Modélisation de la planification de mouvements humains par le contrôle optimal

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Modeling human movements

- Aim: understanding biological motor control in computational terms
  leading framework = optimal control theory
- Evidence of two steps: 1. planning stage, 2. execution stage
  → mainly stage 1 here
- Focus on short motions (arm pointing motions, eye saccades)
  ⇒ no available feedback in general (delay 50-100ms)
  → open-loop optimal control problems

Study of two phenomenon: duration of motions, muscle co-contraction

All in collaboration with Bastien BERRET (CIAMS - Univ. Paris Sud)
The value of time in the neural control of action

1. Cost of time
2. Recovering $g$
3. Inverse optimal control
4. Experimental results

Planning of muscle co-contraction

1. How to model muscle co-contraction?
2. Stochastic Open-loop OC
3. Application and interpretation
Outline

1. The value of time in the neural control of action
   - Cost of time
   - Recovering $g$
   - Inverse optimal control
   - Experimental results

2. Planning of muscle co-contraction
   - How to model muscle co-contraction?
   - Stochastic Open-loop OC
   - Application and interpretation
Human movements are neither too fast nor too slow

Movements are **not too fast** for several reasons

- Accuracy (speed-accuracy trade-off or Fitts’ law)
- Energy (e.g. work)
- Smoothness (e.g. jerk)
Movements are **not too slow**, but why?

- Slow movements seem good: small work, jerk, inaccuracy
- Gravity? Cosmonauts do not drastically increase movement time (MT) in 0g

![Graph A: Upward without weight](image)

![Graph C: Downward without weight](image)

[Papaxanthis et al., 1998]

- Cost of time theory: the brain implicitly penalizes the passage of time.
Movements are **not too slow**, but why?

- Slow movements seem good: small work, jerk, inaccuracy
- Gravity? Cosmonauts do not drastically increase movement time (MT) in 0g

![Graph showing MT (ms) for different flight conditions](image)

[Papaxanthis et al., 1998]

- Cost of time theory: the brain implicitly penalizes the passage of time.
Cost of time theory

Taking time to complete a task:
- delays its achievement and acquisition of reward,
- monopolizes neural and attentional resources.

→ Theory motivated by psycho-economical findings in decision-making

→ Introduced by R. Shadmehr in motor control

→ Duration results from a compromise
- Cost of the motion (**in fixed time**):
  - large literature [e.g. Todorov, Flash, Shadmehr, Berret, ...]
  - systematic approach (inverse optimal control)
- Cost of the time: lot of different modelling,
  - psychologists/economists $\rightarrow$ hyperbolic costs (concave functions)
  - other interpretations $\rightarrow$ exponential costs, convex functions

Only interpretations/intuitions, no quantitative results.

![Diagram](attachment:image.png)
Modelling

Dynamics of the motion: \( \dot{x} = f(x, u) \)

Paradigm

Any registered trajectory \( x(\cdot) \) from \( x^0 \) to \( x^f \) is an optimal solution of

\[
\min_u \int_0^{t_u} (g(t) + L(x_u(t), u(t))) \, dt,
\]

among all \( u(\cdot) \) defined on \([0, t_u]\) s.t. \( x_u(0) = x^0, x_u(t_u) = x^f \).

- \( \int_0^T g(t) \, dt \): cost of the time \( T \)
- \( \int_0^T L(x_u, u) \): cost of the motion in fixed time \( T \)
Modelling

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\[
\int_0^T g(t) \, dt : \text{cost of the time } T \quad \leftarrow \text{what we are looking for!}
\]

\[
\int_0^T L(x_u, u) : \text{cost of the motion in fixed time } T
\]
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Necessary condition

Fix $x^f$. Value function of the motion cost in **fixed time**:

$$V(t, x) = \inf \left\{ \int_0^t L(x_u, u) : \text{for } x_u \text{ joining } x \text{ to } x^f \text{ in time } t \right\}$$

Set $T =$ movement time from $x^0$ to $x^f$. Then

$$T \in \arg\min_{t \in [0, +\infty)} \left( \int_0^t g(s)ds + V(t, x^0) \right),$$

and so $g(T) = -\frac{\partial V}{\partial t} (T, x^0)$.

More precisely,

$$g(T) = -H_0(x^*(T), p^*(T), u^*(T))$$

where:

- $H_0(x, p, u) = \langle p, f(x, u) \rangle + L(x, u)$ normal Hamiltonian in fixed time,
- $(x^*(\cdot), u^*(\cdot))$ optimal solution in time $T$ with adjoint vector $p^*(\cdot)$.
Remark 1.
Requires some technical assumptions on the fixed time problem, e.g.,
- existence of minimizers,
- no abnormal minimizers (property of the dynamics).

