OPTIMAL CONTROL ON LIE GROUPS AND INTEGRABLE HAMILTONIAN SYSTEMS

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Synopsis

This series of four lectures will focus on the role of optimal control in the theory of integrable systems. We will begin with bilinear systems with quadratic costs and their liftings to Lie groups. The lifted systems on Lie groups make a natural connection with mechanical systems, elastic curves , elastic rods and problems of quantum control.

We will show that there is a distinguishable class of left (or right) invariant optimal control problems on Lie groups whose Hamiltonians oversee all of the above mentioned systems. We will single out the subclass of these Hamiltonians that admit Lax pair representation with a spectral parameter. The resulting spectral invariants are sufficient in number to guarantee complete integrability of these Hamiltonians on each coadjoint orbit in the dual of the Lie algebra.

This exposition will provide a self contained account of the theory of integrable systems on Lie algebras inspired by the seminal work of S.M. Manakov on the n-dimensional Euler top. We will then show the applications of this theory for the mechanical systems on spheres and the classic geodesic problem on the ellipsoid of Jacobi. This material will be presented as follows:

1 . Lecture 1. Bilinear quadratic systems and left (right) invariant optimal problems on Lie groups. Euler's top, Serret-Frenet systems, Kirchhoff's elastic problem, Bonnard's problem. Left (right) trivialization of the cotangent bundle of a Lie group and the Hamiltonianb systems.

2 . Lecture 2. Lie goups with an involutive automorphisms. Cartan decomposition and canonical control problems.

3 .Lecture 3. Cotangent bundles as coadjoint orbits. Isospectral representations.

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4 . Lecture 4. Jacobi's geodesic problem on the ellipsoid and other cool problems.