Reproducing sensory induced visual hallucinations via neural fields

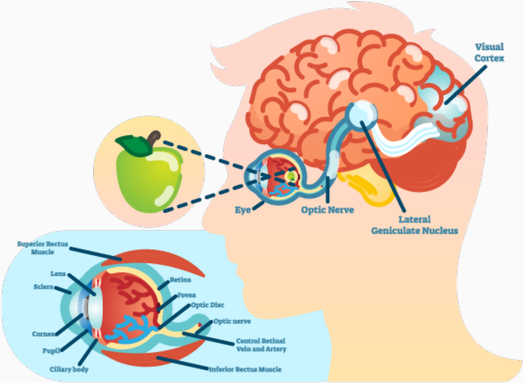
Dario Prandi
Université Paris-Saclay, CNRS, CentraleSupélec

joint work with:
C. Tamekue (L2S), Y. Chitour (L2S), E. Baspinar (NeuroPSI), M. Bertalmío (UPF), L. Calatroni (CNRS, I3S), V. Franceschi (UniPD), B. Franceschiello (FAA-LINE), A. Gomez-Villa (UPF)

Séminaire McTAO
May 25th 2022

Modelling the human visual system

Visual data is first processed in the primary visual cortex (V1).



Q: How can we model cortical activity in V1? Can this inform us on aberrant visual behavior (hallucinations, illusions, etc...) ?

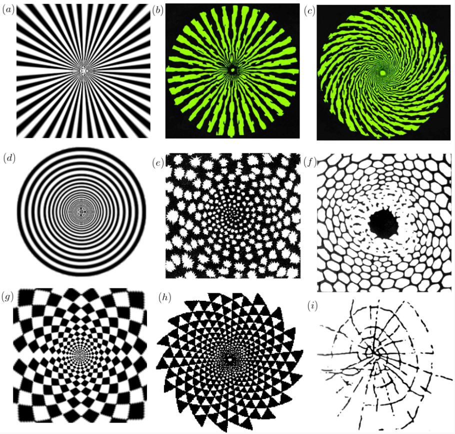
Table of contents

Geometric hallucinations

Introduction

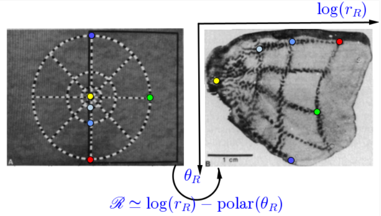
Visual hallucination ⇆ perception of an image which does not exist.

We focus on *geometric visual hallucinations* or *form constants* (Klüver, 1967).



Artist's depictions of geometric visual hallucinations. Reproducing from Oster (1970), Siegel(1977), Patterson (1992), Clottes & Lewis-Williams (1998).

Retinotopic structure of V1



Adapted from (Tootell *et al*, 1982)

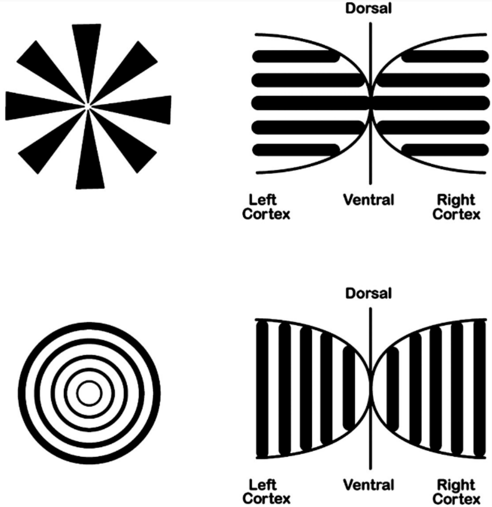
The retinal visual field is mapped in a non-trivial way on the surface of V1.

Retino-cortical map (Schwartz 1977; Cowan 1977)

Using polar coordinates on the retinal plane, we have

$$\begin{aligned} \mathcal{R} : \quad & \mathbb{R}^2 \longrightarrow \mathbb{R}^2 \\ & re^{i\theta} \longmapsto (x_1, x_2) = (\log r, \theta). \end{aligned} \tag{1}$$

Retinotopic structure of V1



Visual illustration of the retino-cortical map. Reproduced from Billock and Tsou (PNAS, 2007).

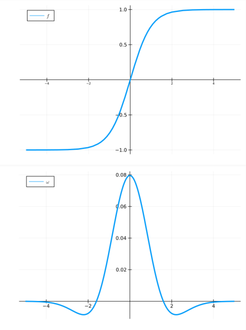
Neuronal Activity in V1

Wilson-Cowan equation

The cortical activity a on the cortical surface evolves according to

$$\frac{\partial}{\partial t} a(x,t) = -\alpha a(x,t) + \mu \int_{\mathbb{R}^2} \omega(\|x - x'\|) f(a(x',t)) dx' + I_{\text{ext}}(x,t). \qquad \text{(WC)}$$

- $\alpha, \mu > 0$
- $f : \mathbb{R} \to \mathbb{R}$ non-linear response function
- I_{ext} cortical representation of visual stimulus
- ω interaction kernel (typicalli a DoG)



Low-pass filter + interaction

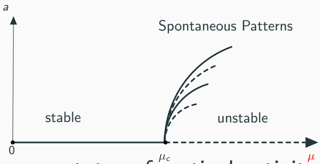
Remark

(WC) *commutes with the natural action of* $\mathbf{E}(2) := \mathbb{R}^2 \rtimes \mathcal{O}(2)$ *when* $I_{ext} \equiv 0$.

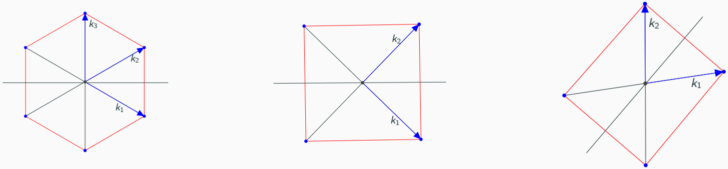
Spontaneous Patterns (SP) and Hallucinatory Patterns (HP)

Spontaneous patterns $\to a \sim 0$ and $I_{ext} = 0$

- 0 is stationary state of (WC) when $I_{ext} = 0$
- For $\mu > \mu_c$ marginally stable stationary states appear (symmetry $\mathbf{E}(2)$)



- **SP**: Paroxismic stationary states of cortical activity in V1 when $I_{ext} = 0$;

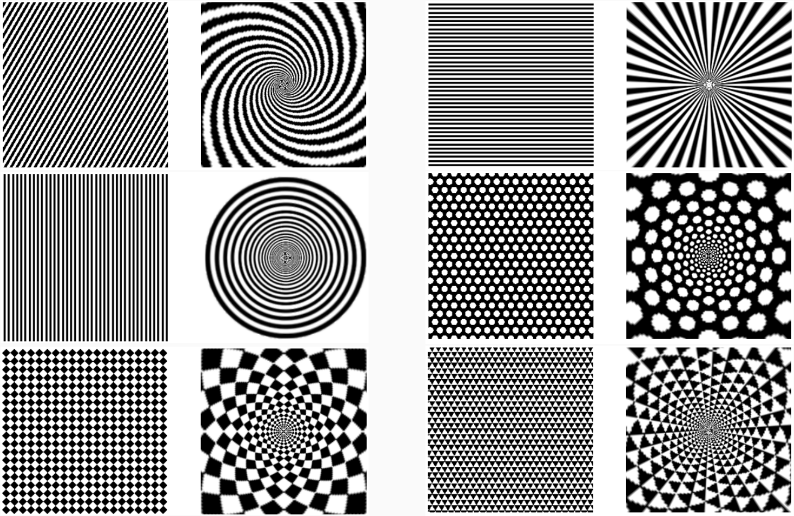


$$SP(x) = \sum_{j=1}^{N} \cos(2\pi k_j \cdot x), \qquad k_j = (\cos\phi_j, \sin\phi_j).$$

- **HP**: "images" of **SP** by the inverse retino-cortical map

$$re^{i\theta} = (\exp(x_1), x_2)$$

Spontaneous Patterns (SP) and Hallucinatory Patterns (HP)



SP in left and HP in the right. Ermentrout & Cowan (1979)

“Mackay effects”, and Billock and Tsou psychophysical tests

→ Complex hallucinatory-like patterns can arise also in normal state

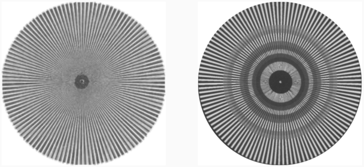


Figure 1: The MacKay effect: the presentation of the stimulus to the left (“MacKay rays”) induces the perception of the image (Artist depiction by Isia Léviant) on the right. Adapted from MacKay (Nature, 1957) and Zeki *et al* (Bio. Sci., 1993).

