R. Weltz*

? Bifurcation.

1. Kuznetsov 2023
2. Haragus & Iooss 2010
local bifurcations..

Orbite periodique

Poincaré

Ana. fonct.

~ Iooss

→ Bifurcation. OP

× Bifurcations équations scalaires.

ODE.

$$\dot{u} = f(u,p) \in \mathbb{R}, \quad p \in \mathbb{R}.$$

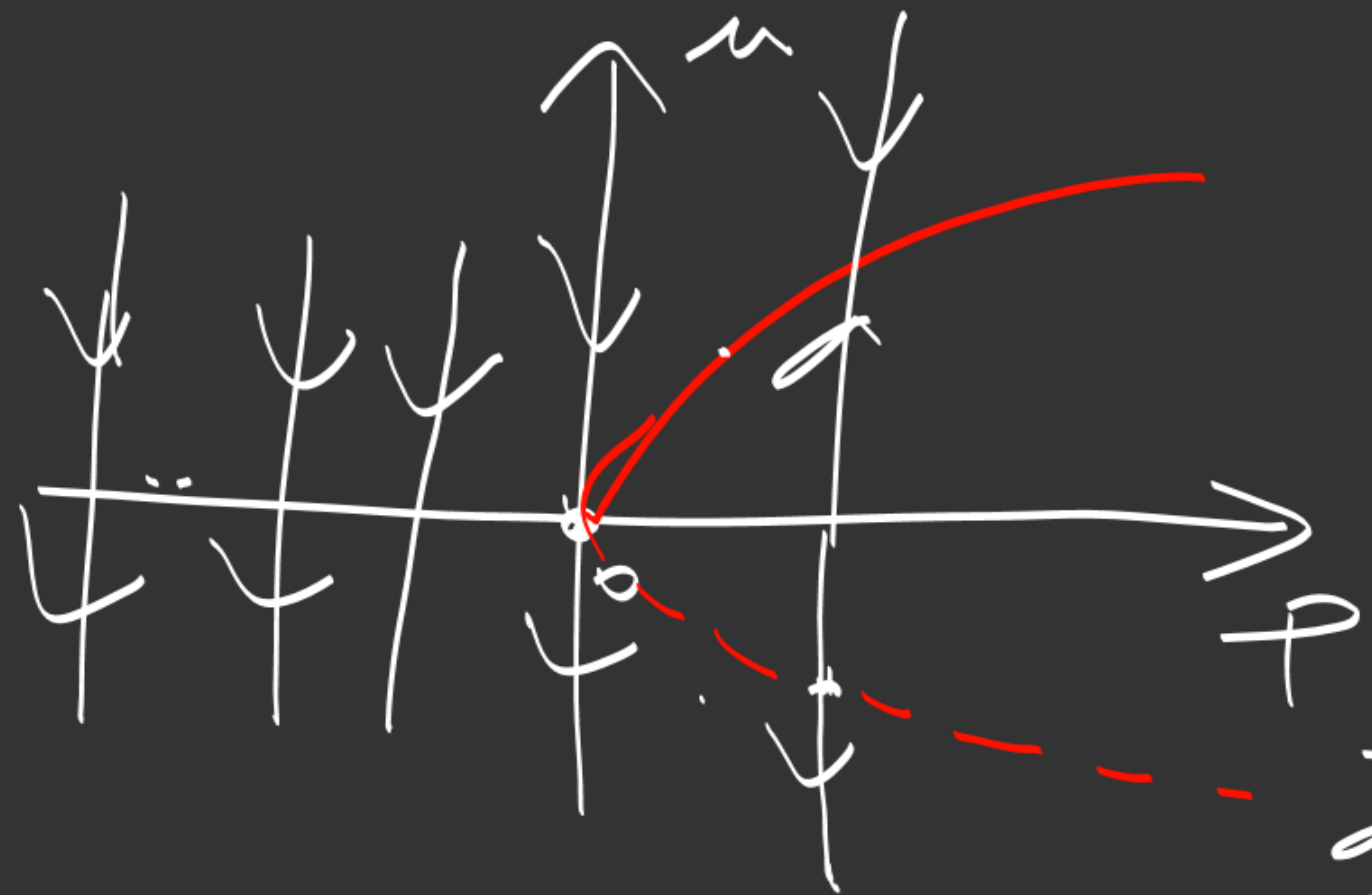
* $u(t) = 0$ solution, $p_i = 0$. $f(0,0) = 0$.

