Sensorless control of electric motors

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December 1st, 2021

Outline

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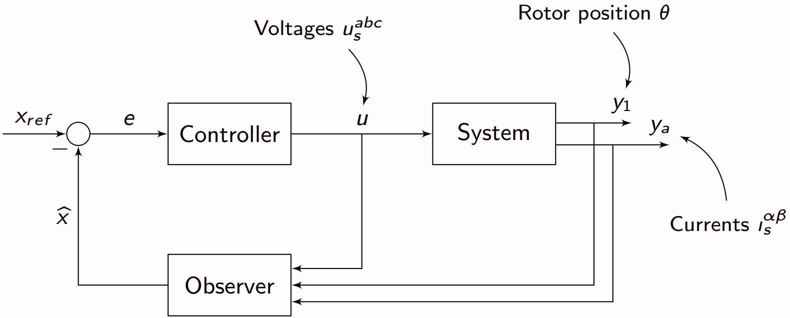
Introduction – Sensorless control of electric motors

Main target: sensorless control of electric motors



– System is <u>observable</u> with both currents and rotor position measurements

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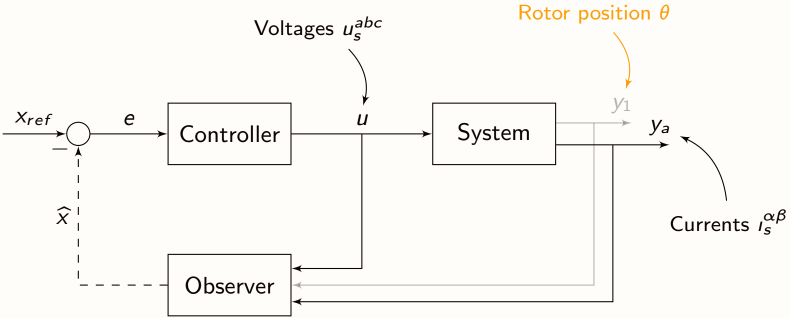

- System is observable with both currents and rotor position measurements
- Without mechanical sensor: degeneracy of observability at low speed

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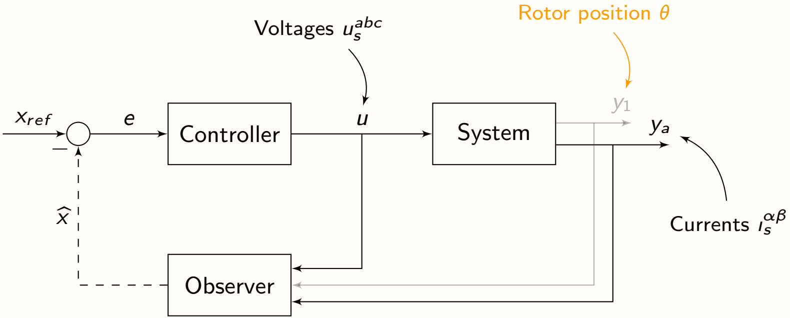
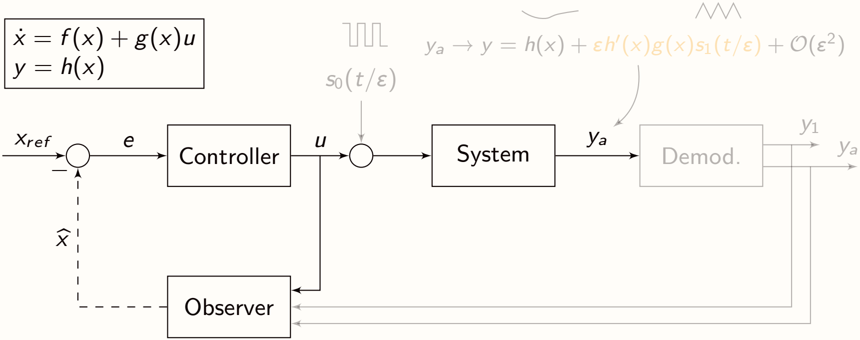


- System is <u>observable</u> with both currents and rotor position measurements
- <u>Without mechanical sensor</u>: degeneracy of observability at low speed
- To bypass this issue: **signal injection technique**

Introduction — Signal injection technique and position estimation



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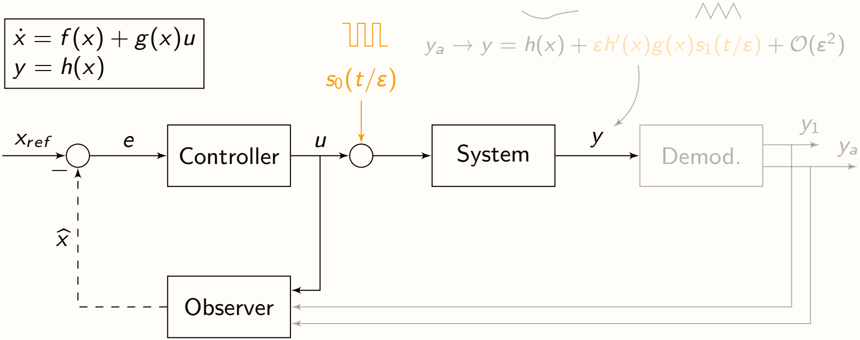

- Injection of a high-frenquency signal $s_0(t/\varepsilon)$
- Perturbation of the measurement y

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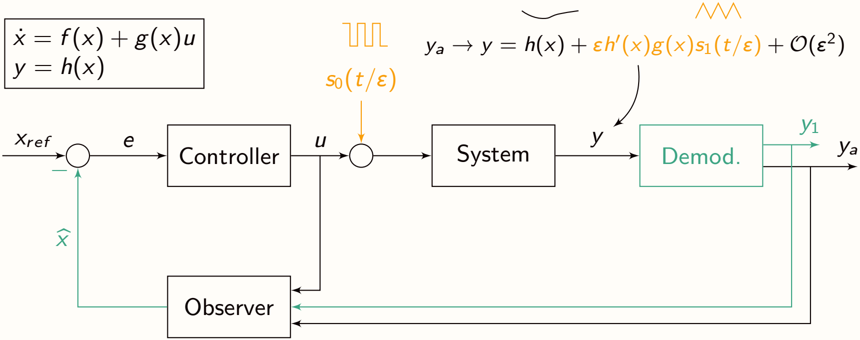

- Injection of a high-frenquency signal $s_0(t/\varepsilon)$
- Perturbation of the measurement y
- Demodulation procedure for extracting y_a and y_1
- System is now observable with those "virtual measurements"

Outline

Periodic averaging = Near-identity transformation + Comparison result

System with injection

$$\boxed{\begin{aligned}\dot{x} &= \varepsilon f^1(x,t) + \ldots + \varepsilon^k f^k(x,t) \\ &\quad + \varepsilon^{k+1} f^{[k+1]}(x,t,\varepsilon)\end{aligned}}$$

Periodic averaging = Near-identity transformation + Comparison result

Near-identity transformation

$$\check{x} = x + \varepsilon\varphi^1(x,t) + \ldots + \varepsilon^k\varphi^k(x,t)$$

with $g^1(x) = \int_0^1 f(x,\sigma)\,d\sigma$

System with injection

$$\dot{x} = \varepsilon f^1(x,t) + \ldots + \varepsilon^k f^k(x,t) + \varepsilon^{k+1} f^{[k+1]}(x,t,\varepsilon)$$

$\longrightarrow$

(Full) Averaged state

$$\dot{\check{x}} = \varepsilon g^1(\check{x}) + \ldots + \varepsilon^k g^k(\check{x}) + \varepsilon^{k+1} g^{[k+1]}(\check{x},t,\varepsilon)$$

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Dropping the last term $g^{[k+1]}$ (instationary)

Truncated averaged state

$$\boxed{\dot{\overline{x}} = \varepsilon g^1(\overline{x}) + \ldots + \varepsilon^k g^k(\overline{x})}$$

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Dropping the last term $g^{[k+1]}$ (instationary)