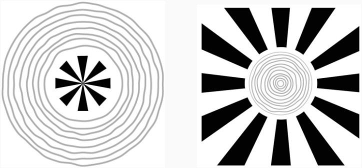
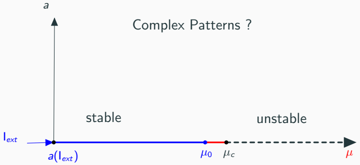
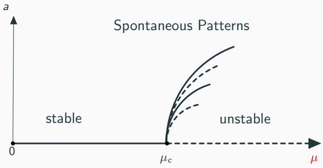


Figure 2: Artist’s depictions of some subjects report biasing stimuli and hallucinatory percepts. Reproduced from Billock & Tsou (PNAS, 2007).

Complex Patterns (CP) via controllability of (WC) equation



Complex Patterns via controllability?



Ermentrout & Cowan.

$$\begin{cases} \partial_t a(x,t) + \underbrace{\alpha a(\xi,t) - \mu \int_{\mathbb{R}^2} \omega(\|x-x'\|) f(a(x',t)) dx'}_{\text{non-local \& nonlinear operator}} = \underbrace{I_{ext}(x,t)}_{\text{Control term}}, \\ a(x,0) = a_0(x). \end{cases} \tag{2}$$

Meaning of Controllability

Let $\mu < \mu_c$ and $T \gg 1$. Given two states a_0 and a_1, is there a control I_{ext} such that the solution a of the above Cauchy problem satisfies

$$a(\cdot, T) \approx a_1(\cdot) \quad ?$$

SP can not induce CP in linear regime

It is established since Ermentrout and Cowan (1979) that

$$\mu_c := \frac{\alpha}{f'(0) \max_{r \geq 0} \widehat{\omega}(r)}$$

Theorem (Tamekue, Chitour, P)

Consider the linear equation

$$\begin{cases} \partial_t a(x,t) & = -\alpha a(x,t) + \mu \int_{\mathbb{R}^2} \omega(\|x - x'\|) a(x',t) dx' + SP(x), \\ a(x,0) & = a_0(x). \end{cases}$$

Let $a_0 \in L^\infty(\mathbb{R}^2)$ *and* $\mathrm{I}_{ext} = SP$. *Then the unique solution of the above equation satisfies*

$$a(\cdot, t) \xrightarrow[t \to \infty]{} \frac{1}{\alpha} \frac{\mu_c}{\mu_c - \mu} SP(\cdot), \qquad \textit{exponentially in} \qquad L^\infty(\mathbb{R}^2),$$

provided that

$$\mu < \mu_0 := \frac{\alpha}{f'(0) \|\omega\|_{L^1(\mathbb{R}^2)}} \qquad (\leq \mu_c)$$

$\Longrightarrow$ There is no MacKay effect in the linear regime via SP

Theorem (T, Chitour, Prandi)

Consider the (WC) *equation*

$$\begin{cases} \partial_t a(x,t) & = -\alpha a(x,t) + \mu \int_{\mathbb{R}^2} \omega(\|x-x'\|) f(a(x',t)) dx' + \mathrm{I}_{ext}(x), \\ a(x,0) & = a_0(x). \end{cases}$$

Let $1 \leq p \leq \infty$, $a_0 \in L^p(\mathbb{R}^2)$ *and* $\mathrm{I}_{ext} \in L^p(\mathbb{R}^2)$. *If* $\mu < \mu_0$, *then the solution* $a(\cdot,t)$ *of* (WC) *converges exponentially to the stationary solution* $a_{\mathrm{I}_{ext}}(\cdot)$ *in* $L^p(\mathbb{R}^2)$ *when* $t \longrightarrow \infty$.

We introduce for every $1 \leq p \leq \infty$, the map

$$\Psi(\mathrm{I}_{ext}) = \frac{\mu}{\alpha} \int_{\mathbb{R}^2} \omega(\|x-y\|) f\left(\Psi(\mathrm{I}_{ext})(y)\right) dy + \frac{1}{\alpha} \mathrm{I}_{ext}\,.$$

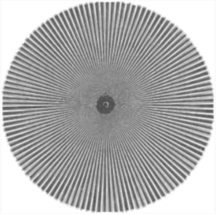
We let $P_T(x) = \cos(\lambda x_1)$ and $P_F(x) = \cos(\lambda x_2)$, $x = (x_1, x_2) \in \mathbb{R}^2$ and $\lambda > 0$.

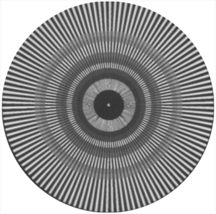
Theorem

Under the assumption $\mu < \mu_0/2$, *the zeros of* P_T *(resp.* P_F*) coincide with those of* $\Psi(P_T)$. *(resp.* $\Psi(P_F)$*)*

$\Longrightarrow$ There is no MacKay effect in the non-linear regime via P_T and P_F

Mackay effects with “MacKay rays”





- Due to above Theorem, we have to take

$$I_{ext} = SP + \varepsilon \mathbb{1}_{\Omega} u, \qquad \varepsilon > 0$$

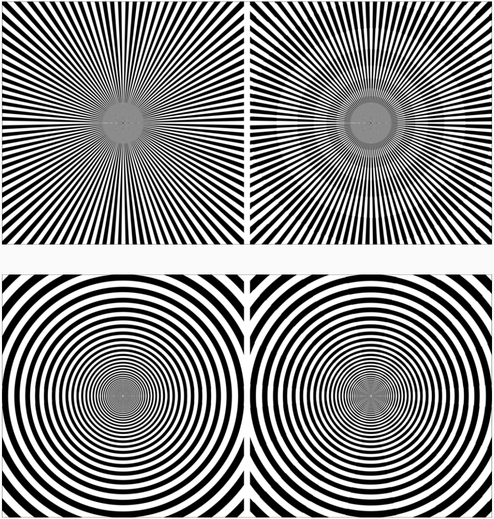
  where Ω is a neighbourhood of the fovea and u is a control function;
- The goal is then to find a convenient control function u such that $a_{I_{ext}}$ will be the superposition of patterns, say,

$$a_{I_{ext}} = SP + \varepsilon \widetilde{SP_j}, \qquad \varepsilon > 0, \; i \neq j.$$

$\rightarrow$ We want to obtain a global complex patterns with a local control

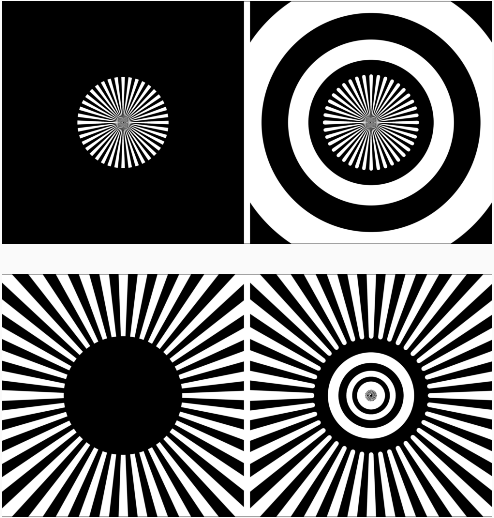
Mackay effects with "MacKay rays", numerical results

Up to now, only partial theoretical results in this direction. However, numerical implementation yields the desired results.



