On the Convergence of Sequential Convex Programming for Non-Linear Optimal Control

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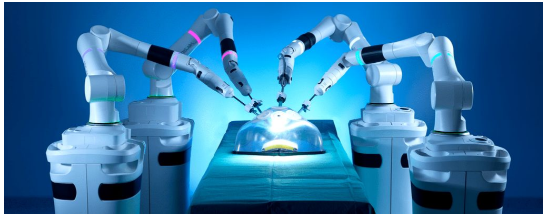
Complex autonomous systems are everywhere



Goal

Select the best available strategy and make robust decisions under uncertainty

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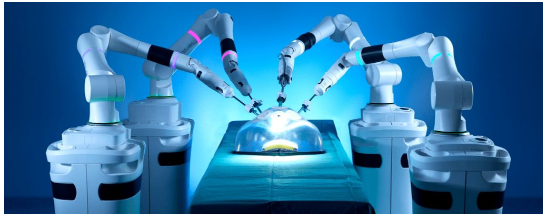

Stochastic Optimal Control

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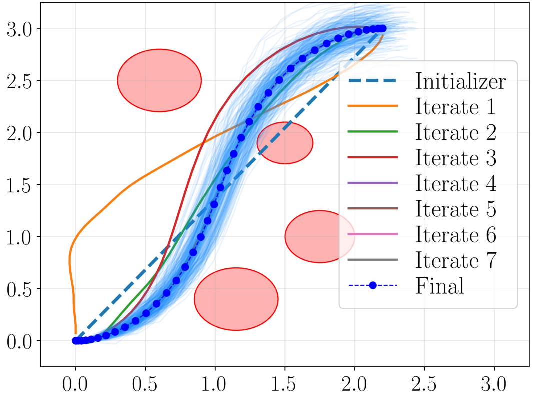


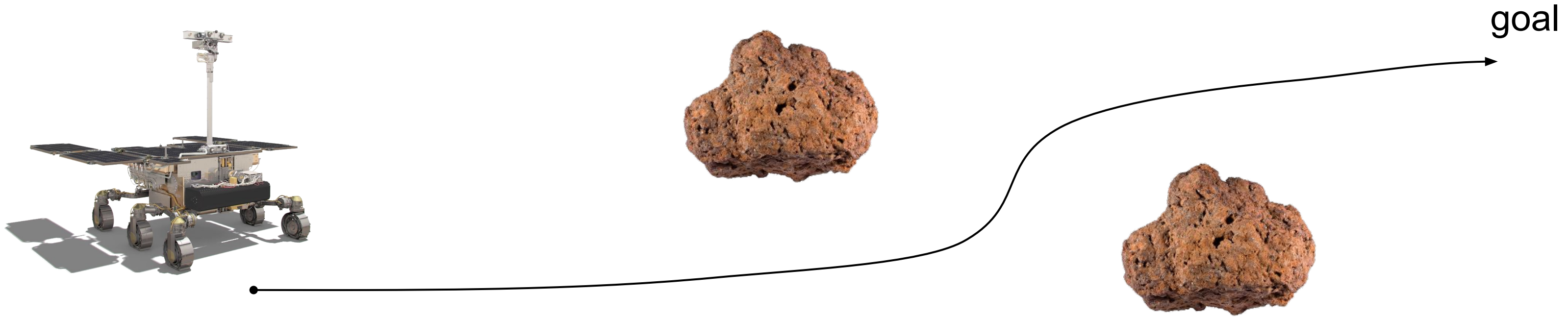
Optimal control of finite-dimensional dynamical systems

$$\min_{u \in \mathcal{U}} \quad \int_0^{t_f} f^0(s, u(s), x(s))\,\mathrm{d}s$$ ← Cost

