



**Barcelona  
Supercomputing  
Center**  
*Centro Nacional de Supercomputación*

# Towards an HPC tool for simulation of 3D CSEM surveys: and edge-based approach

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Computer Applications in Science & Engineering Department

# My country



Universidad Veracruzana



# Scholarship CONACyT



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PARA EL DESARROLLO



# Barcelona Supercomputing Center



# Marenostrum

92nd at Top500 (November 2015)  
77th at Top500 (June 2015)



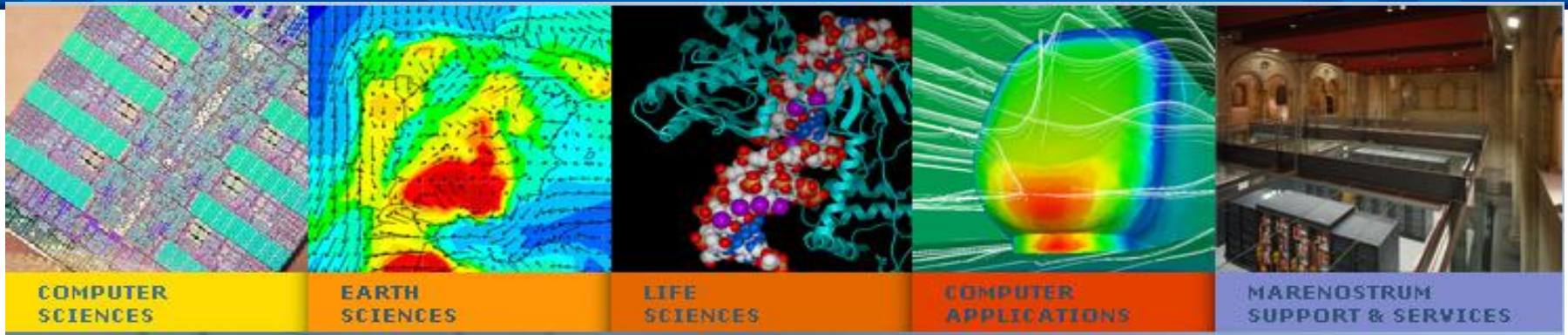
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1,1 Petaflops as peak performance

155.5 TiB of main memory

3,056 nodes

# CASE department



## Optimized applications for science and engineering:

- Biomechanics (Alya Red): coupled electrical/mechanical/fluid in very large grids ( $10^8$  elements)
- Geophysics (BSIT): pioneers in acoustic RTM, elastic FWI with real data, electromagnetic methods
- Coupled Multiphysics (ALYA): Coupling fluids, solids, electromagnetics, particles, chemistry on grids with  $10^9$  elements. Several CFD modelization: free surface, compressible/incompressible, subsonic/supersonic, RANS/LES/DNS, ...

# My research thesis

## ☞ Main target:

- Implement an edge-based FE code on HPC platforms to simulate CSEM in geophysics.

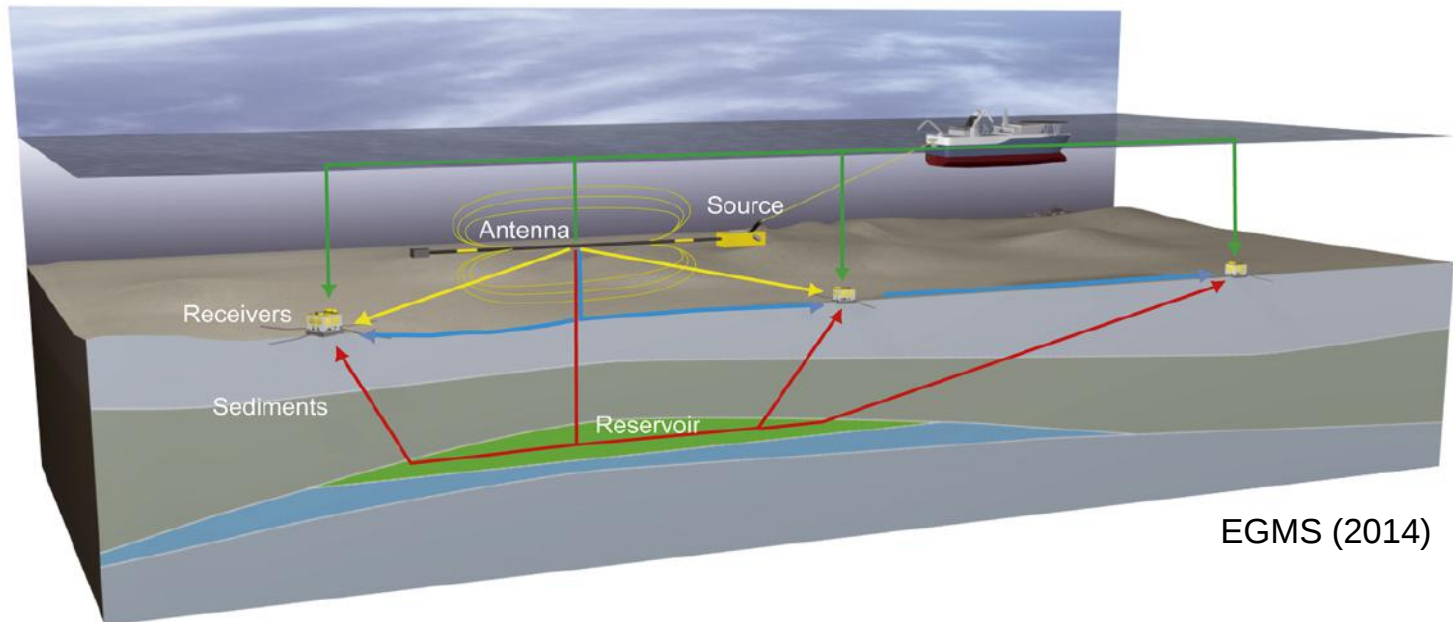
## ☞ Team at BSC has experience with FD, FE but not with Edge Elements (Nédélec elements, curl-conforming elements).