Inputs in left and steady states in the right.

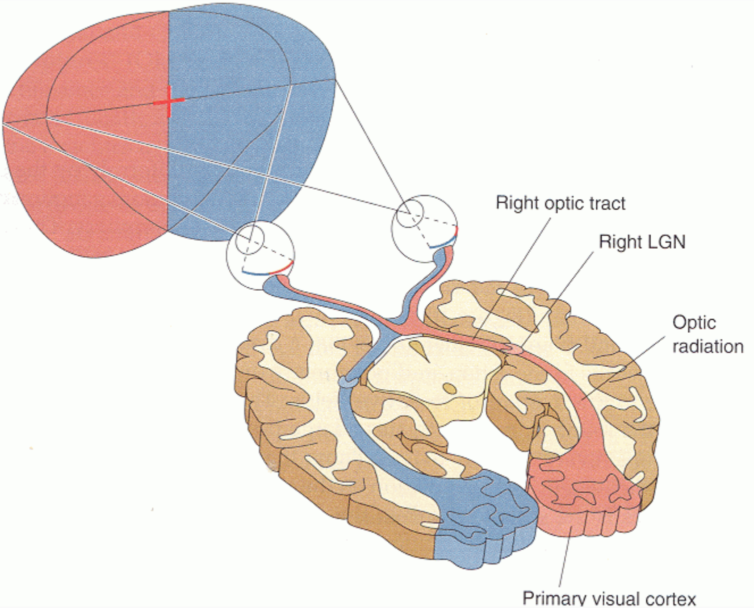
Billock & Tsou psychophysical test, numerical results



Inputs in left and steady states in the right.

Mesoscopic model of V1

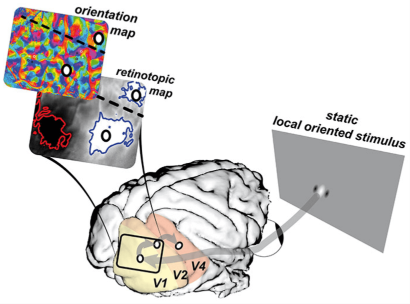
Understanding the visual stream



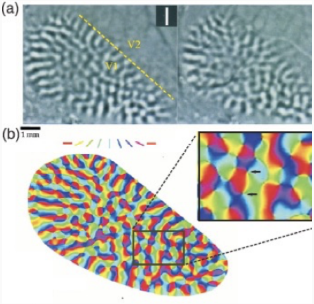
Understanding the visual stream

In 1981 Hubel and Wiesel won the Nobel Prize observing that:

Neurons in V1 are sensitive to both spatial locations **AND** local orientations



From stimulus to V1

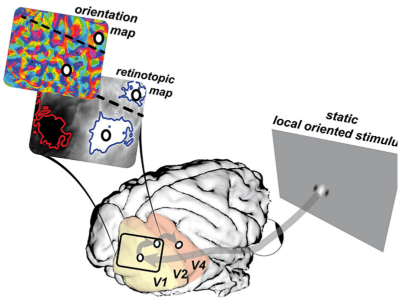


Tree shrew orientation sensitivity

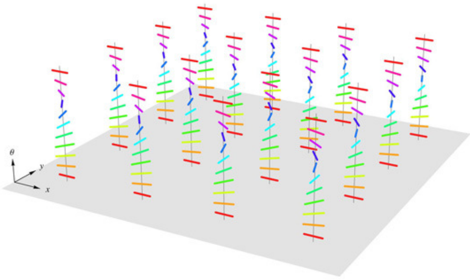
Understanding the visual stream

In 1981 Hubel and Wiesel won the Nobel Prize observing that:

> Neurons in V1 are sensitive to both spatial locations **AND** local orientations



From stimulus to V1

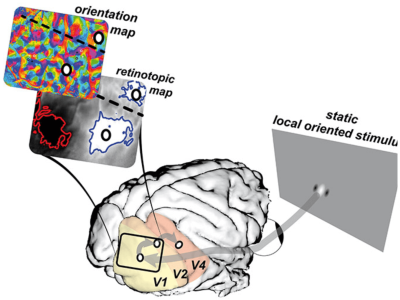


Hypercolumn structure

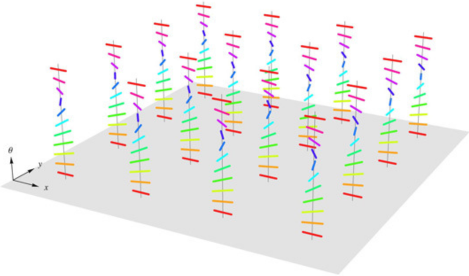
Understanding the visual stream

In 1981 Hubel and Wiesel won the Nobel Prize observing that:

Neurons in V1 are sensitive to both spatial locations **AND** local orientations



From stimulus to V1



Hypercolumn structure

How to build up orientation-dependent models?

Mathematical modelling:

- Neurons in V1 $\leftrightarrow$ points in the Lie group $SE(2) = \mathbb{R}^2 \rtimes \mathbb{S}^1$;
- "Lift" of the 2D image to a 3D object via the operator[1]

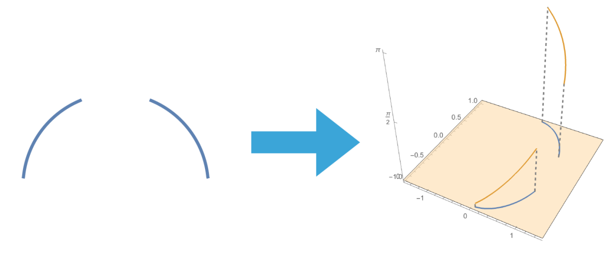
$$L : L^2(\mathbb{R}^2) \to L^2(SE(2))$$

[1] **Mathematical framework**: Petitot, '94, Citti, Sarti, '06, Duits, '05, Boscain, Gauthier, P, '13-'19.

Mathematical modelling:

- Neurons in V1 $\leftrightarrow$ points in the Lie group $SE(2) = \mathbb{R}^2 \rtimes \mathbb{S}^1$;
- "Lift" of the 2D image to a 3D object via the operator[1]

$$L : L^2(\mathbb{R}^2) \to L^2(SE(2))$$



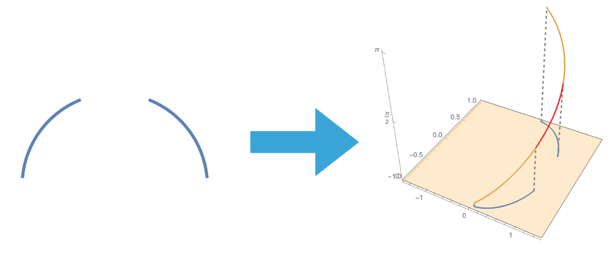
[1] **Mathematical framework**: Petitot, '94, Citti, Sarti, '06, Duits, '05, Boscain, Gauthier, P, '13-'19.