$$\mathrm{d}x(s) = f(s, u(s), x(s))\,\mathrm{d}s$$ ← Dynamics

$$x(0) = x^0, \qquad g(x(t_f)) = 0$$ ← Initial / Final Conditions

$$h(x(s)) \leq 0, \quad s \in [0, t_f]$$ ← State Constraints



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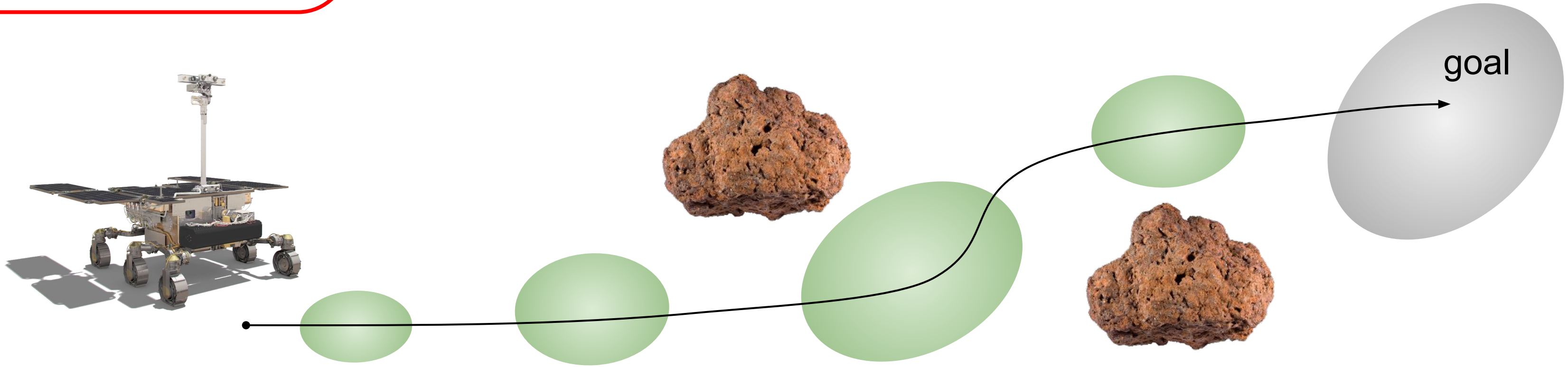

Several approaches have already been proposed

Original LQR papers

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- **J.M. Bismut, *Linear quadratic optimal stochastic control with random coefficients*, SIAM J. Control, 14(3): 419-444, 1976.**
- S. Chen, X. Li, and X. Zhou, *Stochastic linear quadratic regulators with indefinite control weight costs*, SIAM J. Control, 36(5): 1685-1702, 1998.
- T.E. Duncan, B. Pasik-Duncan, *Stochastic linear-quadratic control with state dependent fractional brownian noise and stochastic coefficients*, IFAC-PapersOnLine, 50(2): 199-202, 2017.

Nonlinear systems:

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Remark

Leverage existing works on linear systems: **Sequential Convex Programming (SCP)**

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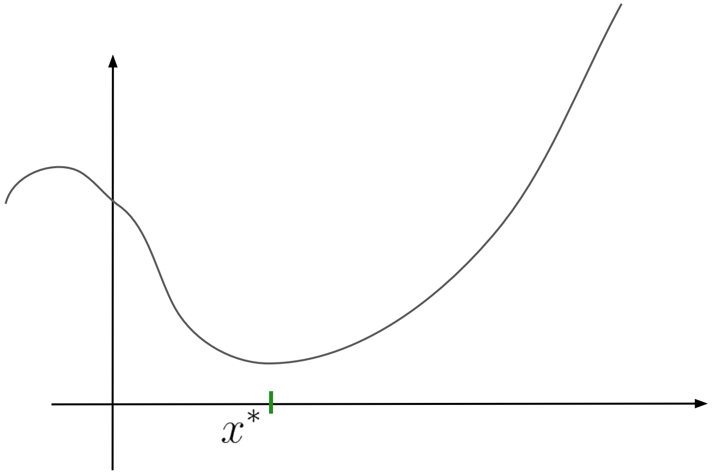
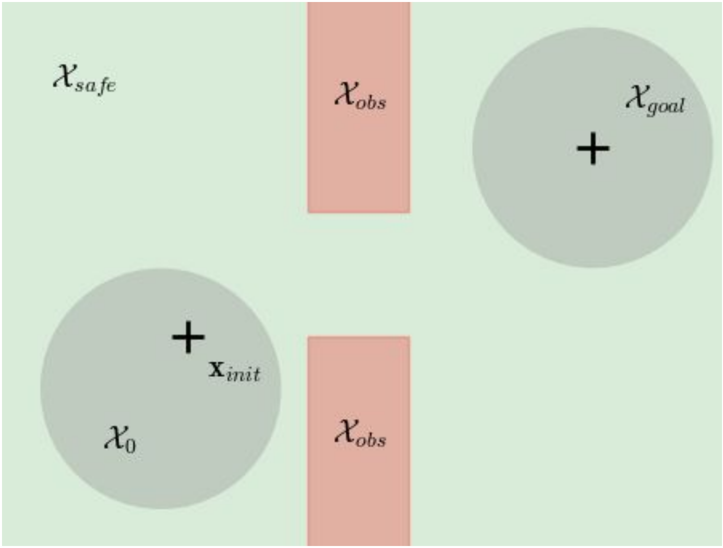
Intuitive introduction to SCP