Remark 2.
Extends to stochastic setting:
- stochastic dynamics: \( dx = f(x, u)dt + \sigma(x, u)dw \)
- \( V = \textbf{expected} \text{ value function} \)

\[ g(T) = -\frac{\partial V}{\partial t}(T, x^0) \]
(computable in the LQG case)
Recovering $g$ from experimental data:

- Fix $x^f$ and choose a set of initial conditions $x^0$.
- Experiments $\rightarrow T(x^0) =$ time of motion from $x^0$ to $x^f$.
- For every $t = T(x^0)$, solve the Optimal Control problem in fixed time.

Then $g(t) = -H_0(x^*(T), p^*(T), u^*(T))$
Conclusion

Given the cost of motion $L(x, u)$, the cost of time $g$ can be deduced from simple experiments.

**Problems:** how to determine $L(x, u)$? 
Robustness of the construction of $g$?
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Inverse optimal control

(Direct) Optimal control problem

Given a dynamic $\dot{x} = f(x, u)$, a cost $C(x_u)$ and a pair of points $x_0, x_1$, find a trajectory $x_u^*$ solution of

$$\inf \{ C(x_u) : x_u \text{ traj. s.t. } x_u(0) = x_0, x_u(T) = x_1 \}.$$ 

Inverse optimal control problem

Given $\dot{x} = f(x, u)$ and a set $\Gamma$ of trajectories, find a cost $C(x_u)$ such that every $\gamma \in \Gamma$ is solution of

$$\inf \{ C(x_u) : x_u \text{ traj. s.t. } x_u(0) = \gamma(0), x_u(T) = \gamma(T) \}.$$ 

Applications to analysis/modelling of human motor control

$\rightarrow$ looking for optimality principles
**Inverse problem:** Choose a class $\mathcal{C}$ of reasonable costs and let

$$\Phi : \mathcal{C} \in \mathcal{C} \mapsto \Gamma$$

optimal synthesis.

Inverse optimal control problem $=$ find an inverse $\Phi^{-1}$.

Well-posed problem?

- $\Phi$ injective?
- Continuity (and stability) of $\Phi^{-1}$?

$\rightarrow$ subject of the PhD thesis of Sofya Maslovskaya

- Linear-Quadratic case (see also [Kalmann 64, Nori-Frezza 04])
- sub-Riemannian case
- control-affine systems with quadratic costs
Practical point of view

- Linear dynamics: \( \dot{x} = Ax + Bu, \ x \in \mathbb{R}^n. \)
- Class of admissible costs \( \equiv \) class of quadratic costs,

\[
\mathcal{C} = \{ L(x, u) = u^\top Qu + x^\top Rx + 2x^\top Su, \ Q \succ 0, \ L \text{ sym } \succeq 0, \ (\& \text{ no pure imaginary eigenvalues in the Hamiltonian}) \}
\]

\( \rightarrow \) optimal controls in time \( T \) of the form \( u(t) = K_T(t)x(t). \)

Remark: \( \{K_T(\cdot), \ T > 0\} \) uniquely determined by a pair \( (K_-, K_+) \),

optimal solutions in time \( T \) of the form:

\[
x(t) = e^{(A+BK_+)t}y_+ + e^{(A+BK_-)(t-T)}y_-
\]
Main properties (S. Maslovskaya, FJ, 2018)

For $L$ in an open dense subset of $C$, there exists a unique pair $(K, R)$ s.t.

$$\Phi(L) = \Phi\left((u + Kx)^\top R (u + Kx)\right) \quad \text{with} \quad \det(R) = 1$$

and $A - BK$ stable. Moreover, $(K_-, K_+) \leftrightarrow (K, R)$ continuous.

$\rightarrow$ the inverse optimal control problem is well-posed.

Application to the computation of $g$

- Find $(K_-, K_+)$ by identification from experimental data [unique];
- Construct $L(x, u) = (u + Kx)^\top R (u + Kx)$ [explicit procedure];
- compute the function $\frac{\partial V}{\partial t}(t, x)$ using the Hamiltonian;
- set $g(t) = -\frac{\partial V}{\partial t}(t, x^0)$.

$\rightarrow$ Method robust w.r.t. perturbations of the data and w.r.t. the choice of cost.
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Two kind of motions:

- Saccadic eye movements.
- Reaching tasks,

In both cases:

- the dynamic is of the form:

  \[ \theta^{(n)} + c_{n-1}\theta^{(n-1)} + \cdots + c_0\theta = u, \]

  \[ \rightarrow \text{linear with a state } x = (\theta, \dot{\theta}, \ldots, \theta^{(n-1)}) \quad (n = 2 \text{ or } 3 \text{ in general}) \]

- Initial and final states are equilibria, typically:

  \[ x^0(a) = (a, 0, \ldots, 0) \quad \text{and} \quad x^f = 0. \]
Saccades

cost of time is hyperbolic, as expected

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The value of time in the neural control of action

Experimental results

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Reaching tasks (arm pointing motion with 1 DoF)

Observations: duration $\approx$ affine function of the amplitude
Reaching tasks (arm pointing motion with 1 DoF)

cost of time neither convex nor concave: sigmoidal shape
Experimental results

Consistency of sigmoidal shape

... w.r.t. the fitting of the data (affine, affine + log,...)