## ☞ My first job:

- Review the state of the art
- Understand the concept of Edge elements
- Understand the physical meaning
- Identify data structures needed

# Controlled-source Electromagnetic Method (CSEM)

- ❧ CSEM uses a dipole to transmit a time-varying electromagnetic field into the earth.
- ❧ The field is modified by the presence of subsurface resistive layers
- ❧ Used as a complementary tool to seismic surveys and well data analysis

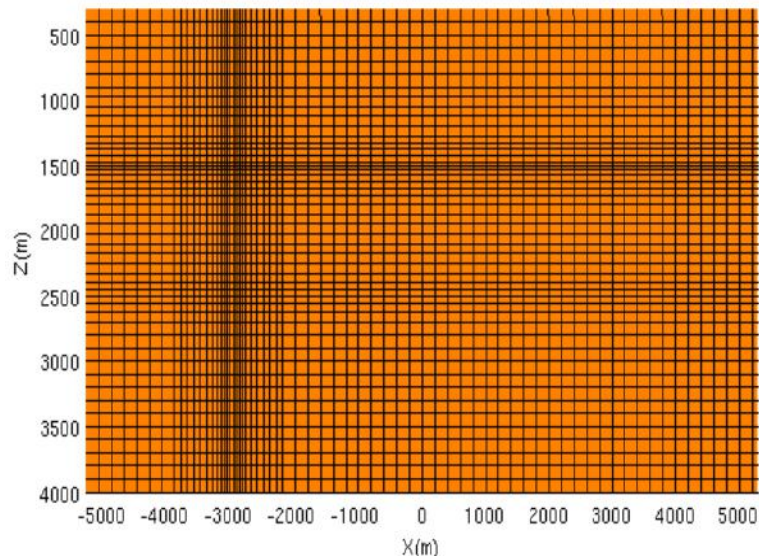




# Controlled-source Electromagnetic Method (CSEM)

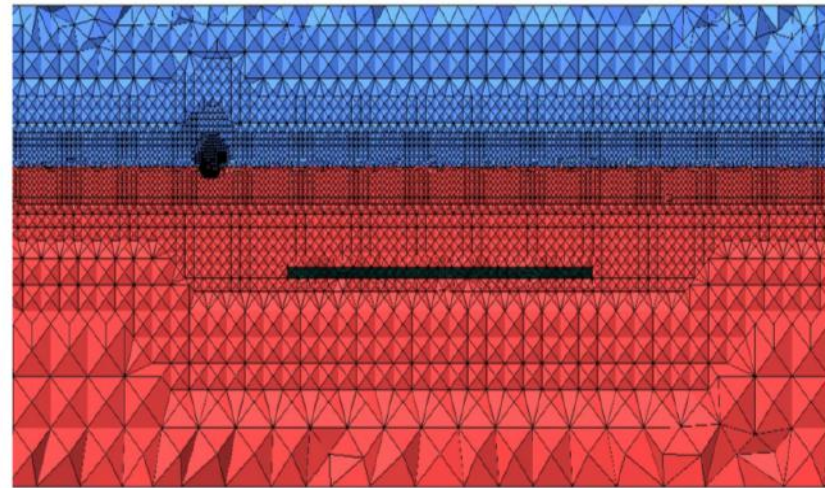
## Finite difference (FD)

- Structured meshes
- Difficult to adapt to complex geometries
- Very fast approach

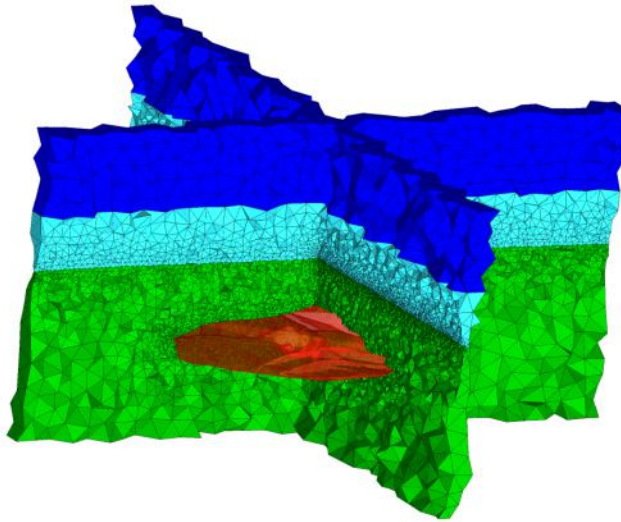


## Finite Element (FE)

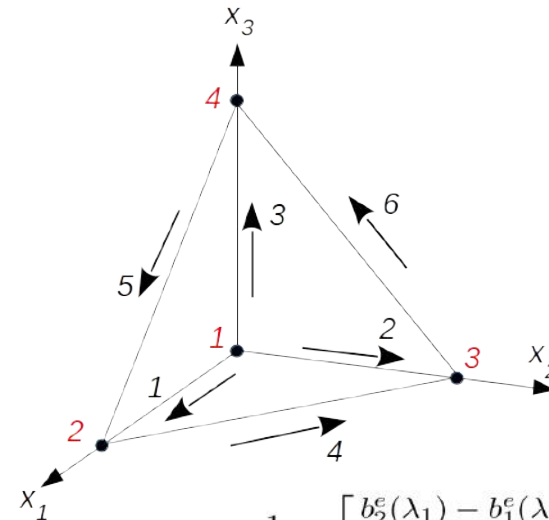
- Unstructured meshes
- Local refinement
- Nodal approach produces spurious solutions



# Edge finite element method: basis functions



- DOF are each edge in the mesh
- Linear vector basis functions (VBF)
- VBS are divergence free but not curl free (useful for the representation of the electric field)



$$W_{12}\ell_1^e = \frac{1}{(6Ve)^2} \begin{bmatrix} b_2^e(\lambda_1) - b_1^e(\lambda_2) \cdot \hat{i} \\ c_2^e(\lambda_1) - c_1^e(\lambda_2) \cdot \hat{j} \\ d_2^e(\lambda_1) - d_1^e(\lambda_2) \cdot \hat{k} \end{bmatrix} \ell_1$$

$$W_{13}\ell_2^e = \frac{1}{(6Ve)^2} \begin{bmatrix} b_3^e(\lambda_1) - b_1^e(\lambda_3) \cdot \hat{i} \\ c_3^e(\lambda_1) - c_1^e(\lambda_3) \cdot \hat{j} \\ d_3^e(\lambda_1) - d_1^e(\lambda_3) \cdot \hat{k} \end{bmatrix} \ell_2$$

