Deterministic observer design for vision-aided inertial navigation

T. Hamel, M-D. Hua and C. Samson

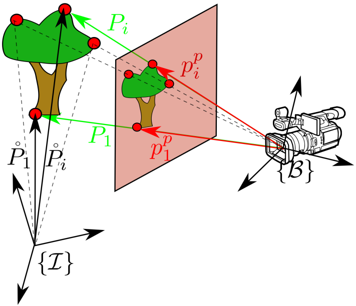
Outline

Motivation

- Autonomous navigation
- Visual Servoing - PBVS
- Autonomous landing
- Station keeping
- Augmented Reality
- Visual Odometry and Visual SLAM

Computer Vision Algorithms for Pose Estimation

- Homography matrix
  - Observed Planar Scene.
  - Pose (up to a scalar) retrieved by decomposing the Homography matrix.
- Essential Matrix
  - General 3D Scene.
  - Pose (up to a scalar) retrieved by decomposing the Essential matrix.
- Perspective-n-Point
  - General 3D Scene.
  - Points location known in an inertial frame.



Vision-aided inertial navigation

Algebraic approaches and iterative algorithms based on gradient descent:

- No filtering.
- No temporal correlation for the video sequence.
- No dynamics.

Observers:

- Constructive observer design methods that exploit invariance and equivariance
    - Kinematic Observer
    - Rely on group velocity measurements
- EKF, UKF and Particle Filters
    - Kinematic or dynamic systems
    - Limitations in terms of robustness

Novelty: Riccati observer framework that generalises the MEKF for vision-aided navigation.

Riccati Framework (Hamel and Samson 2017)

Consider the following class of nonlinear systems

$$\begin{cases} \dot{x} = A(x_1,t)x + \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + O(|x_1|^2) + O(|x_1||u_1|), \; A = \begin{bmatrix} A_{11}(t) & 0_{n_1 \times n_2} \\ A_{21}(x_1,t) & A_{22}(t) \end{bmatrix} \\ y = C_1(x_1,\hat{x}_2,t)x_1 + C_2(x_1,\hat{x}_2,t)x_2 + O(|x_1|^2) + O(|x_1||\tilde{x}_2) + O(|\tilde{x}_2|^2). \end{cases}$$

with $x_1 \in \mathcal{B}_r^{n_1}$ and $x_2 \in \mathbb{R}^{n_2}$. Let $\hat{x}_2$ be an estimate of x_2 and consider the following observer

$$\begin{cases} u_1 = -K_1(y - C_2\hat{x}_2) \\ \dot{\hat{x}}_2 = A_{22}\hat{x}_2 + u_2 + K_2(y - C_2\hat{x}_2) \\ \dot{P} = AP + PA^\top - PC^\top Q(t)CP + V(t) \\ K = k(t)PC^\top Q. \end{cases} \tag{1}$$

Then the origin is locally exponentially stable if $Q(t)$ and $V(t)$ are both larger than some positive matrix and the pair $(A(0,t), C(0,x_2,t))$ is uniformly observable.

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Uniform Observability Condition

- LTV System

$$\begin{cases} \dot{x} = \bar{A}(t)x + \bar{B}(t)u \\ y = \bar{C}(t)x \end{cases}$$

- The *Riccati observability Gramian* associated with the triplet $(\bar{A} := A(0, t), \bar{C} := C(0, x_2, t), Q)$ is the non-negative definite matrix-valued function defined by

$$W_Q^{\bar{A},\bar{C}}(t, t+\delta) := \frac{1}{\delta} \int_t^{t+\delta} \Phi^\top(s,t)\bar{C}^\top(s)Q(s)\bar{C}(s)\Phi(s,t)ds$$

where $\Phi(t, t_0)$ is the transition matrix associated with $\bar{A}(t)$.

- If $\bar{A}(t)$ and $\bar{C}(t)$ are bounded and if there exists $\delta > 0$ and $\epsilon > 0$ such that $W_{I_n}^{\bar{A},\bar{C}}(t, t+\delta) > \epsilon I_n$ for all $t \geq 0$, then we say that the pair $(\bar{A}, \bar{C})$ is uniformly observable.