Modelling V1 architecture by functional lifting

Mathematical modelling:

- Neurons in V1 $\leftrightarrow$ points in the Lie group $SE(2) = \mathbb{R}^2 \rtimes \mathbb{S}^1$;
- "Lift" of the 2D image to a 3D object via the operator[1]

$$L : L^2(\mathbb{R}^2) \to L^2(SE(2))$$



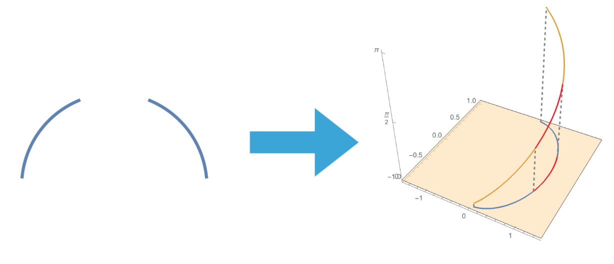
[1] **Mathematical framework**: Petitot, '94, Citti, Sarti, '06, Duits, '05, Boscain, Gauthier, P, '13-'19.

Modelling V1 architecture by functional lifting

Mathematical modelling:

- Neurons in V1 $\leftrightarrow$ points in the Lie group $SE(2) = \mathbb{R}^2 \rtimes \mathbb{S}^1$;
- "Lift" of the 2D image to a 3D object via the operator[1]

$$L : L^2(\mathbb{R}^2) \to L^2(SE(2))$$



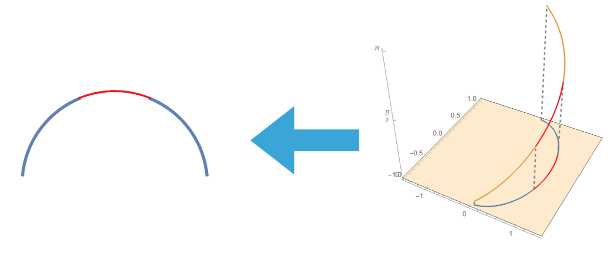
[1] **Mathematical framework**: Petitot, '94, Citti, Sarti, '06, Duits, '05, Boscain, Gauthier, P, '13-'19.

Modelling V1 architecture by functional lifting

Mathematical modelling:

- Neurons in V1 $\leftrightarrow$ points in the Lie group $SE(2) = \mathbb{R}^2 \rtimes \mathbb{S}^1$;
- "Lift" of the 2D image to a 3D object via the operator[1]

$$L : L^2(\mathbb{R}^2) \to L^2(SE(2))$$



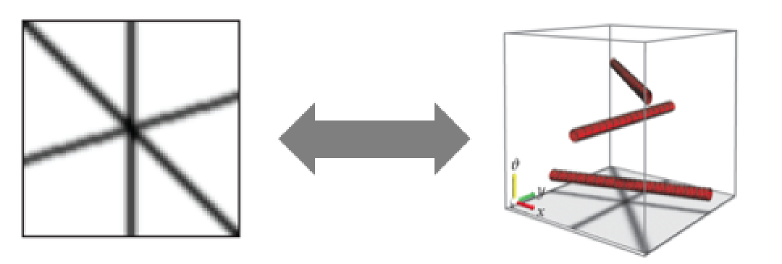
[1] **Mathematical framework**: Petitot, '94, Citti, Sarti, '06, Duits, '05, Boscain, Gauthier, P, '13-'19.

Modelling V1 architecture by functional lifting

Mathematical modelling:

- Neurons in V1 $\leftrightarrow$ points in the Lie group $SE(2) = \mathbb{R}^2 \rtimes \mathbb{S}^1$;
- "Lift" of the 2D image to a 3D object via the operator[1]

$$L : L^2(\mathbb{R}^2) \to L^2(SE(2))$$



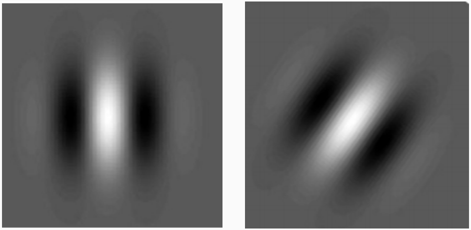
No crossing! (Image from: Bekkers, Duits, Berendschot, ter Haar Romeny, '14)

[1] **Mathematical framework**: Petitot, '94, Citti, Sarti, '06, Duits, '05, Boscain, Gauthier, P, '13-'19.

Mathematical construction

Let $(z, \theta) \in SE(2)$ a neuron in V1.

For $w \in \mathbb{R}^2$ in the **retinal plane**, the **receptive profile** $\psi_{(z,\theta)}(w)$, $w \in \mathbb{R}^2$ models the cortical activation of (z, θ) when a stimulus is applied in location w.



Gabor RP's Daugman, '85

Mathematical construction

Let $(z, \theta) \in SE(2)$ a neuron in V1.

For $w \in \mathbb{R}^2$ in the **retinal plane**, the **receptive profile** $\psi_{(z,\theta)}(w), w \in \mathbb{R}^2$ models the cortical activation of (z, θ) when a stimulus is applied in location w.

Given a visual stimulus u, the cortical output (**lift, orientation score**) is defined as:

$$U(z, \theta) = Lu(z, \theta) = \int_{\mathbb{R}^2} u(w)\psi_{(z,\theta)}(w)\, dw$$

Mathematical construction

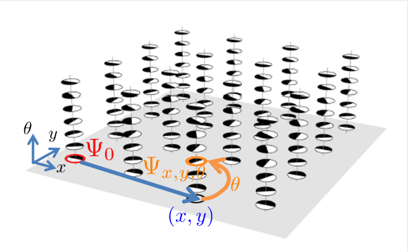
Let $(z, \theta) \in SE(2)$ a neuron in V1.

For $w \in \mathbb{R}^2$ in the **retinal plane**, the **receptive profile** $\psi_{(z,\theta)}(w)$, $w \in \mathbb{R}^2$ models the cortical activation of (z, θ) when a stimulus is applied in location w.

Given a visual stimulus u, the cortical output (**lift, orientation score**) is defined as:

$$U(z,\theta) = Lu(z,\theta) = \int_{\mathbb{R}^2} u(w)\psi_{(z,\theta)}(w)\, dw \; = \int_{\mathbb{R}^2} u(w)\overline{\Psi(R_{-\theta}(w-z))}\, dw$$

for a mother wavelet $\Psi \in L^2(\mathbb{R}^2)$ and $R_{-\theta}$ a rotation matrix.



Mathematical construction

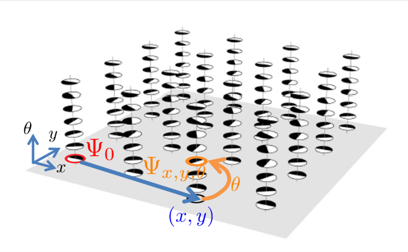
Let $(z, \theta) \in SE(2)$ a neuron in V1.

For $w \in \mathbb{R}^2$ in the **retinal plane**, the **receptive profile** $\psi_{(z,\theta)}(w)$, $w \in \mathbb{R}^2$ models the cortical activation of (z, θ) when a stimulus is applied in location w.

Given a visual stimulus u, the cortical output (**lift, orientation score**) is defined as:

$$U(z, \theta) = Lu(z, \theta) = \int_{\mathbb{R}^2} u(w)\psi_{(z,\theta)}(w)\, dw = \int_{\mathbb{R}^2} u(w)\overline{\Psi(R_{-\theta}(w - z))}\, dw$$

for a mother wavelet $\Psi \in L^2(\mathbb{R}^2)$ and $R_{-\theta}$ a rotation matrix.



