is a multiple of the cost of the linear programming problem for Optimal Transportation.

C2 (Benamou, Carlier, Nenna) Extension of the ALG2 CFD algorithm [BB00] to the Barycenter problem of Gangbo-Swiech [GŚ98].

C3 (Agueh Carlier Pass Mérigot) Quantified stability for multi-marginal, barycenter and matching for teams problems: In order to prove convergence and rate of convergence results for numerical methods, it is pivotal to establish stability results with respect to perturbations of the densities and weights. This is intimately related to the (uniform) convexity of the barycenter functional. The only quantitative stability result for optimal transport plans is due to Gigli [Gig11]. It relies on Caffarelli's regularity theory and is therefore restricted to target measures with density. An extension to more general target measures would have impact in the complexity of numerical schemes for optimal transport such as [OP88].

D1 (Agueh Benamou Carlier Blanchet) Splitting methods for kinetic equations, the idea is to use one JKO step to deal with the non-linear velocity advection part of kinetic equations [CG04]. This seems to be relevant to granular media equation [Agu13], and also may offer a completely new method for Liouville equations arising from Geometrical Optics [BCKP02].

D2 (Benamou Carlier Merigot Oudet Blanchet) A Steepest descent projected gradient algorith for JKO type computations. The algorithm performs the optimization in the displacement (optimal map) space. This offers nice convexity properties for the line search but requires convexity constraints (objective E) projections. We think that this may be the cheapest way of computing economic equilibrium such as Cournot-Nash [BC].

E1 (Benamou, Carlier, Dupuis, Oudet, Mérigot, Oberman) Convexity constraint algorithms need to be compared and assessed. The Principal Agent problem will be numerically investigated.

NOTA : Objective B is completely open and very challenging, we will discuss it but this is prospective.

3 Budget

- Based on our successful focused research group experience in Banff this summer, we plan to bring everybody together and hold one research meeting (one week) per year. This will consist in morning informal thematic presentations and research in groups in the afternoon. This will be held alternately in France and Canada. The 20KEuro budget will be devoted to fund travel and accommodation. Next meeting will be in France the last one being in Banff. Budget permitting, seniors scientists of our networks may also be invited (Brenier, Ekeland, McCann, Caffarelli, Gangbo, Villani, Figalli, ...)

- Participants already collaborate on regular bi-lateral basis and use their local funds for this :

  For the Mokaplan team, the ISOTACE contract (ANR-12-MONU-0013 (2012-2016)) [3]. `https://project.inria.fr/isotace/`

  Oudet and Mérigot are active members of ANR Tommi : Transport optimal, mthodes multiphysiques et image. `http://tommi.imag.fr/`

  A. Oberman will apply for matching funds from the Applied Mathematics Laboratory at the U. of Montreal Centre Recherche Mathematique `http://www.crm.umontreal.ca/en/` to bring researchers from the group to work in Canada.

- We are also currently applying to organize a large BIRS Banff Meeting on Optimal Transportation in 2015.

4 Added value

The team will naturally fit and strengthen the Mokaplan INRIA group of Carlier and Benamou.

It will be most helpful to the students (Nenna, Dupuis, Saumier, Salvador) as their research subjects are embedded in the objectives and the associate team brings some of the best international specialists in the field.

This project will unite mathematical economists, analysts, and numerical experts, whose combined expertise should lead to numerical solvers for several difficult and important open problems in economic theory. The developed numerical method will also be useful in other applicative domains such as image processing, biology ... This should have a profound impact on all the participants' research, exposing those with a more theoretical background to state-of-the-art numerical techniques, while introducing those with computational expertise to both new problems and new insights into old problems, which are ripe for numerical exploration.

[3] It funds in particular the Post-Doc of X. Dupuis

5 Other remarks

6 References

6.1 Joint publications of the partners

References

[1] Jean-David Benamou, Brittany D. Froese, and Adam M. Oberman. Two numerical methods for the elliptic Monge-Ampère equation. *M2AN Math. Model. Numer. Anal.*, 44(4):737–758, 2010.