$h^k(x,t)$ periodic with zero mean

$$\dot{\tilde{x}} = \varepsilon f^1(\tilde{x},t) + \ldots + \varepsilon^k\left(f^k(\tilde{x},t) + h^k(\tilde{x},t)\right) + \varepsilon^{k+1}\tilde{f}^{[k+1]}(\tilde{x},t,\varepsilon)$$

Truncated averaged state

$$\dot{\bar{x}} = \varepsilon g^1(\bar{x}) + \ldots + \varepsilon^k g^k(\bar{x})$$

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Comparison result: $x(t) = \widetilde{x}(t) + \mathcal{O}(\varepsilon^k)$

on a timescale $1/\varepsilon$

$h^k(x,t)$ periodic with zero mean

Dropping the last term $g^{[k+1]}$ (instationary)

$$\dot{\widetilde{x}} = \varepsilon f^1(\widetilde{x},t) + \ldots + \varepsilon^k\left(f^k(\widetilde{x},t) + h^k(\widetilde{x},t)\right) + \varepsilon^{k+1}\tilde{f}^{[k+1]}(\widetilde{x},t,\varepsilon)$$

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$$\tilde{x} = \overline{x} + \varepsilon\varphi^1(\overline{x},t) + \ldots + \varepsilon^{k-1}\varphi^{k-1}(\overline{x},t)$$

$$h(x) = h(\overline{x}) + \varepsilon dh(\overline{x})\cdot\varphi^1(\overline{x},t) + \tfrac{1}{2}\varepsilon^2 d^2h(\overline{x})\cdot\varphi^1(\overline{x},t)\cdot\varphi^1(\overline{x},t) + \varepsilon^2 dh(\overline{x})\cdot\varphi^2(\overline{x},t) + \mathcal{O}(\varepsilon^3)$$

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Third-order averaging theory – Exogenous signal injection – Linear dynamics

SISO system, linear dynamics, nonlinear measurement

$$\dot{x} = Ax + Bu, \quad y = h(x) \qquad \textbf{Exogenous injection:}\ u \to u + s_0(\tfrac{t}{\varepsilon})$$

Set of **virtual measurements**

$$H(x) = \begin{pmatrix} h(x) & \boxed{\varepsilon h'(x)B} & \boxed{\frac{\varepsilon^2}{2} h''(x)(B,B)} & \boxed{\varepsilon^2 h'(x)AB} \end{pmatrix}.$$

We assume that, with these additional measurements, the original system is <u>observable</u>

Non-perturbed system

$$\dot{\overline{x}} = A\overline{x} + B\alpha(\overline{\eta}, H(\overline{x}), t)$$
$$\dot{\overline{\eta}} = a(\overline{\eta}, H(\overline{x}), t)$$

System with signal injection

$$\dot{x} = Ax + B\alpha(\eta, \overline{H}(x), t) + Bs_0(\tfrac{t}{\varepsilon})$$
$$\dot{\eta} = a(\eta, \overline{H}(x), t)$$
$$\overline{H}(x) = H(x - \varepsilon Bs_1(\tfrac{t}{\varepsilon}) - \varepsilon^2 ABs_2(\tfrac{t}{\varepsilon}))$$

- s_0 : 1-periodic signal with zero mean.
- s_{i+1} is the primitive of s_i with zero mean.

Third-order averaging theorem – Exogenous signal injection

Theorem (Third-order averaging for exogenous signal injection, SCMR, 2019)

Let $x(t)$ (resp. $\overline{x}(t)$) be the solution of the perturbed (resp non-perturbed) system, with $x(0) = \overline{x}(0) + \varepsilon B s_1(0) + \varepsilon^2 AB s_2(0)$. Assume the original system is locally exponentially stable. Then for $t \geq 0$,

$$x(t) = \overline{x}(t) + \varepsilon B s_1(\tfrac{t}{\varepsilon}) + \varepsilon^2 AB s_2(\tfrac{t}{\varepsilon}) + \mathcal{O}(\varepsilon^3)$$

$$\eta(t) = \overline{\eta}(t) + \mathcal{O}(\varepsilon^3)$$

$$y(t) = \underbrace{h(\overline{x}(t))}_{Y_0(t)} + \underbrace{\varepsilon h'(\overline{x}(t))B}_{Y_1(t)}\, s_1(\tfrac{t}{\varepsilon})$$

$$+ \underbrace{\tfrac{\varepsilon^2}{2} h''(\overline{x}(t))(B, B)}_{Y_3(t)}\, s_1(\tfrac{t}{\varepsilon})^2 + \underbrace{\varepsilon^2 h'(\overline{x}(t))AB}_{Y_2(t)}\, s_2(\tfrac{t}{\varepsilon}) + \mathcal{O}(\varepsilon^3)$$

- <u>Locally exponentially stability hypothesis</u>: for the extension to infinity of the result (otherwise only valid on a timescale $1/\varepsilon$)
- <u>Idea of the proof</u>: change of coordinates (the so-called *near-identity transformation*), then use of a comparison theorem

Third-order averaging theory for exogenous signal injection – Numerical example

Dynamic system

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3$$
$$\dot{x}_3 = u + d$$
$$y = x_1 x_2 + \frac{x_3^3}{3}$$

- u: input
- d: (unknown) disturbance
- **Objective**: controlling x_1 while rejecting d

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We (momentarily) assume we can extract Y_i with an accuracy in ε^3!

Virtual measurements

$$Y_1 := h'(x)B = x_3^2$$
$$Y_2 := \frac{1}{2}h''(x)(B, B) = x_3$$
$$Y_3 := h'(x)AB = x_1$$

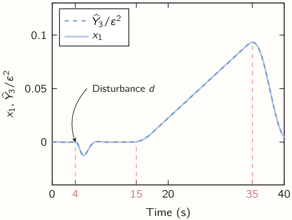
- Second-order averaging theory is not enough here
- With Y_3, the system is controllable with a linear controller

→ **Third-order averaging theory is paramount**

Third-order averaging theory for exogenous signal injection – Scenario

Scenario

- System at rest at $t = 0$; step $d = -0.25$ at $t = 4$
- Filtered ramp (slope 5×10^{-2}) for $15 \leq t \leq 35$
- At $t = 35$: filtered step to go back to 0
- **Injection**: square wave with $\varepsilon = 10^{-3}$



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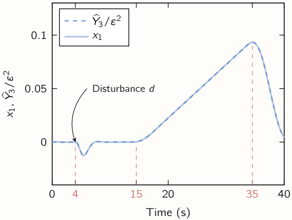
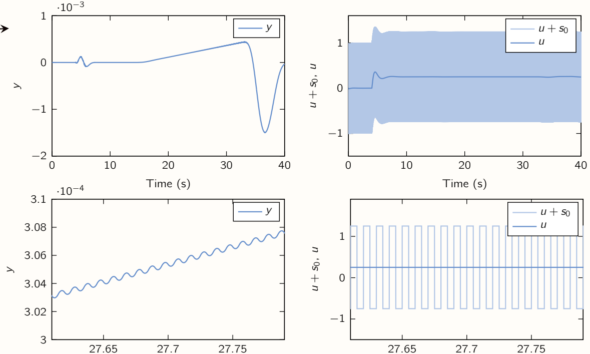
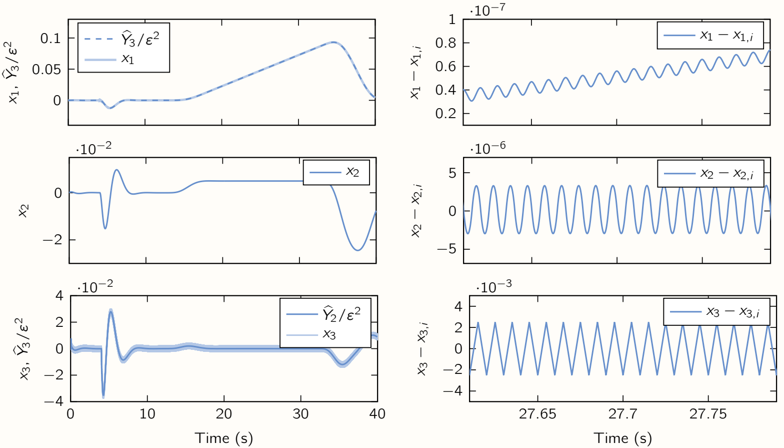