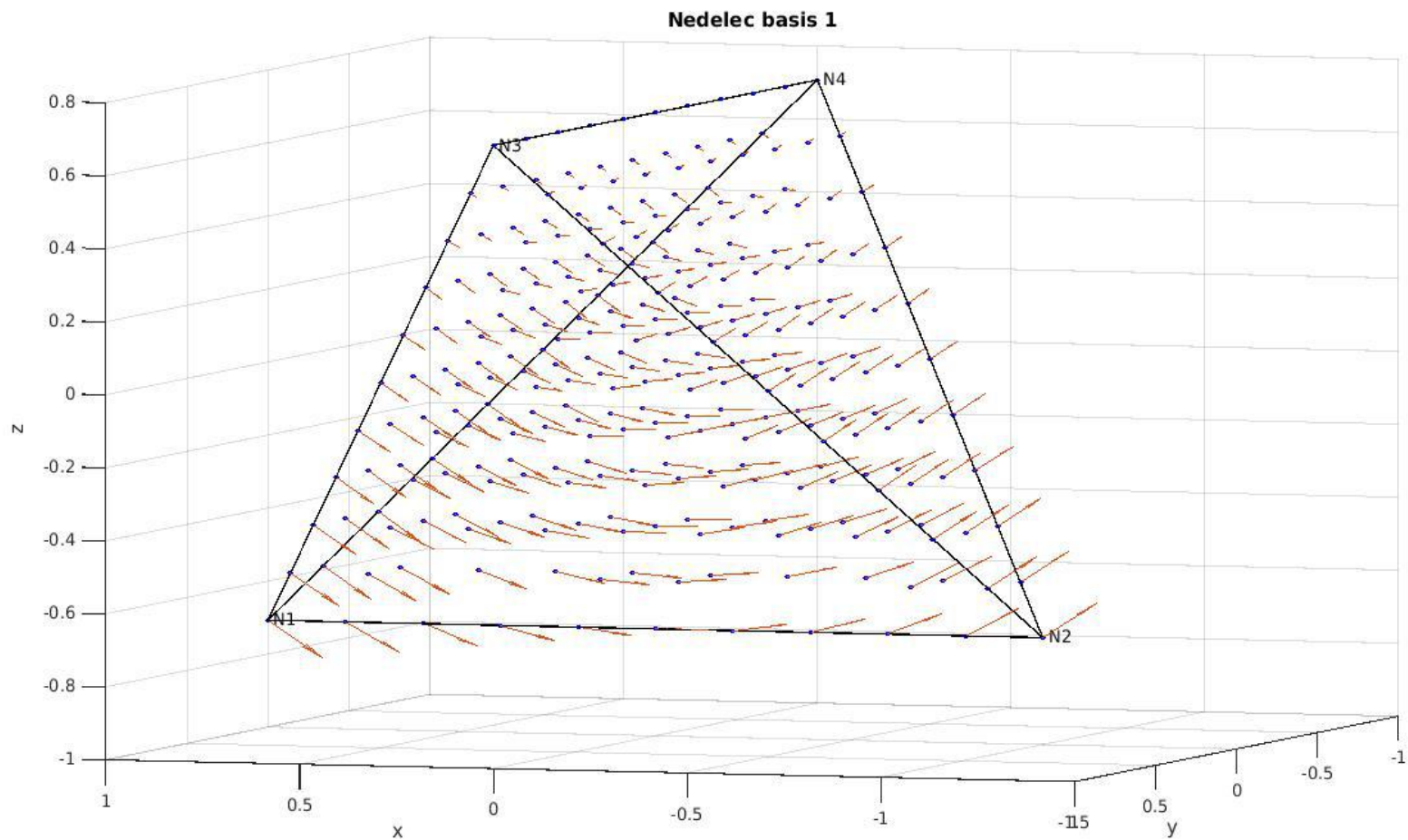
$$W_{14}\ell_3^e = \frac{1}{(6Ve)^2} \begin{bmatrix} b_4^e(\lambda_1) - b_1^e(\lambda_4) \cdot \hat{i} \\ c_4^e(\lambda_1) - c_1^e(\lambda_4) \cdot \hat{j} \\ d_4^e(\lambda_1) - d_1^e(\lambda_4) \cdot \hat{k} \end{bmatrix} \ell_3$$

$$W_{23}\ell_4^e = \frac{1}{(6Ve)^2} \begin{bmatrix} b_3^e(\lambda_2) - b_2^e(\lambda_3) \cdot \hat{i} \\ c_3^e(\lambda_2) - c_2^e(\lambda_3) \cdot \hat{j} \\ d_3^e(\lambda_2) - d_2^e(\lambda_3) \cdot \hat{k} \end{bmatrix} \ell_4$$

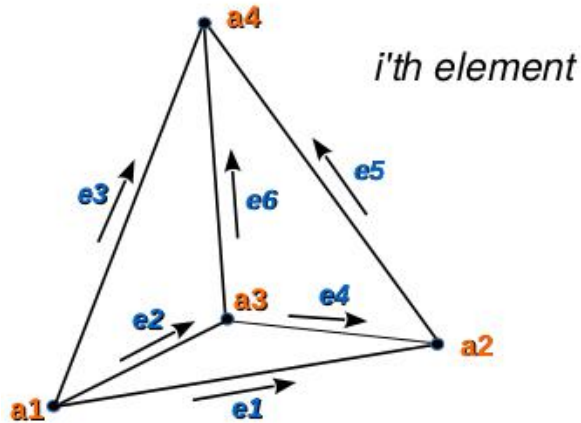
$$W_{42}\ell_5^e = \frac{1}{(6Ve)^2} \begin{bmatrix} b_2^e(\lambda_4) - b_4^e(\lambda_2) \cdot \hat{i} \\ c_2^e(\lambda_4) - c_4^e(\lambda_2) \cdot \hat{j} \\ d_2^e(\lambda_4) - d_4^e(\lambda_2) \cdot \hat{k} \end{bmatrix} \ell_5$$

$$W_{34}\ell_6^e = \frac{1}{(6Ve)^2} \begin{bmatrix} b_4^e(\lambda_3) - b_3^e(\lambda_4) \cdot \hat{i} \\ c_4^e(\lambda_3) - c_3^e(\lambda_4) \cdot \hat{j} \\ d_4^e(\lambda_3) - d_3^e(\lambda_4) \cdot \hat{k} \end{bmatrix} \ell_6$$

# Edge finite element method: basis functions



# Edge finite element method: data structures



$$\text{nodes} = \begin{pmatrix} \dots & \text{a1} & \dots & \text{a2} & \dots & \text{a3} & \dots & \text{a4} & \dots \\ \dots & x & \dots & x & \dots & x & \dots & x & \dots \\ \dots & y & \dots & y & \dots & y & \dots & y & \dots \\ \dots & z & \dots & z & \dots & z & \dots & z & \dots \end{pmatrix}$$

$$\text{elements} = \begin{pmatrix} \dots & i & \dots \\ \dots & \text{a1} & \dots \\ \dots & \text{a2} & \dots \\ \dots & \text{a3} & \dots \\ \dots & \text{a4} & \dots \end{pmatrix}$$