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Consistency of sigmoidal shape

\[ \ldots \text{w.r.t. the trajectory cost} \]
Consistency of sigmoidal shape

... w.r.t. the dynamic model (addition of muscle dynamics)
Consistency of sigmoidal shape

... w.r.t. signal-dependent motor noise (stochastic model)
Conclusion

- Automatic and robust method for identification of the cost of time $g$

- $\neq$ motions $\Rightarrow$ $\neq$ functions $g$?
  or same function on different time intervals?

- Two interpretations:
  - temporal discounting of reward
  - neural metabolic cost

Sigmoidal shape $\iff$ juxtaposition of the two reasons?
Planning of muscle co-contraction

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Muscle co-contraction

- used by the brain in tasks requiring stability, robustness or accuracy
- modulate the mechanical impedance of the neuromusculoskeletal system (e.g. increase the apparent joint stiffness)
  → effect only seen in presence of perturbations
- evidence of feedforward mechanism (descending motor commands)
- Humans and animals generate relatively stable motor behaviors even in the absence of feedback circuitry (deafferecence)
- accurate estimates of the current system state limited in some cases (too fast motion in particular, delay of 50-100ms)
  → open-loop motion planning

What kind of optimal control (OC) to use?
Deterministic OC

- focus on planning stage
- explanation of average motor behaviors $\rightarrow$ inverse OC problems
- but... no random perturbation in the model

$\Rightarrow$ no functional gain of co-contracting (waste of energy)

Example (1 dof arm with two antagonist muscles)

$$\min \int_0^T \|u\|^2 dt \quad \text{with} \quad I\ddot{\theta} = T(u_1 - u_2) + mgl_c \sin(\theta), \quad u_1, u_2 \geq 0$$

torque $= u_1 - u_2$, co-contraction $= u_2$ if torque $\geq 0$, $u_1$ if torque $\leq 0$

$\rightarrow$ zero co-contraction at the optimum
Stochastic OC

- account for the variability of biological movement
- take into account noise that affects the neuromusculoskeletal system

But . . .
- feedback control ⇒ requires state estimates (even if it can explain motion planning through feedback gains, no feedforward control)
- produce reciprocal activations less costly than co-contraction

⇒ not compatible with planned co-contraction

+ computational difficulties (curse of dimensionality) besides LQG case
→ necessity of a new framework: stochastic + open-loop
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Dynamics = Stochastic differential equation (SDE)

\[ dx_t = f(x_t, u(t)) \, dt + G(x_t, u(t)) \, dW_t \]

with \( u(\cdot) \) deterministic function of the time, for instance in \( L^2 \)

Cost expressed as an expectation,

\[ C(u) = \mathbb{E} \left[ \int_0^T L(x_t, u(t)) \, dt + \psi(x_T) \right] \]

possibly terminal constraints such as: \( \mathbb{E}[\phi(x_T)] \in \mathcal{S} \)

\[ \text{(SOOC)} \]

\text{minimize } C(u) \text{ among all controls } u(t), \ t \in [0, T], \text{ s.t. the solution } x^u_t \text{ with } x^u_0 = x^0 \text{ satisfies } \mathbb{E}[\phi(x^u_T)] \in \mathcal{S} \]
In general: few theoretical tools – approach of Annunziato-Borzi:

- characterize $x_t$ by its density $\rho_t$ solution of a Fokker-Planck PDE
- express cost and constraints as functions of $\rho_t$

$\rightarrow$ SOOC replaced by deterministic OC problem on a PDE

Give results on existence of solutions but very difficult to solve

Remark: in applications in neurosciences and robotics

- costs $L$ and $\psi$ are quadratic (for simplicity)
- constraints expressed in terms of mean value and covariance (to specify intended target and precision)

$$C(u) = \int_0^T \left( L(m, u) + \text{tr}(QP) \right) dt + \psi(m(T)) + \text{tr}(Q_f P(T))$$

only function of the mean $m(t)$ and the covariance $P(t)$ of $x_t$

[$Q$ and $Q_f$ are the quadratic part in $x$ of $L$ and $\psi$]
In general: few theoretical tools – approach of Annunziato-Borzi:

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[$Q$ and $Q_f$ are the quadratic part in $x$ of $L$ and $\psi$]
Planning of muscle co-contraction