Measured output y



Input u

Exogenous injection – Simulation, results



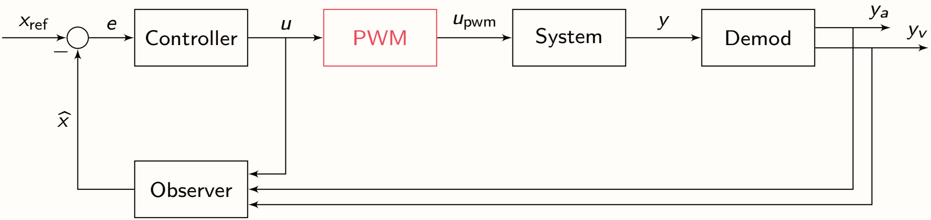
States x_1, x_2, x_3, Virtual measurements $\mathbf{Y}_2$, $\mathbf{Y}_3$ (left); zoom on the error $x - x_i$ (right)

→ Control with the virtual measurements as effective as with the actual state x_1

Outline

Endogenous signal injection for PWM-controlled systems

– Many electrical motors are controlled using **Pulse-Width Modulation (PWM)**, and limits the use of an exogenous injection



– The periodic modulation introduces a perturbation in the system

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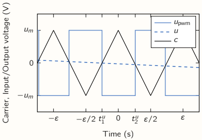
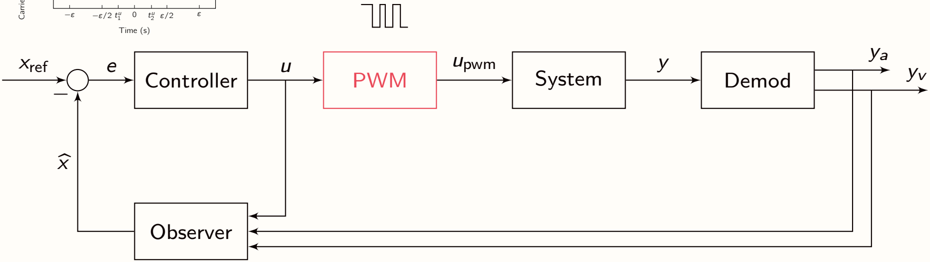


Computation: comparison to a fast-varying carrier c



- The periodic modulation introduces a perturbation in the system
- For nonlinear systems, this perturbation may carry additional information...
- ...**just as in the exogenous case!**

Endogenous signal injection for PWM-controlled systems

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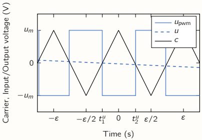
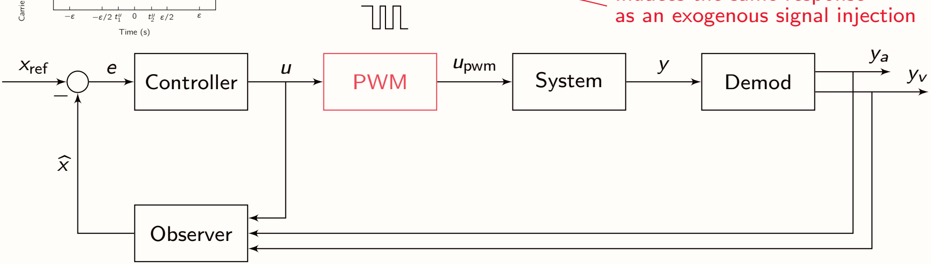


Computation: comparison to a fast-varying carrier c

Induces the same response
as an exogenous signal injection



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Endogenous signal injection – PWM operating principle

Principle: the input u is compared to an ε-periodic carrier c to produce a PWM signal u_{pwm} (with $\varepsilon << 1$ is a small parameter)

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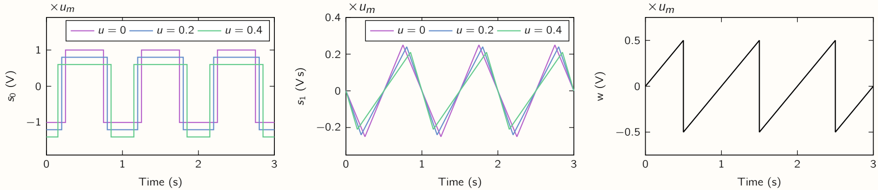
Expression for u_{pwm}, with $s_0 =:$ endogenous injection

$$u_{pwm}(t) = u(t) + s_0\left(u(t), \frac{t}{\varepsilon}\right),$$



s_0 (top), s_1 (middle), w (right)

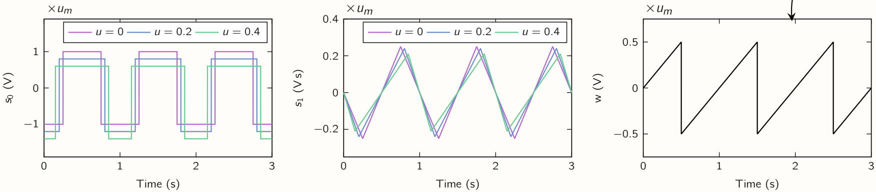
Endogenous signal injection – PWM operating principle

Principle: the input u is compared to an ε-periodic carrier c to produce a PWM signal u_{pwm} (with $\varepsilon << 1$ is a small parameter)

Expression for u_{pwm}, with $s_0 =:$ endogenous injection

$w(\sigma) = u_m \operatorname{mod}(\sigma + \frac{t}{2}, 1) - u_m/2)$

$$u_{pwm}(t) = u(t) + s_0(u(t), \tfrac{t}{\varepsilon}),$$



s_0 (top), s_1 (middle), w (right)

s_0 1-periodic with zero mean in the second argument

$$s_0(u,\sigma) = u_m - u + u_m \operatorname{sign}\left(\frac{u-u_m}{4} - w(\sigma)\right) + u_m \operatorname{sign}\left(\frac{u-u_m}{4} + w(\sigma)\right)$$

$s_1 :=$ zero-mean primitive of s_0 in the second argument

$$s_1(u,\sigma) := \left(1 - \frac{u}{u_m}\right) w(\sigma) - \left|\frac{u-u_m}{4} - w(\sigma)\right| + \left|\frac{u-u_m}{4} + w(\sigma)\right|$$

Second-order averaging theorem for endogenous signal injection – 1

SISO nonlinear system

$$\dot{x} = f(x) + g(x)u$$
$$y = h(x)$$

Set of **virtual measurements**

$$H(x) = \begin{pmatrix} h(x) & \varepsilon h'(x)g(x) \end{pmatrix}$$

We assume that, with this additional measurement $(\varepsilon h'(x)g(x))$, the original system is <u>observable</u>

Non-perturbed system

$$\dot{\overline{x}} = f(\overline{x}) + g(\overline{x})u(t)$$
$$\dot{\overline{\eta}} = a(\overline{\eta}, H(\overline{x}), t)$$
$$u(t) = \alpha(\overline{\eta}, H(\overline{x}), t)$$

PWM-controlled system

$$\dot{x} = f(x) + g(x)(u + s_0(u(t), \tfrac{t}{\varepsilon}))$$
$$\dot{\eta} = a\big(\eta, \overline{H}(x, \eta, \tfrac{t}{\varepsilon}, t), t\big)$$
$$u = \alpha\big(\eta, \overline{H}(x, \eta, \tfrac{t}{\varepsilon}, t), t\big)$$
$$\overline{H}(x, \eta, \sigma, t) := H\Big(x - \varepsilon g(x) s_1\big(\alpha(\eta, H(x), t), \sigma\big)\Big)$$

- $s_0(u, \tau)$: 1-periodic signal with zero mean in τ.
- $s_1(u, \tau)$ is the primitive wrt. τ of s_0 with zero mean.