Transformation  
stage in  
pre-processing

$$\text{edges} = \begin{pmatrix} \dots & \text{e1} & \dots & \text{e2} & \dots & \text{e3} & \dots & \text{e4} & \dots & \text{e5} & \dots & \text{e6} & \dots \\ \dots & \text{a1} & \dots & \text{a1} & \dots & \text{a1} & \dots & \text{a3} & \dots & \text{a2} & \dots & \text{a3} & \dots \\ \dots & \text{a2} & \dots & \text{a3} & \dots & \text{a4} & \dots & \text{a2} & \dots & \text{a4} & \dots & \text{a4} & \dots \end{pmatrix}$$

$$\text{elements} = \begin{pmatrix} \dots & i & \dots \\ \dots & \text{e1} & \dots \\ \dots & \text{e2} & \dots \\ \dots & \text{e3} & \dots \\ \dots & \text{e4} & \dots \\ \dots & \text{e5} & \dots \\ \dots & \text{e6} & \dots \end{pmatrix}$$

$$\text{signs} = \begin{pmatrix} \dots & i & \dots \\ \dots & \text{+e1} & \dots \\ \dots & \text{+e2} & \dots \\ \dots & \text{+e3} & \dots \\ \dots & \text{+e4} & \dots \\ \dots & \text{+e5} & \dots \\ \dots & \text{+e6} & \dots \end{pmatrix}$$

# Edge finite element method: data structures

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## Algorithm 1: Edge conductivity computation

---

**Input:** A finite set of conductivities values per element:

$$\nu_\sigma = \{\nu_1, \nu_2, \dots, \nu_t\}.$$

**Output:** A finite set of conductivities values per edge:

$$\varrho_\sigma = \{\varrho_1, \varrho_2, \dots, \varrho_s\}.$$

```
1 begin
2   for  $i = 1 : s$  do Loop over edges
3      $\varrho$  computation of edge  $i$  given by:
4
5       
$$\varrho(i) = \frac{\sum \nu \in i}{\text{Number of } \nu \in i}$$

6   end
7 end
```

---

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## Algorithm 1: Primary field computation

---

**Input:** A finite set of nodes expressed by their 3D coordinates:

$$N(p_{i1}, p_{i2}, p_{i3}) \quad i = 1, 2, \dots, r.$$

A finite set of edges expressed by their

$$\text{two nodes } e(n_{i1}, n_{i2}) \quad i = 1, 2, \dots, s$$

**Output:** A finite set of projected primary electric fields in each edge:

$$\epsilon = \{\epsilon_1, \epsilon_2, \dots, \epsilon_m\}$$

```
1 begin
2   for  $i = 1 : s$  do Loop over edges
3     Mid-points computation of edge  $i$  given by:
4
5       
$$h[x_i, y_i, z_i] = \begin{bmatrix} N(e(n_{i1},1)+N(e(n_{i2},1)) \\ N(e(n_{i1},2)+N(e(n_{i2},2)) \\ N(e(n_{i1},3)+N(e(n_{i2},3)) \end{bmatrix} * 0.5$$

6
7     Depending the source orientation, the primary field
      computation of edge  $i$  in his mid-point is given by:
8
9       
$$E_{pi}(h) = \text{Equation 6, 7 or 8 of paper}$$

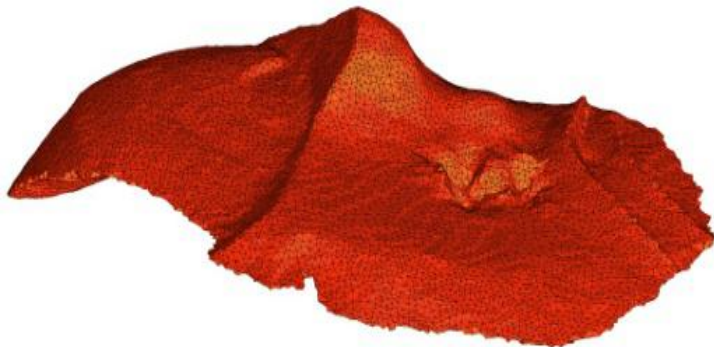
10
11     Electric field projection  $\epsilon_i$ , which is given by:
12
13       
$$\epsilon_i(E_{pi}) = \text{Equation 10 of paper}$$

14   end
15 end
```

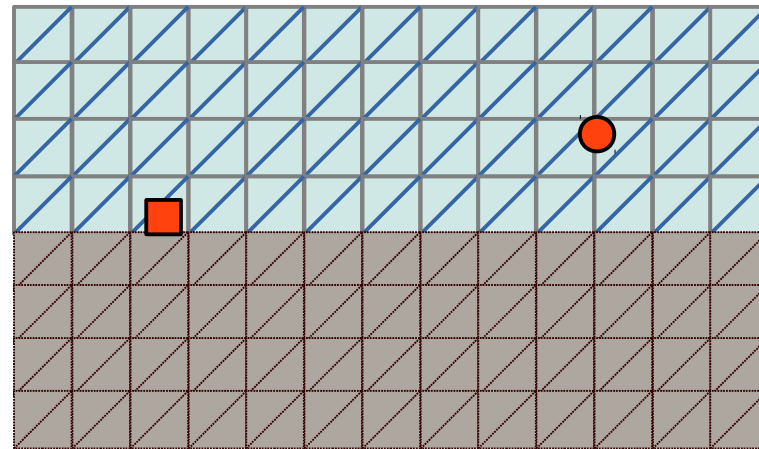
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# Edge finite element method: PDE

- ⌋ In our approach, the electric field is splitted into primary and secondary field
- ⌋ Primary field is calculated analytically for a background layered-earth model
- ⌋ Secondary field is discretized using Edge elements
- ⌋ Maxwell's equations



Repsol-BSC Research Center 2014



Slice of a CSEM mesh in X-Z plane

● Transmitter      ■ Receiver

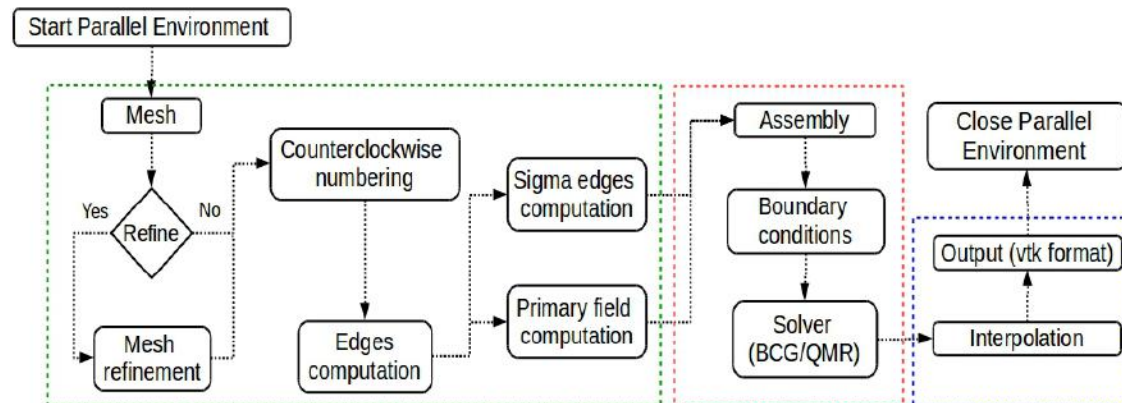
# My research thesis

## Matlab prototype as proof of concept:

- Modest parallel approach
- Support for different mesh formats
- Direct and iterative solvers

