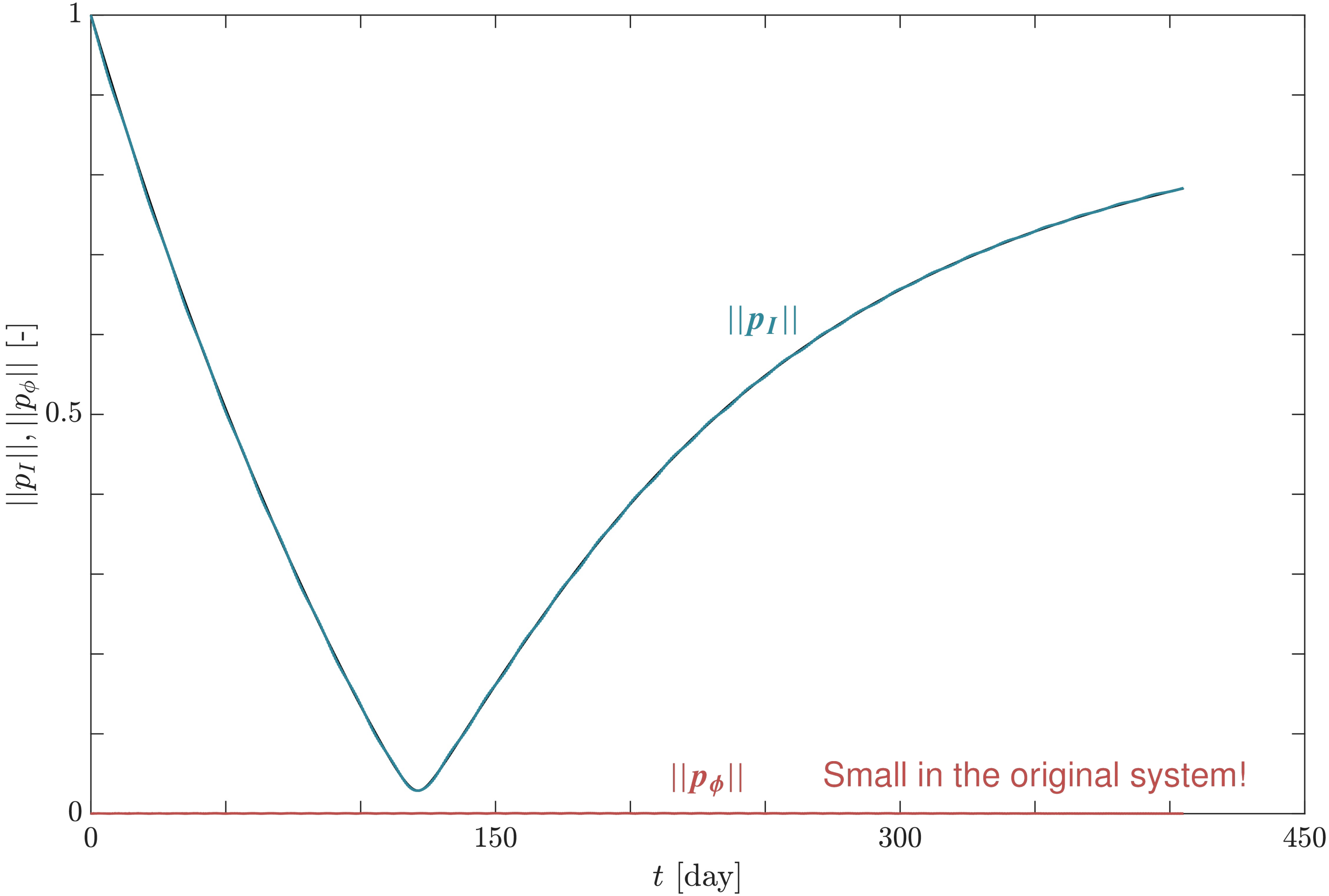
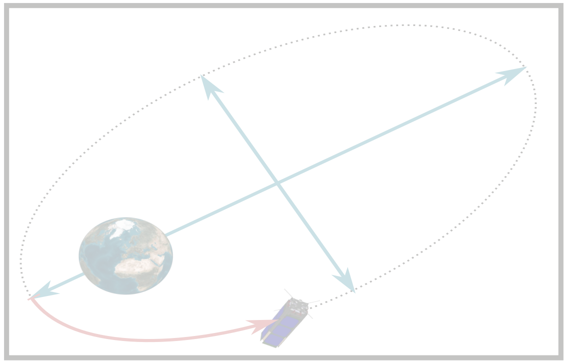
Osculating adjoints of fast variables remain small



