Magnetic propel artificial micro-swimmers

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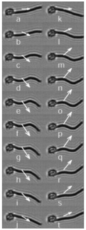
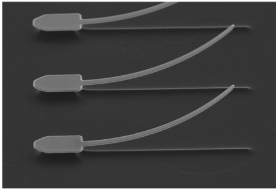
Microswimming

- Displacement of micro-swimmer
- Well establish domain
- Applications to Biology / Robotic
- Emerging of artificial mechanisms :
  How to obtain a micro-robot self-propelled and controlled ?



ESPCI (2005)



Spintec Lab (2014)

Natural world

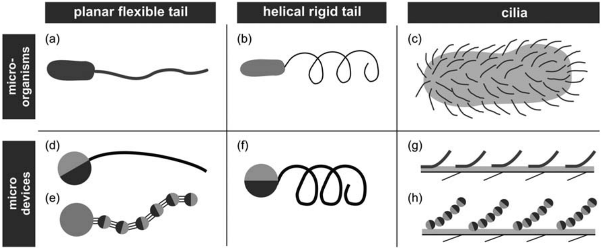
Definition

Swimming is the ability of moving in or under water with suitable body deformation.

- Numerous organisms are able to swim at this scale.

Magnetic swimming microrobots

- Biomimetics
- Magnetic field for deforming the micro-robot body



- The controllability of such devices ?

Content

1. Modeling

2. Numerical results

3. Controllability result
   $\hookrightarrow$ Return method

Modeling : main ingredients

- Hydrodynamics : interaction between the fluid and the body.
- Elasticity (el) : take into account the rigidity of the micro-robot structure.
- Magnetism (mag) : action of the magnetic field on the charged body swimmer.

Rescaling

The fluid is governed by the Navier-Stokes equation

$$Re\left(\tau\frac{\partial \mathbf{u}_*}{\partial t_*} + \mathbf{u}_*\cdot\nabla_*\right)\mathbf{u}_* - \Delta_*\mathbf{u}_* + \nabla_* p_* = \frac{Re}{F}\mathbf{g}_*, \quad \text{div } \mathbf{u} = 0 \quad \text{in } \mathcal{F}.,$$

where, $Re = \frac{\rho_f UL}{\mu}$, $F = \frac{U^2}{LG}$, $\tau = \frac{TU}{L}$.
with the boundary condition,

$$\mathbf{u} = \dot{\mathbf{q}} + \mathbf{u}_d(\mathbf{q}, \text{mag,el}) \quad \text{on } \partial\mathcal{N},$$

with the Newton law

$$\begin{cases} \displaystyle\int_{\partial\mathcal{N}} \boldsymbol{\sigma}(\mathbf{u}_*, p_*)\cdot\mathbf{n}\,\mathrm{ds} = & -\frac{\rho_m}{\rho_f} Re\left(\frac{1}{F}\mathbf{g}_* + \frac{1}{\tau^2}\ddot{\mathbf{q}}_*\right) \\ & +\mathbf{F}_{\text{mag}} + \mathbf{F}_{\text{el}}\,, \\ \displaystyle\int_{\partial\mathcal{N}} \boldsymbol{\sigma}(\mathbf{u}_*, p_*)\cdot\mathbf{n}\times(\mathbf{x}-\mathbf{q})\,\mathrm{ds} = & -\frac{\rho_m}{\rho_f} Re\left(\frac{1}{F}\mathbf{q}_*\times\mathbf{g}_* + \frac{1}{\tau^2}\dot{\Omega}_*\right) \\ & +\mathbf{T}_{\text{mag}} + \mathbf{T}_{\text{el}}\,, \end{cases}$$

At micro scale $Re := \frac{\rho_f \mathbf{UL}}{\mu} \sim 10^{-6}$

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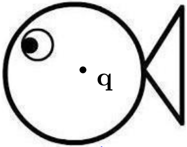
To sum up

A swimmer is parametrized by

- $\mathbf{q} \in \mathbb{R}^3 \times \mathrm{SO}_3$: position and orientation.
- $\boldsymbol{\xi} \in \mathbb{R}^k$: shape.