* $\frac{\partial f}{\partial u}(0,0) = 0$

* $f(u,p) = ap + bu^2 + \dots$ $a, b \in \mathbb{R}$

* $\dot{u} = p - u^2$ $p \geq 0$ $u^{eq}_{\pm} = \pm\sqrt{p}$

$p < 0$ $u^{eq} = \emptyset$



$$\dot{u} = p - u^2$$

$$\Sigma\left(\frac{\partial f}{\partial u}(u^{eq}, p)\right)$$

$$\vee$$

$$\frac{\partial f}{\partial u} = -2u^{eq}_{\pm}$$

$$= \mp 2\sqrt{p}$$

$$\dot{u} = p - u^2 + \underbrace{g(u)}_{O(u^3)}$$

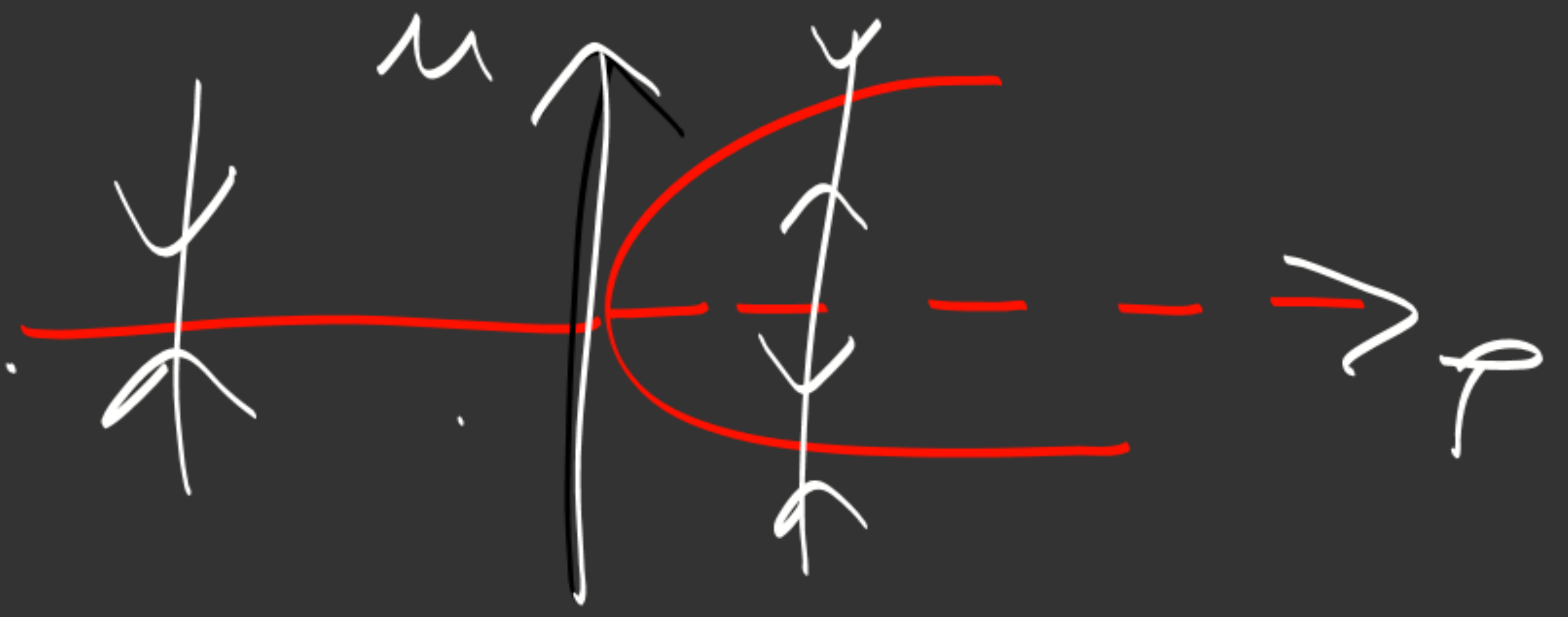
Fold bifurcation

Saddle node

$$\dot{u} = pu - u^3 + \text{h.o.t.}$$

pitchfork bif.