Mathematical construction

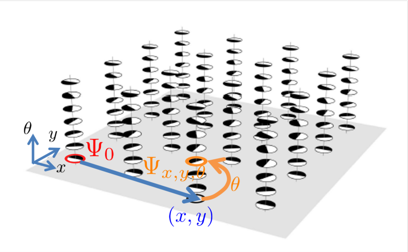
Let $(z, \theta) \in SE(2)$ a neuron in V1.

For $w \in \mathbb{R}^2$ in the **retinal plane**, the **receptive profile** $\psi_{(z,\theta)}(w)$, $w \in \mathbb{R}^2$ models the cortical activation of (z, θ) when a stimulus is applied in location w.

Given a visual stimulus u, the cortical output (**lift, orientation score**) is defined as:

$$U(z,\theta) = Lu(z,\theta) = \int_{\mathbb{R}^2} u(w)\psi_{(z,\theta)}(w)\, dw \; = \int_{\mathbb{R}^2} u(w)\overline{\Psi(R_{-\theta}(w-z))}\, dw$$

for a mother wavelet $\Psi \in L^2(\mathbb{R}^2)$ and $R_{-\theta}$ a rotation matrix.



Mathematical construction

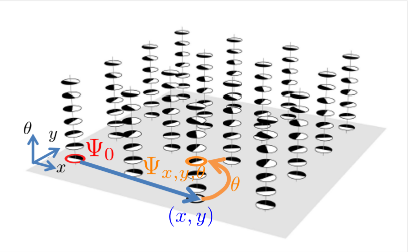
Let $(z, \theta) \in SE(2)$ a neuron in V1.

For $w \in \mathbb{R}^2$ in the **retinal plane**, the **receptive profile** $\psi_{(z,\theta)}(w)$, $w \in \mathbb{R}^2$ models the cortical activation of (z, θ) when a stimulus is applied in location w.

Given a visual stimulus u, the cortical output (**lift, orientation score**) is defined as:

$$U(z,\theta) = Lu(z,\theta) = \int_{\mathbb{R}^2} u(w)\psi_{(z,\theta)}(w)\, dw = \int_{\mathbb{R}^2} u(w)\overline{\Psi(R_{-\theta}(w-z))}\, dw$$

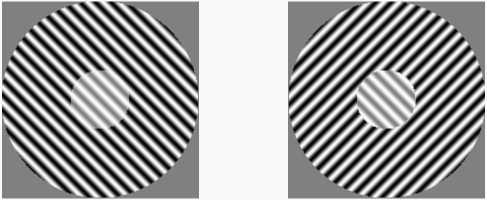
for a mother wavelet $\Psi \in L^2(\mathbb{R}^2)$ and $R_{-\theta}$ a rotation matrix.



Cortical-inspired extension of **WC** to space of positions and orientations:

$$\frac{\partial}{\partial t}U(x,\theta,t) = -\alpha U(x,\theta,t) + \mu \int_{SE(2)} \omega(x,\theta \| y,\theta') f\big(U(y,\theta',t)\big)\, dy\, d\theta' + I_{ext}(x,\theta)$$

- Allowed to recover some missing geometric hallucitations[2]
- Could not reproduce satisfactorily some common visual illusions



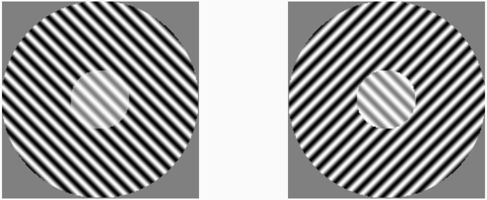
Tilt illusion

[2] Bressloff, Cowan, 2002

Cortical-inspired extension of **WC** to space of positions and orientations:

$$\frac{\partial}{\partial t}U(x,\theta,t) = -\alpha U(x,\theta,t)+\mu\int_{SE(2)}\omega(x,\theta\|y,\theta')f\big(U(y,\theta',t)\big)\,dy\,d\theta'+\mathrm{I}_{\mathrm{ext}}(x,\theta)$$

- Allowed to recover some missing geometric hallucitations[2]
- Could not reproduce satisfactorily some common visual illusions



Tilt illusion

Issue : WC does not admit a variational counterpart!

[2] Bressloff, Cowan, 2002

A variational approach to cortical activity

Variational VS. evolution approaches for efficient representation & coding

Efficient representation[3]: encoding visual information in the most efficient way

- **Ecological viewpoint**: optimisation problem [4] involving natural image statistics & biological constraints → **minimise redundancy of resources**

$$\min_{u} \ \mathcal{E}(u) \tag{3}$$

- **Neuro-physiology viewpoint**: transmission, diffusion & interaction phenomena of stimuli in the visual cortex [5] → **stationary states**

$$\begin{cases} \frac{\partial u}{\partial t} & = F(u) \\ u(0) & = u_0 \end{cases} \tag{4}$$

[3] Attneave, '54, Barlow, '61
[4] Olshausen, '00, Atick, '92
[5] Beurle, '56, Wilson, Cowan, '73

Variational VS. evolution approaches for efficient representation & coding

Efficient representation[3]: encoding visual information in the most efficient way

- **Ecological viewpoint**: optimisation problem [4] involving natural image statistics & biological constraints → **minimise redundancy of resources**

$$\min_u \ \mathcal{E}(u) \tag{3}$$

- **Neuro-physiology viewpoint**: transmission, diffusion & interaction phenomena of stimuli in the visual cortex [5] → **stationary states**

$$\begin{cases} \frac{\partial u}{\partial t} & = F(u) \\ u(0) & = u_0 \end{cases} \tag{4}$$

Via **gradient descent**, it is always true that (3)⇒(4) with $F = -\delta\mathcal{E}$.

However, there exist evolution processes not minimising any $\mathcal{E}$

→ they are **sub-optimal** in reducing redundancy!

[3] Attneave, '54, Barlow, '61
[4] Olshausen, '00, Atick, '92
[5] Beurle, '56, Wilson, Cowan, '73

A WC variant from imaging: Local Histogram Equalisation (LHE)

The following variation of WC has been proposed[6] for contrast/colour enhancement:

$$\frac{\partial u(x,t)}{\partial t} = -\alpha u(x,t) + \mu \int_{\mathbb{R}^2} \underbrace{\omega(x,y) f\left(u(x,t) - u(y,t)\right)}_{\omega(x,y)\, \cancel{f(u(y,t))}} dy + \mathrm{I}_{\text{ext}}$$

→ Non-linear behaviour on local contrast, **NOT** on local activation!

[6] Bertalmío, Caselles, Provenzi, Rizzi, '07, Pierre, Aujol, Bugeau, Steidl, Ta, '17

A WC variant from imaging: Local Histogram Equalisation (LHE)

The following variation of WC has been proposed[6] for contrast/colour enhancement:

$$\frac{\partial u(x,t)}{\partial t} = -\alpha u(x,t) + \mu \int_{\mathbb{R}^2} \underbrace{\omega(x,y) f\big(u(x,t) - u(y,t)\big)}_{\omega(x,y)\, \cancel{f(u(y,t))}} dy + \mathrm{I}_{\text{ext}}$$

→ Non-linear behaviour on local contrast, **NOT** on local activation!

Theorem (Calatroni, Franceschi, P, et al. '20)

LHE complies with a variational principle, **WC doesn't**.