Observer equation and model adaptation

Equations of motion of the camera and the proposed observer:

$$\begin{cases} \dot{R} = R\Omega_\times \\ \dot{\xi} = -\Omega_\times \xi + V \end{cases} \qquad \begin{cases} \dot{\hat{R}} = \hat{R}\Omega_\times - \hat{R}\sigma_{R\times} \\ \dot{\hat{\xi}} = -\Omega_\times \hat{\xi} + V - \sigma_\xi \end{cases}$$

where:

- $R \in \mathrm{SO}(3) : \{\mathcal{B}\} \to \{\mathcal{I}\}$, $\xi = R^\top \mathring{\xi} \in \{\mathcal{B}\}$,
- $\Omega, V \in \mathbb{R}^3$, the rotational and translational velocities, expressed in $\{\mathcal{B}\}$
- $\sigma_R, \sigma_\xi \in \mathbb{R}^3$ the innovation terms,

Observer equation and model adaptation

Define the attitude error matrix

$$\tilde{R} := \hat{R}^\top R = I_3 + \tilde{\lambda}_\times + O(|\tilde{\lambda}|^2)$$

with $\tilde{\lambda} \in \mathcal{B}_1^3$ equal to twice the vector part of the quaternion associated with the attitude error matrix $\tilde{R}$, and whose convergence to zero 'implies' the convergence of $\hat{R}$ to R.
One then deduces that:

$$\dot{\tilde{\lambda}} = -\Omega_\times \tilde{\lambda} + \sigma_R + O(|\tilde{\lambda}|^2) + O(|\tilde{\lambda}||\sigma_R|)$$

By setting $x = [x_1^\top, x_2^\top]^\top := [\tilde{\lambda}^\top, \xi^\top]^\top$, $u_1 := \sigma_R$ and $u_2(t) := V$, one obtains:

$$\begin{cases} \dot{x} = A(x_1, t)x + \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + O(|x_1|^2) + O(|x_1||u_1|), A = A(0,t) := \begin{bmatrix} -\Omega_\times & 0_3 \\ 0_3 & -\Omega_\times \end{bmatrix} \\ y = C_1(x_1, \hat{x}_2, t)x_1 + C_2(x_1, \hat{x}_2, t)x_2 + O(|x_1|^2) + O(|x_1||\tilde{x}_2) + O(|\tilde{x}_2|^2). \end{cases}$$

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System output associated with the PnP problem

- Source points $\mathring{P}_i$ are known, the bearing $p_i := \frac{R^\top(\mathring{P}_i - \mathring{\xi})}{|\mathring{P}_i - \mathring{\xi}|} = \frac{R^\top \mathring{P}_i - \xi}{|R^\top \mathring{P}_i - \xi|} \in \mathbb{S}^2$ are measured.
  Using the fact the fact that $\Pi_{p_i} p_i = 0$, one has:

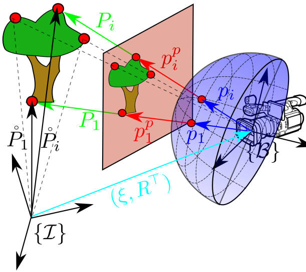
$$\begin{aligned} 0 &= -|\mathring{P}_i - \mathring{\xi}|\Pi_{p_i} p_i \\ &= \Pi_{p_i}\xi - \Pi_{p_i}\hat{R}^\top \mathring{P}_i - \Pi_{p_i}(\hat{R}^\top \mathring{P}_i)_\times \tilde{\lambda} + O(|\tilde{\lambda}|^2) \end{aligned}$$

- By setting $y_i := \Pi_{p_i}\hat{R}^\top \mathring{P}_i$:

$$y_i = \Pi_{p_i}\xi - \Pi_{p_i}(\hat{R}^\top \mathring{P}_i)_\times \tilde{\lambda} + O(|\tilde{\lambda}|^2)$$

  and by defining $y := [y_1^\top, \ldots, y_n^\top]^\top$, one gets:

$$C_1 = \begin{bmatrix} -\Pi_{p_1}(\hat{R}^\top \mathring{P}_1)_\times \\ \vdots \\ -\Pi_{p_n}(\hat{R}^\top \mathring{P}_n)_\times \end{bmatrix}, \quad C_2 = \begin{bmatrix} \Pi_{p_1} \\ \vdots \\ \Pi_{p_n} \end{bmatrix}$$



Observability issues for the PnP problem

The system is not uniformly observable if:

- The number of source points is less than or equal to two.
- All source points are aligned ($n \geq 3$).
- In case of three non-aligned source points:
  - **Static Case**: The camera lies on the dangerous cylinder.
  - **Moving Case**: The camera moves along a straight line orthogonal to the plane containing the source points and passing through a source point.
- In case of four and more non-aligned source point ($n \geq 4$):
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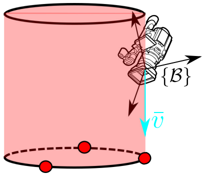