[2] Jean-David Benamou, Brittany D. Froese, and Adam M. Oberman. Numerical solution of the Optimal Transportation problem using the Monge-Ampère equation. 2012. submitted, available at https://wiki.sfu.ca/personal/aoberman/images/9/9d/OT_Num.pdf and http://hal.inria.fr/hal-00703677/PDF/bof.pdf

[3] Jean-David Benamou, Brittany D. Froese, and Adam M. Oberman. Numerical solution of the optimal transportation problem via viscosity solutions for the Monge-Ampère equation. 2012. submitted, available at https://wiki.sfu.ca/personal/aoberman/images/9/9d/OT_Theory.pdf and http://hal.inria.fr/hal-00703677/PDF/bof.pdf

[4] Brittany D. Froese and Adam M. Oberman. Convergent finite difference solvers for viscosity solutions of the elliptic Monge-Ampère equation in dimensions two and higher. *SIAM J. Numer. Anal.*, 49(4):1692–1714, 2011.

[5] Brittany D. Froese and Adam M. Oberman. Fast finite difference solvers for singular solutions of the elliptic Monge-Ampère equation. *J. Comput. Phys.*, 230(3):818–834, 2011.

[6] Brittany D. Froese and Adam M. Oberman. Convergent filtered schemes for the Monge-Ampère partial differential equation. *submitted*, 2012, http://arxiv.org/abs/1204.5798

[7] Adrien Blanchet, Guillaume Carlier Optimal Transport and Cournot-Nash Equilibria. *preprint*, 20012.

[8] Quentin Mérigot and Édouard Oudet. Discrete optimal transport: complexity, geometry and applications. Preprint, 2012.

[9] Quentin Mérigot and Édouard Oudet. Handling convexity-like constraints in variational problems. Preprint, 2013.

[10] M. Agueh and G. Carlier. A class of total variation minimization problems on the whole space. *Nonlinear Anal.*, 70(6):2356–2365, 2009.

[11] Martial Agueh and Guillaume Carlier. Barycenters in the Wasserstein space. *SIAM J. Math. Anal.*, 43(2):904–924, 2011.

[ABC10] M. Agueh, A. Blanchet, and J. A. Carrillo. Large time asymptotics of the doubly nonlinear equation in the non-displacement convexity regime. *Journal of Evolution Equations*, 10(1):59–84, 2010.

6.2 Main publications of the participants relevant to the project

List the main publications of the participants that are relevant for the project. List at most 5 publications for each partner.

[12] Adam M. Oberman. Convergent difference schemes for degenerate elliptic and parabolic equations: Hamilton-Jacobi equations and free boundary problems. *SIAM J. Numer. Anal.*, 44(2):879–895 (electronic), 2006.

[13] Adam M. Oberman. A Numerical Method for Variational Problems with Convexity Constraints. *accepted SIAM J. Scientific Computing.* arXiv:1107.5290v3

[14] J.-D. Benamou. A domain decomposition method for the polar factorization of vector-valued mappings. *SIAM J. Numer. Anal.*, 32(6):1808–1838, 1995.

[15] J. D. Benamou and Y. Brenier. Mixed L^2-Wasserstein optimal mapping between prescribed density functions. *J. Optim. Theory Appl.*, 111(2):255–271, 2001.

[16] Jean-David Benamou and Yann Brenier. A computational fluid mechanics solution to the Monge-Kantorovich mass transfer problem. *Numer. Math.*, 84(3):375–393, 2000.

[17] Jean-David Benamou, Yann Brenier, and Kevin Guittet. Numerical analysis of a multiphasic mass transport problem. In *Recent advances in the theory and applications of mass transport*, volume 353 of *Contemp. Math.*, pages 1–17. Amer. Math. Soc., Providence, RI, 2004.

[18] Brittany D. Froese. A numerical method for the elliptic Monge-Ampère equation with transport boundary conditions. *SIAM J. Sci. Comput.*, 34(3):A1432–A1459, 2012.

[19] Fethallah Benmansour, Guillaume Carlier, Gabriel Peyré, and Filippo Santambrogio. Numerical approximation of continuous traffic congestion equilibria. *Netw. Heterog. Media*, 4(3):605–623, 2009.

[20] Lorenzo Brasco, Guillaume Carlier, and Filippo Santambrogio. Congested traffic dynamics, weak flows and very degenerate elliptic equations [corrected version of mr2584740]. *J. Math. Pures Appl. (9)*, 93(6):652–671, 2010.

[21] Guillaume Carlier. A general existence result for the principal-agent problem with adverse selection. *J. Math. Econom.*, 35(1):129–150, 2001.

[22] Carlier, Guillaume, Dacorogna, Bernard . Rsolution du problme de Dirichlet pour l'quation du Jacobien prescrit via l'quation de MongeAmpre. *C. R. Math. Acad. Sci. Paris* 350 (2012), no. 7-8, 371–374.