[6] Bertalmío, Caselles, Provenzi, Rizzi, '07, Pierre, Aujol, Bugeau, Steidl, Ta, '17

A WC variant from imaging: Local Histogram Equalisation (LHE)

The following variation of WC has been proposed[6] for contrast/colour enhancement:

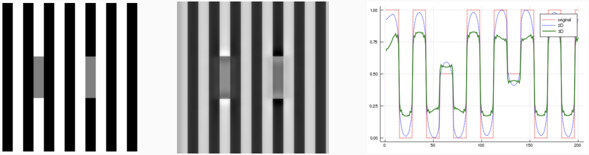
$$\frac{\partial u(x,t)}{\partial t} = -\alpha u(x,t) + \mu \int_{\mathbb{R}^2} \underbrace{\omega(x,y) f\left(u(x,t) - u(y,t)\right)}_{\omega(x,y)\,\cancel{f(u(y,t))}} \, dy + \mathrm{I}_{\mathrm{ext}}$$

→ Non-linear behaviour on local contrast, **NOT** on local activation!

Theorem (Calatroni, Franceschi, P, et al. '20)

LHE complies with a variational principle, **WC doesn't**.

APPLICATION: understanding contrast perception phenomena



RECONSTRUCTION = **PERCEPTION** in this context!

[6] Bertalmío, Caselles, Provenzi, Rizzi, '07, Pierre, Aujol, Bugeau, Steidl, Ta, '17

Variational LHE

LHE model proposed originally [7]:

$$\min_{u} \left\| u - \frac{1}{2} \right\|_2^2 + \frac{\lambda}{2} \| u - u_0 \|_2^2 - \frac{1}{4M} \int_Q \int_Q \omega(x, y) \Sigma\big(u(x) - u(y)\big) \, dx \, dy$$

- **Gray World principle**: models the 'reference' mean for $u \in [0, 1]$
- **Fidelity**: $\lambda > 0$, u_0 is the given initial image
- **Local contrast perception measure**: inspired by the neurophysiology of the Human Visual System
- Σ is an even convex primitive function of a non-linear (odd) sigmoid σ

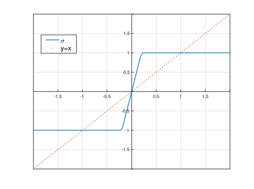
[7] Bertalmío, Caselles, Provenzi, Rizzi, '11

Variational LHE

LHE model proposed originally [7]:

$$\min_u \left\| u - \frac{1}{2} \right\|_2^2 + \frac{\lambda}{2} \|u - u_0\|_2^2 - \frac{1}{4M} \int_Q \int_Q \omega(x,y) \Sigma\big(u(x) - u(y)\big)\, dx\, dy$$

- **Gray World principle**: models the 'reference' mean for $u \in [0,1]$
- **Fidelity**: $\lambda > 0$, u_0 is the given initial image
- **Local contrast perception measure**: inspired by the neurophysiology of the Human Visual System
- Σ is an even convex primitive function of a non-linear (odd) sigmoid σ



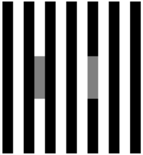
$\sigma(x) = \min\{1, \max\{\alpha x, -1\}\}$

$|u(x,\cdot) - u(y,\cdot)| < \frac{1}{\alpha}$: contrast increasing

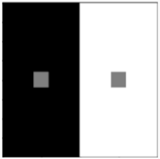
$|u(x,\cdot) - u(y,\cdot)| \geq \frac{1}{\alpha}$: saturation

[7] Bertalmío, Caselles, Provenzi, Rizzi, '11

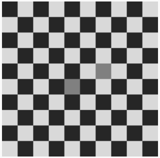
Orientation-independent tested illusions



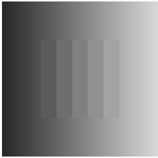
White



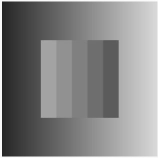
Brightness



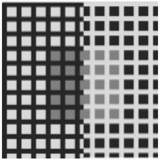
Checkerboard



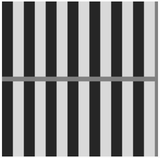
Chevreul



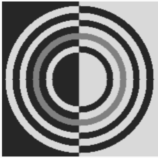
Chevreul canc.



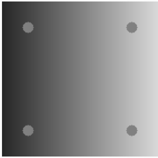
Dungeon



Grating



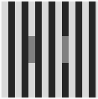
Hong-Shevell



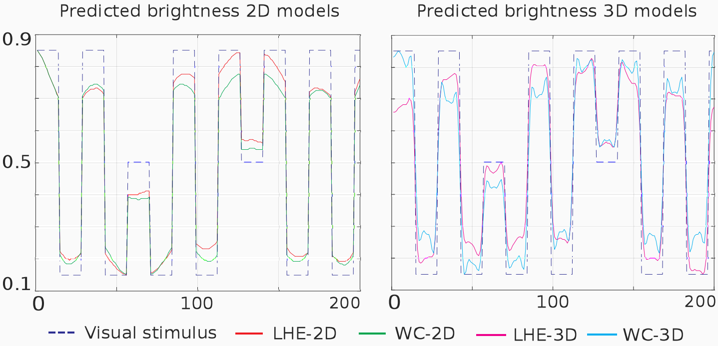
Luminance

NOTE: hard to compare model performance **quantitatively**. Look at **line profiles**!