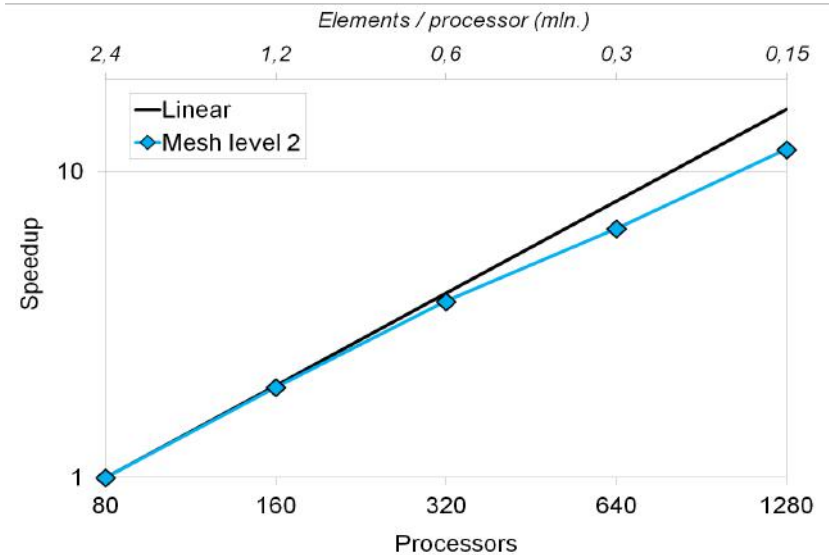
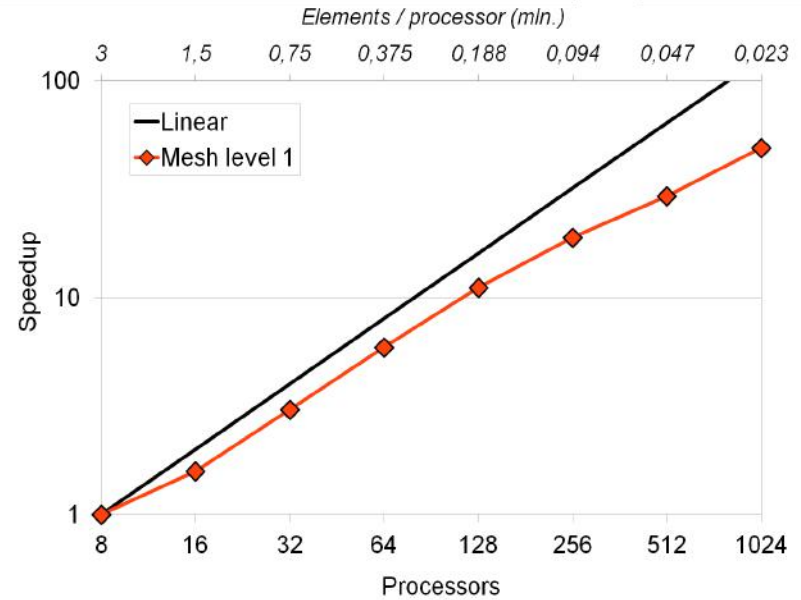
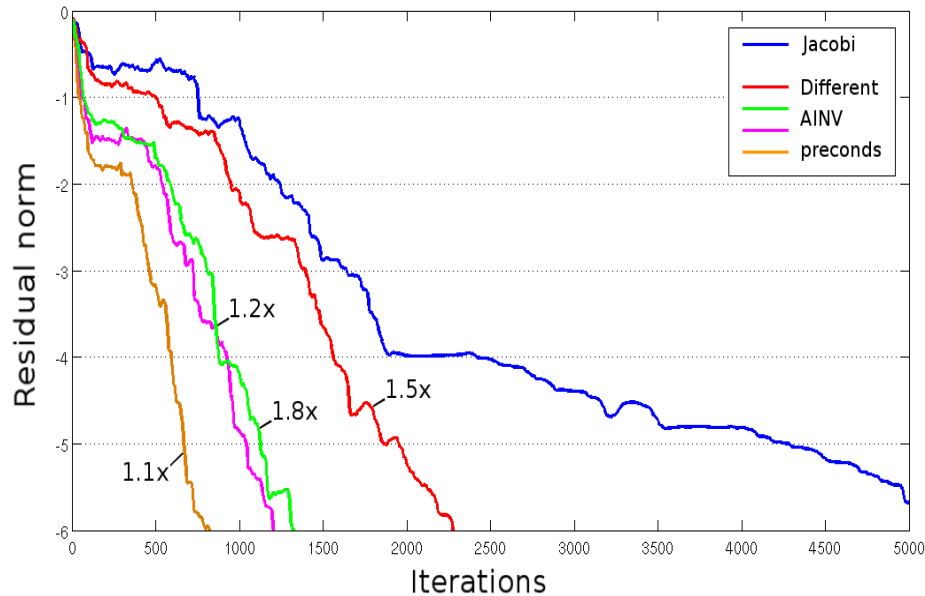
## Useful to study:

- Convergence
- Properties of elemental matrices
- Errors (L1, L2, Linf)
- Speed-up