$$\begin{cases} -\mu\Delta\mathbf{u} + \nabla p = 0 \\ \operatorname{div}\mathbf{u} = 0 \end{cases}$$



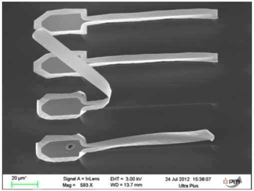
$\mathcal{N}$: deformable solid

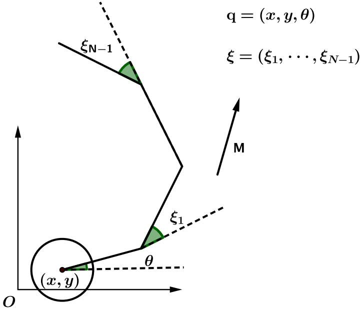
$$\mathbf{u} = \dot{\mathbf{q}} + \sum_{i=1}^{k} c_k(\mathbf{q}, \mathrm{mag}, \mathrm{el})\, \dot{\xi}_i \quad \text{sur} \quad \partial\mathcal{N}$$

speed of swimmer (unknown)

speed of deformation ("controlled" by magnetic field)

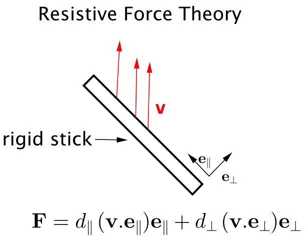
A particular micro-robots





$\hookrightarrow$ Constructed by CEA laboratory (Spintec) in Grenoble

1. Hydrodynamics : approximation



Resistive Force Theory

If a segment has a given speed at its extremities, noted **v**, then the associated distribution of hydrodynamic forces, called **F**, is given by

$$\mathbf{F} = d_\parallel(\mathbf{v}.\mathbf{e}_\parallel)\mathbf{e}_\parallel + d_\perp(\mathbf{v}.\mathbf{e}_\perp)\mathbf{e}_\perp$$

J. Gray and J. Hancock
The propulsion of sea-urchin spermatozoa, Journal of Experimental Biology, 1955.

2. Rigidity of the structure : discrete beam theory

The rigidity of the structure is given by an elastic torque compute on each extremities of the segment, called $\mathbf{x}_i$, as

$$\mathbf{T}^{el}_{i,\mathbf{x}_i} = \kappa(\xi_{i+1} - \xi_i)\mathbf{e}_z$$

where,

κ : the spring constant.

$\hookrightarrow$ This torque tends to align all of the segments.

3. Magnetism

Each segment is magnetized and it experiences a magnetic torque due to the external field denoted by **M**

$$\mathbf{T}_i^m = m_i(\mathbf{e}_{i,\parallel} \times \mathbf{M})$$

where,

m_i : the (total) magnetization of the $i-$th segment

$\mathbf{M}$: the external magnetic field

Equations of motion

- 3 unknowns corresponding to the position and the orientation of the swimmer
- (N-1) unknowns corresponding to the shape of the swimmer

Newton law $\rightarrow$ 3 equations.
The balance of the torque component of each subsystem, we get an invertible system of N+2 equations

$$\begin{cases} \mathbf{F} = \sum_{i=1}^{N} \mathbf{F}_i^h = 0\,, \\ \mathbf{e}_z \cdot \sum_{i=1}^{N} \left(\mathbf{T}_{i,\mathbf{x}_1}^h + \mathbf{T}_i^m \right) = 0\,, \\ \mathbf{e}_z \cdot \sum_{i=2}^{N} \left(\mathbf{T}_{i,\mathbf{x}_2}^h + \mathbf{T}_i^m \right) = -\kappa(\xi_2 - \theta)\,, \\ \vdots \\ \mathbf{e}_z \cdot \sum_{i=k}^{N} \left(\mathbf{T}_{i,\mathbf{x}_k}^h + \mathbf{T}_i^m \right) = -\kappa(\xi_k - \xi_{k-1})\,, \\ \vdots \\ \mathbf{e}_z \cdot \left(\mathbf{T}_{N,\mathbf{x}_N}^h + \mathbf{T}_N^m \right) = -\kappa(\xi_N - \xi_{N-1})\,. \end{cases}$$

