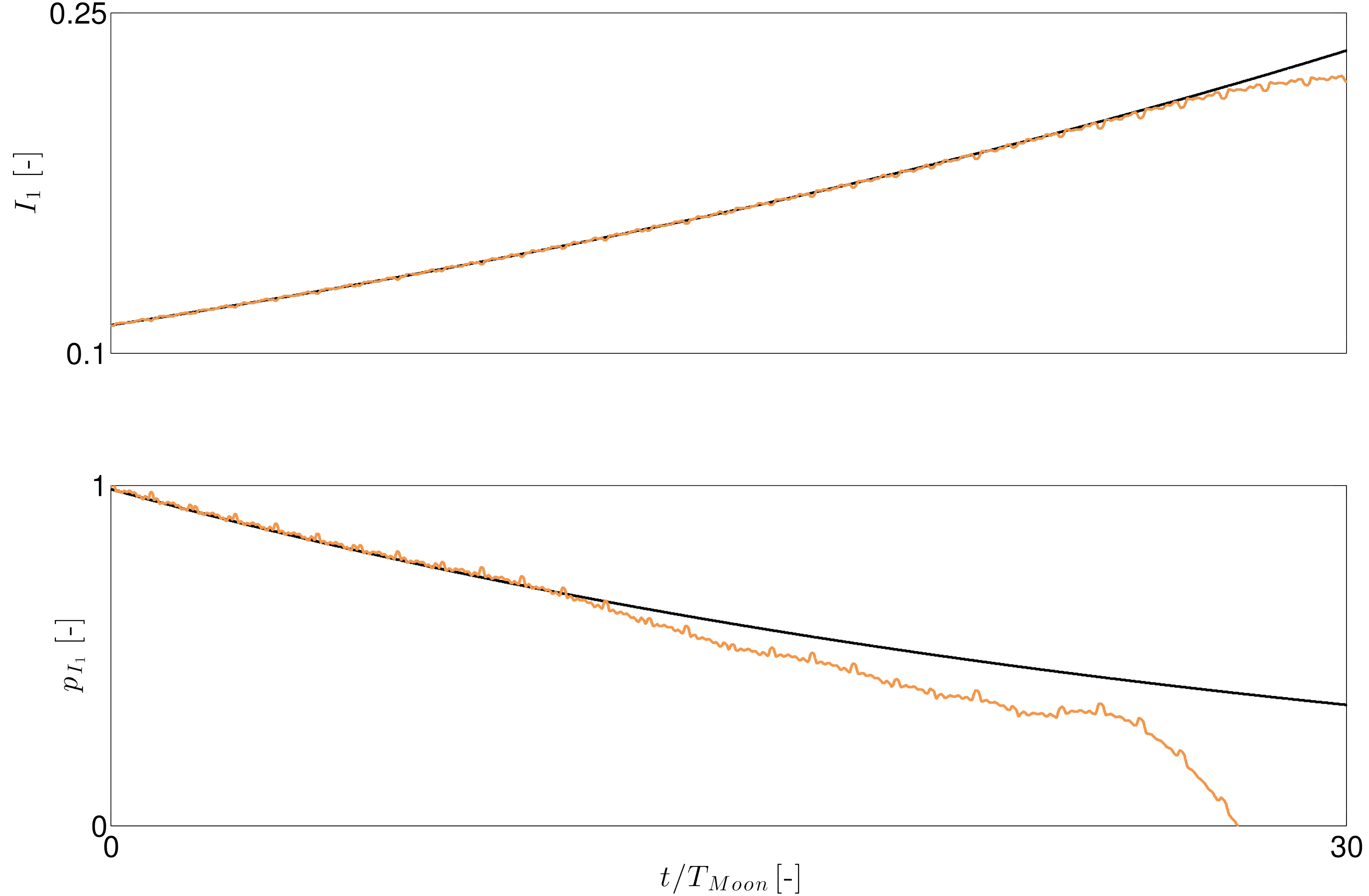
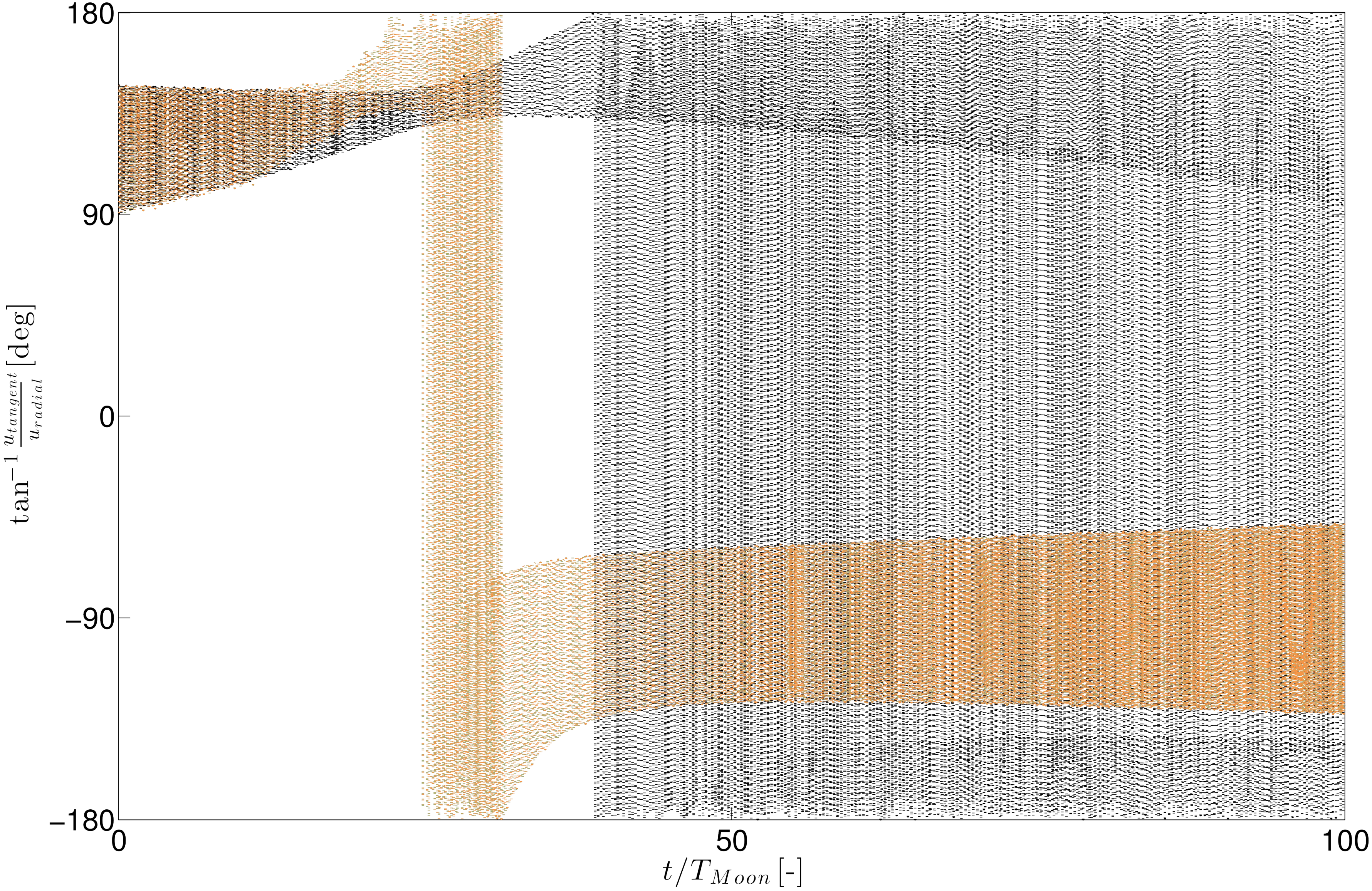
