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Project-Team MCTAO

Mathematics for Control, Transport and Applications

IN COLLABORATION WITH: Institut Mathématique de Bourgogne, Laboratoire Jean-Alexandre Dieudonné (JAD)

RESEARCH CENTER
Sophia Antipolis - Méditerranée

THEME
Optimization and control of dynamic systems

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Project-Team MCTAO

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B2.6. - Biological and medical imaging
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B5.2.4. - Aerospace
B5.6. - Robotic systems

1. Personnel

Research Scientists

Jean-Baptiste Pomet [Team leader, Inria, Senior Researcher, HDR]
Laetitia Giraldi [Inria, Researcher]

Faculty Members

Bernard Bonnard [Univ de Bourgogne, Professor, HDR]
Jean-Baptiste Caillau [Univ Côte d'Azur, Professor, HDR]
Ludovic Rifford [Univ Côte d'Azur, Professor, HDR]

Post-Doctoral Fellows

Lamberto Dell'Elce [Inria, from Jan 2017]
Walid Djema [Inria, from Dec 2017, part-time with project-team BIOCORE]

PhD Students

Zeinab Badreddine [Univ de Bourgogne, until Dec 2017]
Alice Nolot [Univ de Bourgogne]
Karine Sérier [Univ de Bourgogne, from Sep 2017]
Sébastien Fueyo [Univ Nice Sophia Antipolis, part time with project-team APICS (FACTAS as of 2018)]
Yacine El Alaoui-Faris [Inria, from Oct 2017]
Clément Moreau [Ecole Normale Supérieure Cachan, from Sep 2017]
Michael Orieux [Univ Dauphine Paris, from Sep 2015]

Intern

Yacine El Alaoui-Faris [Inria, from Mar 2017 until Sep 2017]

Administrative Assistant

Claire Senica [Inria]

External Collaborators

Thierry Dargent [Thales, from Oct 2013]
Joseph Gergaud [ENSEEIHT, from Oct 2013]
Jérémy Rouot [EPF Troyes, from Oct 2017]

2. Overall Objectives

2.1. Overall Objectives

Our goal is to develop methods in geometric control theory for finite-dimensional nonlinear systems, as well as in optimal transport, and to transfer our expertise through real applications of these techniques.

Our primary domain of industrial applications in the past years is space engineering, namely designing trajectories in space mechanics using optimal control and stabilization techniques: transfer of a satellite between two Keplerian orbits, rendez-vous problem, transfer of a satellite from the Earth to the Moon or more complicated space missions. A second field of applications is quantum control with applications to Nuclear Magnetic Resonance and medical image processing. A third and more recent one is the control of micro-swimmers, i.e. swimming robots where the fluid-structure coupling has a very low Reynolds number.

There is also a form of transfer to other mathematical fields: some problems in dynamical systems are being solved thanks to control theory techniques.

3. Research Program

3.1. Control Problems

McTAO's major field of expertise is control theory in the large. Let us give an overview of this field.

3.1.1. Modelling

Our effort is directed toward efficient methods for the control of real (physical) *systems*, based on a *model* of the system to be controlled. Choosing accurate models yet simple enough to allow control design is in itself a key issue. The typical continuous-time model is of the form $\mathrm{d}x/\mathrm{d}t = f(x,u)$ where x is the *state*, ideally finite dimensional, and u the *control*; the control is left free to be a function of time, or a function of the state, or obtained as the solution of another dynamical system that takes x as an input. Deciding the nature and dimension of x, as well as the dynamics (roughly speaking the function f). Connected to modeling is identification of parameters when a finite number of parameters are left free in "f".

3.1.2. Controllability, path planning

Controllability is a property of a control system (in fact of a model) that two states in the state space can be connected by a trajectory generated by some control, here taken as an explicit function of time.. Deciding on local or global controllability is still a difficult open question in general. In most cases, controllability can be decided by linear approximation, or non-controllability by "physical" first integrals that the control does not affect. For some critically actuated systems, it is still difficult to decide local or global controllability, and the general problem is anyway still open.

Path planning is the problem of constructing the control that actually steers one state to another.

3.1.3. Optimal control

In optimal control, one wants to find, among the controls that satisfy some contraints at initial and final time (for instance given initial and final state as in path planning), the ones that minimize some criterion.

This is important in many control engineering problems, because minimizing a cost is often very relevant. Mathematically speaking, optimal control is the modern branch of the calculus of variations, rather well established and mature [70], [41], [29], but with a lot of hard open questions. In the end, in order to actually compute these controls, ad-hoc numerical schemes have to be derived for effective computations of the optimal solutions.

See more about our research program in optimal control in section 3.2.

3.1.4. Feedback control

In the above two paragraphs, the control is an explicit function of time. To address in particular the stability issues (sensitivity to errors in the model or the initial conditions for example), the control has to be taken as a function of the (measured) state, or part of it. This is known as closed-loop control; it must be combined with optimal control in many real problems.

On the problem of stabilization, there is longstanding research record from members of the team, in particular on the construction of “Control Lyapunov Functions”, see [59], [72].

3.1.5. Classification of control systems

One may perform various classes of transformations acting on systems, or rather on models... The simpler ones come from point-to-point transformations (changes of variables) on the state and control, and more intricate ones consist in embedding an extraneous dynamical system into the model, these are dynamic feedback transformations, they change the dimension of the state.

In most problems, choosing the proper coordinates, or the right quantities that describe a phenomenon, sheds light on a path to the solution; these proper choices may sometimes be found from an understanding of the modelled phenomenons, or it can come from the study of the geometry of the equations and the transformation acting on them. This justifies the investigations of these transformations on models for themselves.

These topics are central in control theory; they are present in the team, see for instance the classification aspect in [48] or [18], or —although this research has not been active very recently— the study [69] of dynamic feedback and the so-called “flatness” property [62].

3.2. Optimal Control and its Geometry

Let us detail our research program concerning optimal control, evoked in section 3.1.3. Relying on Hamiltonian dynamics is now prevalent, instead of the Lagrangian formalism in classical calculus of variations. The two points of view run parallel when computing geodesics and shortest path in Riemannian Geometry for instance, in that there is a clear one-to-one correspondance between the solutions of the geodesic equation in the tangent bundle and the solution of the Pontryagin Maximum Principle in the cotangent bundle. In most optimal control problems, on the contrary, due to the differential constraints (not all direction can be the tangent of a fesable trajectory in the state space), the Lagrangian formalism becomes more involved, while the Pontryagin Maximum Principle keeps the same form, its solutions still live in the cotangent bundle, their projections are the extremals, and a minimizing curve must be the projection of such a solution.

