MOKALIEN Inria/McGill Associate Team - Montreal Meeting - October 20-24, 2014

Schedule and Abstracts :

Monday oct. 20

– *9:30 AM* - Filippo Santambrogio (U. Orsay).
Approximation of Steiner and length-minimization problems under connectedness constraints.
Given a finite number of points $D := \{x_i\}_{i=1,\dots,N} \subset \Omega \subset \mathbb{R}^2$, the so-called Steiner problem consists in solving

$$\min \left\{ \mathcal{H}^1(K) \quad : \quad K \subset \mathbb{R}^2 \text{ compact, connected, and containing } D \right\}.$$

It is known that minimizers do exist, need not to be unique, and are trees composed by a finite number of segments joining with only triple junctions at 120°, whereas computing a minimizer is very hard.
In the talk, I will propose a way to approximate the problem, and present a convergence result, obtained in collaboration with M. Bonnivard and A. Lemenant (Paris 7). The strategy is to approximate the length by an elliptic energy of Modica-Mortola type, but the novelty consists in adding a term taking care of the connectedness constraint. This term relies on the weighted geodesic distance d_φ, defined as

$$d_\varphi(x, y) := \inf \left\{ \int_\gamma \varphi(x) d\mathcal{H}^1(x); \ \gamma \text{ curve in } \Omega \text{ connecting } x \text{ and } y \right\}.$$

The idea to approximate Steiner problem is then to minimize

$$S_\varepsilon(\varphi) := \frac{1}{4\varepsilon} \int_\Omega (1 - \varphi)^2 dx + \varepsilon \int_\Omega \|\nabla \varphi\|^2 + \frac{1}{\varepsilon} \sum_{i=1}^{N} d_\varphi(x_i, x_1),$$

among all functions $\varphi \in \mathcal{A} := H^1(\Omega) \cap C^0(\overline{\Omega}) \cap \{0 \leq \varphi \leq 1 \text{ and } \varphi = 1 \text{ on } \partial\Omega\}$.
The first two terms are a simple variant of the standard Modica-Mortola functional: as $\varepsilon \to 0$, they force φ to tend to 1 a.e. and, if φ_ε stays small (close to 0) on a thin region (with measure tending to 0), they force to pay the transition between the value 1 and the value 0 by means of the length of the transition set. In arbitrary dimension, these very terms would converge rather to an $(n-1)-$dimensional measure, which is the reason to stick to $\mathbb{R}^2$ if we want to approximate a length term. Finally, it is worth noticing that the distance $d_\varphi(\cdot, x_1)$ can be treated numerically by the so-called *fast-marching* method and that recent improvements of this algorithm allow to compute at the same time d_φ and its gradient with respect to φ, which is useful every time one needs to optimize w.r.t. φ a functional involving d_φ.
In the talk I will present the idea behind this approximation, and possibly discuss applications to other length-minimization problems among connected sets.

– *10:30 AM* Quentin Merigot (CNRS Ceremade U. Dauphine).
Discretization of functionals involving the Monge-Ampr̀e operator.
Gradient flows in the Wasserstein space have become a powerful tool in the analysis of diffusion equations, following the seminal work of Jordan, Kinderlehrer and Otto (JKO). The numerical applications of this formulation have been limited by the difficulty to compute the Wasserstein distance in dimension ≥ 2. One step of the JKO scheme is equivalent to a variational problem on the space of convex functions, which involves the Monge-AmpÃ¨re operator. Convexity constraints are notably difficult to handle numerically, but in our setting the internal energy plays the role of a barrier for these constraints. This enables us to introduce a consistent discretization, which inherits convexity properties of the continuous variational problem. We show the effectiveness of our approach on nonlinear diffusion and crowd-motion models.

Tuesday oct. 21

– *9:30 AM* Yann Brenier (CNRS CMLS Ecole Polytechnique).
Two examples of numerical applications of the OT problem with quadratic cost .
We explain two possible applications of the OT problem with quadratic cost: a) Computation of minimizing geodesics along the group of volume preserving diffeomorphisms; b) Simulation and inverse problem for the "early universe" gravitational model, following Zeldovich, Peebles and, more recently, Frisch and coworkers, by using the Euler-Monge-Ampère approximation to the general relativity equations.