$$O\left(u|p| + u^3\right)$$



Theoreme de Poincaré-Hopf

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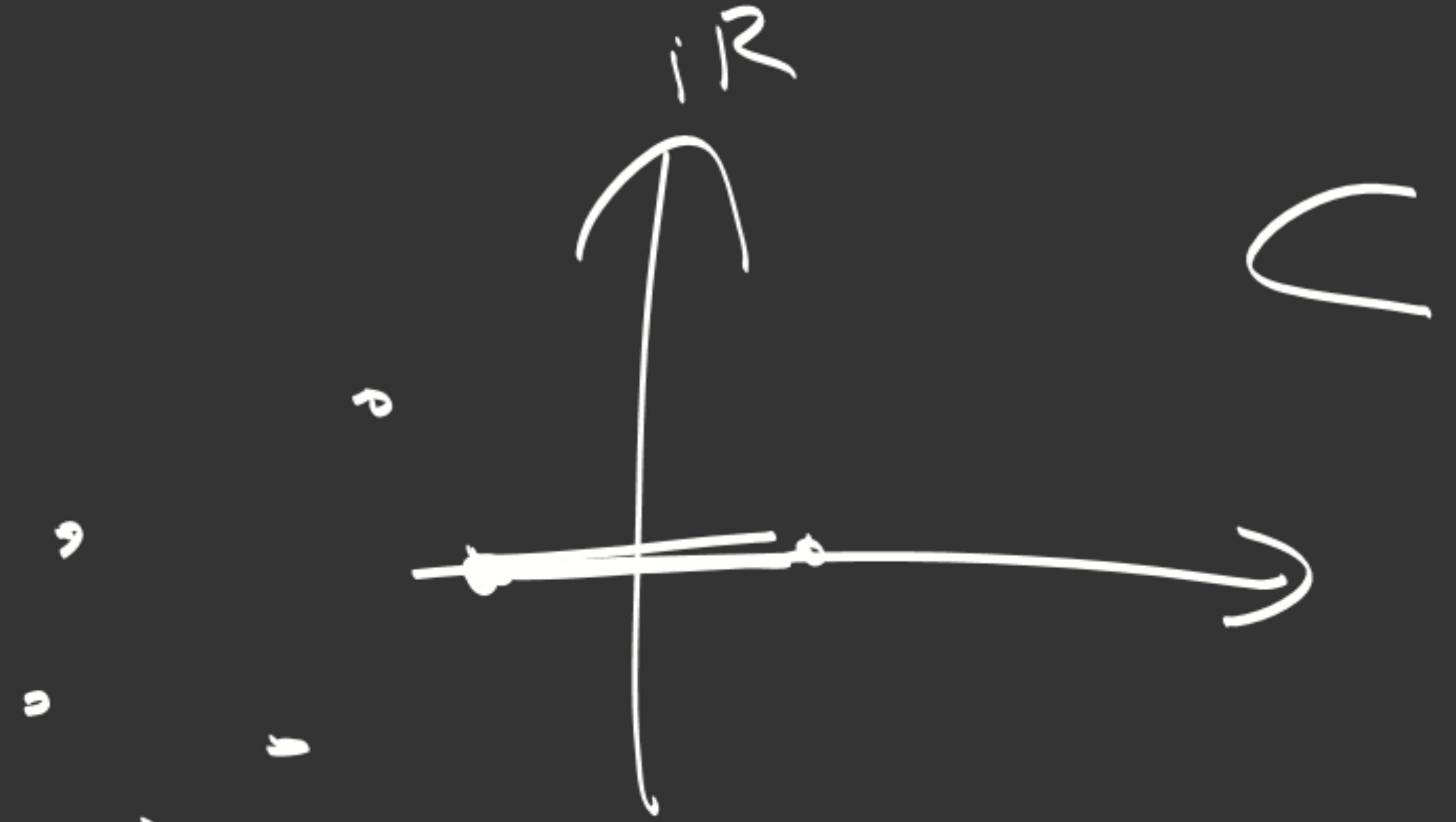
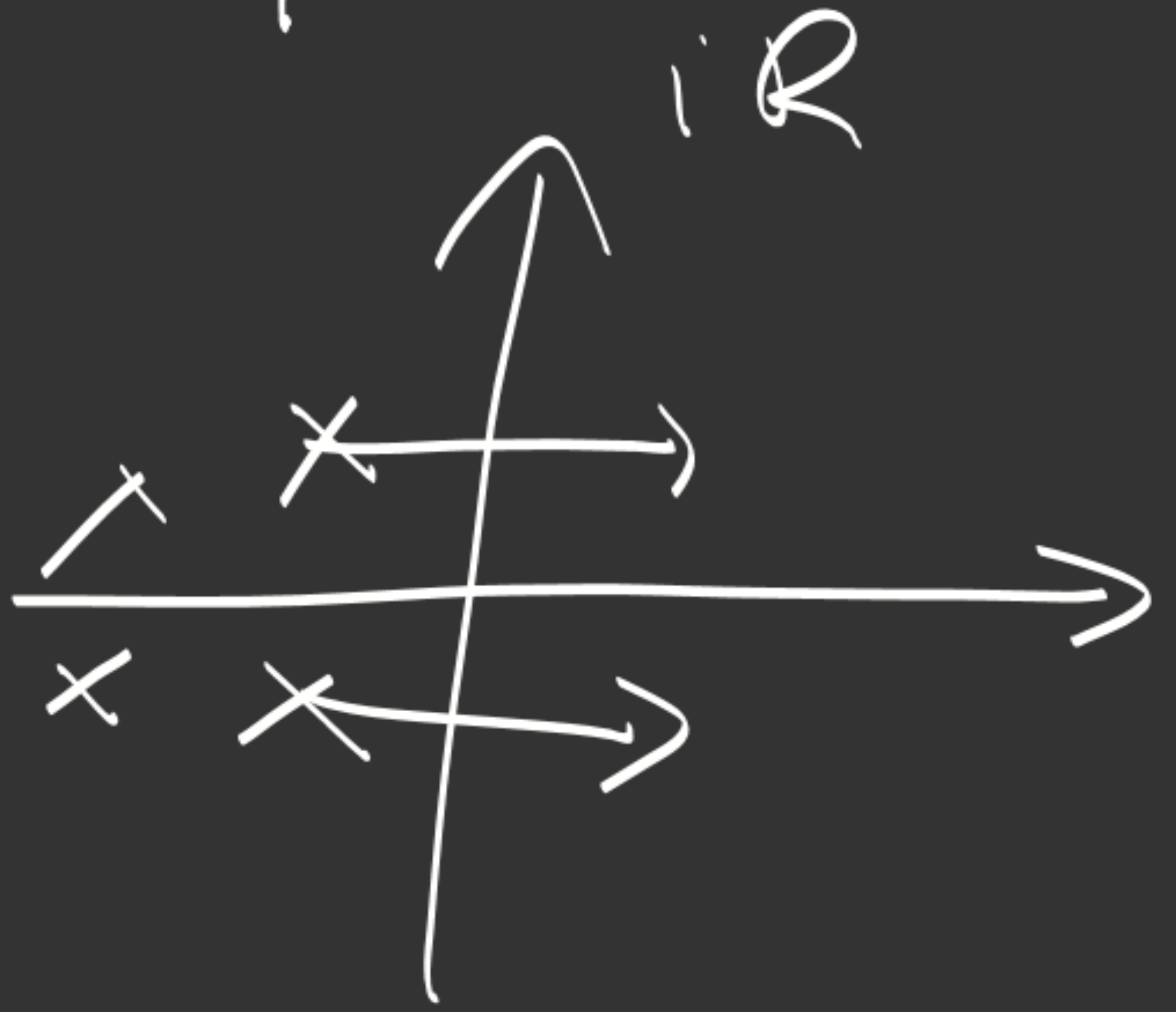