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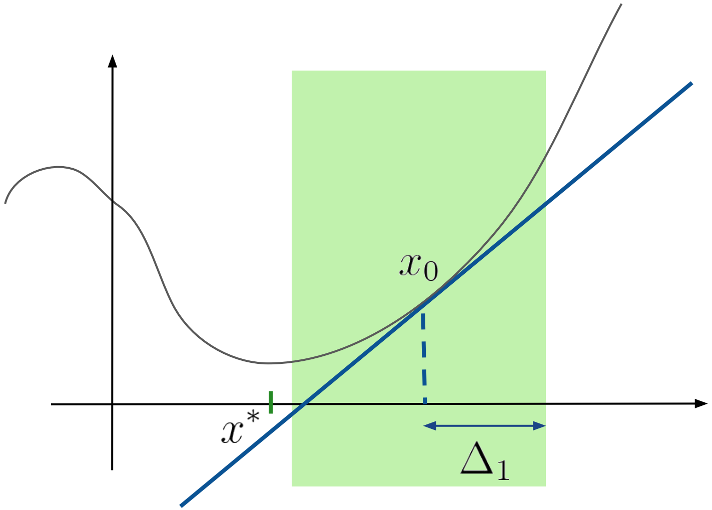

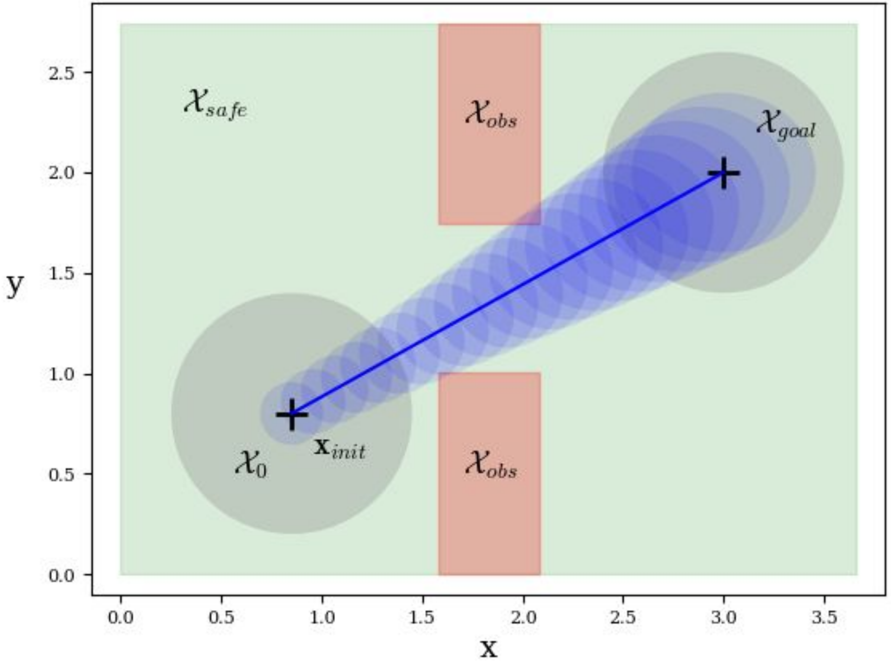