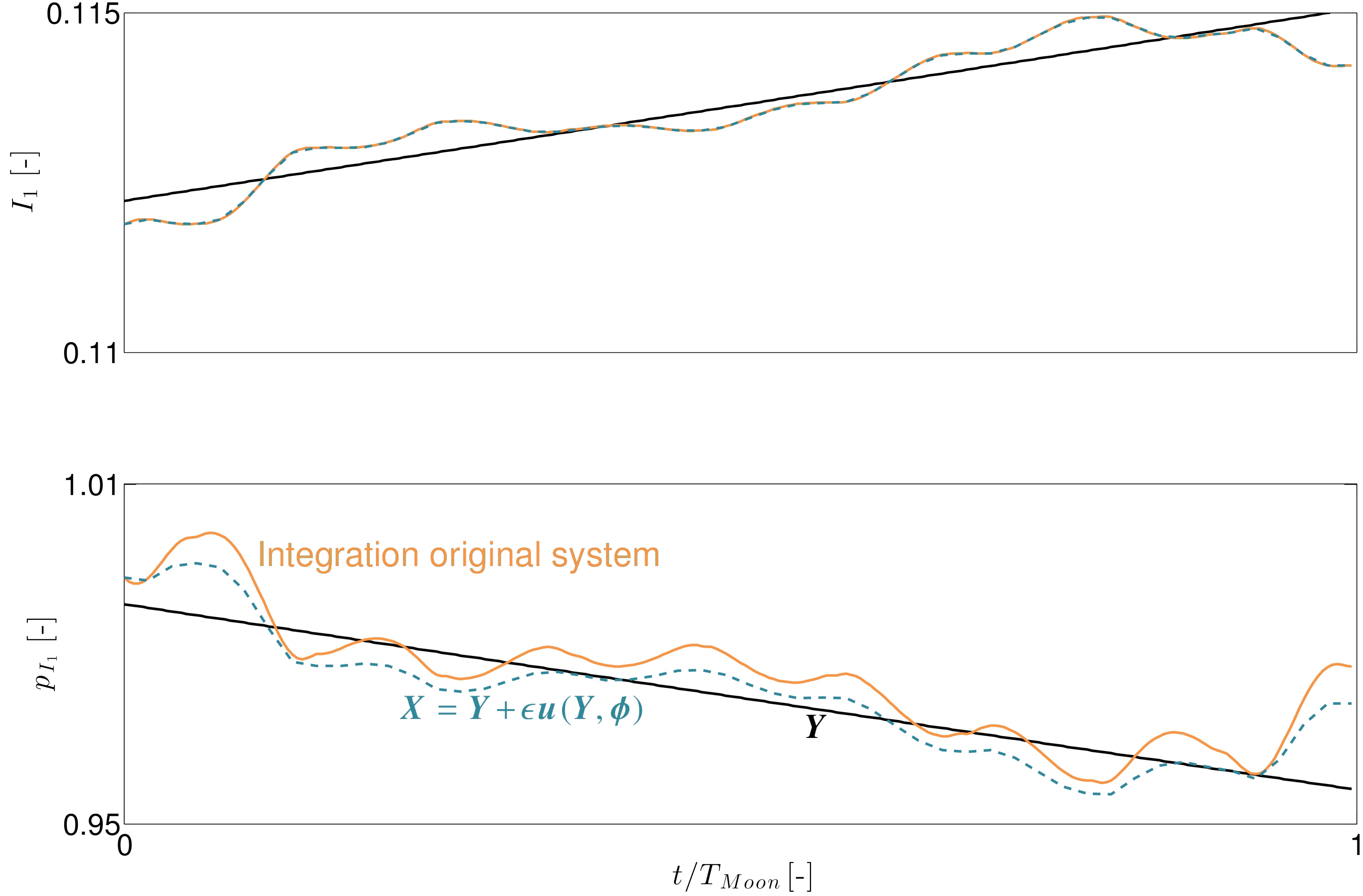
Adjoints drift faster slow states