$$\pm i\omega \in \Sigma\big(df(0,0)\big) \cap i\mathbb{R}$$

$$\begin{cases}\dot{x} = \rho x - \omega y - x(x^2+y^2) \\ \dot{y} = \omega x + \rho y - y(x^2+y^2)\end{cases}$$

b

$+ \dots$

$u = v + P(v)$

Theorie formes normales

$$\begin{bmatrix}\rho & -\omega \\ \omega & \rho\end{bmatrix} \longrightarrow \rho \pm i\omega$$

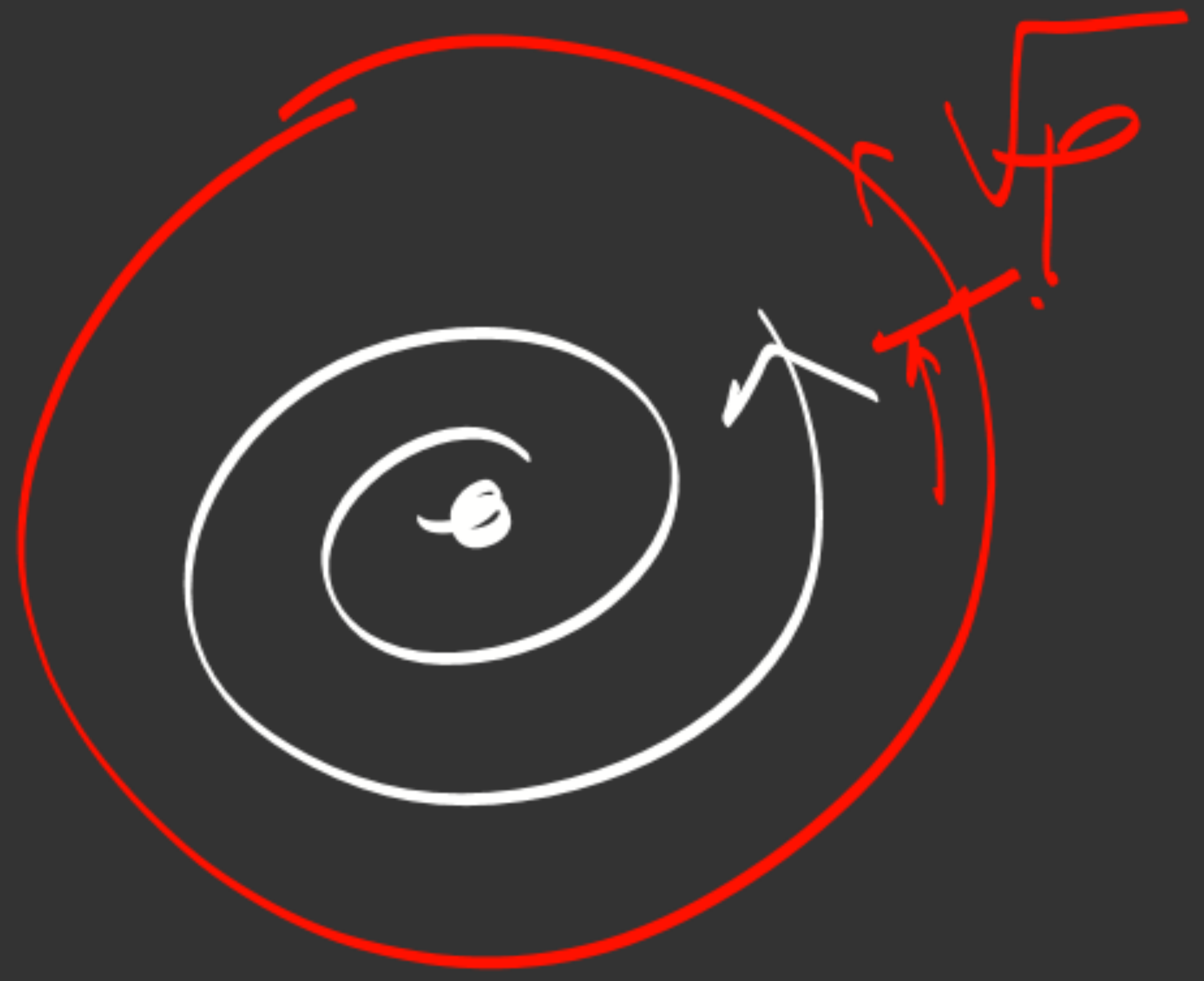
$b \in \mathbb{C}$

$$\dot{z} = \rho z + i\omega z - b z|z|^2$$

$z = 0$ eq

$\boxed{\rho < 0}$