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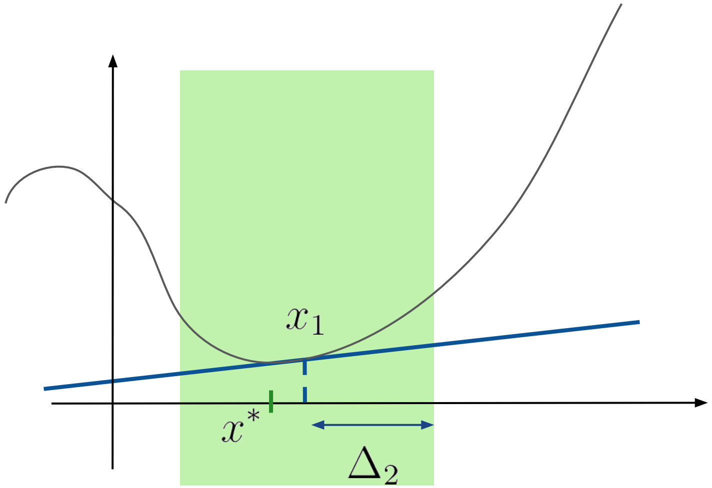
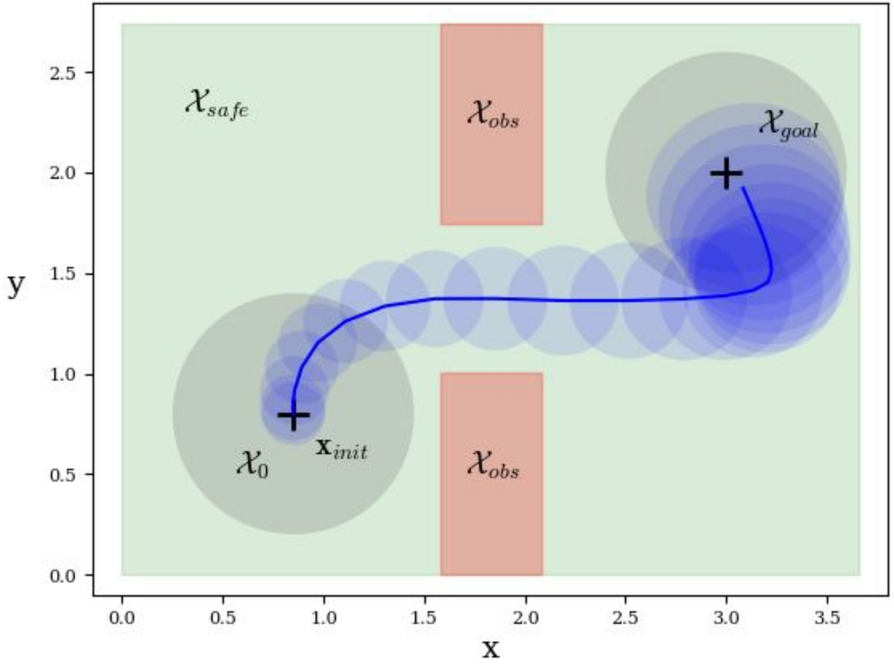
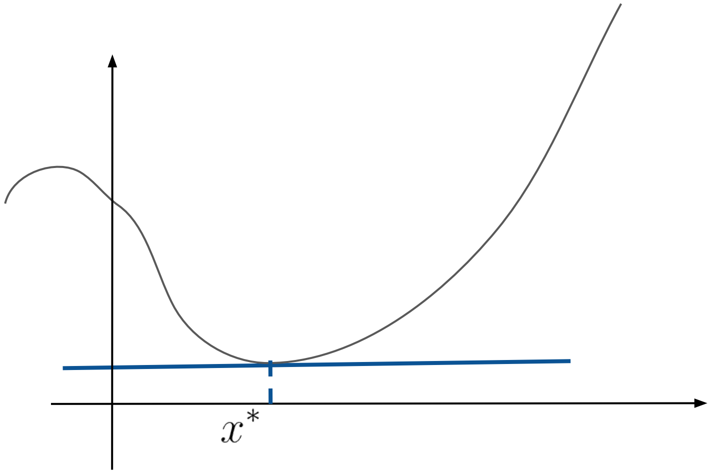