The perturbation is the trigger, the control yields the drift



The near-identity transformation of $\boldsymbol{p_I}$ is not adequate



The term $\omega'(I)p_\phi$ has to be included in the transformation

Previous transformation:

$$\boldsymbol{u}\left(\boldsymbol{Y},\boldsymbol{\phi}\right)=-i\sum_{0<|\boldsymbol{k}|\leq N}\frac{\Delta\boldsymbol{f}_{\boldsymbol{k}}}{\boldsymbol{k}\cdot\boldsymbol{\omega}(\boldsymbol{Y})}\exp\left(i\boldsymbol{k}\cdot\boldsymbol{\phi}\right)$$

Where $\Delta\boldsymbol{f}_{\boldsymbol{k}}$ are Fourier coefficients of $\Delta\boldsymbol{f}\doteqdot\boldsymbol{f}\left(\boldsymbol{Y},\boldsymbol{\phi},\epsilon\right)-\overline{\boldsymbol{f}}\left(\boldsymbol{Y}\right)$ and $\boldsymbol{Y}=\left\{\boldsymbol{J},\ \boldsymbol{p}_{\boldsymbol{J}},\ p_\alpha=0,\ p_\beta\right\}$

The term $\omega'(\boldsymbol{I})\boldsymbol{p}_{\boldsymbol{\phi}}$ has to be included in the transformation