Second-order averaging theorem for endogenous signal injection – 2

Theorem (Second-order averaging for PWM-controlled systems, SCMR20)

*Let $(x(t), \eta(t))$ be the solution of the PWM-controlled system starting from (x_0, η_0), and define $u(t) := \alpha\big(\eta(t), H\big(x(t)\big), t\big)$ and $y(t) := H\big(x(t)\big)$;
let $(\overline{x}(t), \overline{\eta}(t))$ be the solution of the original system starting from $\big(x_0 - \varepsilon g(x_0) s_1\big(u(0), 0\big), \eta_0\big)$, and define $\overline{u}(t) := \alpha\big(\overline{\eta}(t), H\big(\overline{x}(t)\big), t\big)$. Then, for all $t \geq 0$,*

$$
\begin{aligned}
x(t) &= \overline{x}(t) + \varepsilon g\big(\overline{x}(t)\big) s_1\big(\overline{u}(t), \tfrac{t}{\varepsilon}\big) + \mathcal{O}(\varepsilon^2)\\
\eta(t) &= \overline{\eta}(t) + \mathcal{O}(\varepsilon^2)\\
y(t) &= \underbrace{h(\overline{x}(t))}_{:=y_a(t)} + \underbrace{\varepsilon h'(\overline{x}(t)) g(\overline{x}(t))}_{:=y_v(t)} s_1\big(\overline{u}(t), \tfrac{t}{\varepsilon}\big) + \mathcal{O}(\varepsilon^2).
\end{aligned}
$$

- Idea of the proof: change of coordinates (the so-called *near-identity transformation*), then use of a comparison theorem (similar to the Lipschitz case)
- The proof requires a slight adaptation of the classic averaging theorem for systems with state-discontinuities

Endogenous signal injection – Virtual measurements

Dynamic system

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3$$
$$\dot{x}_3 = u + d$$
$$y = x_2 + x_1 x_3$$

- u: input
- d: (unknown) disturbance
- Objective: controlling x_1 while rejecting d

Still assume y_a and y_v are known with a $\mathcal{O}(\varepsilon^2)$ accuracy

Virtual measurements

$$y_a := x_2 + x_1 x_3$$
$$y_v := \varepsilon h'(x) g(x) = \varepsilon x_1$$

- Controlling the system with PWM gives access to $y_v = \varepsilon x_1$
- With y_v, the system is controllable with a linear controller

Endogenous signal injection – Scenario, results

Scenario

- System at rest at $t = 0$; step $d = -0.25$ at $t = 2$ s
- Filtered unit step at $t = 14$ s
- PWM period $\varepsilon = 10^{-3}$ s

Endogenous signal injection – Scenario, results

Scenario

- System at rest at $t = 0$; step $d = -0.25$ at $t = 2$ s
- Filtered unit step at $t = 14$ s
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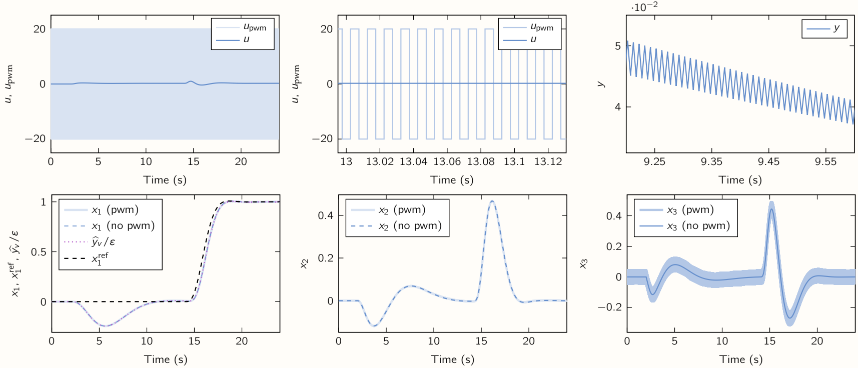
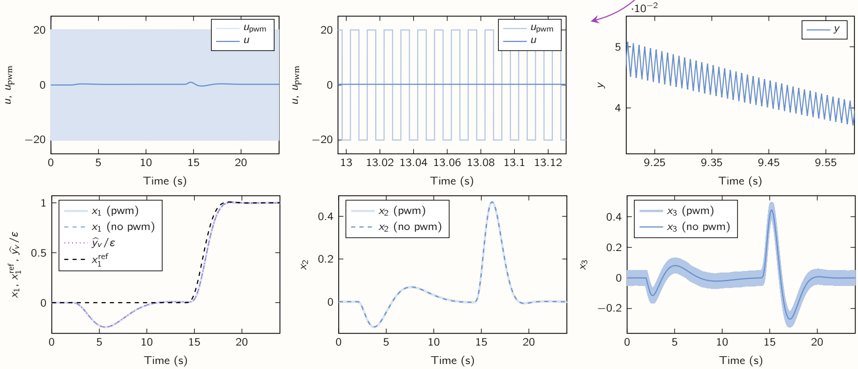


Endogenous signal injection – Scenario, results

Scenario

- System at rest at $t = 0$; step $d = -0.25$ at $t = 2$ s
- Filtered unit step at $t = 14$ s
- PWM period $\varepsilon = 10^{-3}$ s

Recovery as good as in the exogenous scenario!



Outline

Demodulation procedure of multiplexed signals

Extraction of the coordinates z from the physical measurement y

$$y = \sum_{i=1}^{N} z_i s_i, \qquad z_i \in \mathbb{R}, s_i \in \mathbb{R}^N$$

→ **Objective:** Recovery of each z_i with an arbitrary accuracy in $\mathcal{O}(\varepsilon^k)$

- Simple projection using **Gram-Schmidt** orthogonalization process: exact recovery

Demodulation procedure of multiplexed signals

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$$y = \sum_{i=1}^{N} z_i s_i\left(\frac{t}{\varepsilon}\right), \qquad s_i \in L^2_{\text{per}} \text{ 1-periodic with zero mean}$$

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- Projection (on L^2_{per}), still using G-S: exact recovery → procedure for recovering the "exogenous" virtual measurements!

Demodulation procedure of multiplexed signals

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Demodulation procedure of multiplexed signals

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- Carriers with a **slow-time dependency**: is it possible to design a **uniform** estimate...

Demodulation procedure of multiplexed signals

Extraction of the coordinates z from the physical measurement y

$$y = \sum_{i=1}^{N} z_i(t) s_i\left(u(t), \tfrac{t}{\varepsilon}\right) + \mathcal{O}(\varepsilon^p) + \nu + d\left(t, \tfrac{t}{\varepsilon}\right)$$

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- ...that also mitigates both the **measurement noise** ν and the **physical disturbance** d...

Demodulation procedure of multiplexed signals

Extraction of the coordinates z from the physical measurement y

$$y = \Sigma\Delta\left[Z(t)S\left(u(t), \tfrac{t}{\varepsilon}\right)^T + \mathcal{O}(\varepsilon^p) + d\left(t, \tfrac{t}{\varepsilon}\right)\right] + \nu_{\Sigma\Delta}$$

→ **Objective:** Recovery of each z_i with an arbitrary accuracy in $\mathcal{O}(\varepsilon^k)$

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$\rightarrow$ **Synchronous detection problem on analog/$\Sigma\Delta$ signals** (with exotic considerations)

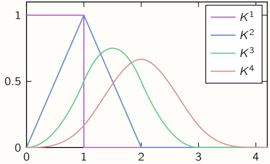
Demodulation procedure – Reconstruction kernel $\widetilde{K}_k$

(Without disturbance $d = 0$ nor slow dependency on $S\left(t, \frac{t}{\varepsilon}\right)$)

Definition of the **kernel** K_k

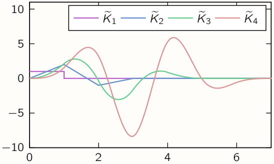
$$K_k = \left(\frac{1}{\varepsilon}1_{[0,\varepsilon]}\right)^{*k} = \frac{1}{\varepsilon^k}1_{[0,\varepsilon]} * \ldots * 1_{[0,\varepsilon]}$$

$\to$ $k = 2$: triangle/Bartlett window; $k = 4$: Parzen/de la Vallée Poussin



Modified kernels $\widetilde{K}_k$ $\to$ $\boxed{\widetilde{K}_k * \varphi = \varphi + \mathcal{O}(\varepsilon^k)}$

$$\widetilde{K}_1(t) := K_1(t)$$
$$\widetilde{K}_2(t) := 2K_2(t) - 1K_2(t-\varepsilon),$$
$$\widetilde{K}_3(t) := \tfrac{17}{4}K_3(t) - 5K_3(t-\varepsilon) + \tfrac{7}{4}K_3(t-2\varepsilon).$$