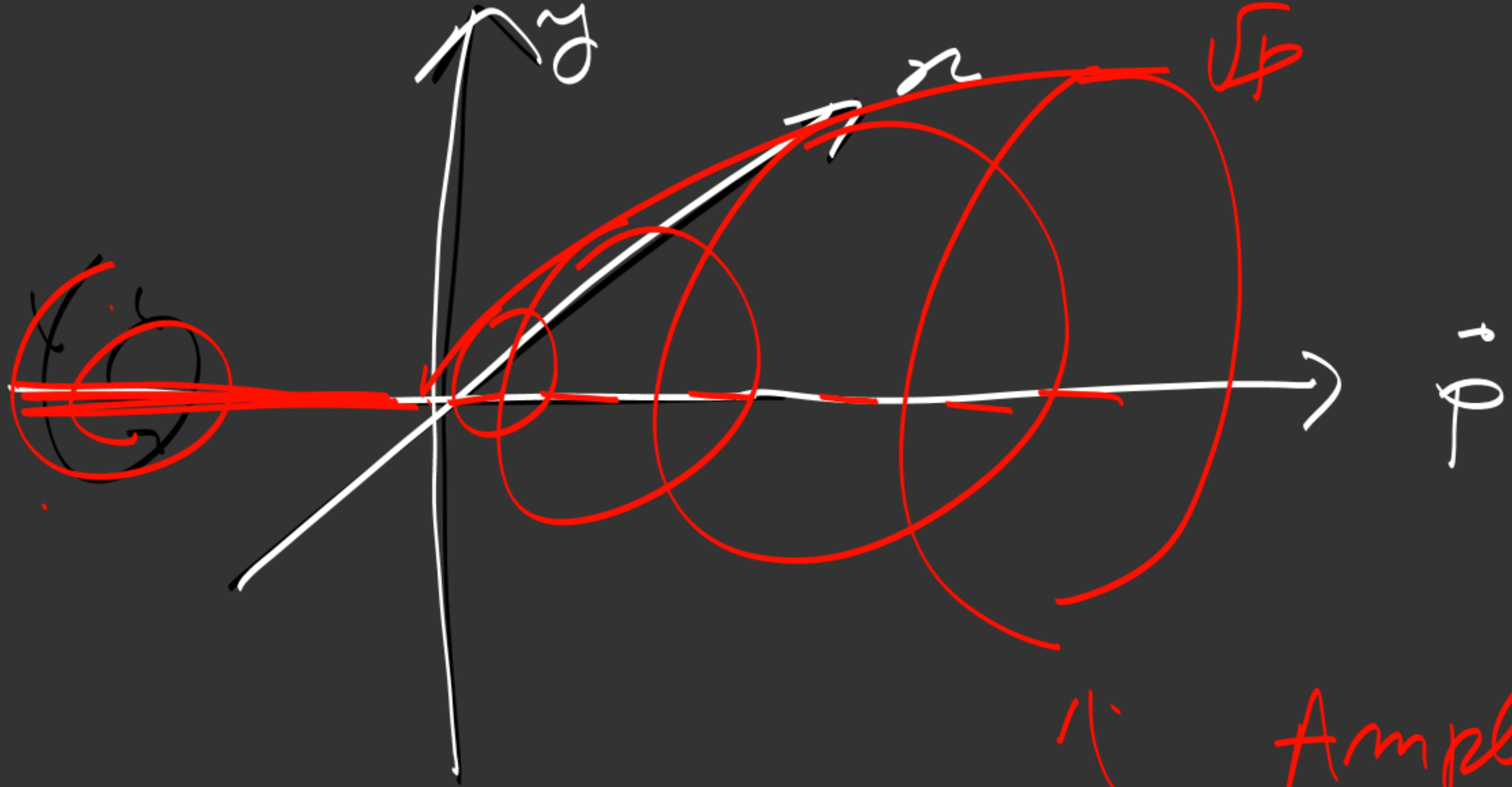
$\underline{p > 0}$

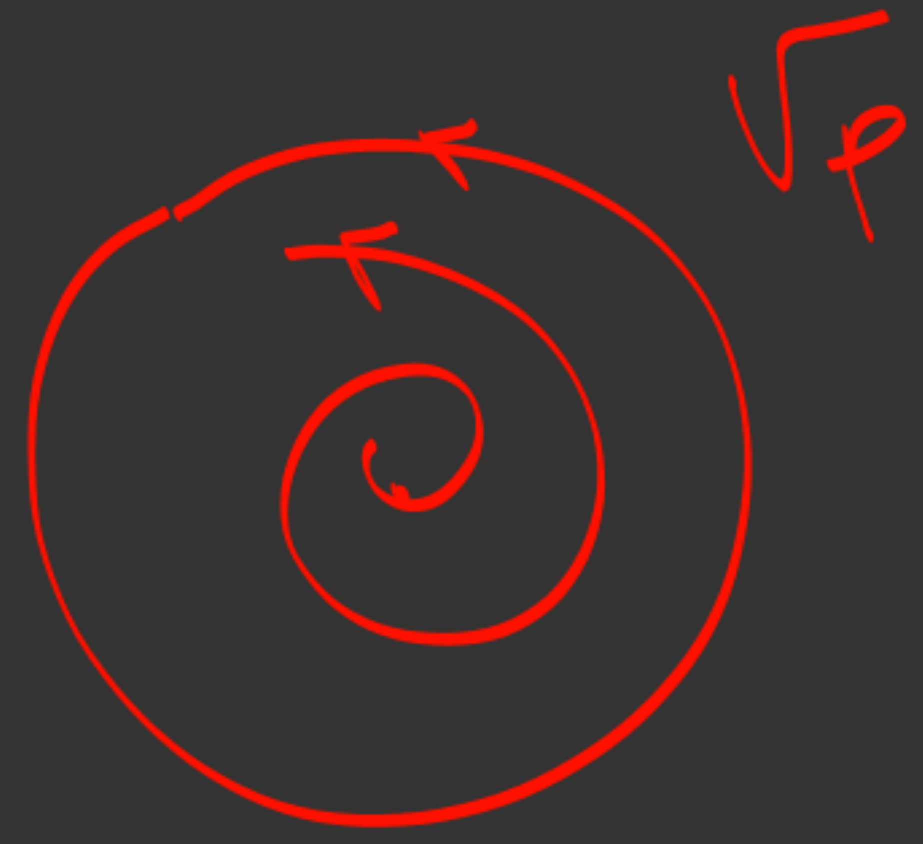


$$\boxed{\begin{aligned} \dot\rho &= \rho(p - \rho^2) \\ \dot\theta\rho &= \omega\rho \end{aligned}}$$

$$* \quad z = \rho e^{i\theta}$$

$$(\dot\rho + i\dot\theta\rho)e^{i\theta} = e^{i\theta}(\underbrace{p + i\omega} - \rho^2)\rho$$





↳ Amplitude of

$\cancel{A}$ $2\sqrt{p}$

freq: ω

— $0 \in \Sigma\left(\frac{\partial f}{\partial u}(0,0)\right) \longrightarrow$ creation eq