Dynamics

The dynamics of the micro-robot is governed by an ODE linear with respect to the external magnetic field (i.e., control function $\mathbf{M} = (M_x, M_y)$) with a drift term,

$$\begin{pmatrix} \dot{\mathbf{q}} \\ \dot{\boldsymbol{\xi}} \end{pmatrix} = \underbrace{\mathbf{F}_0(\theta, \boldsymbol{\xi})}_{\text{Restoring force}} + \mathbf{F}_1(\theta, \boldsymbol{\xi}) M_x(t) + \mathbf{F}_2(\theta, \boldsymbol{\xi}) M_y(t)\,,$$

Equilibrium states : : $\{(x, y, 0, 0, \cdots, 0\}$

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Questions :

Is it possible to control the swimmer by acting on the magnetic field (M_x, M_y) ?

Numerical answer

By prescribing the magnetic sinusoidal field such as $(M_x, M_y \sin(\omega t))$ we get

Numerical simulations

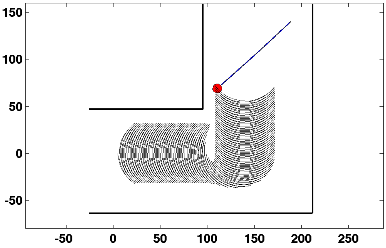
By prescribing now

$$\mathbf{M} = M_x \mathbf{e}_{\theta(t)} + M_y \sin(\omega t) \mathbf{e}_{\theta(t)}^{\perp}$$

with θ as

$$\theta(t) = \frac{\pi}{4}\left(1 + \tanh\left(30\left(\frac{t}{T_{\max}} - \frac{1}{2}\right)\right)\right) .$$

the swimmer navigates inside a narrow pipe (length are in μm.).



Numerical simulations

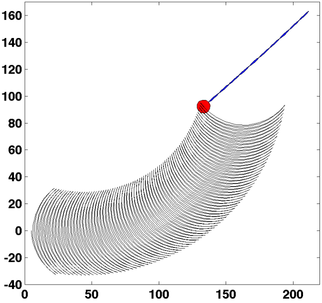
By prescribing

$$\mathbf{M} = M_x \mathbf{e}_{\theta(t)} + M_y \sin(\omega t)\mathbf{e}^{\perp}_{\theta(t)}$$

with θ as

$$\theta(t) = 2\pi t / T_{\max} ,$$

the swimmer "makes a circle"



Negative answer

Definition [Small Time Locally Controllable (STLC)]

For all $T > 0$, there exists $\eta > 0$ such that for all initial and final states η-closed to the equilibrium point, we could control the system from one to the other state by using control functions bounded by T.

STLC : The system is controllable closed to the equilibrium state with "small" control function

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STLC : The system is controllable closed to the equilibrium state with "small" control function

Theorem [L.G, P. Lissy, C. Moreau, J.-B. Pomet, 2016]

The 2-link (resp. 3-link) magnetized swimmer is not STLC.

Ideas of the proof

- Main idea : we prove that small control do not allow the swimmer to follow small closed-loop.
- $\hookrightarrow$ An asymptotic expansion leads to obtain a new parameter ζ (as linear combination of the states) such that

$$\forall \gamma \text{ close loop,} \qquad \int_\gamma \zeta > 0$$

whereas this integral should vanish.

Controllability result

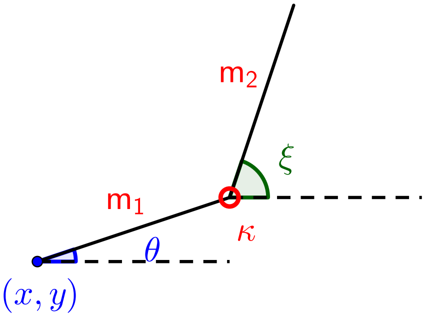
Definition : "weak" small time local controllability

$\forall$ equilibrium points, $\forall$ time $T > 0$, $\exists$ a neighborhood $\mathcal{V}$, such that any couples $((\mathbf{q}_i, \boldsymbol{\xi}_i), (\mathbf{q}_f, \boldsymbol{\xi}_f)) \in \mathcal{V}^2$, $\exists$ a bounded control $\mathbf{M}$ associated with the solution of the swimmer dynamics which starts at $(\mathbf{q}_i, \boldsymbol{\xi}_i)$ and ends at time T at $(\mathbf{q}_f, \boldsymbol{\xi}_f)$.