3.2.1. Cut and conjugate loci

The cut locus —made of the points where the extremals lose optimality— is obviously crucial in optimal control, but usually out of reach (even in low dimensions), and anyway does not have an analytic characterization because it is a non-local object. Fortunately, conjugate points —where the extremal lose *local* optimality— can be effectively computed with high accuracy for many control systems. Elaborating on the seminal work of the Russian and French schools (see [75], [30], [31] and [49] among others), efficient algorithms were designed to treat the smooth case. This was the starting point of a series of papers of members of the team culminating in the outcome of the *cotcot* software [40], followed by the *HamPath* [51] code. Over the years, these codes have allowed for the computation of conjugate loci in a wealth of situations including applications to space mechanics, quantum control, and more recently swimming at low Reynolds number.

With in mind the two-dimensional analytic Riemannian framework, a heuristic approach to the global issue of determining cut points is to search for singularities of the conjugate loci; this line is however very delicate to follow on problems stemming from applications in three or more dimensions (see *e.g.* [52] and [37]).

Recently, computation of conjugate points was conducted in [16], [2] to determine the optimality status in swimming at low Reynolds number; because of symmetries, and of the periodicity constraint, a tailor-made notion of conjugate point had to be used, and some additional sign conditions must be checked for local minimality, see more in [63]. In all these situations, the fundamental object underlying the analysis is the curvature tensor. In Hamiltonian terms, one considers the dynamics of subspaces (spanned by Jacobi fields) in the Lagrangian Grassmannian [28]. This point of view withstands generalizations far beyond the smooth case: In L^1-minimization, for instance, discontinuous curves in the Grassmannian have to be considered (instantaneous rotations of Lagrangian subspaces still obeying symplectic rules [56]).

The cut locus is a central object in Riemannian geometry, control and optimal transport. This is the motivation for the a series of conferences on “The cut locus: A bridge over differential geometry, optimal control, and transport”, co-organized by team members and Japanese colleagues. The first one took place during the summer, 2016, in Bangkok; the second one will take place the first week of September, 2018, in Sapporo.

3.2.2. Riemann and Finsler geometry

Studying the distance and minimising geodesics in Riemannian Geometry or Finsler Geometry is a particular case of optimal control, simpler because there are no differential constraints; it is studied in the team for the following two reasons. On the one hand, after some tranformations, like averaging (see section Section 3.4), and/or reduction, some more difficult optimal control problems lead to a Riemann or Finsler geometry problem, that have been much studied and known facts from these areas are useful. On the other hand, optimal control, mostly the Hamiltonian setting, brings a fresh viewpoint on problems in Riemann and Finsler geometry.

On Riemaniann ellipsoids of revolution, the optimal control approach allowed to decide on the convexity of the injectivity domain, which, associated with non-negativity of the Ma-Trudinger-Wang curvature tensor, ensures continuity of the optimal transport on the ambient Riemannian manifold [61], [60]. The analysis in the oblate geometry [38] was completed in [54] in the prolate one, including a preliminary analysis of non-focal domains associated with conjugate loci.

Averaging in systems coming from space mechanics control (see sections 3.4 and 4.1) with L^2-minimization yields a Riemannian metric, thoroughly computed in [35] together with its geodesic flow; in reduced dimension, its conjugate and cut loci were computed in [39] with Japanese Riemaniann geometers. Averaging the same systems for minimum time yields a Finsler Metric, as noted in [34]. In [47], the geodesic convexity properties of these two types of metrics were compared. When perturbation (other than the control) are considered, they introduce a “drift”, *i.e.* the Finsler metric is no longer symmetric.

3.2.3. Sub-Riemannian Geometry

Optimal control problems that pertain to sub-Riemannian Geometry bear all the difficulties of optimal control, like the role of singular/abnormal trajectories, while having some useful structure. They lead to many open problems, like smoothness of minimisers, see the recent monograph [66] for an introduction. Let us detail one open question related to these singular trajectories: the Sard conjecture in sub-Riemannian geometry.

Given a totally non-holonomic distribution on a smooth manifold, the Sard Conjecture is concerned with the size of the set of points that can be reached by singular horizontal paths starting from a given point. In the setting of rank-two distributions in dimension three, the Sard conjecture is that this set should be a subset of the so-called Martinet surface, indeed small both in measure and in dimension. In [33], it has been proved that the conjecture holds in the case where the Martinet surface is smooth. Moreover, the case of singular real-analytic Martinet surfaces was also addressed. In this case, it was shown that the Sard Conjecture holds true under an assumption of non-transversality of the distribution on the singular set of the Martinet surface. It is, of course, very intersting to get rid of the remaining technical assumption, or to go to higher dimension. Note that any that Sard-type result has strong consequences on the regularity of sub-Riemannian distance functions and in turn on optimal transport problems in the sub-Riemannian setting.

3.2.4. Singularities

The analysis of singularities in optimal control yields some more interplay with Hamiltonian dynamics. The Hamiltonian setting, much more than the Lagrangian one used in Riemannian geometry, is instrumental to treat such degeneracies. In fact, the latter do not really create singularities in the Pontryagin Maximum Principle equations. Almost-Riemannian metrics on the two-sphere appear after averaging the Pontryagin Maximum Principle for a quadratic cost (these metrics on the two-sphere are thoroughly described, with their degenracies, in [36]), or in the control of a quantum system with Ising coupling of three spins [45].

Another example comes from the analysis of singularities arising in minimum time systems. Consider a control affine system in dimension four with control on the disc such that the controlled fields together with their first order Lie brackets with the drift have full rank. There is a natural stratification of the codimension two singular set in the cotangent bundle leading to a local classification of extremals in terms of singular and bang arcs. This analysis was done in [52] using the nilpotent model, and extended in [27] by interpreting the singularities of the extremal flow as equilibrium points of a regularized dynamics to prove the continuity of the flow. One can actually treat these singularities as connections of pairs of normally hyperbolic invariant manifolds in order to find a suitable stratification of the flow and prove finer regularity properties. Another issue is to be able to give global bounds on the number of these heteroclinic connections. This work in progress is part of M. Orieux PhD thesis in collaboration with J. Féjoz at U. Paris-Dauphine.