[23] Agueh, Martial, Carlier, Guillaume . Barycenters in the Wasserstein space. *SIAM J. Math. Anal.* 43 (2011), no. 2, 904–924.

[24] Blanchet Adrien, Mossay Pascal, et Santambrogio Filippo Existence and uniqueness of equilibrium for a spatial model of social interactions *Pre-print* (2012).

[25] Blanchet Adrien et Laurenot Philippe The parabolic-parabolic Keller-Segel system with critical diffusion as a gradient flow in $R^d, d \geq 3$. *Pre-print* (2012).

[26] Blanchet Adrien, Carlen Eric, et Carrillo Jos Functional inequalities, thick tails and asymptotics for the critical mass Patlak-Keller-Segel model *Journal of Functional Analysis* 261 (5), pp. 2142-2230 (2012).

[27] Blanchet Adrien, Calvez Vincent, et Carrillo Jos Convergence of the mass-transport steepest descent scheme for the subcritical Patlak-Keller-Segel model *SIAM Journal on Numerical Analysis* 46 (2), pp. 691-721 (2008).

[28] B. Pass. Uniqueness and monge solutions in the multimarginal optimal transportation problem. *SIAM Journal on Mathematical Analysis*, 43(6):2758–2775, 2011.

[29] B. Pass. On the local structure of optimal measures in the multi-marginal optimal transportation problem. *Calculus of Variations and Partial Differential Equations*, 43:529–536, 2012. 10.1007/s00526-011-0421-z.

[30] B. Pass. Optimal transportation with infinitely many marginals. *Journal of Functional Analysis*, 2012. 10.1016/j.jfa.2012.12.002.

[31] Brendan Pass. Convexity and multi-dimensional screening for spaces with different dimensions. *Journal of Economic Theory*, 147(6):2399 – 2418, 2012.

[32] P. Machado Manhaes de Castro, Q. Mérigot, and B. Thibert. Intersection of paraboloids and application to minkowski-type problems. Preprint, 2013.

[33] Quentin Mérigot. A multiscale approach to optimal transport. *Computer Graphics Forum*, 30(5):1583–1592, 2011.

[34] Quentin Mérigot. A comparison of two dual methods for discrete optimal transport. *Geometric Science of Information, LNCS 8085, 389-396*, 2013.

[35] Quentin Mérigot. A comparison of two dual methods for discrete optimal transport. In *Geometric Science of Information*, pages 389–396. Springer Berlin Heidelberg, 2013.

[36] Martial Agueh. Finsler structure in the p-Wasserstein space and gradient flows. *C. R. Math. Acad. Sci. Paris*, 350(1-2):35–40, 2012.

[37] Martial Agueh and Malcolm Bowles. One-dimensional numerical algorithms for gradient flows in the p-Wasserstein spaces. *Acta Appl. Math.*, 125:121–134, 2013.

[38] L.P. Saumier, M. Agueh, and B. Khouider An efficient numerical algorithm for the L^2 optimal transport problem with periodic densities. *IMA Journal of Applied Mathematics* (2013), to appear.

[39] Martial Agueh. Global weak solutions to kinetic models of granular media. preprint, 2013.

[40] G. Buttazzo, C. Jimenez, and E. Oudet. An optimization problem for mass transportation with congested dynamics. *SIAM J. Control Optim.*, 48(3):1961–1976, 2009.

[41] Gabriel Peyre Nicolas Papadakis and Édouard Oudet. Optimal transport with proximal splitting. Preprint, 2013.

6.3 Other references

List the other references used in Section 2.

References

[Agu13] Martial Agueh. Global weak solutions to kinetic models of granular media. 2013.

[BB00] Jean-David Benamou and Yann Brenier. A computational fluid mechanics solution to the Monge-Kantorovich mass transfer problem. *Numer. Math.*, 84(3):375–393, 2000.

[BC] A. Blanchet and G. Carlier. Optimal transport and cournot-nash equilibria.

[BCKP02] Jean-David Benamou, François Castella, Theodoros Katsaounis, and Benoit Perthame. High frequency limit of the Helmholtz equations. *Rev. Mat. Iberoamericana*, 18(1):187–209, 2002.

[BDPGG12] Giuseppe Buttazzo, Luigi De Pascale, and Paola Gori-Giorgi. Optimal-transport formulation of electronic density-functional theory. *Phys. Rev. A*, 85:062502, Jun 2012.