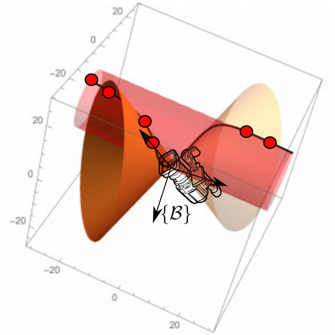

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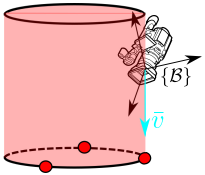

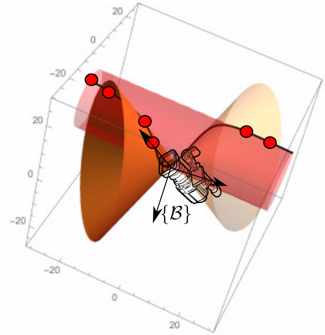

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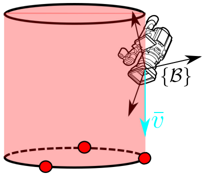

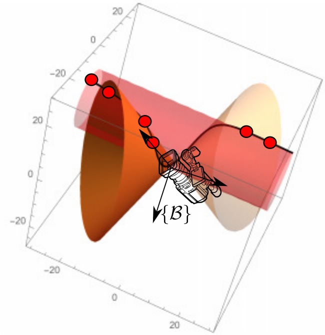

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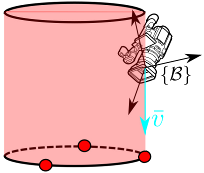
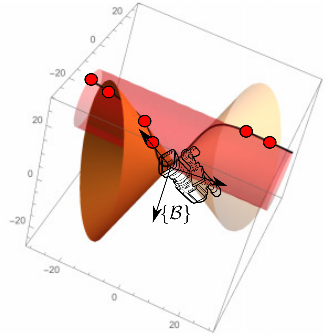
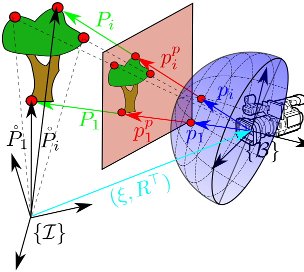




System output when using Epipolar Constraints

- Source points $\mathring{P}_i$ are unknown, the bearing $p_i \in \mathbb{S}^2$ are measured.
  Using the epipolar constraint $\mathring{p}_i^\top R \xi_\times p_i = 0$, one has:

$$\begin{aligned} 0 &= -\mathring{p}_i^\top \hat{R}\tilde{R}\xi_\times p_i \\ &= \mathring{p}_i^\top \hat{R}(\hat{\xi} \times p_i)_\times \tilde{\lambda} + \mathring{p}_i^\top \hat{R} p_{i\times} \xi + O(|\tilde{\lambda}|^2) \\ &\quad + O(|\tilde{\xi}||\tilde{\lambda}|) \end{aligned}$$



- By defining y as the vector in $\mathbb{R}^n$ with zero entries, one gets:

$$C_1 = \begin{bmatrix} \mathring{p}_1^\top \hat{R}(\hat{\xi} \times p_1)_\times \\ \vdots \\ \mathring{p}_n^\top \hat{R}(\hat{\xi} \times p_n)_\times \end{bmatrix}, \quad C_2 = \begin{bmatrix} \mathring{p}_1^\top \hat{R} p_{1\times} \\ \vdots \\ \mathring{p}_n^\top \hat{R} p_{n\times} \end{bmatrix}$$

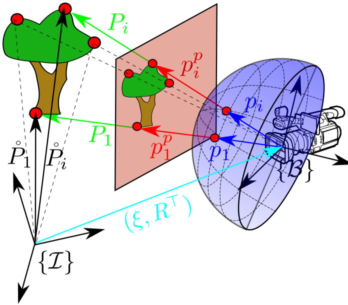
Observability condition when epipolar constraints are involved

Consider a robotic vehicle equipped with a monocular camera observing n ($n \geq 3$) source points with unknown 3D coordinates.