3.2.5. Optimality of periodic solutions/periodic controls.

When seeking to minimize a cost with the constraint that the controls and/or part of the states are periodic (and with other initial and final conditions), the notion of conjugate points is more difficult than with straightforward fixed initial point. In [43], for the problem of optimizing the efficiency of the displacement of some micro-swimmers (see section 4.3) with periodic deformations, we used the sufficient optimality conditions established by Vinter's group [79], [63] for systems with non unique minimizers due to the existence of a group of symmetry (always present with a periodic minimizer-candidate control). This takes place in a long term collaboration with P. Bettiol (Univ. Bretagne Ouest) on second order sufficient optimality conditions for periodic solutions, or in the presence of higher dimensional symmetry groups, following [79], [63].

Another question relevant to locomotion is: how minimizing is it to use periodic deformations ? Observing animals (or humans), or numerically solving the optimal control problem associated with driftless micro-swimmers for various initial and final conditions, we remark that the optimal strategies of deformation seem to be periodic, at least asymptotically for large distances. This observation is the starting point for characterizing dynamics for which some optimal solutions are periodic, and are asymptotically attract other solutions as the final time grows large; this is reminiscent of the "turnpike theorem" (classical, recently applied to nonlinear situations in [77]).

3.2.6. Software

These applications (but also the development of theory where numerical experiments can be very enlightening) require many algorithmic and numerical developments that are an important side of the team activity. The software *HamPath* (see section 5.1) is maintained by former members of the team in close collaboration with McTAO. We also use direct discretization approaches (such as the Bocop solver developed by COMMANDS) in parallel. Apart from this, we develop on-demand algorithms and pieces of software, for instance we have to interact with a production software developed by Thales Alenia Space.

A strong asset of the team is the interplay of its expertise in geometric control theory with applications and algorithms (see sections 4.1 to 4.3) on one hand, and with optimal transport, and more recently Hamiltonian dynamics, on the other.

3.3. Optimal Transport

Given two measures, and calling transport maps the maps that transport the first measure into the second one, the Monge-Kantorovich problem of Optimal Transport is the search of the minimum of some cost on the set of transport maps. The cost of a map usually comes from some point to point cost and the transporte measure. This topic attracted renewed attention in the last decade, and has ongoing applications of many types, see section 4.5. Matching optimal transport with geometric control theory is one originality of our team. Work in the team has been concerned with optimal transport originating from Riemannian geometry ([61] gives strong conditions for continuity of the transport map, [38], [54] checks these conditions on ellipsoids, [60] studies them on more general Riemannian manifolds), sub-Riemannian geometry (see section 6.2.2 and Zeinab Badreddine's PhD) or more general optimal control costs [64].

Let us sketch an important class of open problems. In collaboration with R. McCann [65], we worked towards identifying the costs that admit unique optimizers in the Monge-Kantorovich problem of optimal transport between arbitrary probability densities. For smooth costs and densities on compact manifolds, the only known examples for which the optimal solution is always unique require at least one of the two underlying spaces to be homeomorphic to a sphere. We have introduced a multivalued dynamics induced by the transportation cost between the target and source space, for which the presence or absence of a sufficiently large set of periodic trajectories plays a role in determining whether or not optimal transport is necessarily unique. This insight allows us to construct smooth costs on a pair of compact manifolds with arbitrary topology, so that the optimal transport between any pair of probability densities is unique. We investigated further this problem of uniquely minimizing costs and obtained in collaboration with Abbas Moameni [24] a result of density of uniquely minimizing costs in the C^0-topology. The results in higher topology should be the subject on some further research.

3.4. Small controls and conservative systems, averaging

Using averaging techniques to study small perturbations of integrable Hamiltonian systems is as old an idea as celestial mechanics. It is very subtle in the case of multiple periods but more elementary in the single period case, here it boils down to taking the average of the perturbation along each periodic orbit [32], [74].

This line of research stemmed out of applications to space engineering (see section 4.1): the control of the super-integrable Keplerian motion of a spacecraft orbiting around the Earth is an example of a slow-fast controlled system. Since weak propulsion is used, the control itself acts as a perturbation, among other perturbations of similar magnitudes: higher order terms of the Earth potential (including J_2 effect, first), potential of more distant celestial bodies (such as the Sun and the Moon), atmospheric drag, or even radiation pressure.

Properly qualifying the convergence properties (when the small parameter goes to zero) is important and is made difficult by the presence of control. In [34], convergence is seen as convergence to a differential inclusion; this applies to minimum time; a contribution of this work is to put forward the metric character of the averaged system by yielding a Finsler metric (see section 3.2.2). Proving convergence of the extremals (solutions of the Pontryagin Maximum Principle) is more intricate. In [20], standard averaging ([32], [74]) is performed on the minimum time extremal flow after carefully identifying slow variables of the system thanks to a symplectic reduction. This alternative approach allows to retrieve the previous metric approximation, and to partly address the question of convergence. Under suitable assumptions on a given geodesic of the averaged system (disconjugacy conditions, namely), one proves existence of a family of quasi-extremals for the original system that converge towards the geodesic when the small perturbation parameter goes to zero. This needs to be improved, but convergence of all extremals to extremals of an “averaged Pontryagin Maximum Principle” certainly fails. In particular, one cannot hope for C^1-regularity on the value function when the small parameter goes to zero as swallowtail-like singularities due to the structure of local minima in the problem are expected. (A preliminary analysis has been made in [53].)

3.5. Other topics

The above does not cover all the fundamentals underlying our research.

- There is a wealth of techniques in linear control and linear identification that we have not mentioned in section 3.1 and are used in work described in sections 4.4 and 6.7. Stability analysis of dynamical systems is also present here.
- Stochastic control, or more precisely optimal control with stochastic constraints is covered in section 6.6.1.
- Analysis and structurally stable properties of Hamiltonian dynamics is the domain section 6.1 is most relevant to.

4. Application Domains

4.1. Aerospace Engineering

Space engineering is very demanding in terms of safe and high-performance control laws. It is therefore prone to fruitful industrial collaborations.

McTAO now has an established expertise in space and celestial mechanics. Our collaborations with industry are mostly on orbit transfer problems with low-thrust propulsion. It can be orbit transfer to put a commercial satellite on station, in which case the dynamics are a Newtonian force field plus perturbations and the small control. There is also, currently, a renewed interest in low-thrust missions such as Lisa Pathfinder (ESA mission towards a Lagrange point of the Sun-Earth system) or BepiColombo (joint ESA-JAXA mission towards Mercury). Such missions look more like a controlled multibody system. In all cases the problem involves long orbit transfers, typically with many revolutions around the primary celestial body. When minimizing time, averaging techniques provide a good approximation. Another important criterion in practice is fuel consumption minimization (crucial because only a finite amount of fuel is onboard a satellite for all its “life”), which amounts to L^1-minimization. Both topics are studied by the team.