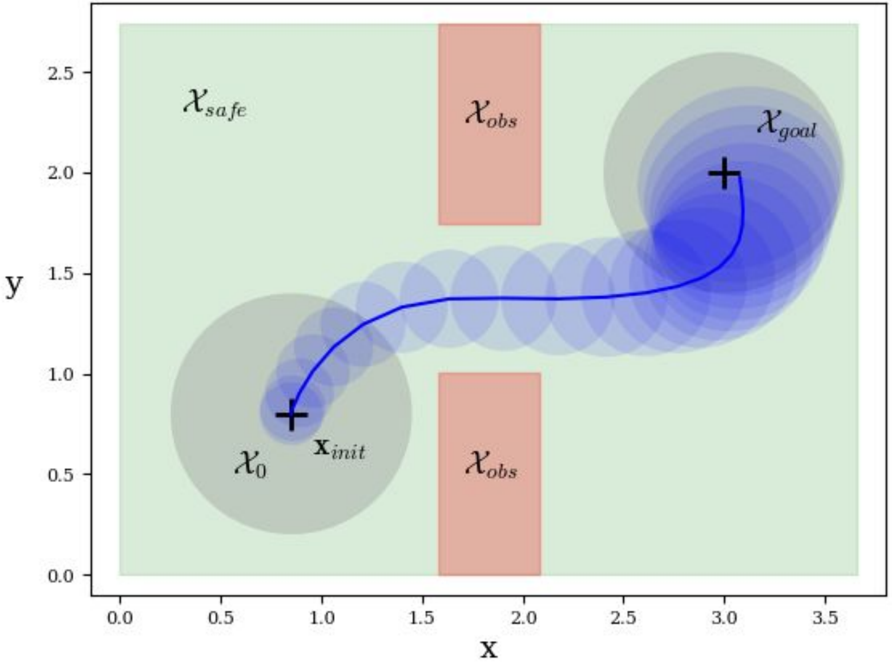


Intuitive introduction to SCP

A few iterations later...



Convergence!



Stochastic SCP formulation

$$
\begin{aligned}
&\min_{u \in \mathcal{U}} \quad \mathbb{E}\left[\int_0^{t_f} u(s)^2 + h(x(s))\,\mathrm{d}s\right] \\
(\mathbf{OCP}) \quad & \mathrm{d}x(s) = b(x(s), u(s))\,\mathrm{d}s + \sigma(x(s))\,\mathrm{d}B_s \\
&\qquad \triangleq \big(f_0(x(s)) + u(s) f_1(x(s))\big)\,\mathrm{d}s + \sigma(x(s))\,\mathrm{d}B_s \\
& x(0) = x^0, \quad \mathbb{E}\left[g(x(t_f))\right] = 0
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& \min_{u \in \mathcal{U}} \quad \mathbb{E}\left[\int_0^{t_f} u(s)^2 + h(x_k(s)) + \frac{\partial h}{\partial x}(x_k(s))(x(s) - x_k(s))\,\mathrm{d}s\right] \\
& \mathrm{d}x(s) = \left(b(x_k(s), u(s)) + \frac{\partial b}{\partial x}(x_k(s), u_k(s))(x(s) - x_k(s))\right)\,\mathrm{d}s \\
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Linearization makes sense only locally. Add trust-region constraints:

$$
\mathbb{E}\left[\int_0^{t_f} \|x(s) - x_k(s)\|^2\, \mathrm{d}s\right] \leq \Delta_{k+1}
$$

$$
\Delta_{k+1} \in \mathbb{R}_+, \quad \Delta_{k+1} \longrightarrow 0
$$

Are we really solving the original problem?

This begs the question: “Are we doing something meaningful? I.e., when convergence is achieved, what is the quantity we come up with?”

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Our answer
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The proof leverages the **continuity** properties of stochastic Itô variational inequalities with respect to **convexification**

The stochastic Pontryagin Maximum Principle

Let $\mathcal{U} = L^2([0,t_f];U)$ or $\mathcal{U} = L^2([0,t_f]\times\Omega;U)$, where $U \subseteq \mathbb{R}$. For $(\mathbf{OCP})$, we define the Hamiltonian

$$H(x,p,p^0,q,u) = p^\top\big(f_0(x) + uf_1(x)\big) + p^0\big(u^2 + h(x)\big) + q^\top\sigma(x).$$