Previous transformation:

$$\boldsymbol{u}\left(\boldsymbol{Y},\boldsymbol{\phi}\right) = -i \sum_{0<|\boldsymbol{k}|\leq N} \frac{\Delta \boldsymbol{f}_{\boldsymbol{k}}}{\boldsymbol{k}\cdot\boldsymbol{\omega}(\boldsymbol{Y})} \exp\left(i\boldsymbol{k}\cdot\boldsymbol{\phi}\right)$$

Where $\Delta \boldsymbol{f}_{\boldsymbol{k}}$ are Fourier coefficients of $\Delta \boldsymbol{f} \doteqdot \boldsymbol{f}\left(\boldsymbol{Y},\boldsymbol{\phi},\epsilon\right) - \overline{\boldsymbol{f}}\left(\boldsymbol{Y}\right)$ and $\boldsymbol{Y} = \left\{\boldsymbol{J},\ \boldsymbol{p}_{\boldsymbol{J}},\ p_\alpha = 0,\ p_\beta\right\}$

The differential equation of $\boldsymbol{p}_{\boldsymbol{I}}$ is:

$$\dot{\boldsymbol{p}}_{\boldsymbol{I}} = -\epsilon\frac{\partial K}{\partial \boldsymbol{I}} - \underbrace{\frac{\partial|\omega|}{\partial \boldsymbol{I}}p_\alpha}_{\mathcal{O}(\epsilon)}$$