— $i\omega \in \Sigma\left(\frac{\partial f}{\partial u}(0,0)\right) \longrightarrow$ OP

$$\dot{u} = Au + R(u) \in \mathbb{R}^n$$

$$\dot{v} = Av + \underbrace{N(v)}_{\deg p} + \underbrace{\rho(u)}_{= o(v^p)}$$

$$N(e^{tA^*} v) = e^{tA^*} N(v)$$

$i\omega \in \Sigma(A)$

Iggs

$\longrightarrow N(z) = z P(|z|^2)$

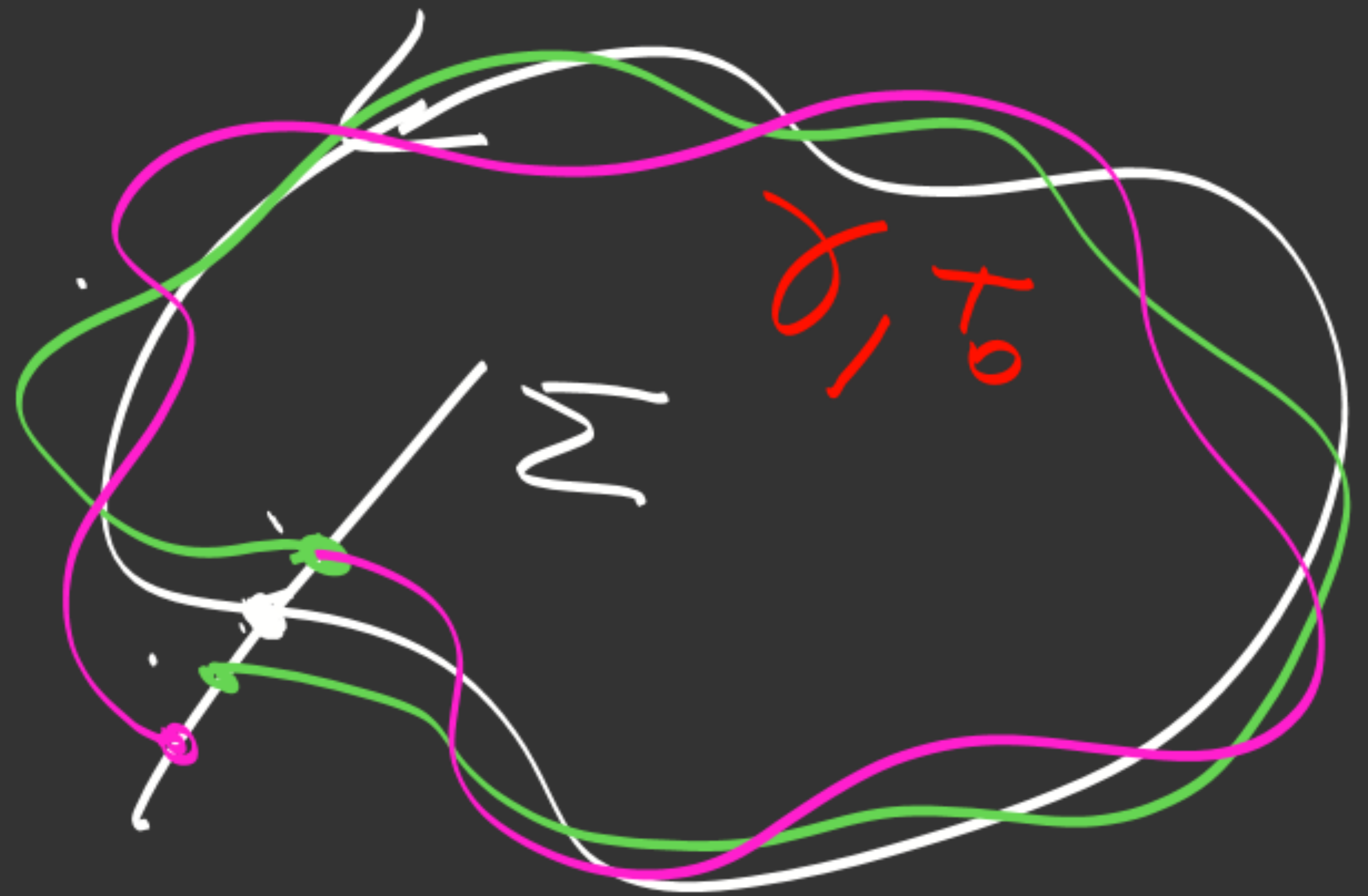
$$\left.\begin{array}{l} \pm i\omega_1 \\ \pm i\omega_2 \end{array}\right\} \in \Sigma(A)$$