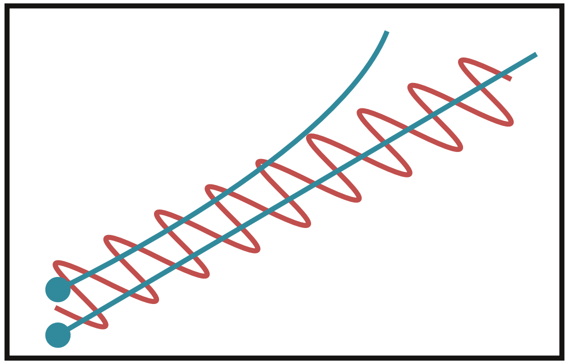
Outline



1. Dynamical systems with slow & fast dynamics

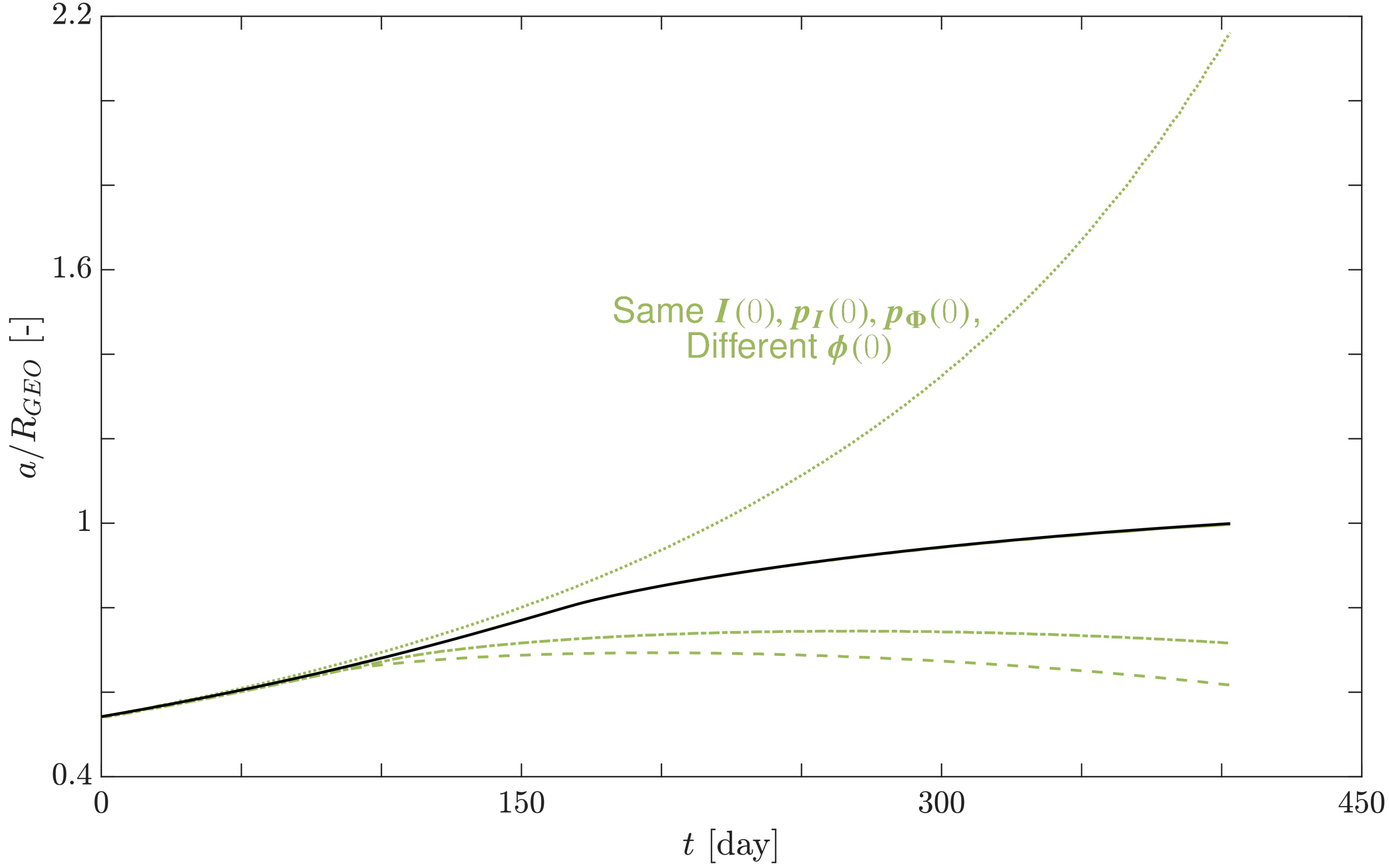
$$\min_{u \in \mathcal{U}} \mathcal{H}(p, q, u)$$