Another application is the optimization of space launchers, this lead to issues in stochastic optimization, see section 6.6.1.

We have a steady relationships with CNES and Thales Alenia Space (Cannes), that have financed or co-financed 3 PhDs and 2 post-docs in the Sophia location of the team in the decade and are a source of inspiration even at the methodological level. Team members also have close connections with Airbus-Safran (Les Mureaux) on launchers.

Some of the authoritative papers in the field were written by team members, with an emphasis on the geometric analysis and on algorithms (coupling of shooting and continuation methods). There are also connections with peers more on the applied side, like D. Scheeres (Colorado Center for Astrodynamics Research at Boulder), the group of F. Bernelli (Politecnico Milano), and colleagues from U. Barcelona (A. Farrès, A. Jorba).

4.2. Magnetic resonance imaging (MRI)

Participants: Bernard Bonnard, Alice Nolot, Jérémy Rouot, Joseph Gergaud, Olivier Cots [ENSEEIHT, Toulouse], Stephen Glaser [TU München, Germany], Dominique Sugny [Univ de Bourgogne].

The starting point of our interest in optimal control for quantum systems was a collaboration with physicist from ICB, University of Burgundy (Dominique Sugny), motivated by an ANR project where we worked on the control of molecular orientation in a dissipative environment using a laser field, and developed optimal control tools, combined with numerical simulations, to analyze the problem for Qubits. This was related to quantum computing rather than MRI.

Using this expertise and under the impulse of Prof. S. Glaser and his group (Chemistry, TU München), we investigated Nuclear Magnetic resonance (NMR) for medical imaging (MRI), where the model is the Bloch equation describing the evolution of the Magnetization vector controlled by a magnetic field, but in fine is a specific Qubit model without decoherence. We worked on, and brought strong contributions to, the contrast problem: typically, given two chemical substances that have an importance in medicine, like oxygenated and de-oxygenated blood, find the (time-dependent) magnetic field that will produce the highest difference in brightness between these two species on the image resulting from Nuclear Magnetic Resonance. This has immediate and important industrial applications in medical imaging. Our contacts are with the above mentioned physics academic labs, who are themselves in contact with major companies.

The team has produced and is producing important work on this problem. One may find a good overview in [46], a reference book has been published on the topic [50], a very complete numerical study comparing different optimization techniques was performed in [44]. We conduct this project in parallel with S. Glaser team, which validated experimentally the pertinence of the methods, the main achievement being the in vivo experiments realized at the Creatis team of Insa Lyon showing the interest to use optimal control methods implemented in modern softwares in MRI in order to produce a better image in a shorter time. A goal is to arrive to a cartography of the optimal contrast with respect to the relaxation parameters using LMI techniques and numerical simulations with the Hamapth and Bocop code; note that the theoretical study is connected to the problem of understanding the behavior of the extremal solutions of a controlled pair of Bloch equations, and this is an ambitious task. Also, one of the difficulties to go from the obtained results, checkable on experiments, to practical control laws for production is to deal with magnetic field space inhomogeneities and poorly known relaxation parameters.

4.3. Swimming at low-Reynolds number

Participants: Bernard Bonnard, Yacine El Alaoui-Faris, Laetitia Giraldi, Clément Moreau, Alice Nolot, Jean-Baptiste Pomet, Jérémy Rouot, Karine Sérier.

Following the historical reference for low Reynolds number locomotion [71], the study of the swimming strategies of micro-organisms is attracting increasing attention in the recent literature. This is both because of the intrinsic biological interest, and for the possible implications these studies may have on the design of bio-inspired artificial replicas reproducing the functionalities of biological systems. In the case of micro-swimmers, the surrounding fluid is dominated by the viscosity effects of the water and becomes reversible. In this regime, it turns out that the infinite dimensional dynamics of the fluid do not have to be retained as state variables, so that the dynamics of a micro-swimmer can be expressed by ordinary differential equations if its shape has a finite number of degrees of freedom. Assuming this finite dimension, and if the control is the rate of deformation, one obtains a control system that is linear (affine without drift) with respect to the controls, *i.e.* the optimal control problem with a quadratic cost defines a sub-Riemannian structure (see section 3.2.3). This is the case where the shape is “fully actuated”, *i.e.* if all the variables describing the shape are angles, there is an actuator on each of these angles. For artificial micro-swimmers, this is usually unrealistic, hence (artificial) magneto-elastic micro-swimmers, that are magnetized in order to be deformed by an external magnetic field. In this case, the control functions are the external magnetic field.

In both cases, questions are controllability (straightforward in the fully actuated case), optimal control, possibly path planning. We collaborate with teams that have physical experiments for both.

- In collaboration with D. Takagi and M. Chyba (Univ of Hawaii), this approach is currently at the experimental level for copepod-like swimmer at the university of Hawaii: on the one hand, this zooplankton and its locomotion can be observed, and a robot micro swimmer mimicking a copepod has been constructed, but in fact large enough for direct actuation to be possible, and the low Reynolds number is achieved by using a more viscous fluid. This gives possibilities, through an inverse optimization problem, to determine what cost can be optimised by these crustaceans, see [2], [76], and to validate models on the robot.
- For magneto-elastic micro-robots, Y. El-Alaoui’s PhD is co-advised with Stéphane Régnier from the robotics lab ISIR, Univ. Paris 6. Magneto-elastic micro-robots and their magnetic actuation are

actually built at ISIR and the aim of the collaboration is to validate models and improve the existing control laws both in performance and in energy; of course, the micro scale does make things difficult.

The questions about optimality of periodic controls raised in section 3.2.5 are related to these applications for periodic deformations, or strokes, playan important role in locomotion.

4.4. Stability of high frequency amplifiers

Participants: Sébastien Fueyo, Jean-Baptiste Pomet, Laurent Baratchart [APICS project-team APICS (FACTAS as of 2018)].

Nonlinear hyper-frequency amplifiers contain on the one hand nonlinear active components and on the other hand “lines”, that induce some sort of delays and make the system infinite-dimensional. Computer assisted design tools are extensively used. They provide frequency responses but fail to provide a reliable estimation of their stability, and this stability is crucial because an unstable response will not be observed in practice and the engineer needs to have this information between building the actual device. It is, of course, of utmost importance to find means to predict stability/instability from simulations in the Computer assisted design tool.