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If $(u(\cdot),x(\cdot))$ is optimal, then there exist $p^0 \leq 0$, $p:[0,t_f]\times\Omega\to\mathbb{R}^n$ and $q \in L^2([0,t_f]\times\Omega;\mathbb{R}^n)$ such that:

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- **Adjoint Equations:**

$$\mathrm{d}x(s) = \frac{\partial H}{\partial p}(x(s),p(s),p^0,q(s),u(s))\,\mathrm{d}s + \sigma(x(s))\,\mathrm{d}B_s,$$

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- **Maximality Condition:**

Deterministic controls: $\mathbb{E}\left[H(x(s), p(s), p^0, q(s), u(s))\right] = \max_{u \in U} \mathbb{E}\left[H(x(s), p(s), p^0, q(s), u)\right]$, a.e.

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- **Transversality Condition:** $p(t_f) \perp \ker \mathbb{E}\left[\frac{\partial g}{\partial x}(x(t_f))\right]$.

Theoretical guarantees for stochastic SCP

Assume that SCP provides a sequence $(\Delta_k, u_k, x_k)_{k\in\mathbb{N}}$ such that:

- $(u_k(\cdot), x_k(\cdot))$ locally solves $(\mathbf{COCP})_k$;
- $\mathbb{E}\left[\int_0^{t_f} \|x_{k+1}(s) - x_k(s)\|^2 \, \mathrm{d}s\right] < \Delta_{k+1}$ where $\Delta_k \to 0$;
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We may adopt weak convergences for deterministic controls

If $x : [0, t_f] \times \Omega \to \mathbb{R}^n$ denotes the solution to the dynamics of $(\mathbf{OCP})$ with control u, then:

1. There exists $(p(\cdot), p^0, q(\cdot))$ such that $(x(\cdot), p(\cdot), p^0, q(\cdot), u(\cdot))$ is a Pontryagin extremal for $(\mathbf{OCP})$;

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Main result of convergence

Theoretical guarantees for stochastic SCP

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1. There exists $(p(\cdot), p^0, q(\cdot))$ such that $(x(\cdot), p(\cdot), p^0, q(\cdot), u(\cdot))$ is a Pontryagin extremal for $(\mathbf{OCP})$;
2. Whenever $k \to \infty$, the following convergences hold:
   (a) $\mathbb{E}\left[\sup_{s\in[0,t_f]} \|x_k(s) - x(s)\|^2\right] \longrightarrow 0$
   (b) $p_k^0 \to p^0$, $\mathbb{E}\left[\sup_{s\in[0,t_f]} \|p_k(s) - p(s)\|^2\right] \to 0$, $\mathbb{E}\left[\int_0^{t_f} \|q_k(s) - q(s)\|^2 \, \mathrm{d}s\right] \to 0$, up to a subsequence

Main result of convergence

where $(x_k(\cdot), p_k(\cdot), p_k^0, q_k(\cdot), u_k(\cdot))$ is a Pontryagin extremal for $(\mathbf{COCP})_k$.

Note: this may be leveraged to accelerate SCP!

Convergence of SCP: sketch of proof

Define the augmented dynamics to be

$$\tilde{b}(x,u) = \big(f_0(x) + u f_1(x), u^2 + h(x)\big) \in \mathbb{R}^{n+1}, \quad \tilde{\sigma}(x) = (\sigma(x), 0) \in \mathbb{R}^{n+1}.$$

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To prove the PMP, we can leverage classical **needle-like variations** on the end-point mapping

$$E : \mathcal{U} \longrightarrow \mathbb{R}^{n+1} : u(\cdot) \mapsto \Big(\mathbb{E}\left[g(x_u(t_f))\right], \mathbb{E}\left[x_u^{n+1}(t_f)\right]\Big), \quad \begin{cases} \mathrm{d}\tilde{x}_u(s) = \tilde{b}(x_u(s), u(s))\,\mathrm{d}s + \tilde{\sigma}(x_u(s))\,\mathrm{d}B_s \\ x_u(0) = x^0, \quad x_u^{n+1}(0) = 0. \end{cases}$$