Hopf-Hopf))

Fold-Hopf $0, \pm i\omega$

* Orbites periodiques.

— application premier retour Π



$\Sigma \ni x \longmapsto \Pi(x) \in \Sigma$

* $x^* \in \gamma \longrightarrow \Pi(x^*) = x^*$

* γ est stable si

$\forall \lambda \in \Sigma(d\Pi(x^*))$

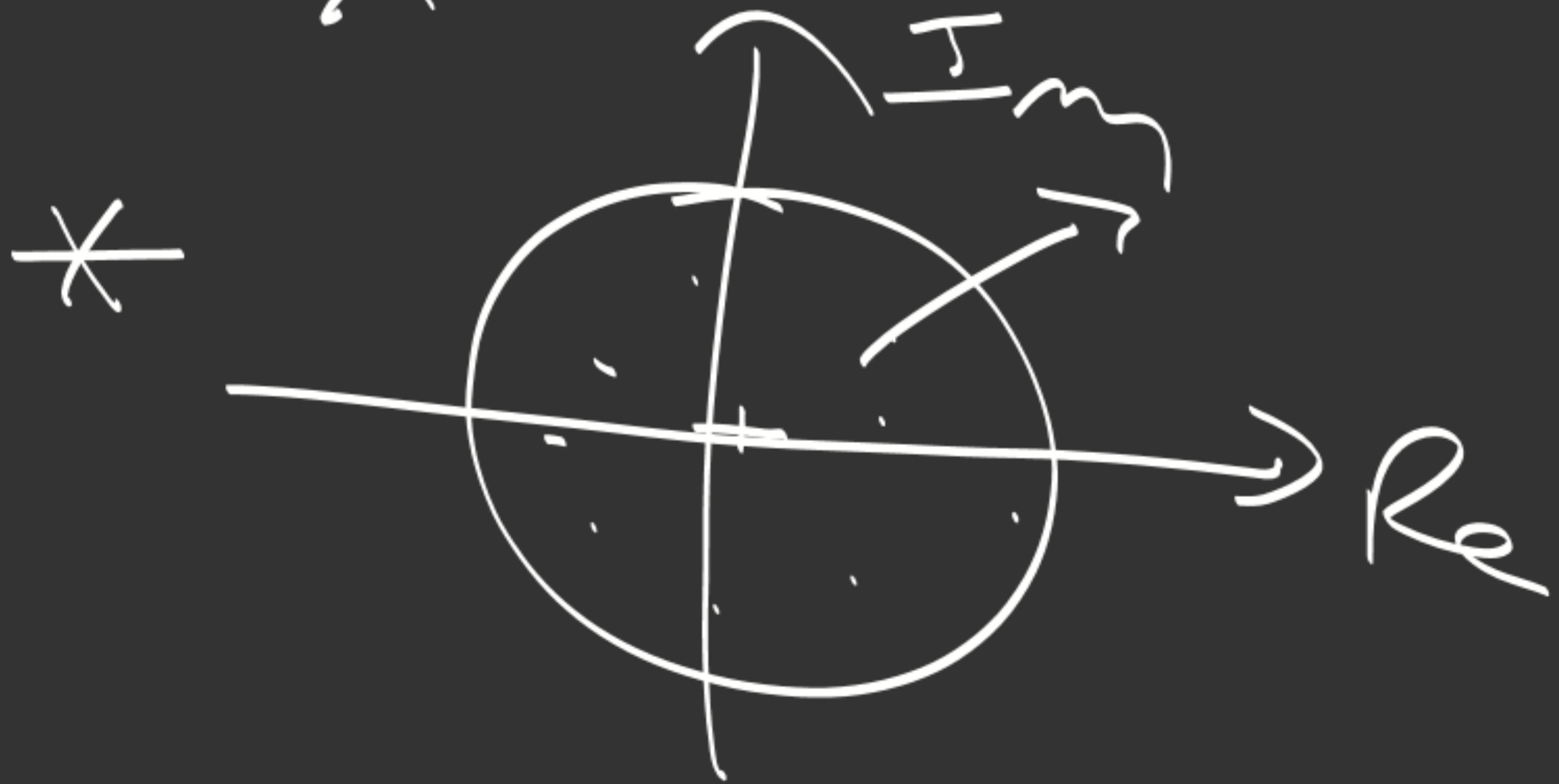
$|\lambda| < 1$

$\varphi^t(x)$

$d\phi^T(x^*)$