Sébastien Fueyo’s PhD is co-advised between McTAO and APICS on this topic. It is an opportunity to build theoretical basis and justification to a stability analysis through harmonic identification; the latter is one of the specialties of APICS, we collaborate on the infinite-dimensional non-linear stability analysis for periodic solutions and how it works with the results of harmonic identification.

Potential transfer is important.

4.5. Applications of optimal transport

Image processing, biology, fluid mechanics, mathematical physics, game theory, traffic planning, financial mathematics, economics are among the most popular fields of application of the general theory of optimal transport. Many developments have been made in all these fields recently. Three more specific examples:

- In image processing, since a grey-scale image may be viewed as a measure, optimal transportation has been used because it gives a distance between measures corresponding to the optimal cost of moving densities from one to the other, see e.g. the work of J.-M. Morel and co-workers [67].
- In representation and approximation of geometric shapes, say by point-cloud sampling, it is also interesting to associate a measure, rather than just a geometric locus, to a distribution of points (this gives a small importance to exceptional “outlier” mistaken points, see [55]). The relevant distance between measures is again the one coming from optimal transportation.
- A fluid motion or a crowd movement can be seen as the evolution of a density in a given space. If constraints are given on the directions in which these densities can evolve, we are in the framework of non-holonomic transport problems, i.e. these where the cost comes from the point-to-point sub-Riemannian distance, or more general optimal control costs.

5. New Software and Platforms

5.1. Hampath

KEYWORDS: Optimal control - Second order conditions - Differential homotopy - Ordinary differential equations

FUNCTIONAL DESCRIPTION: Hampath is a software developped to solve optimal control problems by a combination of Hamiltonian et path following methods. Hampath includes shooting and computation of conjugate points. It is an evolution of the software cotcot (apo.enseeiht.fr/cotcot). It has a Fortran kernel, uses Tapenade (www-sop.inria.fr/tropics/tapenade.html) for automatic differentiation and has a Matlab interface.

- Participants: Jean-Baptiste Caillau, Joseph Gergaud and Olivier Cots
- Contact: Jean-Baptiste Caillau
- URL: http://www.hampath.org

6. New Results

6.1. Stability properties of geodesic flows on Riemannian manifolds

Participants: Ludovic Rifford, Rafael Ruggiero [PUC, Rio de Janeiro, Brazil].

In a paper by Rifford and Ruggiero [25], the C^2-structural stability conjecture from Mañé's viewpoint for geodesics flows of compact manifolds without conjugate points is investigated. The structural stability conjecture is an open problem in the category of geodesic flows because the C^1 closing lemma is not known in this context. Without the C^1 closing lemma, we combine the geometry of manifolds without conjugate points and a recent version of Franks' Lemma from Mañé's viewpoint to prove the conjecture for compact surfaces, for compact three dimensional manifolds with quasi-convex universal coverings where geodesic rays diverge, and for n-dimensional, generalized rank one manifolds.

6.2. Optimal transport and sub-Riemannian geometry

6.2.1. Uniquely minimizing costs for the Kantorovitch problem

Participants: Ludovic Rifford, Robert Mccann [Univ of Toronto, Canada], Abbas Moameni [Carleton Univ, Ottawa, Canada].

In continuation of the work by McCann and Rifford [65], a paper by Moameni and Rifford [24] study some conditions on the cost which are sufficient for the uniqueness of optimal plans (provided that the measures are absolutely continuous with respect to the Lebesgue measure). As a by-product of their results, the authors show that the costs which are uniquely minimizing for the Kantorovitch problem are dense in the C^0-topology. Many others applications and examples are investigated.

6.2.2. The Sard conjecture in sub-Riemannian geometry, optimal transport and measure contraction properties

Participants: Zeinab Badreddine, Ludovic Rifford.

Zeinab Badreddine [13] obtained the first result of well-posedness for the Monge problem in the sub-Riemannian setting in the presence singular minimizing curves. This study is related to the so-called measure contraction property. In collaboration with Rifford [14], Badreddine obtained new classes of sub-Riemannian structures satisfying measure contraction properties.

6.3. Optimal control of fully actuated micro-swimmers

6.3.1. A general geometric approach of optimal strokes for driftless micro-swimmers

Participants: Thomas Chambrion [Univ. Lorraine], Laetitia Giraldi, Alexandre Munnier [Univ. Lorraine].

In [3], we study the control problem associated to the locomotion of a deformable swimmer. we present a unified geometric approach for optimization of the body deformation of the swimmers in a 3D Stokes flow (case of micro-swimmers) and 2D or 3D potential flow. The latter cases correspond to the analysis of the sphere in a sub-Riemannian space. A general framework is introduced, allowing the complete analysis of five usual nonlinear optimization problems to be carried out. The results are illustrated with examples and with a in-depth study of a swimmer in a 2D potential flow. Numerical tests are also provided.

6.3.2. Optimal periodic strokes for the Copepod and Purcell micro-swimmers

Participants: Piernicola Bettiol [Uni. Bretagne Ouest], Bernard Bonnard, Alice Nolot, Jérémy Rouot.

We have analyzed the problem of optimizing the efficiency of the displacement of two micro swimmers with slender links, namely the following two models: the symmetric micro swimmer introduced by Takagi (see [43], this model describes the locomotion of the micro crustaceans named copepod), and the historical three link Purcell swimmer. The problems are studied in the framework of optimal control theory and SR geometry vs the standard curvature control point of view. Our contribution is to determine the optimal solutions combining geometric analysis and adapted numerical scheme. In particular the nilpotent models introduced in SR geometry allow to make a neat analysis of the problem of determining optimal strokes with small amplitudes and numerical continuation methods are then applied to compute more general stroke. This approach is completely original in optimal control. Also necessary and sufficient optimality conditions are applied to select the topology of optimal strokes (simple loops) and to determine the optimal solution in both cases, see [16]. Also note that in collaboration with D. Takagi and M. Chyba, this approach is currently at the experimental level at the university of Hawaii using a robot micro swimmer mimicking a copepod, see above. More theoretical issues in relation with SR geometry are investigated in the framework of A. Nolot's starting PhD (started August, 2016) and K. Sérier's PhD (started September, 2017), see [10], [42] and other publications under review.

6.4. Modelling and controllability of Magneto-elastic Micro-swimmers

6.4.1. Purcell magneto-elastic swimmer controlled by an external magnetic field

Participants: François Alouges [École Polytechnique], Antonio Desimone [SISSA Trieste, Italy], Laetitia Giraldi, Marta Zoppello [Univ. di Padova, Italy].