# My research thesis

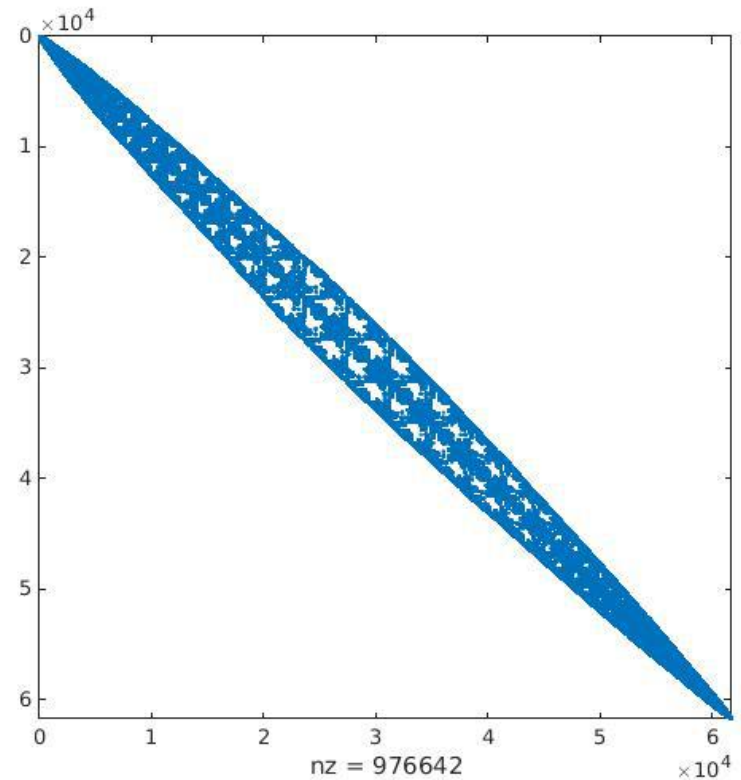
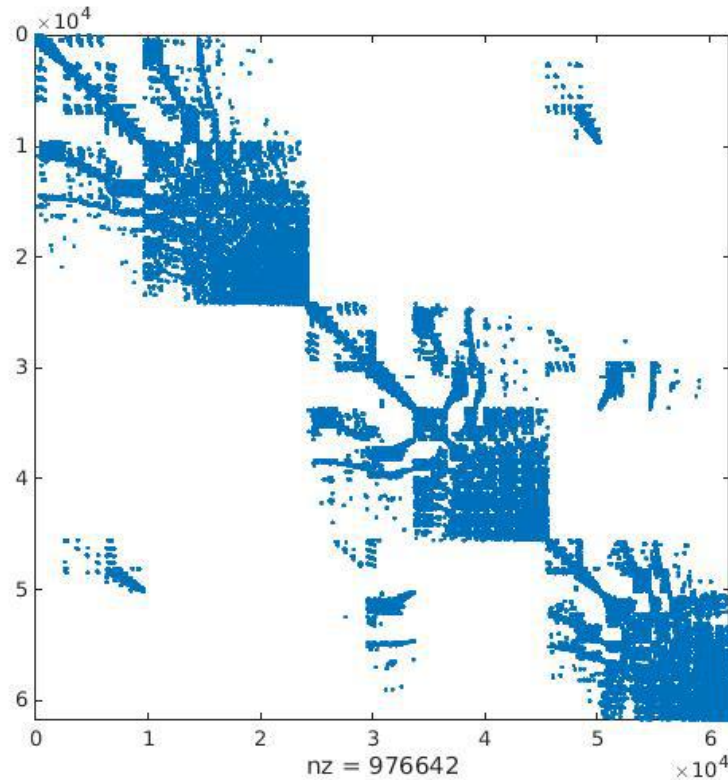
## Convergence of the solvers and scalability





# My research thesis

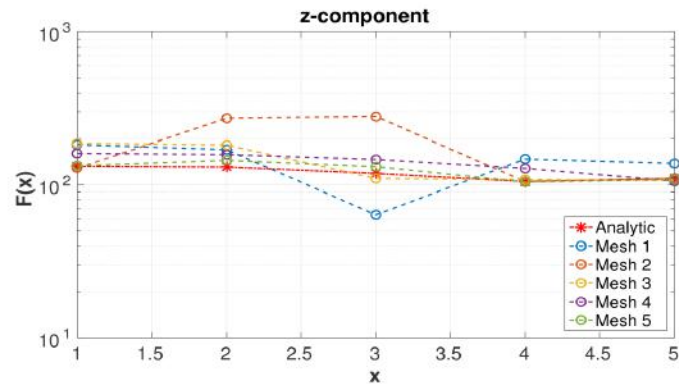
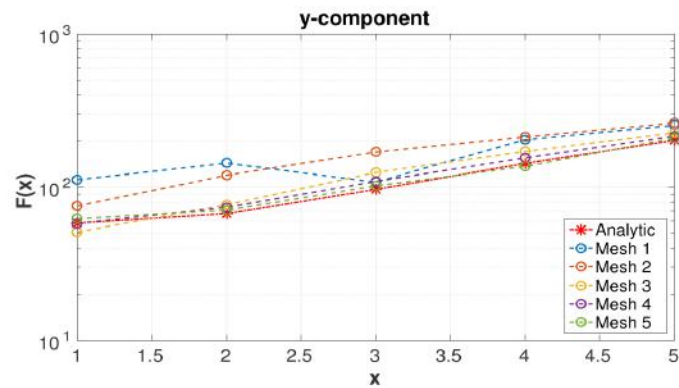
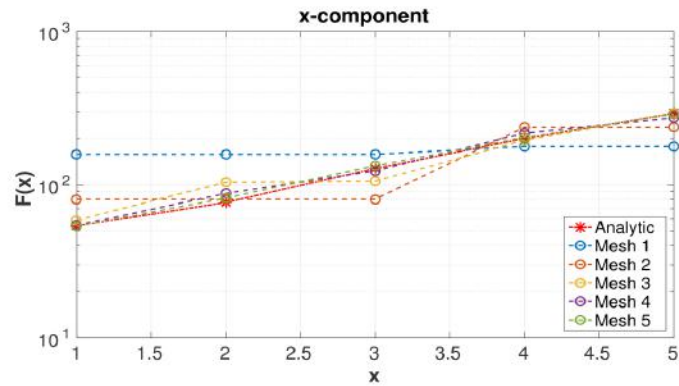
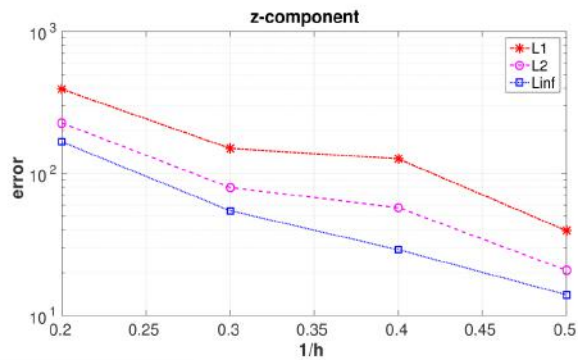
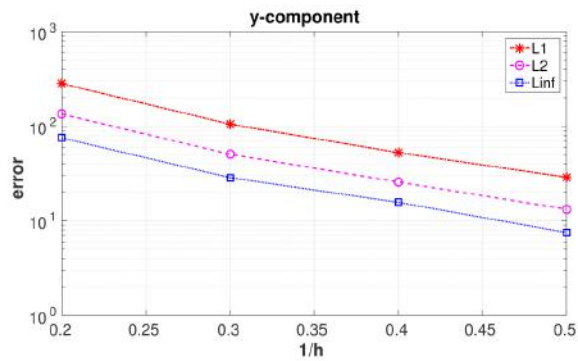
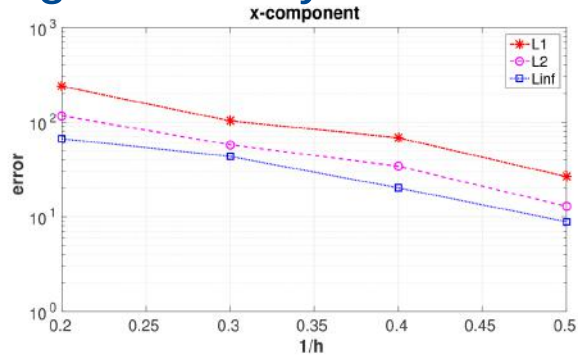
- Useful to study the matrices's properties in order to take advantages of the numerical method