$$x_{n+1} = \Pi(x_n, p)$$

$$x^{eq} = \Pi(x^{eq}, p)$$



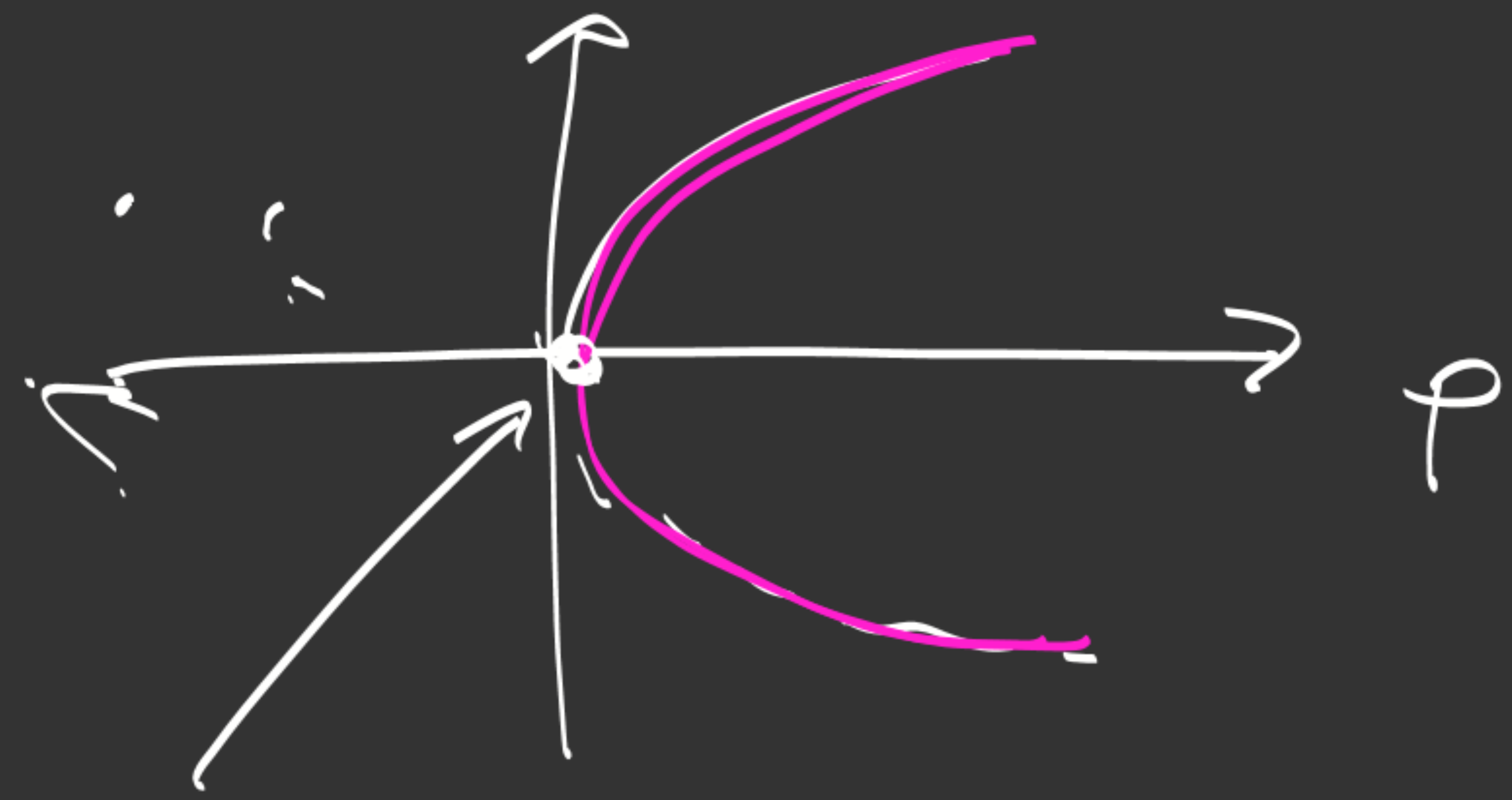
$$\begin{cases} 1 \in \Sigma(d\Pi(x^{eq}, 0)) \\ e^{\pm i\theta} \neq 1 & \text{Neimark-Sacker} \\ -1 \in \Sigma(d\Pi) \end{cases}$$

Hopf

period doubling

TBC...

Pli d'orbite périodique



$$1 \in \Sigma\left(d\Pi(x^{*q}, 0)\right)$$