Stability analysis of high frequency nonlinear amplifiers via harmonic identification

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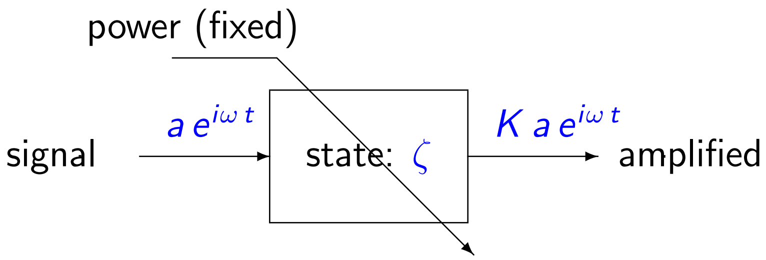
Motivation

- Amplifiers at high frequency are ubqitous (Cell phones, relays...), need to be quick to design and produced in large quantites.
- Computer-assisted design and simulation before production.
- Powerful "frequency simulation" tools give a reliable prediction of the response, but that response might be unstable.
- **Need** for a tool to predict stability/unstability.

Amplifiers (seen at a distance)



State ζ contains many currents and tensions,
possibly distributed (∞-dimensional)

$$\dot{\zeta} = F_{\text{signal}}(\zeta, t)\,, \quad \text{amplified} = g(\zeta) \qquad (*)$$

►► **stable** periodic solution of (*), yielding desired amplified signal.

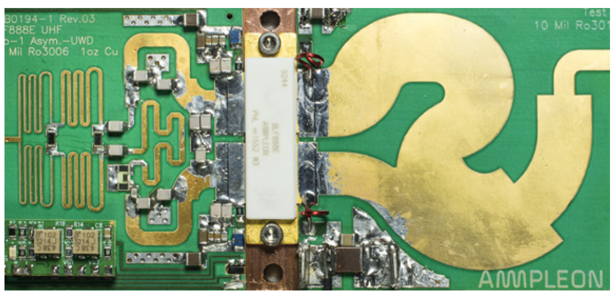
►► In particular, if zero signal,
$\dot{\zeta} = F_0(\zeta)$ must have a stable equilibrium ζ_0, with $g(\zeta_0) = 0$.

Inside

An amplifier is made of interconnected

- resistors, inductors, capacitors,
- diodes/transistors, ► **nonlinear**
- transmission lines.
  ► **infinite dimension** (delays, telegrapher equation...)
  cannot be neglected at high frequencies

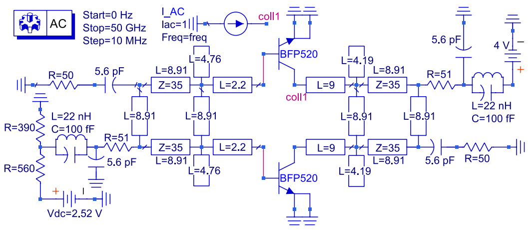
In fact,
transmission lines
have an important role
in the design.



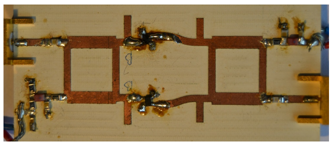
Computer-assisted design

Harmonic Balance (HB): a reliable design and simulation tool.
Simulations take place in the frequency domain.

- Transmission lines are described by their frequency response, sometimes experimental.
- The nonlinear characteristic of the diods (and its linearization) are known numerically.



CAD tool



prototype

Stability from computer-assisted design?

$$\text{“small signal”} = 0 : \quad \dot{\zeta} = F_0(\zeta) \tag{1}$$
$$\text{“large signal”} = a\, e^{i\omega t} : \quad \dot{\zeta} = F_\omega(\zeta, t) \quad (2\pi/\omega\text{-periodic}) \tag{2}$$

Harmonic Balance computes an equilibrium point ζ_0 of (1) or a periodic solution $t \mapsto \zeta_{\text{per}}(t)$ of (2)

Linearized system:

$$\dot{\xi} = \frac{\partial F_0}{\partial \zeta}(\zeta_0)\, \xi \quad \text{or} \quad \dot{\xi} = \frac{\partial F_0}{\partial \zeta}(\zeta_{\text{per}}(t), t)\, \xi\,.$$

Stability is given by the spectrum of $\frac{\partial F_0}{\partial \zeta}(\zeta_0)$,
or the time-var. counterpart (Floquet) of $\frac{\partial F_0}{\partial \zeta}(\zeta_{\text{per}}(\cdot), \cdot)$.

These are out of reach, no “time-domain” modelisation.

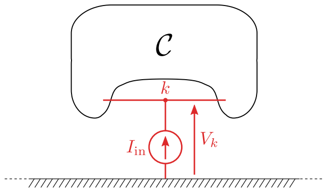
Stability from computer-assisted design

However, *Harmonic Balance* may compute the frequency response of $u \to y$ as in

$$\dot{\xi} = \frac{\partial F_\omega}{\partial \zeta}(\zeta_{\mathrm{per}}(t), t)\, \xi + B\, u\,, \quad u = C\, \zeta \quad \text{or} \quad \cdots \qquad (*)$$

for suitable linear operators B, C. Hence a possibility to reconstruct the singularities of the (harmonic) transfer function of this linear system.

System (*) is built artificially by adding a fictive source of current or tension u and a measure y in the circuit.



Program

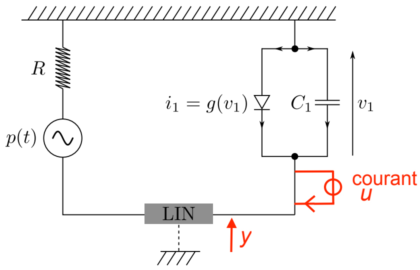
State of the art in electronics is to use the frequency response to perform rational approximation of the (harmonic) transfer function on frequency intervals (it is not rational!), check the location of poles and their stability [Jugo & al 2001, Suarez 2015].

Our program is to improve the estimation of the singularities and to further analyse stability to justify/understand its link with these.

- [Baratchart & al 2017, Cooman & al]: singularities can be determined without rational approximation;
  in the "small signal" case, singularities are poles.
- [Fueyo, PhD]: time-domain modelisation;
  link between: localstability / the spectrum of the generator / the singularities of (harmonic) transfer function;
  transfer function is not meromorphic for "large signals" in general, conjecture: meromorphic in the right-half plane for real life circuits?

A simple circuit [Fueyo & al.]

Example due
to [Hale & al 1993]



Model "$\dot{\zeta} = F_\omega(\zeta, t)$"
with ζ ∞-dimension.
Assume a periodic solution.

- The spectrum of the monodromy operator along the periodic solution is made of:
  - a continuous component or eigenvalues with infinite multiplicity
  - plus a finite number of eigenvalues with finite multiplicity,
- It does give local stability of the nonlinear circuit.
- Modulo a minor lack of observability when some rational ratio, and at least right of a vertical axis with negative real part, the set of singularities of the harmonic transfer function is the spectrum of the monodromy operator.

Perspectives

- General circuits
- Algorithms
- Transfer

Thank you for your attention.