Stochastic Open-loop OC

- Case of a linear SDE:

\[ d\mathbf{x}_t = \left( A(\mathbf{u}(t))\mathbf{x}_t + \mathbf{b}(\mathbf{u}(t)) \right) dt + G(\mathbf{u}(t)) \, d\mathbf{W}_t, \]

\[ \Rightarrow \mathbf{x}_t = \text{Gaussian process, fully determined by } \mathbf{m}(t) \text{ and } P(t) \]

In this case, SOOC equivalent to the following deterministic OC:

\[
\min C(\mathbf{u}) = \int_0^T \left( L(\mathbf{m}, \mathbf{u}) + \text{tr}(QP) \right) dt + \psi(\mathbf{m}(T)) + \text{tr}(Q_f P(T))
\]

among the solutions of

\[
\begin{cases}
\dot{\mathbf{m}}(t) = A(\mathbf{u}(t))\mathbf{m}(t) + \mathbf{b}(\mathbf{u}(t)) \\
\dot{P}(t) = A(\mathbf{u}(t))P(t) + P(t)A(\mathbf{u}(t))^\top + G(\mathbf{u}(t))G(\mathbf{u}(t))^\top
\end{cases}
\]

\[ \rightarrow \text{easy to solve and analyse} \]
General case:

For a general SDE $m(t)$ and $P(t)$ solution of

$$\begin{align*}
\dot{m} &= \mathbb{E} [f(x, u)], \\
\dot{P} &= \mathbb{E} [f(x, u)(x - m)^\top] + \mathbb{E} [(x - m)f(x, u)^\top] + \mathbb{E} [G(x, u)G(x, u)^\top].
\end{align*}$$

Gaussian statistical linearization

$m(t)$ and $P(t)$ may be approximated by the solution of

$$\begin{align*}
\dot{\tilde{m}} &= \tilde{b}(\tilde{m}, \tilde{P}, u), \\
\dot{\tilde{P}} &= \tilde{A}(\tilde{m}, \tilde{P}, u)\tilde{P} + \tilde{P}\tilde{A}(\tilde{m}, \tilde{P}, u)^\top + \tilde{G}(\tilde{m}, \tilde{P}, u)\tilde{G}(\tilde{m}, \tilde{P}, u)^\top.
\end{align*}$$

In the simplest (1st order) version,

$$\begin{align*}
\tilde{b} &= f(m, u), \\
\tilde{A} &= \frac{\partial f}{\partial x}(m, u), \text{ and } \tilde{G}\tilde{G}^\top = G(m, u)G(m, u)^\top.
\end{align*}$$
Conclusion:
An approximation of the optimal solution $u$ of SOOC may be computed as the optimal solution of a deterministic OC,

$$
\min C(u) = \int_0^T \left( L(m, u) + \text{tr}(QP) \right) dt + \psi(m(T)) + \text{tr}(Q_f P(T))
$$

among the solutions of

$$
\begin{cases}
\dot{m} = \tilde{b}(m, P, u), \\
\dot{P} = \tilde{A}(m, P, u)P + P\tilde{A}(m, P, u)^\top + \tilde{G}(m, P, u)\tilde{G}(m, P, u)^\top
\end{cases}
$$

→ seems to give very good numerical approximations (tested by Monte-Carlo procedures)
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Illustration that co-contraction may be an open-loop optimal strategy to reduce variability without feedback

→ experiment to reproduce:
  - Planar reaching task in divergent force field
  - Two-joint arm (shoulder + elbow) with 6 muscles

→ SOOC framework:
  - Muscle-level modeling of the dynamic (non linear model)
  - Cost = trade-off between
    - an integral part penalizing the energy
    - a final cost penalizing the state covariance

→ Two simulations:
  A. zero weight on the final cost
  B. comparable weights on both costs
Planning of muscle co-contraction

Application and interpretation

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Simplistic interpretation (Stoch. OC vs SOOC)

→ 1 dof system with **direct feedback** control of velocity (Stoch. OC)
\[
dx_t = v_t dt + gdw_t
\]

Aim= find the control strategy which minimizes
\[
\mathbb{E} \left[ \int_0^T (v_t^2 + qx_t^2) \, dt + q_f x_T^2 \right]
\]
LQG theory ⇒ \( v_t = k(t)x_t \) \( (k(t) \text{ given by Riccati ODE}) \)

→ 1 dof system with **indirect open-loop** control of velocity (SOOC)
\[
dx_t = u(t)x_t dt + gdw_t,
\]
Aim= find the control law \( u(t) \) which minimizes
\[
\mathbb{E} \left[ \int_0^T ((u(t)x_t)^2 + qx_t^2) \, dt + q_f x_T^2 \right]
\]
⇒ \( u(t) = k(t) \) !!