Extraction of the vector Z (with $R = S$)

$$P_k[y](t) := \left(\widetilde{K}_k * \left(yR_\varepsilon^T\right)\right)(t) \times \left(\widetilde{K}_k * \left(S_\varepsilon R_\varepsilon^T\right)\right)^{-1}(t)$$
$$= Z^T(t) + \mathcal{O}(\varepsilon^k)$$

Demodulation procedure – Main result

In presence of disturbances, one technique consists in **windowing** the location of the perturbation:

→ $c(t, \frac{t}{\varepsilon}) := 0$ when d is active, 1 otherwise

→ $R = S \times c$

With the **slow-time dependency** on S, we assume $SR^T(\upsilon, \sigma)$ are $\mathcal{A}^k$

Definition ($\mathcal{A}_k$ property)

Let $g(t, \sigma)$ be 1-periodic with zero mean. It is said to be $\mathcal{A}_k$, $k \geq 1$, if $g^{(-k)}$ is $k-1$ times differentiable in the first variable, with bounded derivatives at all orders, and $\partial_1^{k-1} g^{(-k)}$ Lipschitz. **Example**: $g(\upsilon, t) := \mathrm{sign}(\upsilon + t) \leftarrow$ PWM signal; $g^{(-1)} = |v + t|$

Demodulation of the components of Z

The following estimator recovers z_k with an accuracy in ε^k

$$P_k[y](t) := \left(\widetilde{K}_k * \left(yR_\varepsilon^T\right)\right)(t) \times \left(\widetilde{K}_k * \left(S_\varepsilon R_\varepsilon^T\right)\right)^{-1}(t).$$

In other words, $Z^T(t) = P_k[y](t) + \mathcal{O}(\varepsilon^k)$.

Same result as before using a modified vector R (*with a trickier proof though*)

Demodulation procedure

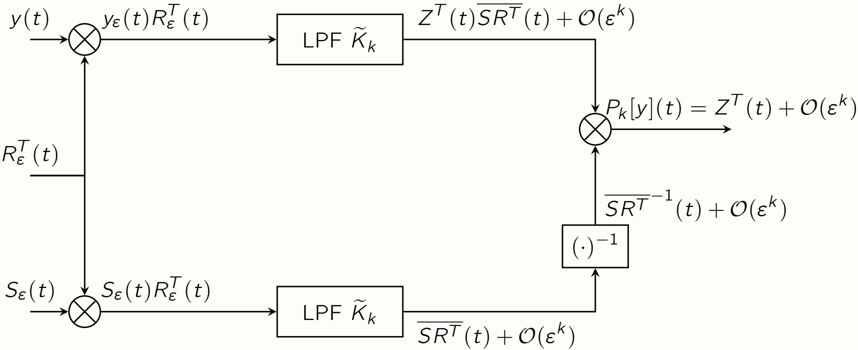


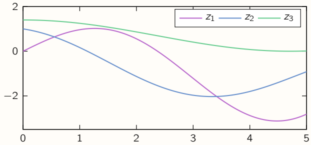
Diagram of the demodulation procedure

Numerical validation of the demodulation procedure

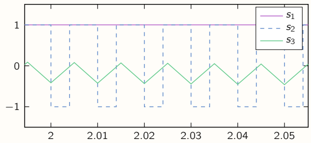
Validation of the theory on a toy problem. **Composite signal**

$$y(t) = z_1(t)s_1(t, \tfrac{t}{\varepsilon}) + z_2(t)s_2(t, \tfrac{t}{\varepsilon}) + z_3(t)s_3(t, \tfrac{t}{\varepsilon}) + d(t, \tfrac{t}{\varepsilon}), \quad \varepsilon = 10^{-3}$$

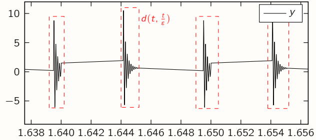
Components z_i



Carriers s_i



Signal y



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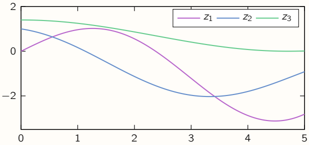

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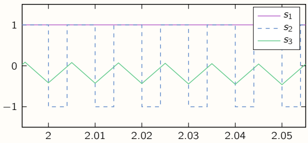

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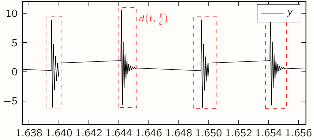
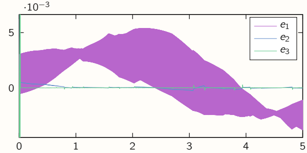
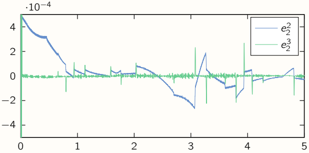
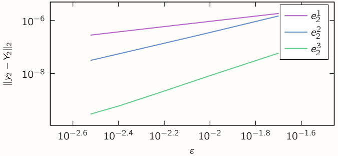


$P_k^2[y] - z_2$



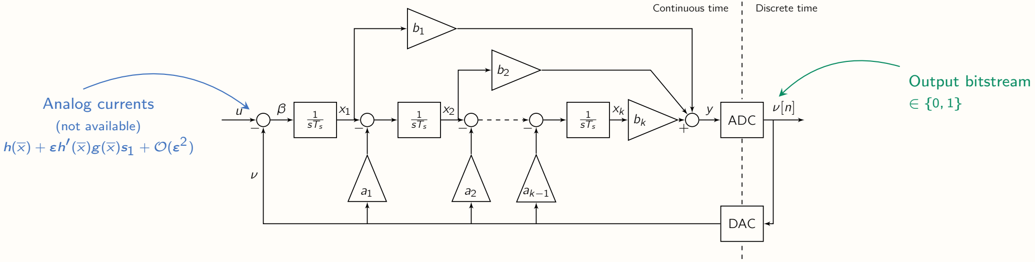


Asymptotic (L^2)



Synchronous detection over Sigma-Delta modulators

→ Modern Variable-Frequency Drives embedd **Sigma-Delta ADC**



State-space model $i = 1, \ldots, k-1$

$$\frac{1}{N}\dot{x}_1(\tau) = u(\tau) - \nu(N\tau)$$

$$\frac{1}{N}\dot{x}_{i+1}(\tau) = x_i(\tau) - a_i\nu(N\tau)$$

$$y(\tau) = \sum_{i=1}^{k} b_i x_i(\tau)$$

- Input: $u(t/\varepsilon)$, ε: PWM period
- Sampling time T_s (15 MHz)
- Normalized time $\tau := t/\varepsilon$
- **Oversampling ratio** $N := \varepsilon/T_s$ (= 3750 for the actual implementation)

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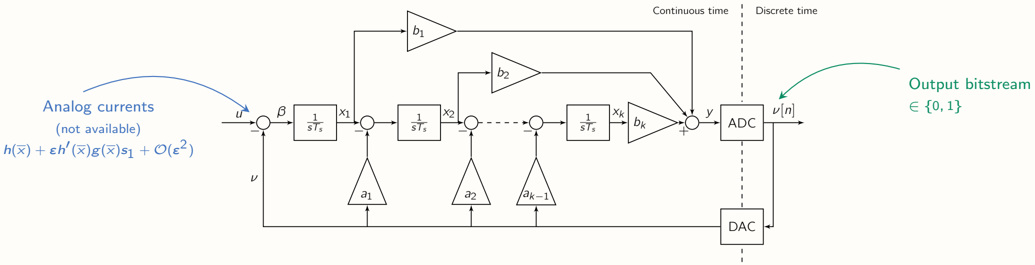

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→ **(Filtered) input-ouput estimate?**

Synchronous detection over continuous ΣΔ modulators – Theorem

Assumptions:

- the input u is selected such that the modulator is stable, *i.e.* the states $x_{1,\ldots,k}$ are bounded. In particular, $\|u\|_\infty < 1$.
- k^{th}-order continuous ΣΔ modulator in **pure feedforward form**: $a_1 = .. = a_{k-1} = 0$

Definition: f p-times differentiable + $f^{(p)}$ absolutely continuous (resp. piecewise) = f AC^p (resp. piecewise).