2. Averaging the two-phase optimal control problem

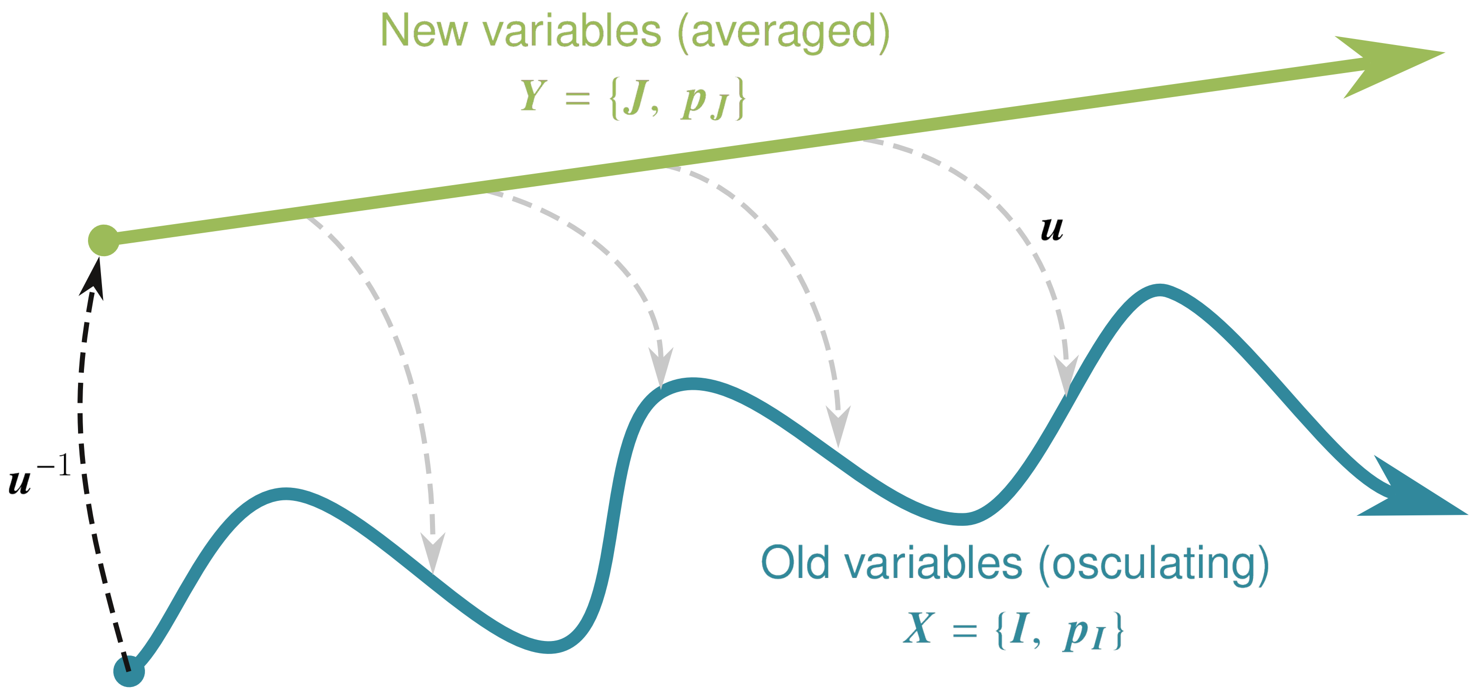


3. Near identity transformation of the initial state

Are initial average states a good guess as is?