[BFO] Jean-David Benamou, Brittany D. Froese, and Adam M. Oberman. Numerical solution of the optimal transportation problem using the monge-ampère equation. *J. Comput. Physics, to appear.*

[BHLP13] Mathias Beiglbck, Pierre Henry-Labordre, and Friedrich Penkner. Model-independent bounds for option pricesa mass transport approach. *Finance and Stochastics*, 17(3):477–501, 2013.

[Car01] Guillaume Carlier. A general existence result for the principal-agent problem with adverse selection. *J. Math. Econom.*, 35(1):129–150, 2001.

[CC95] Luis A. Caffarelli and Xavier Cabré. *Fully nonlinear elliptic equations*, volume 43 of *American Mathematical Society Colloquium Publications*. American Mathematical Society, Providence, RI, 1995.

[CDF11] L. Chacón, G. L. Delzanno, and J. M. Finn. Robust, multidimensional mesh-motion based on Monge-Kantorovich equidistribution. *J. Comput. Phys.*, 230(1):87–103, 2011.

[CE10] G. Carlier and I. Ekeland. Matching for teams. *Econom. Theory*, 42(2):397–418, 2010.

[CFK13] Codina Cotar, Gero Friesecke, and Claudia Klppelberg. Density functional theory and optimal transportation with coulomb cost. *Communications on Pure and Applied Mathematics*, 66(4):548–599, 2013.

[CG04] E. A. Carlen and W. Gangbo. Solution of a model Boltzmann equation via steepest descent in the 2-Wasserstein metric. *Arch. Ration. Mech. Anal.*, 172(1):21–64, 2004.

[CKO99] Luis A. Caffarelli, Sergey A. Kochengin, and Vladimir I. Oliker. On the numerical solution of the problem of reflector design with given far-field scattering data. In *Monge Ampère equation: applications to geometry and optimization (Deerfield Beach, FL, 1997)*, volume 226 of *Contemp. Math.*, pages 13–32. Amer. Math. Soc., Providence, RI, 1999.

[CLRM01] Guillaume Carlier, Thomas Lachand-Robert, and Bertrand Maury. A numerical approach to variational problems subject to a convexity constraint. *Numer. Math.*, 88:299–318, 2001.

[CM10] Luis A. Caffarelli and Robert J. McCann. Free boundaries in optimal transport and Monge-Ampère obstacle problems. *Ann. of Math. (2)*, 171(2):673–730, 2010.

[CO08] L. A. Caffarelli and V. I. Oliker. Weak solutions of one inverse problem in geometric optics. *J. Math. Sci. (N. Y.)*, 154(1):39–49, 2008. Problems in mathematical analysis. No. 37.

[CP84] M. J. P. Cullen and R. J. Purser. An extended Lagrangian theory of semi-geostrophic frontogenesis. *J. Atmospheric Sci.*, 41(9):1477–1497, 1984.

[CS11a] G. Carlier and F. Santambrogio. A continuous theory of traffic congestion and Wardrop equilibria. *Zap. Nauchn. Sem. S.-Peterburg. Otdel. Mat. Inst. Steklov. (POMI)*, 390(Teoriya Predstavlenii, Dinamicheskie Sistemy, Kombinatornye Metody. XX):69–91, 307–308, 2011.

[CS11b] Jean-François Cossette and Piotr K. Smolarkiewicz. A Monge-Ampère enhancement for semi-Lagrangian methods. *Comput. & Fluids*, 46:180–185, 2011.

[CWVB09] Rick Chartrand, Brendt Wohlberg, Kevin R. Vixie, and Erik M. Bollt. A gradient descent solution to the Monge-Kantorovich problem. *Appl. Math. Sci. (Ruse)*, 3(21-24):1071–1080, 2009.

[EM10] Ivar Ekeland and Santiago Moreno. An algorithm for computing solutions of variational problems with global convexity constraints. *Numer. Math.*, 115(1):45–69, 2010.

[Fig10] Alessio Figalli. The optimal partial transport problem. *Arch. Ration. Mech. Anal.*, 195(2):533–560, 2010.

[FMCK11] A. Figalli, R.J. Mc Cann, and Y.H. Kim. When is multi-dimensional screening a convex program? *Journal of Economic Theory*, 2011.