Convergence of SCP: sketch of proof

Define the augmented dynamics to be

$$\tilde{b}(x,u) = \big(f_0(x) + u f_1(x), u^2 + h(x)\big) \in \mathbb{R}^{n+1}, \quad \tilde{\sigma}(x) = (\sigma(x), 0) \in \mathbb{R}^{n+1}.$$

To prove the PMP, we can leverage classical **needle-like variations** on the end-point mapping

$$E : \mathcal{U} \longrightarrow \mathbb{R}^{n+1} : u(\cdot) \mapsto \Big(\mathbb{E}\left[g(x_u(t_f))\right], \mathbb{E}\left[x_u^{n+1}(t_f)\right]\Big), \quad \begin{cases} \mathrm{d}\tilde{x}_u(s) = \tilde{b}(x_u(s), u(s))\, \mathrm{d}s + \tilde{\sigma}(x_u(s))\, \mathrm{d}B_s \\ x_u(0) = x^0, \quad x_u^{n+1}(0) = 0. \end{cases}$$

If $(u(\cdot), x(\cdot))$ is optimal, a contradiction argument entails the existence of $(\mathfrak{p}, p^0) \neq 0$ such that

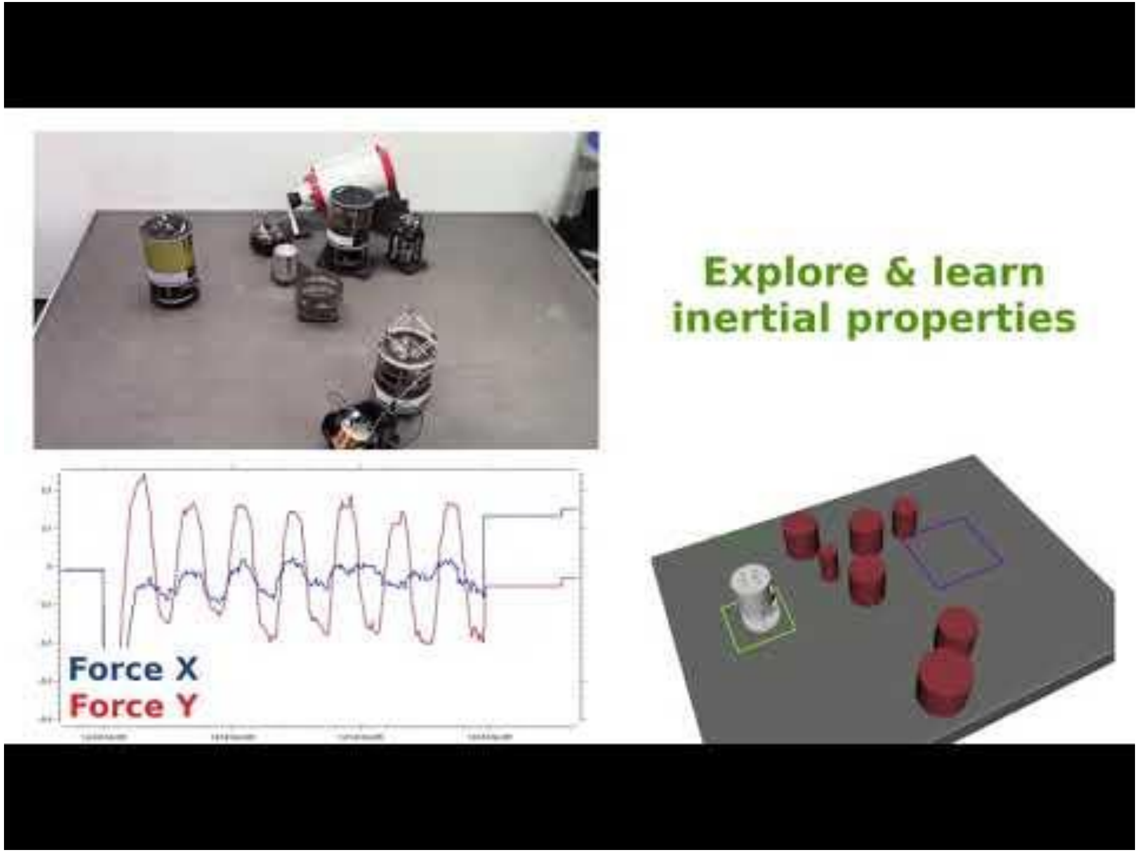
$$\mathfrak{p}^\top \mathbb{E}\left[\frac{\partial g}{\partial x}(x(t_f)) z_{r,v}(t_f)\right] + p^0 \mathbb{E}\left[z_{r,v}^{n+1}(t_f)\right] \le 0, \quad r \text{ Lebesgue point for } u(\cdot), \quad v \in L^2_{\mathcal{F}_r}(\Omega; U)$$