Near-identity transformation of the slow variables



Change of variables

$$\boldsymbol{X} = \boldsymbol{Y} + \epsilon \boldsymbol{u}\,(\boldsymbol{Y}, \boldsymbol{\phi})$$

Formulation for the multi-phase problem

Original system

$$
\begin{aligned}
\dot{X} &= \epsilon\, \boldsymbol{f}(X, \phi, \epsilon) \\
\dot{\phi} &= \epsilon\, \boldsymbol{g}(X, \phi, \epsilon) + \omega(I)
\end{aligned}
$$

Averaged system

$$
\begin{aligned}
\dot{Y} &= \epsilon\, \overline{\boldsymbol{f}}(Y) \\
\overline{\boldsymbol{f}}(Y) &:= \int_{\mathbb{T}^r} \boldsymbol{f}(Y, \phi, 0)\, \mathrm{d}\phi
\end{aligned}
$$

Near identity transformation:

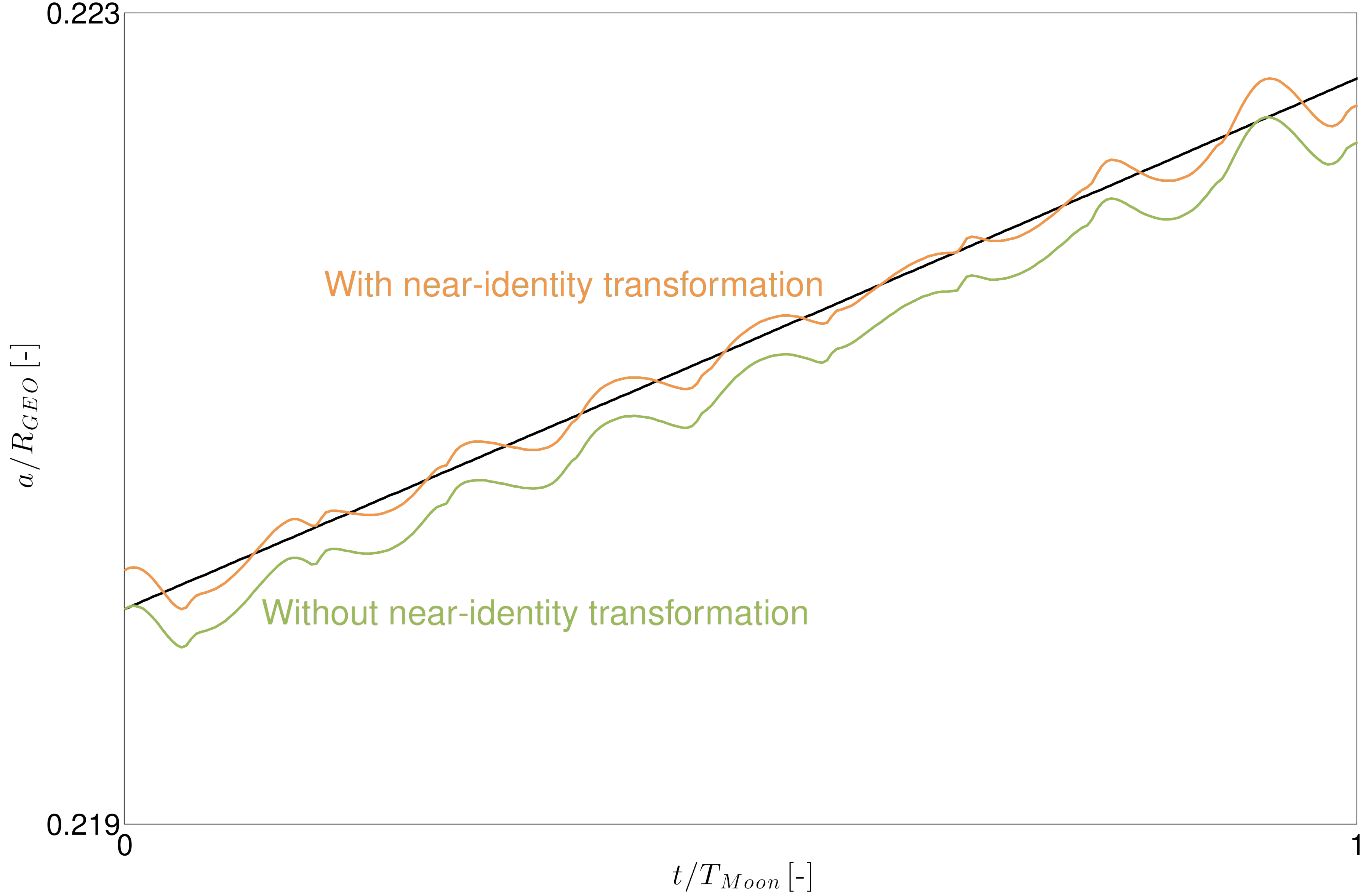
$$
\boldsymbol{u}(Y, \phi) = -i \sum_{0<|\boldsymbol{k}|\leq N} \frac{\Delta \boldsymbol{f}_{\boldsymbol{k}}}{\boldsymbol{k} \cdot \omega(Y)} \exp(i\boldsymbol{k} \cdot \phi)
$$