– *10:30 AM* Jean-Marie Mirebeau (CNRS U. Paris Dauphine) .
Monotone and Consistent discretization of the Monge-Ampere operator.
We introduce a novel discretization of the Monge-Ampere operator, simultaneously consistent and degenerate elliptic, hence accurate and robust in applications. These properties are achieved by exploiting the arithmetic structure of the discrete domain, assumed to be a two dimensional cartesian grid. The construction of our scheme is simple, but its analysis relies on original tools seldom encountered in numerical analysis, such as the geometry of two dimensional discrete lattices, and an arithmetic structure called the Stern-Brocot tree. Numerical experiments illustrate the method's efficiency.

Wenesday oct. 22

– *9:30 AM* - Adrien Blanchet (U. Toulouse).
Nash equilibria ??

– *10:30 AM* - Martial Agueh (U. Victoria).
Weak Solutions to a Fractional Fokker-Planck Equation via Splitting and Wasserstein Gradient Flow.
We study a linear fractional Fokker-Planck equation that models non-local diffusion in the presence of a potential field. The non-locality is due to the appearance of the Ôfractional LaplacianÕ in the corresponding PDE, in place of the classical Laplacian which distinguishes the case of regular diffusion. We prove existence of weak solutions by combining a splitting technique together with a Wasserstein gradient flow formulation. An explicit iterative construction is given, which we prove weakly converges to a weak solution of this PDE. Joint work with Malcolm Bowles.

Thursday oct. 23

– *9:30 AM* - Brendan Pass.
Multi-marginal optimal transport: theory and applications.
Multi-marginal optimal transport is an extension of the classical optimal transport problem; abstractly, it is the problem of aligning three or more probability distributions as efficiently as possible. Recently, interest has arisen in this problem and its

variants due to applications in economics, physics, image processing and mathematical finance.

I will give a general overview of what is known about this problem, and try to provide some intuition about how the uniqueness and structure of the solution(s) depends on the cost function. I will then discuss the interpretation of this theory in the context of applications.

– *10:30 AM* Luca Nenna (MOKAPLAN INRIA)
  An iterative method for regularised optimal transport method
  The aim of this talk is to present a fast numerical method, namely the iterative Bregman projections, and its applications to Optimal Transport. We will illustrate the usefulness of this method to solve several optimal transport problems: the standard Monge-Kantorovich (MK) problem, the MK problem with Coulomb cost (which is strictly related to the strong interaction limit of Density Functional Theory). A generalisation of Bregman projections, the Dijkstra method, can be used for the partial MK problem and their extension to the multi-marginal framework. We will show some numerical results.

Friday oct. 24

– *9:30 AM* Adam Oberman.
  Efficient Linear Programming Approach to Optimal Transportation While the linear programming approach of Kantorovich is provided a very natural numerical method, it is not useful in practice, due to the size of the problems involved. For example, even solving a two dimensional OT problem on a modest size (N^2, N = 30, 40, 50) grid, quickly overwhelms the memory of a typical laptop computer. Because the full problem size scales like $O(N^4)$, or worse, practical implementation on larger grids are impossible using this approach. On the other hand, the Kantorovich approach allows for general (not necessarily quadratic) costs, as well as more general problems (for example the multi-marginal problem).

  In this talk we will we present and justify a simple method which makes the linear programming approach possible, even for problems sizes such as $N = 512$. This preliminary work has been applied to the quadratic cost problem, but it has the potential to deal with very general problems.

– *10:30 AM* JDB/Guillaume.
  Augmented Lagrangian methods for transport optimization, Mean-Field Games and degenerate PDEs.
  Many problems from mass transport can be reformulated as variational problems under a prescribed divergence constraint (static problems) or subject to a time dependent continuity equation which again can also be formulated as a divergence constraint but in time and space. The variational class of Mean-Field Games introduced by Lasry and Lions may also be interpreted as a generalisation of the time-dependent optimal transport problem. Following Benamou and Brenier, we show that augmented Lagrangian methods are well-suited to treat convex but nonsmooth problems. It includes in particular Monge historic optimal transport problem. A Finite Element discretization and implementation of the method is used to provide numerical simulations and a convergence study.