We have studied the mechanism of propulsion of a Purcell swimmer whose segments are magnetized and react to an external magnetic field applied into the fluid. By an asymptotic analysis, we prove that it is possible to steer the swimmer along a chosen direction when the control functions are prescribed as an oscillating field. Moreover, there are obstructions that have to be overcome in order to get classical controllability result for this system. This is exposed in [7] (IFAC World Congress, Toulouse, July 2017).

6.4.2. Local Controllability of the Two-link Magneto-elastic Micro-swimmer

Participants: Laetitia Giraldi, Pierre Lissy [Univ. Paris Dauphine], Clément Moreau, Jean-Baptiste Pomet.

For the smallest magneto-elastic micro-swimmer (2 links), we have been able to prove a strong local controllability result around its straight position of the swimmer. This is exposed in[6]. However, the latter result is weaker than the classical local controllability concept called STLC which means that a system could reach any position around its equilibrium with a small control (as small as the desired displacement of the system). Moreover, we prove in [5] that the 2-link magneto-elastic swimmer is indeed *not* STLC.

6.5. Numerical aspect of the N-link micro-swimmer model

Participants: Hermes Gadhêla [Univ. of York, UK], Laetitia Giraldi, Clément Moreau, Jean-Baptiste Pomet.

This topic was initiated with a 1 year research invitation of Clément Moreau at University of York and further collaboration. The goal is to compare the ODE given by the "N-link swimmer" model with the PDE for an elastic rod.

In [22], we study inertialess fluid-structure interaction of active and passive inextensible filaments. In this work, we compare two different approaches that lead to model the behavior of a microscopic elastic filament immersed into a fluid. The first which derives from a continuous formalism corresponds to solve a PDE, the second method exploits the momentum balance in the asymptotic limit of small rod-like elements which are integrated semi-analytically. The equivalence between the continuous and asymptotic model allows a direct comparison between the two formalisms. The asymptotic model is simple and intuitive to implement, and generalisations for complex interaction of multiple rods. We demonstrate these via four benchmarks: transient dynamics, force-displacement buckling instability, magnetic artificial swimmer and cross-linked filament-bundle dynamics.

6.6. Optimal Control and Averaging in Aerospace Engineering

6.6.1. Chance-constrained optimal control problems in aerospace

Participants: Jean-Baptiste Caillau, Max Cerf [Airbus Safran Launchers], Achille Sassi [ENSTA Paristech], Emmanuel Trélat [Univ. Paris VI], Hasnaa Zidani [ENSTA Paristech].

The aim is to minimize the fuel mass of the last stage of a three-stage launcher. Since the design parameters of the spacecraft are not exactly known prior to the launch, uncertainties have to be taken into account. Although these parameters are supposed to be uniformly distributed on fixed ranges, it is not desirable to use "worst-case" robust optimization as the problem may not even be feasible for some values of the parameters due to very strong sensitivities. The idea is to frame instead a stochastic optimization problem where these parameters are independent stochastic variables. The original constraint becomes a stochastic variable, and one only asks that the desired target is reached with some given probability. A key issue in solving this chance constrained problem is to approximate the probability density function of the constraint. Contrary to Monte-Carlo methods that require a large number of runs, kernel density estimation [68] has the strong advantage to permit to build an estimator with just a few constraint evaluations. This approach allows to treat efficiently uncertainties on several design parameters of the launcher, including the specific impulse and index of the third stage and using a simple affine discretization of the control (pitch angle). In [19], we use the Kernel Density Estimation method to approximate the probability density function of a random variable with unknown distribution, from a relatively small sample, and we show how this technique can be applied and implemented for a class of problems including the Goddard problem (with bang-bang or bang-singular-bang controls) and the trajectory optimization of an Ariane 5-like launcher. This work has been done in collaboration with Airbus Safran Launchers at Les Mureaux.

An involved question in chance constrained optimization is the existence and computation of the derivative of the stochastic constraint with respect to deterministic parameter. This shall be investigated in the light of new results in the Gaussian case [78]. Using a single deterministic control to reach a given target (or a given level of performance) with some fixed probability when the parameters of the system are randomly distributed is very similar to issues of ensemble controllability addressed in the recent work [26]. One expects some insight from the comparison of the two viewpoints.

6.6.2. Metric approximation of minimum time control systems

Participants: Jean-Baptiste Caillau, Lamberto Dell'Elce, Jean-Baptiste Pomet, Jérémy Rouot.

Slow-fast affine control systems with one fast angle are considered in this work [20]. An approximation based on standard averaging of the extremal is defined. When the drift of the original system is small enough, this approximation is metric, and minimum time trajectories of the original system converge towards geodesics of a Finsler metric. The asymmetry of the metric accounts for the presence of the drift on the slow part of the original dynamics. The example of the J_2 effect in the two-body case in space mechanics is examined. A critical ratio between the J_2 drift and the thrust level of the engine is defined in terms of the averaged metric. The qualitative behaviour of the minimum time for the real system is analyzed thanks to this ratio. Work in progress aims at dealing with multiphase averaging for systems driven by several fast angles.

6.6.3. Approximation by filtering in optimal control and applications

Participants: Jean-Baptiste Caillau, Thierry Dargent [Thales Alenia Space], Florentina Nicolau [Univ. Cergy-Pontoise].

Minimum time control of slow-fast systems is considered in this analysis [8]. In the case of only one fast angle, averaging techniques are available for such systems. The approach introduced in [57] and [34] is recalled, then extended to time-dependent systems by means of a suitable filtering operator. The process relies upon approximating the dynamics by means of sliding windows. The size of these windows is an additional parameter that provides intermediate approximations between averaging over the whole fast angle period and the original dynamics. The motivation is that averaging over an entire period may not provide a good enough approximation to initialize a convergent numerical resolution of the original system; considering a continuous set of intermediate approximations (filtering over windows of size varying from the period to zero) may ensure convergence. The method is illustrated on problems coming from space mechanics and has been implemented as an addition to the industrial code `T3D` of Thales Alenia Space.

6.6.4. Higher order averaging

Participants: Jean-Baptiste Pomet, Thierry Dargent [Thales Alenia Space], Florentina Nicolau [Univ. Cergy-Pontoise].