Consider the most difficult case of $n = 3$ and assume that the bearings of the 3 observed source points are linearly independent. Assume that the camera translational motion is sufficiently exciting in the sense that for all time there exist $\delta, \beta > 0$ such that $\forall i = 1, 2, 3$

$$\Pi_i(t, t+\delta) := \frac{1}{\delta} \int_t^{t+\delta} \frac{\mathring{\xi}(s)\mathring{\xi}^\top(s)}{|\mathring{P}_i - \mathring{\xi}(s)|^2} ds \geq \beta I_3 \qquad (2)$$

Assume also that $\mathring{\xi}$, Ω and V remain uniformly bounded for all time. Then, the pair $(\bar{A}, \bar{C})$ is uniformly observable and hence the equilibrium $(\tilde{R}, \tilde{\xi}) = (I_3, 0)$ of the error system is locally exponentially stable.



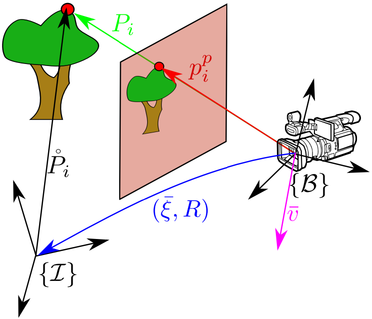
Observer design for pose and linear velocity (PnP problem)

The camera second order kinematics equation are

$$\begin{cases} \dot{R} = R\Omega_\times \\ \dot{\xi} = -\Omega_\times \xi + V \\ \dot{V} = -\Omega_\times V + a_B + gR^\top e_g \end{cases}$$

where:

- a_B specific acceleration expressed in the camera frame $\{\mathcal{B}\}$.
- g gravitational acceleration.
- $e_g \in S^2$ gravitational direction expressed in the inertial frame.



Measurements

- Bearings Measurements:

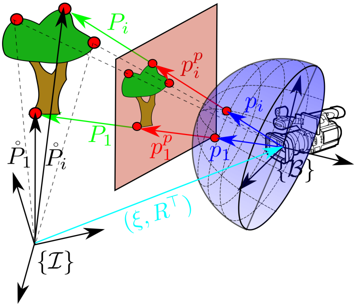
$$p_i := \frac{p_i^p}{|p_i^p|} = -R^\top \frac{\mathring{\xi} - \mathring{P}_i}{|\mathring{\xi} - \mathring{P}_i|}$$

- Gyrometer:

$$\Omega^m = \Omega + \mu_w$$

- Accelerometer:

$$a_B^m = a_B - b_a + \mu_a$$



where:

- μ_w, μ_a denote stochastic additive noises.
- b_a denotes a constant or slowly time-varying bias in the inertial frame ($\dot{b}_a = -\Omega_\times b_a$).

Riccati Observer Design

$$\begin{cases} \dot{\hat{R}} &= \hat{R}\Omega_\times - \hat{R}\sigma_{R\times} \\ \dot{\hat{\xi}} &= -\Omega_\times \hat{\xi} + \hat{V} - \sigma_\xi \\ \dot{\hat{V}} &= -\Omega_\times \hat{V} + \hat{\bar{b}}_a + g\hat{R}^\top e_g + a_{\mathcal{B}} - \sigma_V \\ \dot{\hat{\bar{b}}}_g &= -\Omega_\times \hat{\bar{b}}_a + (\hat{R}^\top a_g^m)_\times \sigma_R - \sigma_b \end{cases}$$

where $\hat{\bar{b}}_a$ is the estimate of a new bias $\bar{b}_a$ ($\bar{b}_a = b_a$, when $\tilde{R} = I_3$)

Riccati Observer Design

By transforming the system in Hamel-Samson Form,

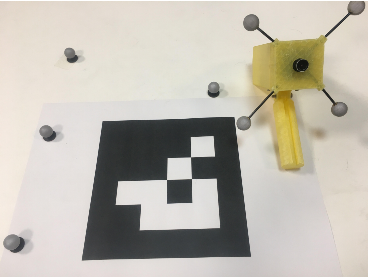
$$\begin{cases} \dot{x} = A(x_1, t)x + \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + O(|x_1|^2) + O(|x_1||u_1|) \\ y = C_1(x_1, \hat{x}_2, t)x_1 + C_2(x_1, \hat{x}_2, t)x_2 + O(|x_1|^2) + O(|x_1||\tilde{x}_2) + O(|\tilde{x}_2|^2). \end{cases}$$

one gets:

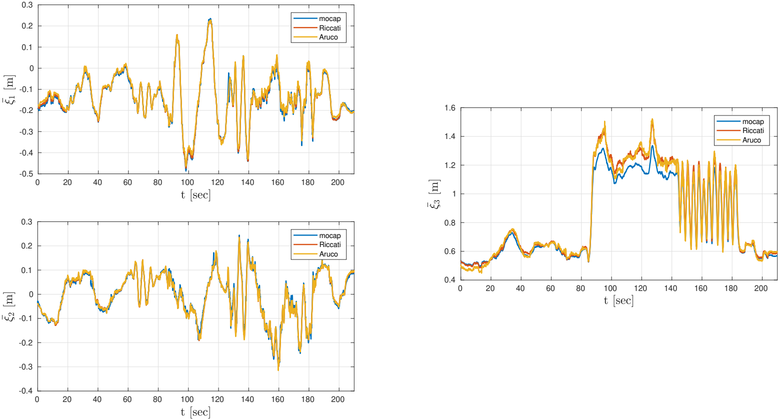
$$\begin{cases} A_{11} = -\Omega_\times, \quad A_{21} = 0_{3\times 9}, \qquad u_1 = \sigma_R \\ A_{22} = \begin{bmatrix} -\Omega_\times & I_3 & 0_3 \\ 0_3 & -\Omega_\times & I_3 \\ 0_3 & 0_3 & -\Omega_\times \end{bmatrix}, \quad u_2 = \begin{bmatrix} 0_{3\times 1} \\ \hat{R}^\top a_g^m + a_{\mathcal{B}} \\ (\hat{R}^\top a_g^m)_\times \sigma_R \end{bmatrix} \\ C_1 = \begin{bmatrix} -\Pi_{p_1}(\hat{R}^\top \mathring{P}_1)_\times \\ \vdots \\ -\Pi_{p_n}(\hat{R}^\top \mathring{P}_n)_\times \end{bmatrix}, \qquad C_2 = \begin{bmatrix} \Pi_{p_1} & 0_3 & 0_3 \\ \vdots & \vdots & \vdots \\ \Pi_{p_n} & 0_3 & 0_3 \end{bmatrix} \end{cases}$$

Experimental Setup

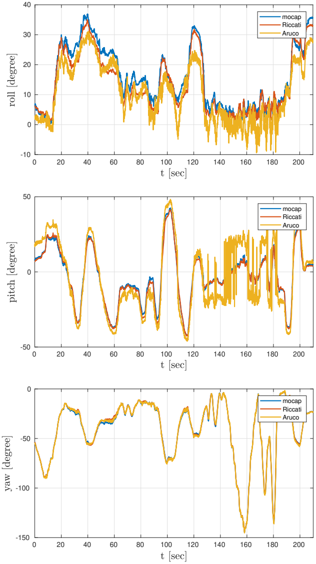
- Low Cost IMU myAHRS+ at 100Hz
- oCam Camera:
  - Resolution of 640×480.
  - Image acquisition at 30 frames per second.
- Point detection is performed with the ArUco library (OpenCV).
- Optitrack Motion Capture system (ground truth).



Experimental Result - Position



Experimental Result - Attitude



Observer design for linear velocity, gravity direction and depth estimation

The camera second order kinematics equation are

$$\begin{cases} \dot{R} = R\Omega_\times \\ \dot{V} = -\Omega_\times V + gR^\top e_g + a_\mathcal{B} \\ \dot{s} = \phi^\perp s \end{cases} \qquad \begin{cases} \dot{\hat{R}} = \hat{R}\Omega_\times - \sigma_{R\times}\hat{R} \\ \dot{\hat{V}} = -\Omega_\times \hat{V} + g\hat{R}^\top e_g + a_\mathcal{B} - \sigma_V \\ \dot{\hat{s}} = \phi^\perp \hat{s} - \sigma_s \end{cases}$$

with:

- a_B specific acceleration expressed in the camera frame $\{\mathcal{B}\}$,
- g gravitational acceleration,
- $e_g \in S^2$ gravitational direction expressed in the inertial frame,
- $s = 1/d$ inverse of the depth,
- $\phi^\perp = -\dot{d}/d$ flow divergence, and $y = \phi = V/d = sV$

The same framework is exploited. A persistent excitation condition on the translational motion is sufficient to ensure the uniform observability of the matrix pair $(A(0,t), C(0,x_2(t),t)$ and therefore the exponential stability of the observer.

Experimental Result - Attitude

Concluding remarks

- Presented a Deterministic Riccati observer design framework for a generic class of nonlinear system and pointed out an adequate uniform observability condition,
- Explained how to exploit this framework to different estimation problems,
- Showed the efficiency of the proposed observers via experimental results,
- Apply the methodology to many other problems such as Visual Odometry and Visual SLAM,
- Derive simple and sufficient observability conditions.

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