Lebesgue points for stochastic controls are correctly introduced via Bochner integration

where $\tilde{z}_{r,v} = (z_{r,v}, z_{r,v}^{n+1}) : [0, t_f] \times \Omega \to \mathbb{R}^{n+1}$ solves

$$\begin{cases} \mathrm{d}\tilde{z}(s) = A(s)\tilde{z}(s)\, \mathrm{d}s + D(s)\tilde{z}(s)\, \mathrm{d}B_s \\ \tilde{z}(s) = 0, \ s \in [0, r), \quad \tilde{z}(r) = \tilde{b}(x(r), v) - \tilde{b}(x(r), u(r)), \end{cases} \qquad \begin{cases} A(s) = \frac{\partial \tilde{b}}{\partial \tilde{x}}(x(s), u(s)) \\ D(s) = \frac{\partial \tilde{\sigma}}{\partial \tilde{x}}(x(s), u(s)). \end{cases}$$

Hardware exp. - Collaboration with Stanford University



Future direction

Solve non-linear coupled ODEs/PDEs

FEM+ROM tend to lose effectiveness

$$
\begin{cases}
\min\limits_{u \in \mathcal{U}} \quad \displaystyle\int_0^{t_f} \|u(t)\|^2 \, \mathrm{d}t \\
\mathrm{d}x = f(x, y, u) \, \mathrm{d}t + \sigma(x, y, u) \, \mathrm{d}B_t \\
\dfrac{\partial y}{\partial t} - \mu \Delta y + \nabla p = -y \cdot \nabla y
\end{cases}
$$

Future direction

Solve non-linear coupled ODEs/PDEs

FEM+ROM tend to lose effectiveness

Could we extend the performance of SCP?

$$\begin{cases} \min\limits_{u \in \mathcal{U}} \quad \int_0^{t_f} \|u(t)\|^2 \, \mathrm{d}t \\ \mathrm{d}x = f(x,y,u)\,\mathrm{d}t + \sigma(x,y,u)\,\mathrm{d}B_t \\ \dfrac{\partial y}{\partial t} - \mu \Delta y + \nabla p = -y \cdot \nabla y \end{cases} \Longrightarrow \begin{cases} \min\limits_{u_{k+1} \in \mathcal{U}} \quad \int_0^{t_f} \|u_{k+1}(t)\|^2 \, \mathrm{d}t \\ \qquad \qquad \dots \\ \dfrac{\partial y^{k+1}}{\partial t} - \mu \Delta y^{k+1} + \nabla p^{k+1} = \\ \quad = -y^k \cdot \nabla y^k - (y^{k+1} - y^k) \cdot \nabla y^k \quad + \quad \text{“trust region” constraints} \\ \quad -y^k \cdot \nabla (y^{k+1} - y^k) \end{cases}$$

Future direction

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Could we extend the performance of SCP?

Thank you for your attention!
Any question?

$$\begin{cases} \min\limits_{u \in \mathcal{U}} \int_0^{t_f} \|u(t)\| \\ \mathrm{d}x = f(x,y,u)\,\mathrm{d}t + \sigma(x,y \\ \dfrac{\partial y}{\partial t} - \mu\Delta y + \nabla p = -y \cdot \nabla y \end{cases}$$

$$= -y^k \cdot \nabla y^k - (y^{k+1} - y^k) \cdot \nabla y^k \quad + \quad \text{"trust region" constraints}$$
$$-y^k \cdot \nabla (y^{k+1} - y^k)$$

References

[1] R. Bonalli, T. Lew, and M. Pavone, *Sequential Convex Programming for Non-Linear Stochastic Optimal Control*. Submitted.
[2] R. Bonalli and Y. Chitour, *Sequential Convex Programming for Infinite Dimensional Non-Linear Optimal Control*. In progress.