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Theorem (Error estimate for k^th-order CT-ΣΔ modulators, SCMR21)

Consider $\beta \in L^\infty[0, +\infty)$ such that the zero-mean primitive $\beta^{(-j)}$ of $\beta^{(-j+1)}$ exists ($j = 0, \dots, k-1$). Consider as well K^k a $(k-1)^{th}$-times differentiable kernel with support in $[0, k]$, and such that $K^k(0) = K^k(k) = (K^k)^{(j)}(0) = (K^k)^{(j)}(k) = 0$ ($j = 0, \dots, k-1$). If s is AC^{k-1}, then for $t \geq 0$,

$$I(t) := \beta s * K^k(t) = o(1/N^k)$$

If s is only piecewise AC^{k-1}, then for $t \geq 0$, $I(t) = \mathcal{O}(1/N^k)$.

Hypotheses on β ← Output-input difference of a ΣΔ CT-MOD

K^k ← satisfied by the previous kernels

Synchronous detection – Numerical results

(Filtered) Output-Input difference, second-order CT-MOD2 in pure FF form

$$I(t) := \beta s * K^k(t) = o(1/N^2), \qquad s\ AC^1$$
$$= O(1/N^2) \qquad s \text{ piecewise } AC^1$$
$$= O(1/N) \qquad s \text{ discontinuous}$$

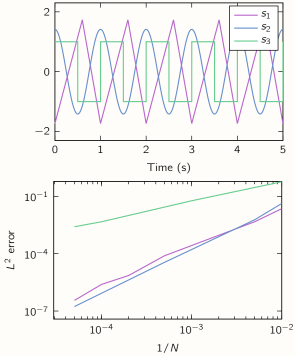
Input $u_i(t) := z(t)s_i(t)$ with $z(t) := 0.04\cos(t/12) + 0.06\sin(t/4\pi)$.
Expression for the different signals s

$$s_1(t) := \frac{1}{\sqrt{0.03}}\left(\tau 1_{[0,0.6]}(\tau) + 1.5(1-\tau)1_{]0.6,1]}(\tau) - 0.3\right),$$

$$s_2(t) := \sqrt{2}\cos(2\pi\tau), \qquad s_3(t) := 1_{[0,0.5]}(\tau) - 1_{]0.5,1]}(\tau),$$

with $\tau = \mathrm{mod}(t,\varepsilon)/\varepsilon$ and $\varepsilon = 1$.

- s_1: piecewise AC^1, s_2 AC^1, s_3 discontinuous



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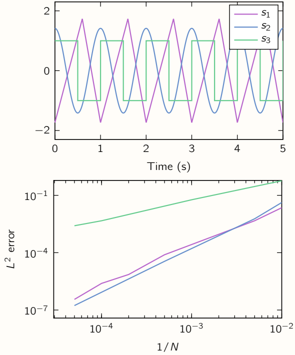
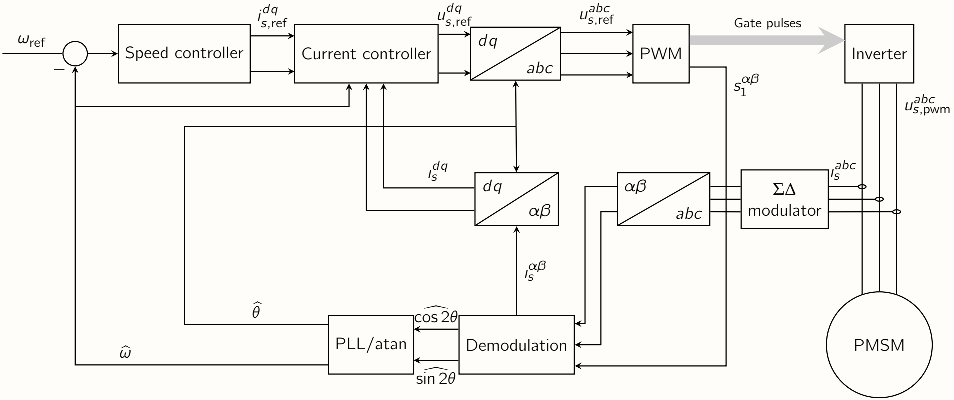
Slope=1 (disc)
2 (piec. AC), 2.3 (AC)



Outline

Sensorless PMSM control scheme



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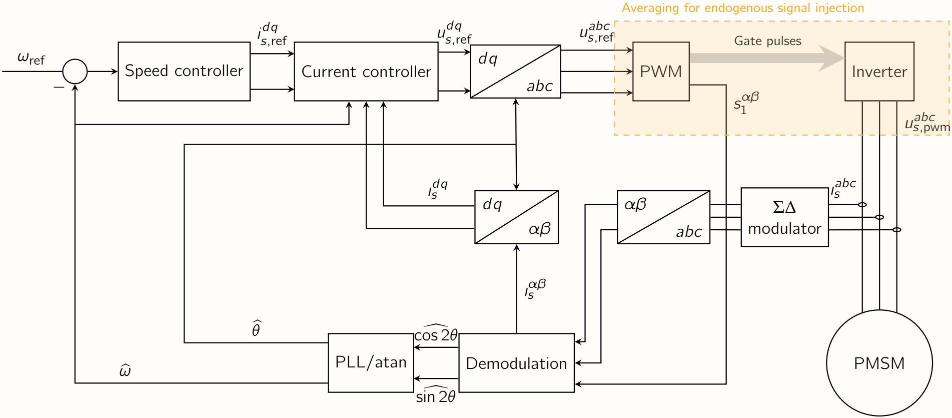

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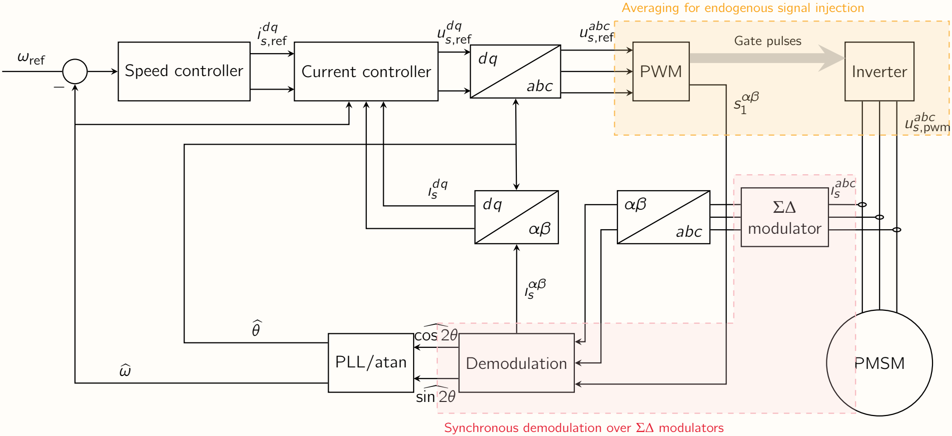

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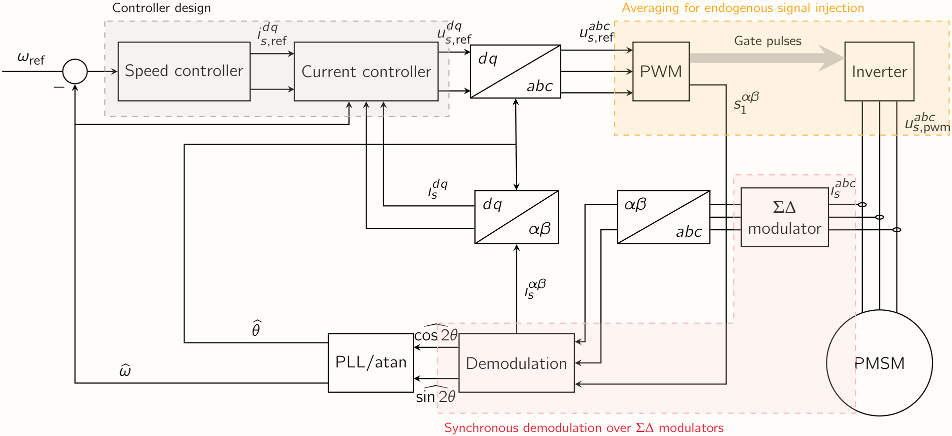