Orientation-independent illusions: White illusion

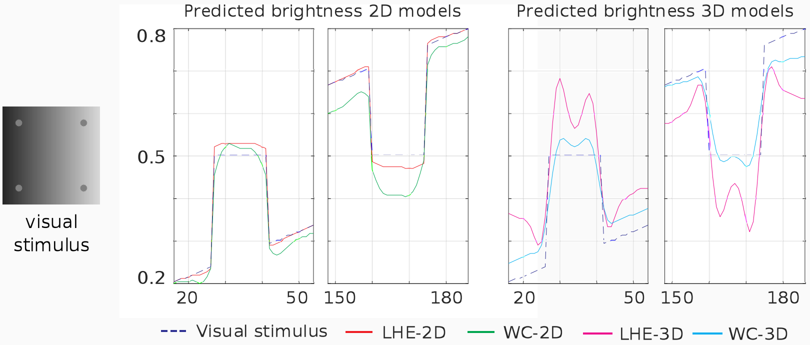


visual
stimulus



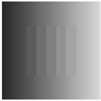
White illusion

Orientation-independent illusions: Luminance illusion

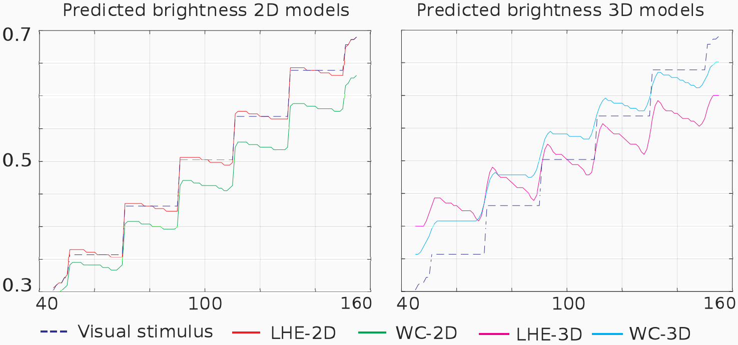


Luminance illusion

Orientation-independent illusions: Chevreul

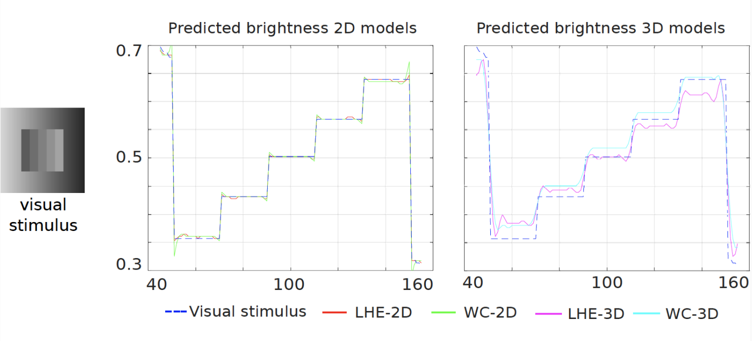


visual stimulus



Chevreul illusion

Orientation-independent illusions: Chevreul cancellation



Chevreul cancellation

Replication results

	WC-2D				LHE-2D				WC-3D				LHE-3D			
Illusion	σ_μ	σ_ω	λ	M	σ_μ	σ_ω	λ	M	σ_μ	σ_ω	λ	M	σ_μ	σ_ω	λ	M
White	10	20	.7	1.4	10	50	.7	1	20	30	.7	1.4	2	50	.7	1
Brightness	2	10	.7	1.4	2	10	.7	1	2	10	.7	1.4	2	10	.7	1
Checkerboard	10	70	.7	1.4	10	70	.7	1	10	70	.7	1.4	10	70	.7	1
Chevreul	2	5	.7	1	2	10	.7	1	2	40	.5	1	5	7	.7	1
Chevreul canc.	2	2	.9	1	5	3	.9	1	2	20	.5	1.4	5	3	.9	1
Dungeon	6	10	.7	1.4	5	40	.7	1	2	50	.7	1.4	5	50	.7	1
Gratings	2	6	.7	1	2	6	.7	1	2	6	.7	1	2	6	.7	1
Hong-Shevell	5	20	.7	1	5	.5	.7	1	10	30	.7	1	10	30	.7	1
Luminance	2	6	.7	1	2	6	.7	1	2	6	.7	1	2	6	.7	1
Poggendorff	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	3	10	.5	1
Tilt	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	15	20	.7	1

Replication results with parameters used in the tests

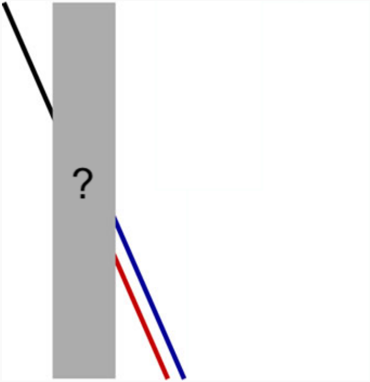
Discussion

All models can replicate simple illusions, but LHE shows **better efficient representation** than WC for orientation-dependent examples.

Modelling the connectivity

Orientation-dependent illusion: Poggendorff illusion

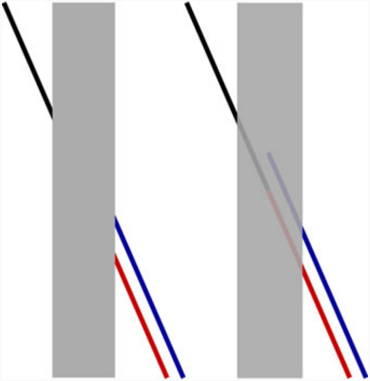
Consider now **orientation dependent** phenomena.



Poggendorff illusion

Orientation-dependent illusion: Poggendorff illusion

Consider now **orientation dependent** phenomena.

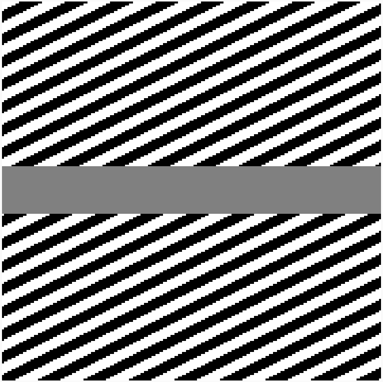


Poggendorff illusion

The central surface induces a misaligned perception of the black line.[8]

[8] Day and Dickinson, '76; Weintraub and Krantz, '71, Westheimer '08.

GOAL: reproducing the **false** perceived connection ($\neq$ inpainting!)

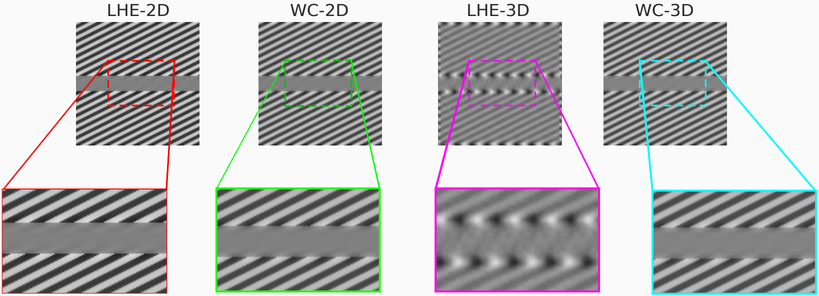


Poggendorff illusion over grating background

Where do the black bottom lines connect?

Poggendorf illusion + grating

GOAL: reproducing the **false** perceived connection ($\neq$ inpainting!)

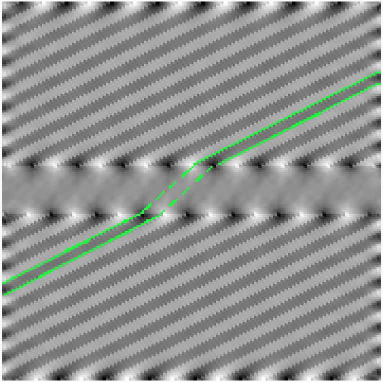


Zoom of the predicted completion for Poggendorff illusion

Not very satisfying, we need a better model

Poggendorf illusion + grating

GOAL: reproducing the **false** perceived connection ($\neq$ inpainting!)



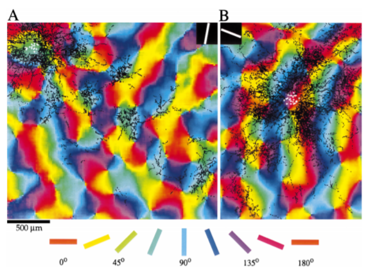
LHE-3D result

Not very satisfying, we need a better model

Connectivity between neurons

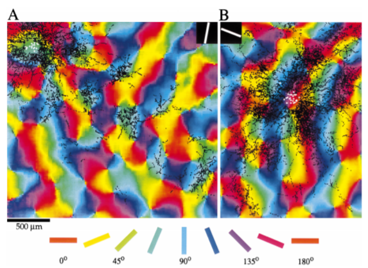
Connections in V1 are of two types[9]:

- Intra-cortical (short range) between neurons sensitive to the same orientation
- Long range connectivity between neurons sensitive to different orientations



[9]Bosking et al. '97

Connectivity between neurons II

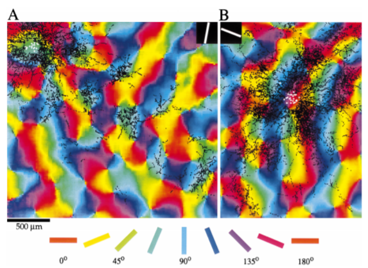


Modelled by the vector fields on $SE(2)$:

$$\Theta = \partial_\theta \qquad \text{and} \qquad X = \cos\theta\partial_x + \sin\theta\partial_y$$

Lie bracket generating family ($\text{span}\{\Theta, X, [\Theta, X]\} = \mathbb{R}^3$).

Connectivity between neurons II



Modelled by the vector fields on $SE(2)$:

$$\Theta = \partial_\theta \qquad \text{and} \qquad X = \cos\theta\partial_x + \sin\theta\partial_y$$

Lie bracket generating family ($\text{span}\{\Theta, X, [\Theta, X]\} = \mathbb{R}^3$).