A further step in defining a suitable approximation of slow-fast oscillating controlled systems is to go beyond the $O(\varepsilon)$ uniform error provided by simple averaging. An original approach has been proposed in [58] and demonstrated numerically; it consists in correcting the boundary values of the slow averaged variables to ensure an $O(\varepsilon^2)$ *average* error, without the difficulties of classical second order averaging [73] (that leads to an $O(\varepsilon^2)$ *uniform* error, that we do not need), and allows an $O(\varepsilon)$ approximation of the angle. It is proved in [9] that it is indeed possible, at least for initial value problems, to compute order one corrections of the initial slow variables to guarantee such an error. From the numerical side, this process is a key to be able to initialize shooting methods on the non-averaged system by averaged solutions when using a model with full perturbations in orbit transfer.

6.7. Stability of nonlinear high frequency amplifiers

Participants: Sébastien Fueyo, Jean-Baptiste Pomet, Laurent Baratchart [APICS project-team APICS (FACTAS as of 2018)].

Sébastien Fueyo's PhD is co-advised between McTAO and APICS on this topic. The problem is presented in section 4.4.

Starting from infinite dimensional time-domain models for these devices, we obtained full justification (with some possible obstructions) to the prediction of stability through transfer function identification on academic examples of simple circuits, and are working on generalisations. A preliminary presentation will be given at a local conference, Université Côte d'Azur Complex Days, in January, 2018.

7. Bilateral Contracts and Grants with Industry

7.1. Bilateral Contract with Industry: CNES - Inria - UB Contract

Contract number: 130777/00. Call Number: R-S13/BS-005-012

"Perturbations and averaging for low thrust" (Poussée faible et moyennation).
Research contract between CNES and McTAO (both the Inria and the Université de Bourgogne parts). It run from 2014 till mid-2017. It concerned averaging techniques in orbit transfers around the earth while taking into account many perturbations of the main force (gravity for the earth considered as circular). The objective was to validate numerically and theoretically the approximations made by using averaging, and to propose methods that refine the approximation. It has co-funded the PhD thesis of Jérémy Rouot (defended in October, 2016, also co-funded by Région PACA) and fully funded the postdoc of Florentina Nicolau and 2016 [9], [8] and the postdoc of Lamberto dell'Elce this year.

8. Partnerships and Cooperations

8.1. National Initiatives

8.1.1. ANR

Weak KAM beyond Hamilton-Jacobi (WKBHJ). Started 2013 (decision ANR-12-BS01-0020 of December 19, 2012), duration: 4 years. L. Rifford is in the scientific committee.

Sub-Riemannian Geometry and Interactions (SRGI). Started 2015 (decision ANR-15-CE40-0018), duration: 4 years. L. Rifford is a member.

Intéractions Systèmes Dynamiques Équations d'Évolution et Contrôle (ISDEEC). Started 2016 (decision ANR-16-CE40-0013), duration: 4 years. L. Rifford is a member.

Maximic: optimal control of microbial cells by natural and synthetic strategies. Started 2017, duration: 4 years. J.-B. Caillau, L. Giraldi, J.-B. Pomet are members.

8.1.2. Others

The McTAO team participates in the GdR MOA, a CNRS network on Mathematics of Optimization and Applications.

PGMO grant (2016-2017) on "Metric approximation of minimizing trajectories and applications" (PI J.-B. Caillau). This project involves colleagues from Université Paris Dauphine and has funding for one year, including one intership (M2 level).

PGMO grant (2017-2019) on "Algebro-geometric techniques with applications to global optimal control for Magnetic Resonance Imaging (MRI)". B. Bonnard, A. Nolot and J. Rouot participate in this project, the PI is O. Cots, from ENSEIHHT, Toulouse.

J.-B. Caillau is associate researcher of the team of the CNRS team Parallel Algorithms & Optimization team at ENSEEIHT, Univ. Toulouse.

Défi InfIniti CNRS project, Control and Optimality of Magnetic Microrobot, (PI L. Giraldi). Started 2017, duration: 1 years. This project involves colleagues from Université Paris 6, from University of York (UK) and University of Padova (Italie). Y. El Alaoui Faris, C. Moreau, L. Giraldi, J.-B. Pomet are members.

8.2. European Initiatives

8.2.1. Collaborations in European Programs, Except FP7 & H2020

Program: FCT (Fundação para a Ciência e a Tecnologia, Portugal)

Project acronym: None

Project title: Extremal spectral quantities and related problems

Duration: 2016-2019

Coordinator: P. Freitas

Other partners: Inria, Univ. Luxembourg, Univ. Lisbon, Prague Czech Technical Univ., Univ. Bern

Abstract: The purpose of this project is to combine analytic, geometric and computational techniques to study extremal values of different spectral quantities, such as individual eigenvalues, functions of these eigenvalues and some global spectral quantities. More specifically, some of the objects under consideration are the possible extremal sets of the first eigenvalue of the Laplacian with Robin boundary conditions, for which team members have recently shown that the ball is no longer an optimiser for large negative values of the boundary parameter, thus providing a counter-example to a 1977 conjecture, finite combinations of eigenvalues of the Laplace and Schrödinger operators, the functional determinant associated with these operators and the spectral abscissa of the (non self-adjoint) operator associated with the damped wave equation. To handle these problems a wide range of methods is required, including those from geometric analysis, functional analysis, control theory, numerical analysis, etc.

8.3. International Research Visitors

8.3.1. Visits of International Scientists

Hermes Gadhêla (Univ. of York), November, 2017. Collaboration on PDE vs ODE models for elastic swimmers.

Izhar Or (Technion), September, 2017. Collaboration on magnetic micro-swimmers.
Sorin Sabau (Tokai Univ.), November, 2017. Collaboration on Finsler geometry.
Romain Serra (Univ. of Strathclyde, Glasgow), October, 2017. Collaboration on space mechanics.
Martha Zoppello (University of Padova), February, 2017. Collaboration on magnetic micro-swimmers.

9. Dissemination

9.1. Promoting Scientific Activities

9.1.1. Scientific Events Organisation

9.1.1.1. General Chair, Scientific Chair

- J.-B. Caillau was member of the 2017 PGMO days scientific committee.

9.1.1.2. Member of the Organizing Committees

- 09/2017: Mini-symposium "Geometric control & applications", 18th French-German-Italian conference on optimization, Paderborn (J.-B. Caillau)
- 11/2017: Mini-symposium "Optimal control & applications to biology", PGMO days 2017, Paris Saclay (J.-B. Caillau)
- 12/2017 (Dijon): Journées McTAO: event organized by the team with thee invited speakers and two from the team.