Sensorless PMSM control scheme

Model and virtual measurements for a PWM-controlled PMSM

State-space model of a PMSM in the *dq*-frame

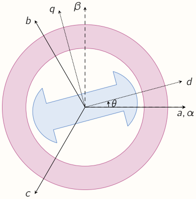
$$\frac{d\phi_s^{dq}}{dt} = u_s^{dq} - R_s \imath_s^{dq} - \omega \mathcal{J} \phi_s^{dq}$$
$$\frac{J}{n}\frac{d\omega}{dt} = n {\imath_s^{dq}}^T \mathcal{J} \phi_s^{dq} - T_l$$
$$\frac{d\theta}{dt} = \omega$$

We assume there is no magnetic saturation

- $L_d \imath_s^d = \phi_s^d - \phi_m$
- $L_q \imath_s^q = \phi_s^q$

Inputs: voltages in *abc*: $u_s^{abc} \to u_{s,\text{pwm}}^{abc}$

Outputs: currents in $\alpha\beta$: $\imath_s^{\alpha\beta} := \mathcal{R}(\theta)\imath_s^{dq}$



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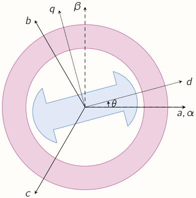

Set of **virtual measurements** for the PWM-controlled PMSM
$\to$ Owing to the **endogenous injection**)

$$
\imath_s^{\alpha\beta} = \overline{\imath_s^{\alpha\beta}} + \varepsilon d h_{\overline{x}}\left(g(\overline{x}) s_1\right) + \mathcal{O}(\varepsilon^2) \Rightarrow y_v(t) = \varepsilon \mathcal{S}\left(\bar{\theta}(t)\right) \underbrace{\overline{s_1^{\alpha\beta} {s_1^{\alpha\beta}}^T}}_{\in \mathcal{M}_2}
$$

Where $\mathcal{S}(\theta)$ is the so-called **saliency matrix**

$$
\mathcal{S}(\theta) := \frac{L_d + L_q}{2 L_d L_q} \begin{pmatrix} 1 + \frac{L_q - L_d}{L_d + L_q} \cos 2\theta & \frac{L_q - L_d}{L_d + L_q} \sin 2\theta \\ \frac{L_q - L_d}{L_d + L_q} \sin 2\theta & 1 - \frac{L_q - L_d}{L_d + L_q} \cos 2\theta \end{pmatrix}.
$$

Model and virtual measurements for a PWM-controlled PMSM

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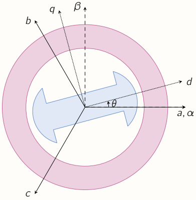
Inputs: voltages in abc: $u_s^{abc} \to u_{s,\mathrm{pwm}}^{abc}$

Outputs: currents in $\alpha\beta$: $\imath_s^{\alpha\beta} := \mathcal{R}(\theta) \imath_s^{dq}$



Set of **virtual measurements** for the PWM-controlled PMSM
→ Owing to the **endogenous injection**)

$$\imath_s^{\alpha\beta} = \overline{\imath_s^{\alpha\beta}} + \varepsilon d h_{\overline{x}}\big(g(\overline{x}) s_1\big) + \mathcal{O}(\varepsilon^2) \Rightarrow y_v(t) = \varepsilon \mathcal{S}\big(\bar{\theta}(t)\big) \underbrace{\overline{s_1^{\alpha\beta} {s_1^{\alpha\beta}}^T}}_{\in \mathcal{M}_2}$$

Where $\mathcal{S}(\theta)$ is the so-called **saliency matrix**

$$\mathcal{S}(\theta) := \frac{L_d + L_q}{2 L_d L_q} \begin{pmatrix} 1 + \frac{L_q - L_d}{L_d + L_q} \cos 2\theta & \frac{L_q - L_d}{L_d + L_q} \sin 2\theta \\ \frac{L_q - L_d}{L_d + L_q} \sin 2\theta & 1 - \frac{L_q - L_d}{L_d + L_q} \cos 2\theta \end{pmatrix}.$$

→ y_v is made available by the previous demodulation method!

Recovery of the position – LSQ on the virtual measurement

$y_v(t) = \varepsilon \mathcal{S}\big(\bar{\theta}(t)\big)\overline{s_1^{\alpha\beta} s_1^{\alpha\beta^T}}$. Standard PWM scheme (i.e. same carrier for the three voltages): **the rank of** $\overline{s_1^{\alpha\beta} s_1^{\alpha\beta^T}}$ **drops** when two input voltages are equal

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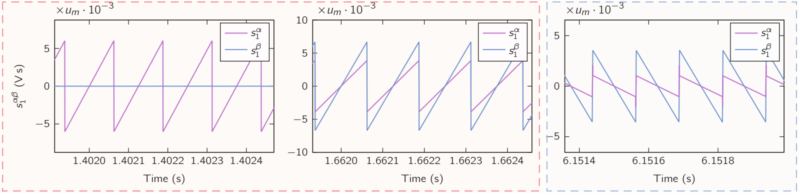
$s_1^{\alpha\beta}$ collinear

$\Downarrow$

rank $= 1$



$s_1^{\alpha\beta}$ ind.

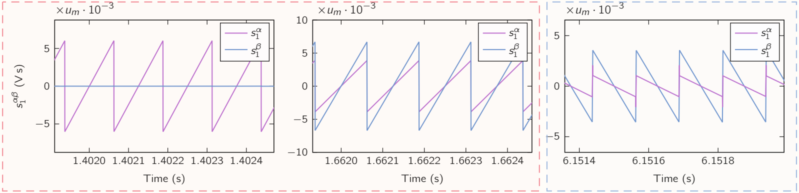
$\Downarrow$

rank $= 2$

Recovery of the position – LSQ on the virtual measurement

$y_v(t) = \varepsilon \mathcal{S}\big(\bar{\theta}(t)\big)\overline{s_1^{\alpha\beta} s_1^{\alpha\beta T}}$. Standard PWM scheme (i.e. same carrier for the three voltages): **the rank of $\overline{s_1^{\alpha\beta} s_1^{\alpha\beta T}}$ drops** when two input voltages are equal

$s_1^{\alpha\beta}$ collinear $\Downarrow$ rank $= 1$



$s_1^{\alpha\beta}$ ind. $\Downarrow$ rank $= 2$

Least-squares method to estimate $\cos 2\theta$ and $\sin 2\theta$. Writing

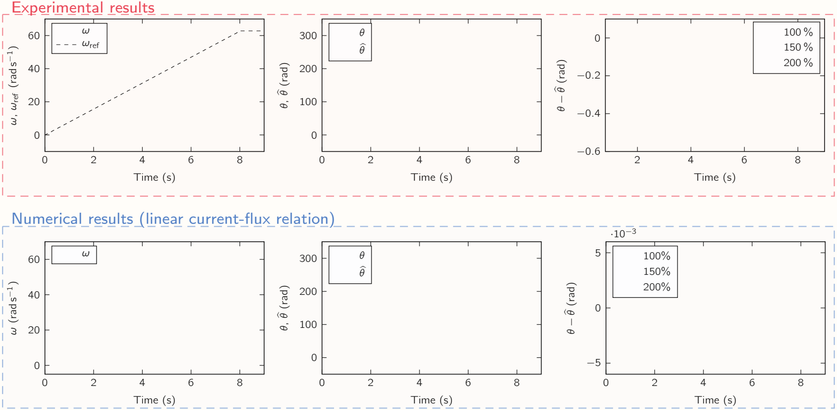
$$\begin{pmatrix} \lambda & \mu \\ \mu & \nu \end{pmatrix} := \overline{s_1^{\alpha\beta} s_1^{\alpha\beta T}}; \qquad \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} := \frac{2L_d L_q}{L_d + L_q} y_v.$$

and $L := \frac{L_d+L_q}{L_q-L_d}$, the *virtual measurement* $y_v = \mathcal{S}(\theta)\overline{s_1^{\alpha\beta} s_1^{\alpha\beta T}}$ reads

$$P =: \begin{pmatrix} \lambda & \mu \\ \mu & \nu \\ -\mu & \lambda \\ -\nu & \mu \end{pmatrix} \begin{pmatrix} \cos 2\theta \\ \sin 2\theta \end{pmatrix} = L \begin{pmatrix} y_{11} - \lambda \\ y_{12} - \mu \\ y_{21} - \mu \\ y_{22} - \nu \end{pmatrix} =: d \Rightarrow \begin{pmatrix} \cos 2\theta \\ \sin 2\theta \end{pmatrix} = L\left[P^T P\right]^{-1} P^T d$$