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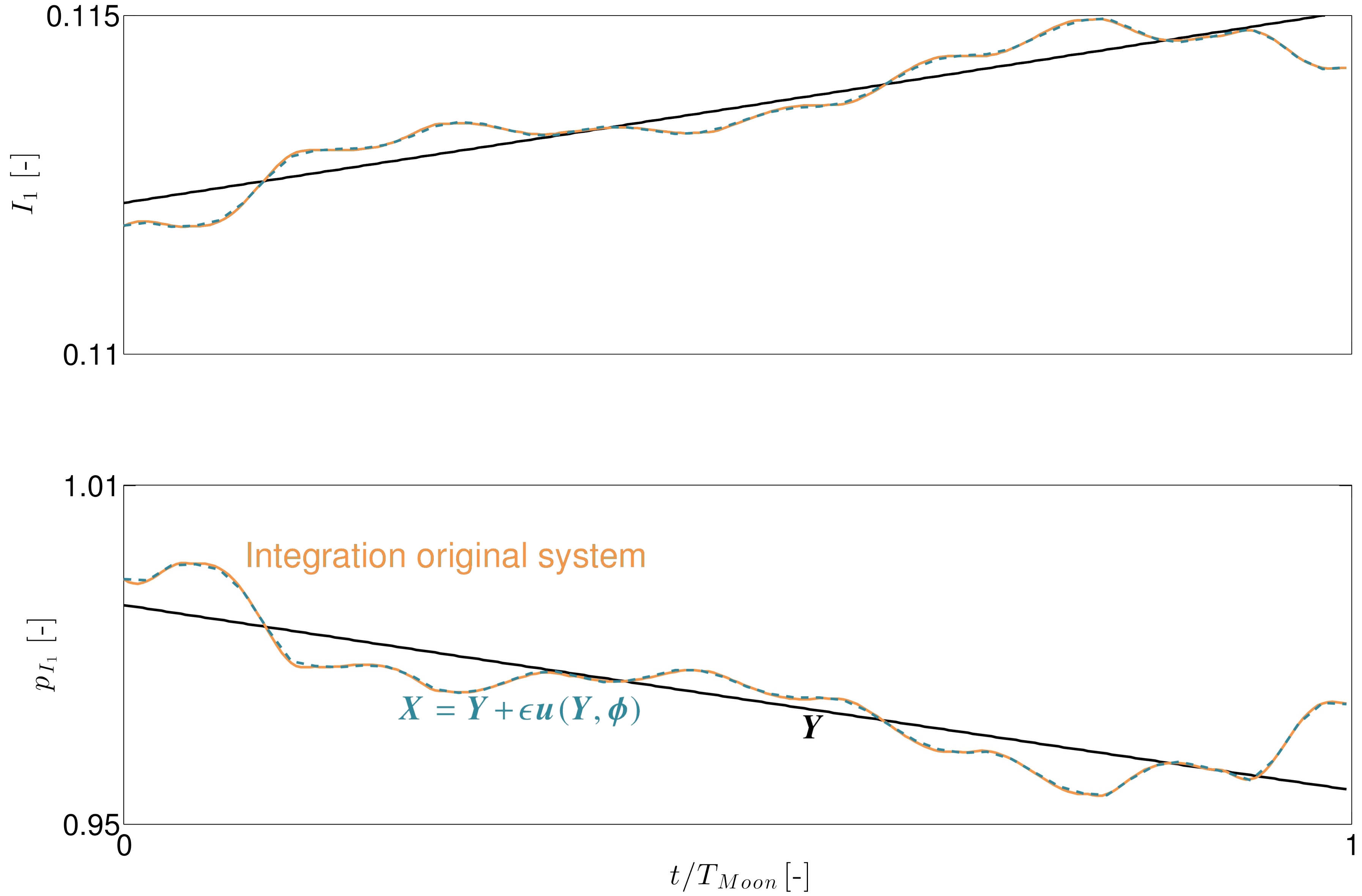
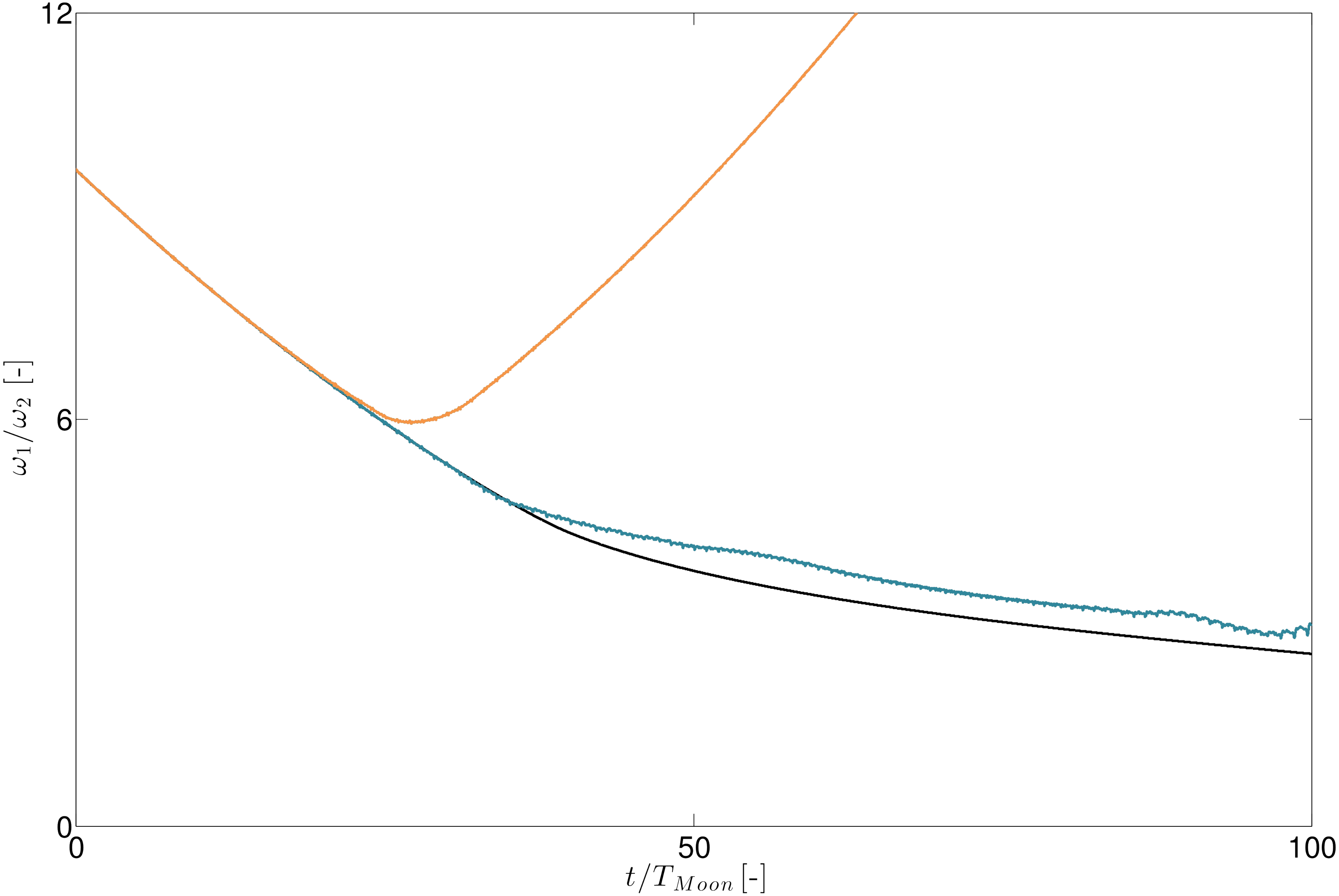
The expansion should be carried out by using:

$$\dot{\boldsymbol{p}}_{\boldsymbol{I}} = -\epsilon\left(\frac{\partial K}{\partial \boldsymbol{I}} - \frac{\partial|\omega|}{\partial \boldsymbol{I}}h(\boldsymbol{J},\boldsymbol{p}_{\boldsymbol{J}},\beta,\boldsymbol{\phi})\right)$$

The term $\omega'(\boldsymbol{I})p_{\boldsymbol{\phi}}$ has to be included in the transformation



Beneficial effect of the enhanced transformation of $\boldsymbol{p_I}$



Conclusions

Simple averaging is sufficient for real-life problems in astrodynamics, e.g., orbit raising

Initial conditions should undergo a **near-identity transformation** to reduce the drift

Key role of the transformation of the **adjoints of slow variables**

Effects of **main resonances** cannot be neglected in relevant astrodynamics applications

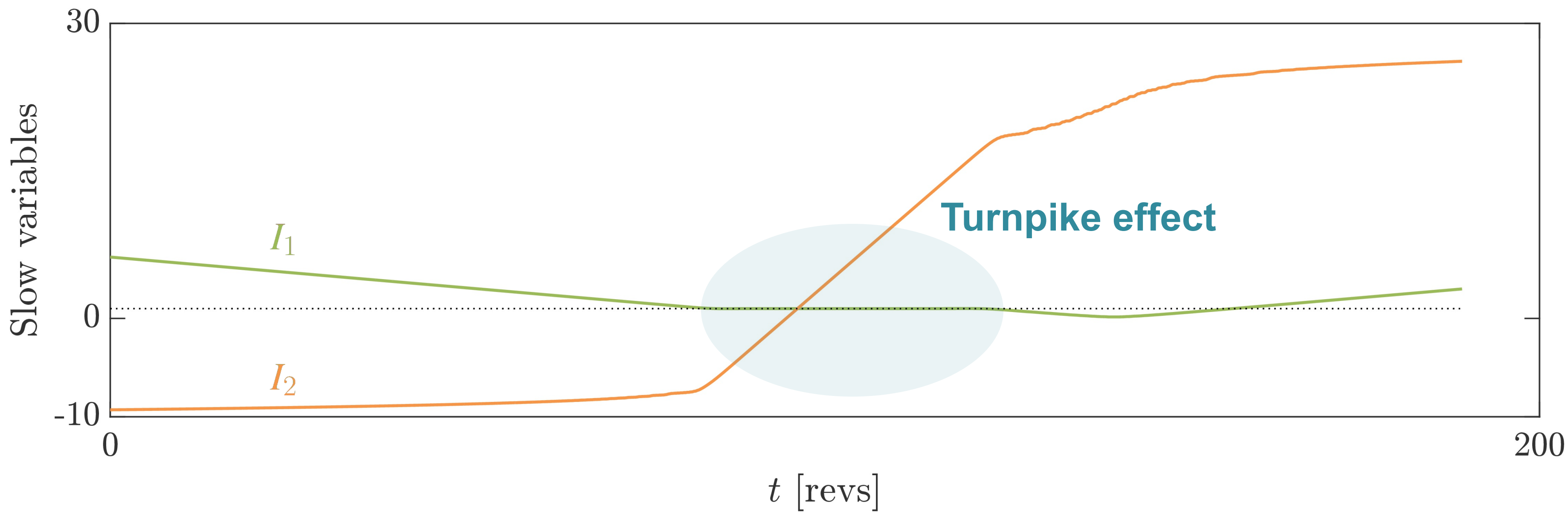
The averaged system might be enriched by using a **composite expansion**

Way forward: exploitation of resonances

$$\min_{||\boldsymbol{u}|| \leq 1} t_f \qquad \text{subject to}$$

$$\begin{aligned}
\dot{I}_1 &= \epsilon\, u_1 \\
\dot{I}_2 &= \epsilon\, f(\phi_1, \phi_2)\, u_2 \\
\dot{\phi}_1 &= I_1 \\
\dot{\phi}_2 &= 1
\end{aligned}$$

where $f(\phi_1, \phi_2) = \left[0.1 + \left(\dfrac{\cos\phi_1 + \cos\phi_2}{2}\right)^2\right]^{-1}$