9.1.2. Scientific Events Selection

- J.-B. Caillau is member of the program committee of the Séminaire de géométrie hamiltonienne of Paris VI - UPMC.

9.1.3. Books

- B. Bonnard and J. Rouot, together with M. Chyba, have written the series of notes [17], soon to appear as Springer briefs Publications. They were the basis of courses at the Phd level given at the University of Burgundy and at the institute of Mathematics for industry at Fukuoka (Japan).
- J.-B. Caillau, together with M. Bergounioux, G. Peyré, C. Schnörr and T. Haberkorn, served as an editor for the volume [11]. With a focus on the interplay between mathematics and applications of imaging, the first part covers topics from optimization, inverse problems and shape spaces to computer vision and computational anatomy. The second part is geared towards geometric control and related topics, including Riemannian geometry, celestial mechanics and quantum control.

9.1.4. Journals

B. Bonnard is a member of the editorial board of the *Pacific Journal of Mathematics for Industry*.

9.1.5. Invited Talks

Jean-Baptiste Caillau:
02/2017: Séminaire ENAC, Toulouse
06/2017: Mathematical Control Theory, Porquerolles
07/2017: New Horizons on Optimal Control, Porto
11/2017: PGMO Days 2017, Paris Saclay (with Barlaud, M.; Gilet, C.)

Laetitia Giraldi:
03/2017: Seminar at University Paris Dauphine
05/2017: Speaker at a meeting of the IPL Algae in Silico
09/2017: Seminar at Gibsa-lab, *Grenoble*
10/2017: Speaker at Interaction Fluide-Structure: Analyse et Contrôle
11/2017: Seminar at Institut de Recherche Mathématiques Avancée, *Strasbourg*

Michaël Orieux:
09/2017: Workshop on Classical Integrability and Perturbations, *Paris*

Jean-Baptiste Pomet:
09/2017: Séminaire de géométrie hamiltonienne of Paris VI - UPMC.

Ludovic Rifford:
07/2017: Plenary speaker at the Pan African Congress of Mathematics, *Rabat (Morocco)*
07/2017: Plenary speaker at the conference New Trends in Control Theory and PDEs, *INdAM, Rome (Italy)*
09/2017: Plenary speaker at the Conférence à la mémoire d'Ahmad El Soufi, *Université François Rabelais, Tours (France)*

9.1.6. Research Administration

Jean-Baptiste Caillau has been the joint head of the CNRS team Statistique, Probabilités, Optimisation & Contrôle at Institut math. Bourgogne. He is member of the Scientific Committee of the GdR Calcul, of the Institut de Mécanique Céleste et de Calcul des Éphémérides (Observatoire de Paris), and of the Programme Gaspard Monge pour l'Optimisation et la recherche opérationnelle de la Fondation Mathématique Jacques Hadamard.

From 2017 Laetitia Giraldi is member of CSD *Comité du suivi Doctoral* at Sophia-Antipolis.

Jean-Baptiste Pomet is a member of the steering committee of the Center for Planetary Origin (C4PO) and of the scientific council of Académie 2 “Complex system”, both for Université Côte d'Azur (UCA). He is an elected member of Commission d'évaluation (Inria permanent evaluation committee).

Ludovic Rifford has been Executive Director of the CIMPA (Centre International de Mathématiques Pures et Appliquées) since September 2016.

9.2. Teaching - Supervision - Juries

9.2.1. Teaching

Licence : Laetitia Giraldi, Colles de Mathématiques en MPSI et MP, 110heures en équivalent TD, niveau (L1, L2, L3), Lycée internationale de Valbonne, France. J.-B. Caillau, Processus Stochastiques, 50 H ETD, Polytech Nice-Sophia.

Master : Laetitia Giraldi, Théorie du Contrôle appliquée à des problèmes de micro-natation, 4 heures en équivalent TD, niveau M2, université de Strasbourg, France. J.-B. Caillau, Commande optimale, 50 H ETD, Polytech Nice-Sophia ; Optimisation, 30 H ETD, Univ. Côte d'Azur.

Doctorat : Laetitia Giraldi, Controlled propulsion of elastico-magnetic micro-swimmer, 4 heures en équivalent TD, université de Padoue, Italie

9.2.2. Supervision

M2: Yacine El Alaoui-Faris: “Machine Learning for Biology ”, supervised by J.-B. Caillau, April to September.

PhD: Zeinab Badreddine, “Mass transportation in sub-Riemannian structures admitting singular minimizing geodesics”, defended December 4, 2017, co-supervised by B. Bonnard and L. Rifford. See bibliography.

PhD in progress : Michaël Orieux, “Minimum time control and applications”, started September, 2015, co-supervised by J.-B. Caillau and J. Féjoz (Univ. Paris Dauphine).

PhD in progress : Alice Nolot, “Sub-Riemannian geometry and optimal swimming at low Reynolds, started October, 2016, supervised by B. Bonnard.

PhD in progress : Sébastien Fueyo, “Testing stability of nonlinear amplifier by frequency-domain methods”, started October, 2016, co-supervised by J.-B. Pomet and L. Baratchart (APICS team).

PhD in progress : Yacine El alaoui-faris, “modeling magnetico-elastic micro-robot from theory to experiment”, started October, 2017, co-supervised by L. Giraldi, J.-B. Pomet and Stephane Régnier (Univ. Paris Sorbonne).

PhD in progress : Clément Moreau, “Contrôlabilité de systèmes en dimension finie ou infinie issus du vivant”, started September, 2017, co-supervised by L. Giraldi, Pierre Lissy and J.-B. Pomet.

PhD in progress : Karine Sérier, “Micro-natation et invariants de la géométrie sous-Riemannienne”, started September, 2017, supervised by B. Bonnard.

9.2.3. Juries

J.-B. Caillau has been reviewer for the Habilitation of Aude Rondepierre (Toulouse), and for the PhD’s of Cécile Carrere (Marseille), Clément Gazzino (Toulouse), and Ouazna Ouchaka (Toulon). He has also been member of the Habilitation jury of Marco Caponigro (Paris).

J.-B. Caillau and L. Giraldi are members of the jury of Agrégation de mathématiques.

Ludovic Rifford has been reviewer for the PhD of Sebastiano Nicolussi Golo (University of Jyvaskyla, Finland).

9.3. Popularization

J.-B. Caillau participates in the MASTIC initiative at Inria and has given conferences at the high school level in Sophia (MathC2+ event, June 2017) and Grasse (Lycée Amiral, Sep. 2017).

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Articles in International Peer-Reviewed Journals

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