Where $\Delta \boldsymbol{f}_{\boldsymbol{k}}$ are Fourier coefficients of $\Delta \boldsymbol{f} \doteqdot \boldsymbol{f}(Y, \phi, \epsilon) - \overline{\boldsymbol{f}}(Y)$

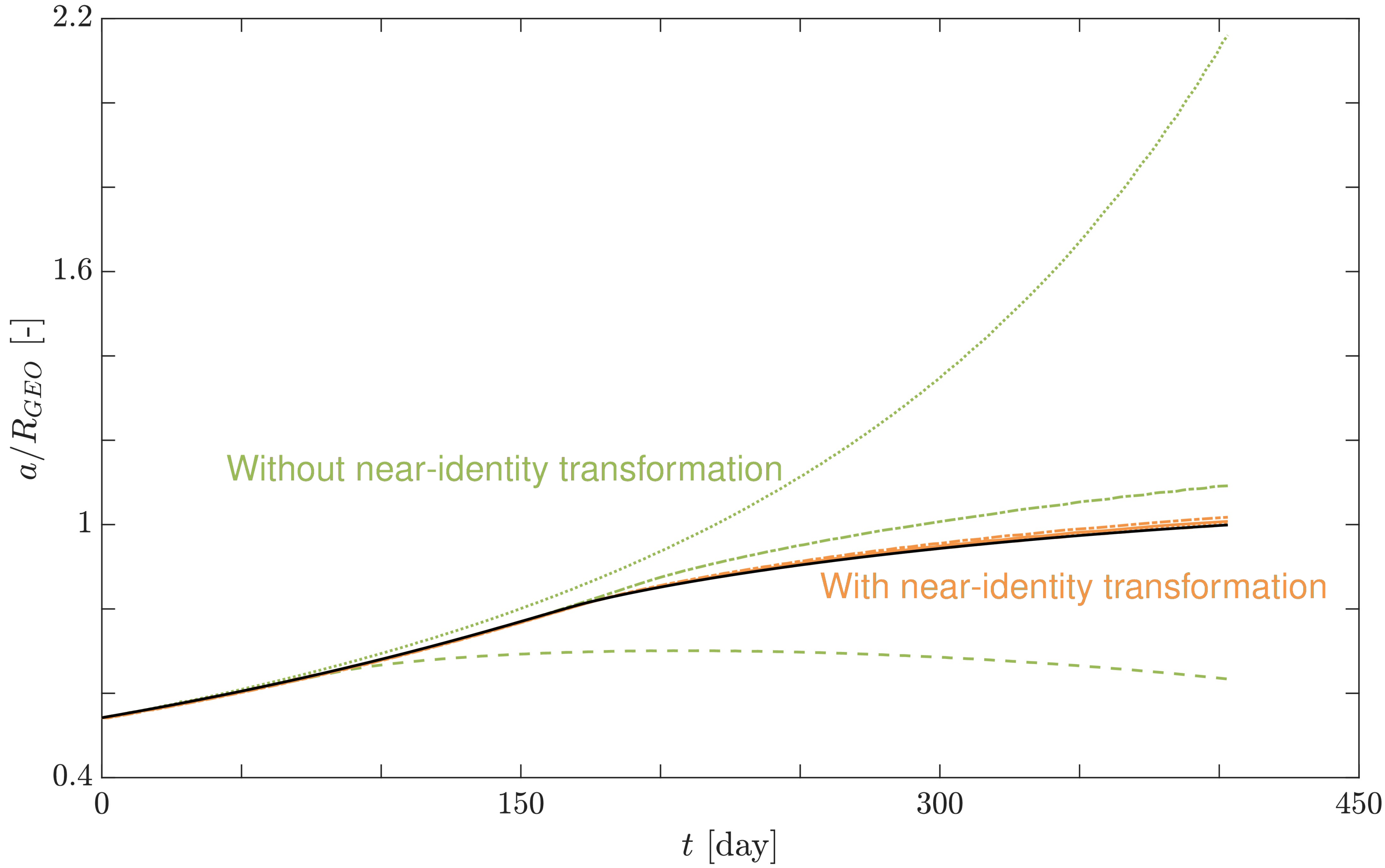
Interpretation: first-order matching of the time derivative

$$
\frac{\mathrm{d}}{\mathrm{d}t} X = \frac{\mathrm{d}}{\mathrm{d}t}\Big(Y + \epsilon \boldsymbol{u}(Y, \phi) \Big) + \mathcal{O}\left(\epsilon^2\right)
$$