[FMMjS02] U. Frisch, S. Matarrese, R. Mohayaee, and 21janA. Sobolevski. Monge-ampre-kantorovitch (mak) reconstruction of the eary universe. *Nature*, 417(260), 2002.

[GHLT] A. Galichon, P. Henry-Labordere, and N. Touzi. A stochastic control approach to non-arbitrage bounds given marginals, with an application to Lookback options. Preprint available at https://sites.google.com/site/alfredgalichon/research.

[GHLT11] A. Galichon, P. Henry-Labordère, and N. Touzi. A stochastic control approach to no-arbitrage bounds given marginals, with an application to loopback options. *submitted to Annals of Applied Probability*, 2011.

[Gig11] Nicola Gigli. On holder continuity-in-time of the optimal transport map towards measures along a curve. *Edinburgh Mathematical Society. Proceedings*, 54(2):401–409, 2011.

[GŚ98] Wilfrid Gangbo and Andrzej Święch. Optimal maps for the multidimensional Monge-Kantorovich problem. *Comm. Pure Appl. Math.*, 51(1):23–45, 1998.

[GS10] A. Galichon and B. Salanié. Matchings with trade-offs: Revealed preferences over competing characteristics. *preprint*, 2010.

[GS11] A. Galichon and B. Salanié. Cupid's invisible hand: Social surplus and identification in matching models. *preprint*, 2011.

[JKO98] Richard Jordan, David Kinderlehrer, and Felix Otto. The variational formulation of the Fokker-Planck equation. *SIAM J. Math. Anal.*, 29(1):1–17, 1998.

[Kan42] L. Kantorovitch. On the translocation of masses. *C. R. (Doklady) Acad. Sci. URSS (N.S.)*, 37:199–201, 1942.

[MÍ3] Quentin Mérigot. A comparison of two dual methods for discrete optimal transport. *Geometric Science of Information, LNCS 8085, 389-396*, 2013.

[MO13] Quentin Mérigot and Édouard Oudet. Handling convexity-like constraints in variational problems. Preprint, 2013.

[MTW05] Xi-Nan Ma, Neil S. Trudinger, and Xu-Jia Wang. Regularity of potential functions of the optimal transportation problem. *Arch. Ration. Mech. Anal.*, 177(2):151–183, 2005.

[Obe13] Adam M. Oberman. A numerical method for variational problems with convexity constraints. *SIAM J. Sci. Comput.*, 35(1):A378–A396, 2013.

[OP88] V. I. Oliker and L. D. Prussner. On the numerical solution of the equation $(\partial^2 z/\partial x^2)(\partial^2 z/\partial y^2) - ((\partial^2 z/\partial x \partial y))^2 = f$ and its discretizations. I. *Numer. Math.*, 54(3):271–293, 1988.

[Ott01] Felix Otto. The geometry of dissipative evolution equations: the porous medium equation. *Comm. Partial Differential Equations*, 26(1-2):101–174, 2001.

[Pas12] Brendan Pass. On the local structure of optimal measures in the multi-marginal optimal transportation problem. *Calc. Var. Partial Differential Equations*, 43(3-4):529–536, 2012.

[Pas13] Brendan Pass. On a Class of Optimal Transportation Problems with Infinitely Many Marginals. *SIAM J. Math. Anal.*, 45(4):2557–2575, 2013.

[RPDB11] J. Rabin, G. Peyré, J. Delon, and M. Bernot. Wassertein barycenter and its applications to texture mixing. In *LNCS, Proc. SSVM'11*, volume 6667, pages 435–446. Springer, 2011.

[Sal97] Bernard Salanié. *The Economics of Contracts: a Primer.* MIT Press, 1997.

[Vil09] Cédric Villani. *Optimal transport*, volume 338 of *Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences].* Springer-Verlag, Berlin, 2009. Old and new.

MOKALIEN WS SCHEDULE

	Sept 3-10	Sept 22-26	Oct 13-17	Oct 20-24
Oudet	OK	OK	-	OK
Brenier	-	OK	OK	OK (partial)
Merigot	-	-	OK	OK
Blanchet	OK	OK	OK	OK
Mirebeau	-	OK	OK	OK
Benamou	OK	OK	OK	OK
Carlier	OK	OK	OK	OK
Santambrogio	-	OK	OK	OK (partial)
Oberman	OK	OK	OK	OK
Froese	-	-	maybe	maybe
Agueh	maybe	maybe	OK	maybe
Pass	-	-	OK	OK