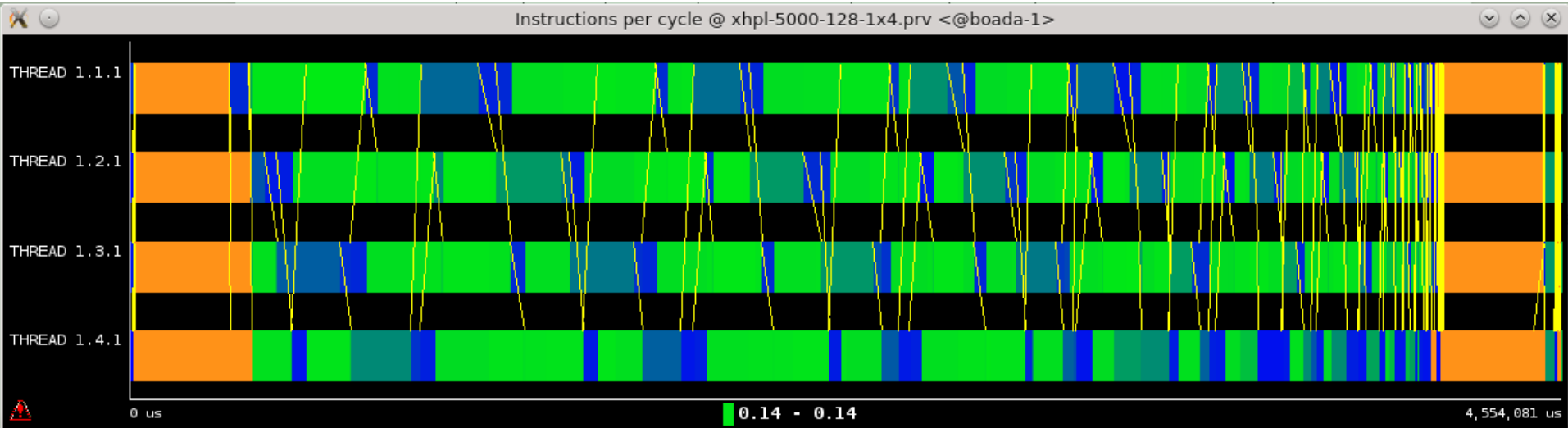
- Improving locality

# My research thesis

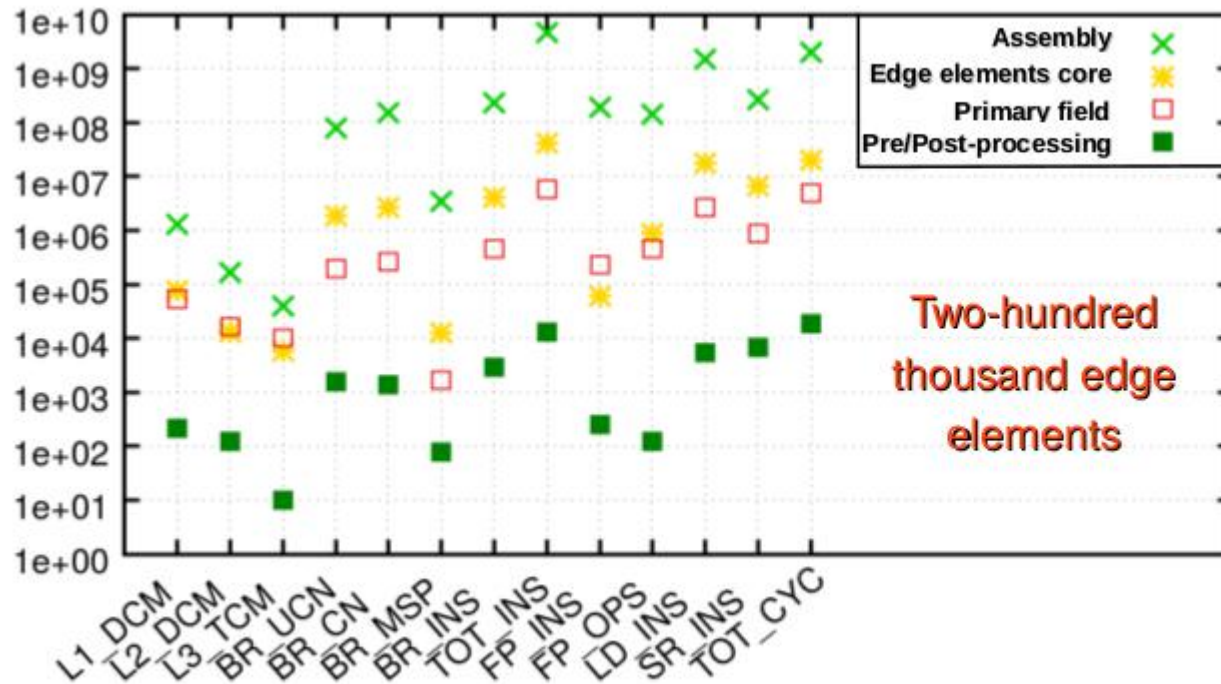
## Convergence analysis



# My research thesis



# My research thesis



Performance  
Analysis

PAPI  
Counters

Automatic generation of Hardware counters

PARAVER: Performance analyzer developed at BSC

# My research thesis

## « Journals & papers:

- Castillo, O., de la Puente, J., Modesto, D., Puzyrev, V., and Cela, J. M. (2015). Parallel edge-based tool for the simulation of 3D electromagnetic surveys in geophysics. *Computación y Sistemas*. National Polytechnic Institute. ISSN 1405-5546
- Castillo, O., de la Puente, J., Puzyrev, V., and Cela, J. M. (2015). Edge-based electric field formulation in 3D CSEM simulations: a parallel approach. In *Proceedings of the 6th International Conference and Workshop on Computing and Communication*. IEEE – 978-1-4799-6908-1. Vancouver, Canada.
- Castillo, O., de la Puente, J., Puzyrev, V., and Cela, J. M. (2015). Parallel and numerical issues of the edge finite element method for 3D controlled-source electromagnetic surveys. In *Proceedings of the International Conference on Computing Systems and Telematics*. IEEE– 978-1-4799-7639-3. Xalapa, Veracruz, México.
- Castillo, O., de la Puente, J., Puzyrev, V., and Cela, J. M. (2015). Assessment of edge-based finite element technique for geophysical electromagnetic problems: efficiency, accuracy and reliability. In *Proceedings of the 1st Pan-American Congress on Computational Mechanics and XI Argentine Congress on Computational Mechanics*, pages 984–995. ISBN 978-84-943928-2-5 CIMNE, Buenos Aires, Argentina.