The transformation yields zero-mean oscillations

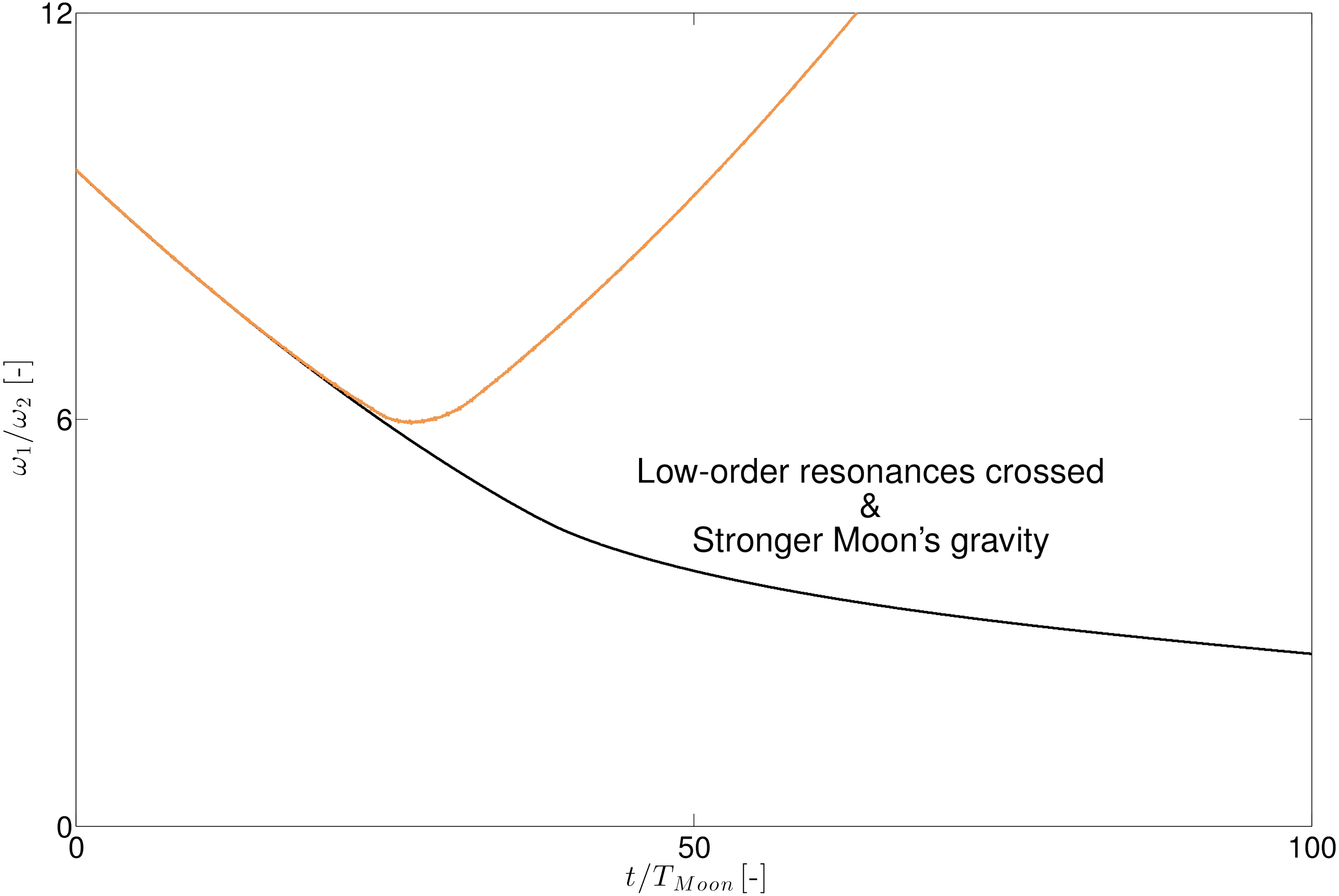


The transformation greatly improves the estimation

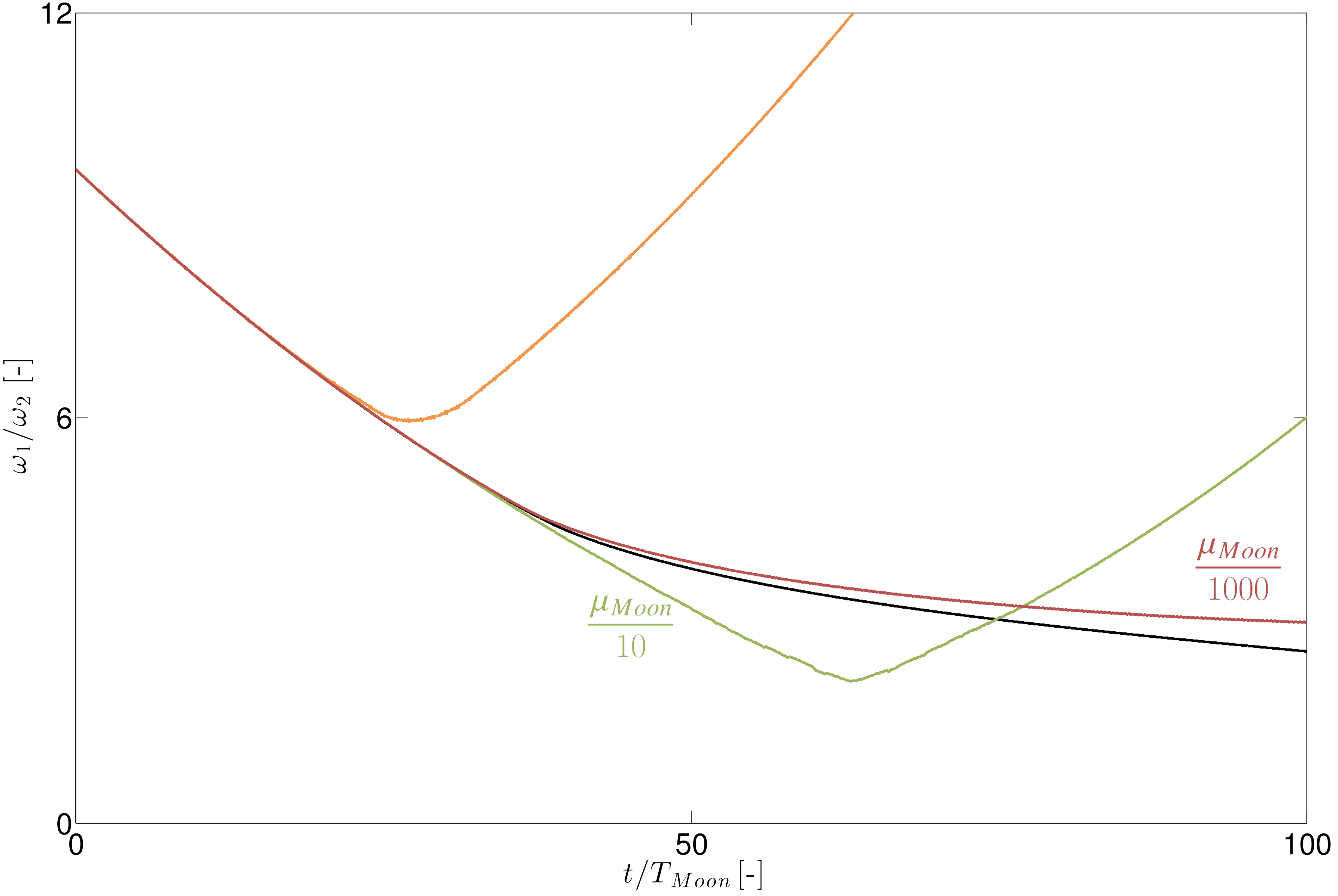


Transforming p_α is the key! Remember: $\dot{\boldsymbol{p}}_{\boldsymbol{I}} = -\frac{\partial|\boldsymbol{\omega}|}{\partial \boldsymbol{I}} p_\alpha + \mathcal{O}(\epsilon)$

But it is not yet enough when the perturbation is stronger



Drift disappears when the perturbation is smaller



Adjoints drift faster slow states