

Post-Doc Proposal : Numerical methods for Mean-Field Games and application to congested crowd motion

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The use of partial differential equations in Economics and more generally in social sciences is a relatively young research field. Optimal transportation and mean field games (MFG) [LL06a] [LL06b] [LL07] have in this context received a lot of attention.

Mean field type models describing the limiting behavior of differential and possibly stochastic game problems as the number of players tends to $+\infty$ have recently been introduced by J-M. Lasry and P-L. Lions. At the limit a system of two coupled equations is obtained: a forward in time Hamilton-Jacobi-Bellman for a value function and a backward in time Fokker-Planck equation for a probability measure.

Numerical methods to solve the system are scarce. Yves Achdou and coauthors [ACCD12] [ACCD13] [ACD10] [AP12] have developed numerical methods using a Newton algorithm to solve the non-linear systems arising from the discretisation of the continuous MFG system.

The computational fluid dynamics formulation of optimal transportation (CFDOT) can be interpreted as a deterministic variational MFG, and in this context, the extension of the popular Augmented Lagrangian numerical method ALG2 [FG83] as been recently proposed by Benamou and Carlier [BC14]

While much slower than the Newton method, the ALG2 iterative algorithm is less sensitive to singular data and energies. Its extension to stochastic MFGs is possible but needs a robust Bi-Laplacian solver.

We propose a one year post-doctoral position on the development of numerical algorithms for the simulation of mean field games with applications to social sciences. The candidate will work on both approaches. She/he will use the 4th order multi-grid numerical method developed by Achdou and Perez to extend the ALG2 approach to stochastic MFGs. She/he will also compare both approaches and possibly use the ALG2 approach to build an initial guess for the fast Newton approach.

The solvers will in particular be tested on applications to mean field game models for the motion of crowds of pedestrians proposed in the framework of the ANR ISOTACE. The tests objectives will be

1. to understand and compare the modern congestion models, on a theoretical viewpoint
2. to become a prototype for industrial collaborations.

Concerning the programming aspect, the goal is to improve a code written in C++ by Yves Achdou. The ALG2 MFG code deterministic code is currently written in FreeFem++.

The young researcher will be supervised by Y. Achdou (U. Paris-Diderot) and J-D. Benamou (INRIA). The research will take place either at Université Paris Diderot or at INRIA Rocquencourt.

She/he should be talented, with a taste for designing algorithms and for implementing them, open minded and eager to work hard. Experience in C++ will be helpful.

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References

- [ACCD12] Yves Achdou, Fabio Camilli, and Italo Capuzzo-Dolcetta. Mean field games: numerical methods for the planning problem. *SIAM J. Control Optim.*, 50(1):77–109, 2012.
- [ACCD13] Yves Achdou, Fabio Camilli, and Italo Capuzzo-Dolcetta. Mean field games: convergence of a finite difference method. *SIAM J. Numer. Anal.*, 51(5):2585–2612, 2013.

- [ACD10] Yves Achdou and Italo Capuzzo-Dolcetta. Mean field games: numerical methods. *SIAM J. Numer. Anal.*, 48(3):1136–1162, 2010.]
- [AP12] Yves Achdou and Victor Perez. Iterative strategies for solving linearized discrete mean field games systems. *Netw. Heterog. Media*, 7(2):197–217, 2012.
- [BB00] Jean-David Benamou and Yann Brenier. A computational fluid mechanics solution to the Monge-Kantorovich mass transfer problem. *Numer. Math.*, 84(3):375–393, 2000.
- [BC14] Jean-David Benamou and Guillaume Carlier. Augmented Lagrangian methods for transport optimization, Mean-Field Games and degenerate PDEs *Preprint* , 2014.
- [FG83] Michel Fortin and Roland Glowinski. *Augmented Lagrangian methods*, volume 15 of *Studies in Mathematics and its Applications*. North-Holland Publishing Co., Amsterdam, 1983. Applications to the numerical solution of boundary value problems, Translated from the French by B. Hunt and D. C. Spicer.
- [LL06a] Jean-Michel Lasry and Pierre-Louis Lions. Jeux à champ moyen. I. Le cas stationnaire. *C. R. Math. Acad. Sci. Paris*, 343(9):619–625, 2006.
- [LL06b] Jean-Michel Lasry and Pierre-Louis Lions. Jeux à champ moyen. II. Horizon fini et contrôle optimal. *C. R. Math. Acad. Sci. Paris*, 343(10):679–684, 2006.
- [LL07] Jean-Michel Lasry and Pierre-Louis Lions. Mean field games. *Jpn. J. Math.*, 2(1):229–260, 2007.