Sensorless position estimation – Numerical and experimental results

Scenario: **Salient** PMSM $L_d = 43.25\,\mathrm{mH}$, $L_q = 69.05\,\mathrm{mH}$
Speed ramp: $0 - 10\,\mathrm{Hz}$ in $8\,\mathrm{s}$; $T_l = 100, 150, 200\%$ of the rated torque



Speed ω (rad s^{-1}) Position θ (rad) Error $\theta - \hat{\theta}$ (rad)

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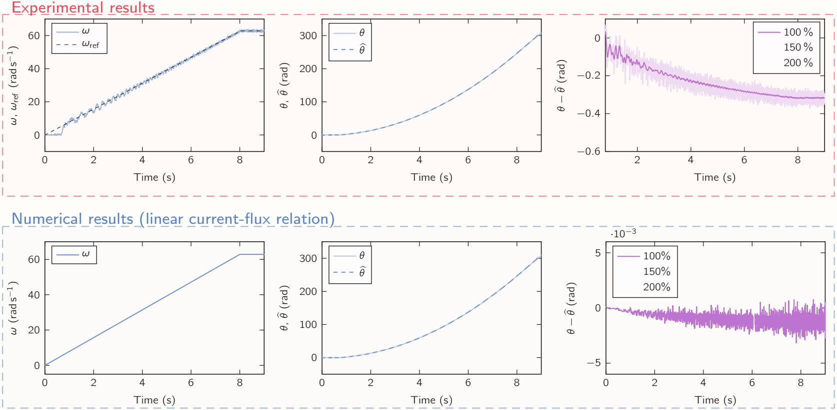

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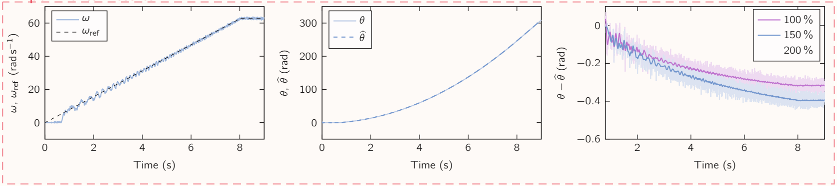
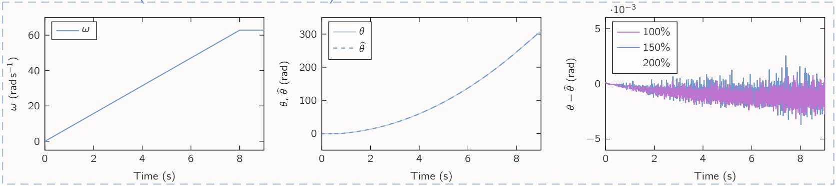
Experimental results



Numerical results (linear current-flux relation)



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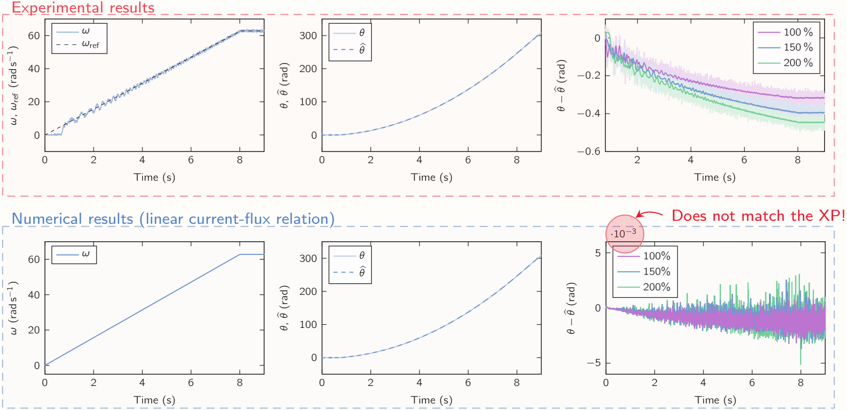

Speed ω (rad s^{-1}) Position θ (rad) Error $\theta - \hat{\theta}$ (rad)

– **Discrepancy** between the numerical and experimental values for $\theta - \hat{\theta}$?

Sensorless position estimation – Saturation model – Results

Energy-based modelling (p_{ij}: magnetic parameters)

$$\mathcal{H}_m^{dq}(\phi_s^{dq}) := p_{10}\phi_s^d + p_{20}(\phi_s^d)^2 + p_{30}(\phi_s^d)^3 + p_{40}(\phi_s^d)^4 + p_{02}(\phi_s^q)^2 + p_{04}(\phi_s^q)^4 + p_{12}\phi_s^d(\phi_s^q)^2 + p_{22}(\phi_s^d)^2(\phi_s^q)^2$$

Linear current-flux relation

$$\imath_s^d = \frac{\phi_s^d - \phi_m}{L_d}, \quad \imath_s^q = \frac{\phi_s^q}{L_q}$$

Saturated current-flux relation

$$\imath_s^d = \frac{\partial \mathcal{H}_m^{dq}}{\partial \phi_s^d}(\phi_s^{dq}), \quad \imath_s^q = \frac{\partial \mathcal{H}_m^{dq}}{\partial \phi_s^q}(\phi_s^{dq})$$

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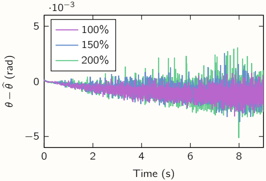
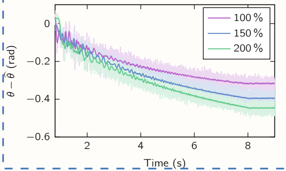
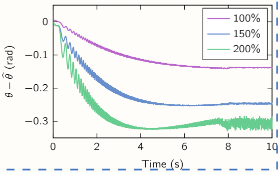
Numerical







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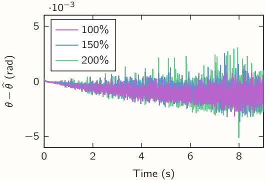

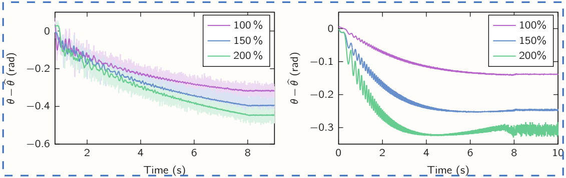

Numerical

– **Behavior** of the error wrt. numerically reproduced thanks to the magnetic model!

Outline

Conclusion

Summary

- Higher-order **averaging theory for exogenous/endogenous** signal injection: **external HF probing signals** or **PWM harmonics** induce nonlinear responses *carrying additional knowledge on the system*
- Generic **demodulation procedure** for extracting the so-called virtual measurements *with an arbitrary accuracy in* $\mathcal{O}(\varepsilon^p)$
- Several **error estimates** for k^{th}-order CT-$\Sigma\Delta$ MOD: "the demodulation process commutes with the $\Sigma\Delta$ modulator"
- **Experimental validations on a PMSM**: successful **sensorless position estimation** from the $\Sigma\Delta$ current bitstreams

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- Third/**Higher-order averaging theory for endogenous** signal injection
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Thank you!

\end{presentation}