## « Conferences:

- Castillo, O., de la Puente, J., Puzyrev, V., and Cela, J. M. (2015). HPC and edge elements for geophysical electromagnetic problems: an overview. In *BSC Doctoral Symposium (2nd: 2015: Barcelona)*. Barcelona Supercomputing Center.
- Castillo, O., de la Puente, J., Puzyrev, V., and Cela, J. M. (2015). Edge-elements for geophysical electromagnetic problems: a new implementation challenge. In *PRACEdays15. Partnership for Advanced Computing in Europe*. Dublin, Ireland.
- Castillo, O. (2014). Soluciones HPC para el sector energético: Desafíos y oportunidades. In *IV Simposio Becarios CONACyT en Europa*. Casa Universitaria Franco-Mexicana - Consejo Nacional de Ciencia y Tecnología de México. Strasbourg, France.

# My stay at INRIA

## Investment of my time (during this month):

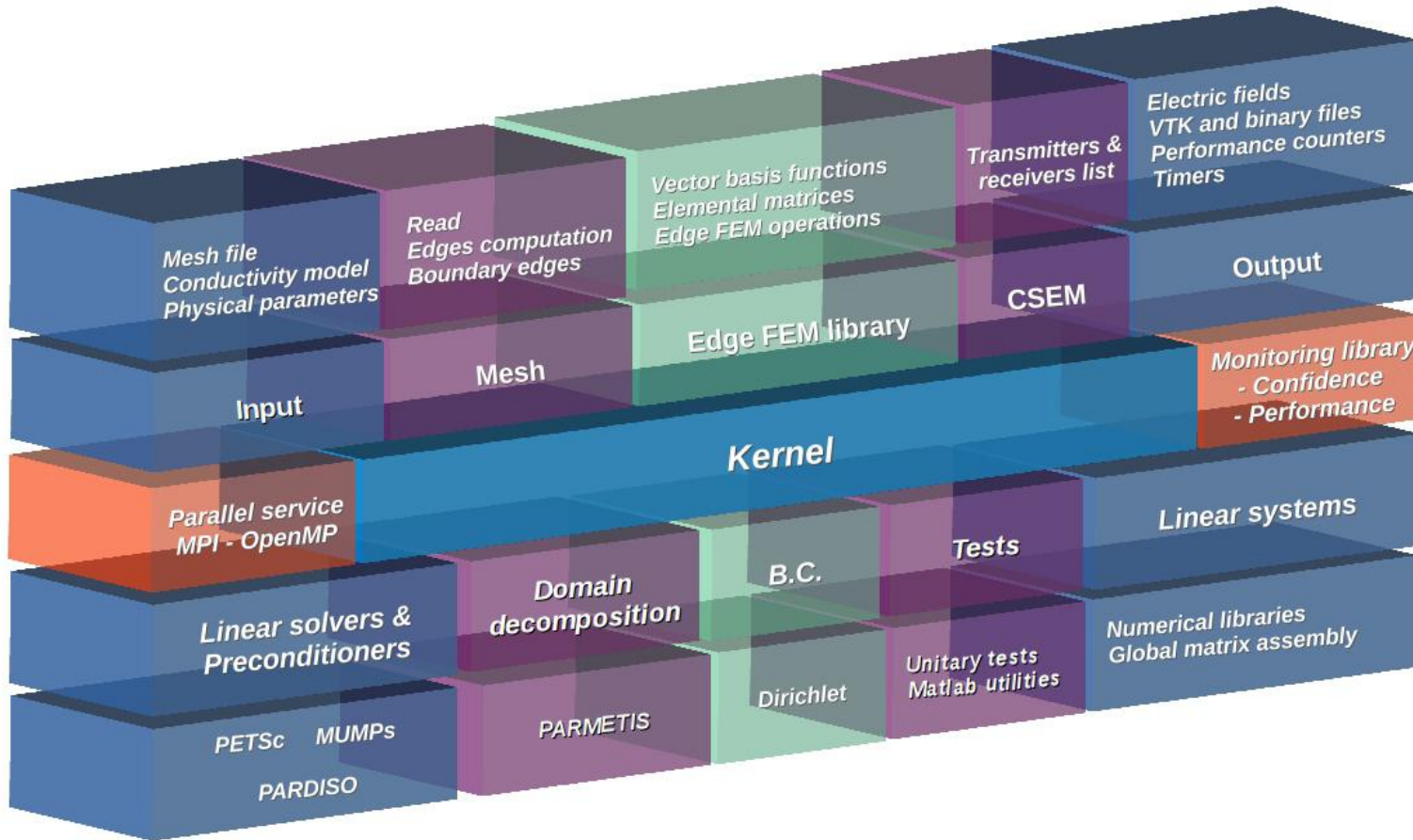
- Finding a bug in RHS assembly
- The proposal of the HPC software stack
- Writing thesis
- Two proposals for conferences (Deadline: november 30 )

### GROUP SUPPORTERS



This Project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 644202

# HPC software stack: a proposal



Accuracy



reliability



efficiency



# My stay at INRIA

« I would like receive comments, opinions and new ideas about my prototype

- Mathematical and physical formulation
- Open to share the code

« I met the team and some of your research topics...

« Fruitful personal experience, so, I would like return next year...





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**Thank you!**

For further information please contact  
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