Theorem [L.G, J.-B. Pomet, IEEE TAC, 2016]

A swimmer with 1-shape-parameter is "weak" small time locally controllable.

Remarks



The swimmer with 1-shape-parameter

- The bound of the control functions depends on magnetization and bending stiffness of the swimmer

$$\|M\|_\infty < 2\,\kappa \left| \frac{m_2 + m_1}{m_2\, m_1} \right| + |\mathcal{V}| \ ,$$

Sketch of the proof

Difficulties :

- Local controllability at an equilibrium point does not hold.
  $\hookrightarrow$ the linearized test using the Kalmann rank condition
- The specific Hermes-Sussmann conditions does not hold.
  $\hookrightarrow$ sufficient conditions for driftless system to get small time locally controllable.

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- Construct a return trajectory
- Ensure that the linearized is controllable along a part of it
- The end-point mapping is a submersion

Sketch of the proof

$\hookrightarrow$ Construct a return trajectory :

- We choose any control M_x and M_y between $[0, \frac{T}{2}]$
- "Turn back" with the same path :

$$\dot{\mathbf{z}}(t) = \dot{\mathbf{z}}(T - t), \quad \forall t \in [\frac{T}{2}, T],$$

  where $\mathbf{z} = (\mathbf{q}, \xi)$.
  $\hookrightarrow$ gives 2 equations with 2 unknowns M_x and M_y.
- $\mathbf{M}$ which solved the system is bounded.

The method cannot be generalized if the number of shape parameters increase.

When the shape parameters increase

To overcome the difficulties due to the large number of shape parameters : $\hookrightarrow$ Local Partial Controllability Concept ([M. Duprez, thesis, 2015])

$\hookrightarrow$ controlling *only* the position of the swimmer (x, y)

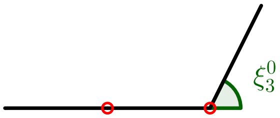
$\hookrightarrow$ leads to get a system with 2 equations and 2 unknowns,

$$\underbrace{\begin{pmatrix} F_{1x}(\boldsymbol{\xi}) & F_{2x}(\boldsymbol{\xi}) \\ F_{1y}(\boldsymbol{\xi}) & F_{2y}(\boldsymbol{\xi}) \end{pmatrix}}_{\mathcal{F}(\boldsymbol{\xi})} \begin{pmatrix} M_x \\ M_y \end{pmatrix} = G(\boldsymbol{\xi}, \theta).$$

Problem : $\mathcal{F}(\boldsymbol{\xi})$ is not invertible around the straight position (the equilibrium one).

For a twist magnetized swimmer

For a given $\xi_3^0 \in [0, \frac{\pi}{2}]$, the equilibrium states : : $\{(x, y, 0, 0, \xi_3^0)\}$



A bended 3-link swimmer at rest

Theorem [L.G, P. Lissy, C. Moreau, J.-B. Pomet, 2016]

A bended 3-link magnetized swimmer is STLC around the equilibrium states.

Proof : Kalmann

And, we track locally the swimmer along a prescribed trajectory

Numerical results : A straight line

Numerical results : A trajectory

Numerical results : Obstructions

Perspectives

- Global controllability - Tracking of trajectories ?
- Optimal control
- When the shape variables increase ?

References

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[2] L. G., J.-B. Pomet, *Local controllability of the two-link magneto-elastic swimmer.*, IEEE TAC, 2015.

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[4] L. G., P. Lissy, C. Moreau, J.-B. Pomet, *Remarks on a 2-link magnetized microswimmer*, working paper, 2016.

Thank you for your attention.