Variational principle: Neural activity strive to minimize the associated energy

$$E(u) = \int_{SE(2)} |Xu|^2 + |\Theta u|^2 \, dp.$$

$\implies$ neural activities are modelled via the kernel w_τ of the sub-Riemannian heat equation

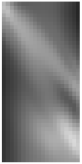
$$\partial_\tau f = (X^2 + \Theta^2)f.$$

Poggendorff illusion revisited

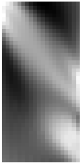
Implementing this kernel in LHE allows to (almost) recover the Poggendorff illusion.

(a) (LHE)

(b) (sR-LHE)



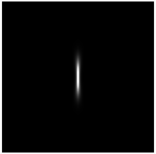
(c) (LHE), zoomed.



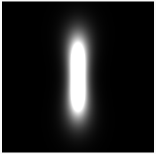
(d) (sR-LHE), zoomed

Inpainting vs perceptual completion

Changing the size of the kernel w_τ changes the strength of the interactions.



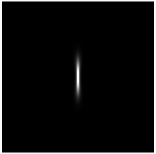
(a) $\tau = 0.5$



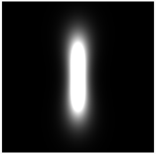
(b) $\tau = 2.5$

Inpainting vs perceptual completion

Changing the size of the kernel w_τ changes the strength of the interactions.

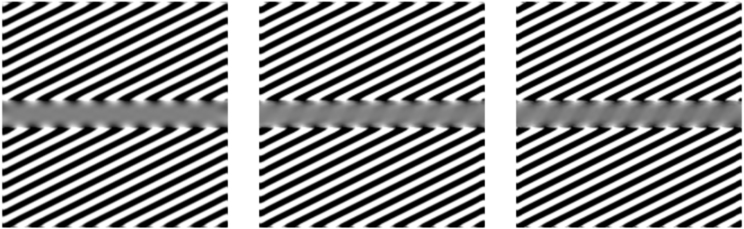


(a) $\tau = 0.5$



(b) $\tau = 2.5$

This allows to pass from an inpainting-like reconstruction to a perceptual completion :



(a) $(\tau = 0.1)$ (b) $(\tau = 0.5)$ (c) $(\tau = 2.5)$

Conclusions & Outlook

Conclusions

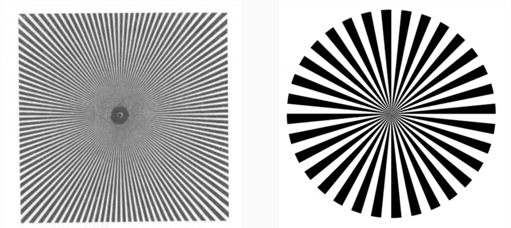
Take-home messages

- **Goal**: unify vision models with image processing (somehow new. . .)
- Cortical-inspired architecture for WC and LHE models.
- Toolbox for replicating visual perception by neural dynamics (codes in Julia, GitHub page: `https://github.com/dprn/WCvsLHE`)
- Better **efficient representation** of LHE for orientation-dependent illusions, in agreement with theory.

Outlook

Project JCJC AAP2020: RUBIN-VASE. Coordinator: Dario Prandi (L2S)

- Explain hallucinatory-like effects

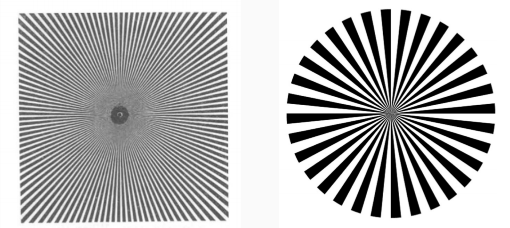


MacKay effect and flickering wheeel illusion

Outlook

Project JCJC AAP2020: RUBIN-VASE. Coordinator: Dario Prandi (L2S)

- Explain hallucinatory-like effects

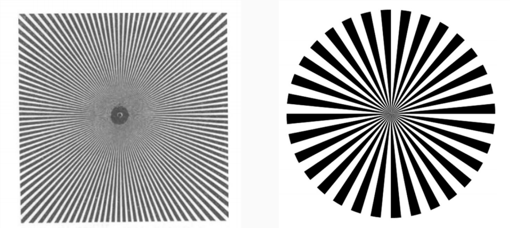


MacKay effect and flickering wheeel illusion

- Predictive control of hallucinatory states

Project JCJC AAP2020: RUBIN-VASE. Coordinator: Dario Prandi (L2S)

- Explain hallucinatory-like effects



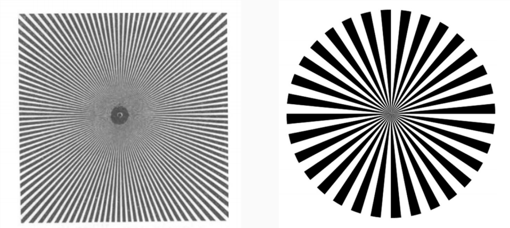
MacKay effect and flickering wheeel illusion

- Predictive control of hallucinatory states
- LHE as the basis for a new framework for Intrinsically Non-Linear Receptive fields

Outlook

Project JCJC AAP2020: RUBIN-VASE. Coordinator: Dario Prandi (L2S)

- Explain hallucinatory-like effects



MacKay effect and flickering wheeel illusion

- Predictive control of hallucinatory states
- LHE as the basis for a new framework for Intrinsically Non-Linear Receptive fields
- Psycho-physical experiments for **validation!**

References I

E. Baspinar, L. Calatroni, V. Franceschi, D. Prandi, *A cortical-inspired sub-Riemannian model for Poggendorff-type visual illusions*, Journal of Imaging (2021).

M. Bertalmío, L. Calatroni, V. Franceschi, B. Franceschiello, D. Prandi, *A cortical-inspired model for orientation-dependent contrast perception: a link with Wilson-Cowan equations*, Scale Space and Variational Methods in Computer Vision LNCS conference proceedings 2019, Springer (2019).

M. Bertalmío, L. Calatroni, V. Franceschi, B. Franceschiello, A. Gomez-Villa, D. Prandi, *Visual illusions via neural dynamics: Wilson-Cowan type models and the efficient representation principle*, Journal of Neurophysiology, 123 (5), (2020).

M. Bertalmío, L. Calatroni, V. Franceschi, B. Franceschiello, D. Prandi, *Cortical-inspired Wilson-Cowan-type equations for orientation-dependent contrast perception modelling*, Journal of Mathematical Imaging and Vision, (2020).

References II

Variational models for histogram equalisation

M. Bertalmío, V. Caselles, E. Provenzi, A. Rizzi, *Perceptual Color Correction Through Variational Techniques*, IEEE Transactions on Image Processing, 16 (4), 2007.

M. Bertalmío, *From image processing to computational neuroscience: a neural model based on histogram equalization*, Frontiers in Computational Neuroscience, 8 (71), 2014.

Cortical models for imaging

J. Petitot, *Elements of Neurogeometry: Functional Architectures of Vision*, Springer, 2017.

G. Citti, A. Sarti, *A cortical based model of perceptual completion in the roto-translation space*, JMIV, 24(3), 2006.

R. Duits, E. Franken, *Left-invariant parabolic evolutions on* $SE(2)$ *and contour enhancement via invertible orientation scores. Part I: linear left-invariant diffusion equations on* $SE(2)$, Quart. Appl. Math., 68 (2), 2010.

D. Prandi, J.P. Gauthier, *A semidiscrete version of the Citti-Petitot-Sarti model as a plausible model for anthropomorphic image reconstruction & pattern recognition*, SpringerBriefs in Mathematics, 2018.

Thank you for your attention!

Questions?

dario.prandi@centralesupelec.fr `https://dprn.github.io`