Way forward: composite expansion to model captures

$$\dot{\boldsymbol{J}} = \dot{\boldsymbol{J}}_O + \dot{\boldsymbol{J}}_I - \dot{\boldsymbol{J}}_{IO}$$

$$\dot{\boldsymbol{J}} = \epsilon \bar{\boldsymbol{f}}\left(\boldsymbol{J}\right)$$

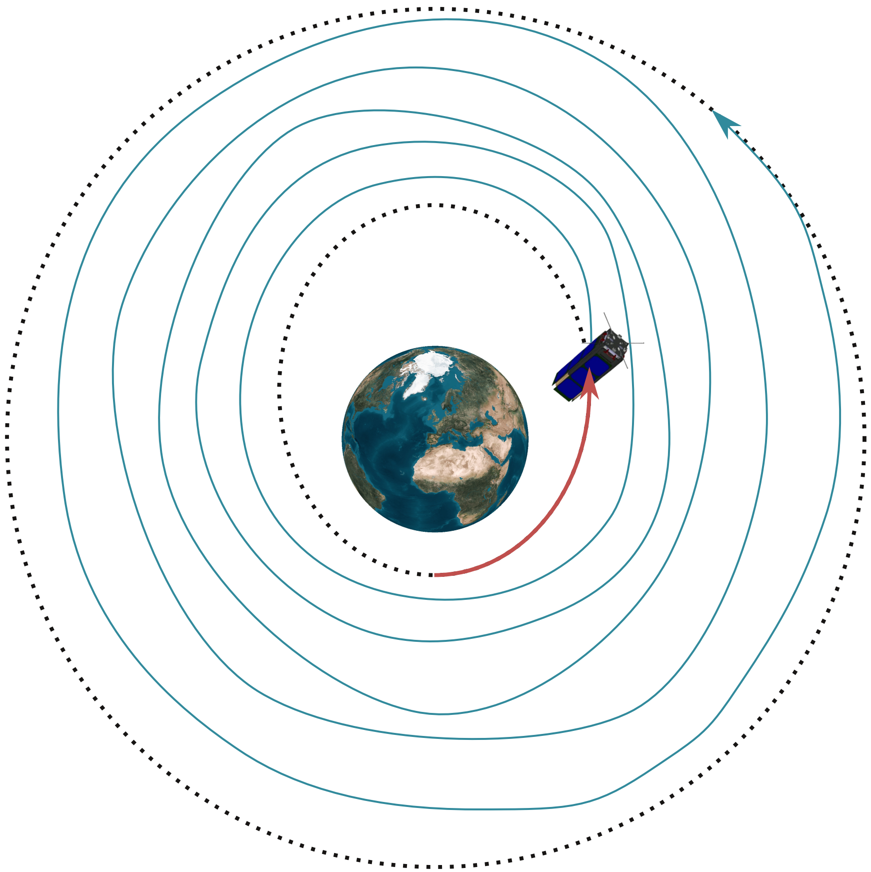
$$\begin{aligned} \dot{\boldsymbol{J}} &= \epsilon \boldsymbol{f}\left(\hat{\boldsymbol{J}}, \boldsymbol{\phi}\right) \\ \dot{\boldsymbol{\phi}} &= \omega'\left(\hat{\boldsymbol{J}}\right)\boldsymbol{J} \end{aligned}$$

$$\begin{aligned} \dot{\boldsymbol{J}} &= \epsilon \bar{\boldsymbol{f}}\left(\hat{\boldsymbol{J}}\right) \\ \dot{\boldsymbol{\phi}} &= \omega'\left(\hat{\boldsymbol{J}}\right)\boldsymbol{J} \end{aligned}$$

$\exists\, \boldsymbol{k} \in \mathbb{Z}^0$ such that $\boldsymbol{k} \cdot \hat{\boldsymbol{J}} = 0$

J.A. Sanders, F. Verhulst, J. Murdock, *Averaging Methods in Nonlinear Dynamical Systems*, Springer, 2007.

Two-Frequency Averaging of Optimal Control Problems with Application to Time-Optimal Orbital Transfer



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McTAO Days, Dijon, 05/12/2017