Anisotropic elastic waves or how to model the subsurface more realistically

GdT INRIA Magique3D, 24 November 2014



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2 Math contribution

Math. contribution

- Introduction
- ABC for iso/VTI
- ABC for TTI
- Results

3 HPC contribution

- Introduction
- DIVA issue in MPI
- Task-based programming
- Results

Outline

DIP context

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Geophysics context

Upstream:

- Searching for underground oil and gas (Seismic Imaging)
- Drilling and operating the wells (Reservoir simulation)



- Midstream: transportation, storage and wholesale marketing
- Downstream: refining and purifying, marketing and distribution

Main SI techniques

Kirchhoff / Gaussian Beam

- old techniques (pre-study, approximated)
- depth migration algorithms
- simplified: vertical propagation

Reverse Time Migration (RTM)

- current widely used technique
- depth migration algorithm
- accurate: full wave equation

Full Waveform Inversion (FWI)

- upcoming technique
- Inverse problem -based
- complete: nature of the layers

RTM result



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Anisotropic elastic wave modeling

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From the simplest to the most realistic



type	(pseudo-)acoustic	elastic	poor-elastic
waves	Р	$P, S_1, S_2(3D)$	
isotropy	$ ho, {m c}$	ρ, V_{p}, V_{s}	$\rho, V_{\rho}, V_{s}, \kappa, \mu$
VTI	ε	$arepsilon,\delta,\gamma$ (3D)	
TTI	$ heta, \phi$ (3D)		

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Equations and implementations

Space discretization

- Finite Diff., Discontinuous Galerkin \Rightarrow (multi-, block-)diagonal matrix
- Finite Element, Finite Volume \Rightarrow random sparse matrix

Time-dependent or harmonic

- Time schemes: Leap-Frog, Runge-Kutta, ...
- Freq. solvers: MUMPS, multigrid...
- 1st or 2nd order (auxiliary variable)
- source or initial condition (e.g. Ricker)
- Hou10ni: DG, time & freq., Gar6more: analytic (Julien)
- Montjoie: FEM, time & freq. (Marc)
- DIVA-DG (TOTAL)
 - TMBM (Simon, Lionel): DG, time
 - THBM (Marie, Théo., Flo.): DG, freq.

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HPC context



Seismic Depth Imaging Methods & HPC evolution

From one-way to full-wave

- more computations
- more storage

From isotropy to TTI

- more computations
- more storage

Miscellaneous

- big data issues (imaging)
- uncertainty quantification

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Introduction

What TTI is?

Geophysics anisotropy

Earth's crust (geological layers of rocks) is assumed to be locally polar anisotropic, also called transversely isotropic (TI).



туре	isotropy	Vertical II	Tilted TI
tensor	sparse ref.	sparse modif.	full dense
wavefronts	circles	non-circles	non-circles
3D S waves	coupled	decoupled	decoupled

TTI example



Introduction

The problem: PML



Introduction

Idea: ABC



Enquist-Majda methodology

Transparent Boundary Condition (TBC)

- Fourier transform in the directions parallel to the boundary
- ODE: $\partial_x \widehat{W} + M \widehat{W} = 0$, *M* invertible
- (Λ_i, P_i) eigenvalues/eigenvectors of M
- $\widehat{W} = \widehat{W}_0 e^{M_X} \Rightarrow \mathsf{TBC}$

From TBC to ABC: approximation of a pseudo-differential operator

2D isotropic eigenvalues

$$\begin{cases} \lambda_p &= \sqrt{k^2 - w^2/V_p^2} \approx \frac{iw}{V_p} \\ \lambda_s &= \sqrt{k^2 - w^2/V_s^2} \approx \frac{iw}{V_s} \end{cases}$$

Enquist-Majda methodology

Transparent Boundary Condition (TBC)

- Fourier transform in the directions parallel to the boundary
- ODE: $\partial_x \widehat{W} + M \widehat{W} = 0$, *M* invertible
- (Λ_i, P_i) eigenvalues/eigenvectors of M

•
$$\widehat{W} = \widehat{W}_0 e^{M_X} \Rightarrow \mathsf{TBC}$$

From TBC to ABC: approximation of a pseudo-differential operator

2D VTI eigenvalues

$$\lambda_{P/S} = \sqrt{\alpha k^2 - \beta \rho w^2 \pm \sqrt{\gamma k^4 - \eta \rho k^2 w^2 + \xi \rho^2 w^4}}$$

and TTI is worse... \Rightarrow this methodology seems not usable!

P-waves and S-waves splitting

Elastic sub-systems

VTI P-waves:

$$\begin{cases} \rho \partial_t v_x = \partial_x \sigma_{xx} \\ \rho \partial_t v_z = \partial_z \sigma_{zz} \\ \partial_t \sigma_{xx} = C_{11} \partial_x v_x + C_{13} \partial_z v_z \\ \partial_t \sigma_{zz} = C_{13} \partial_x v_x + C_{33} \partial_z v_z \end{cases}$$

VTI S-waves:

$$\begin{cases}
\rho \partial_t v_x = \partial_x \sigma_{xx} + \partial_z \sigma_{xz} \\
\rho \partial_t v_z = \partial_x \sigma_{xz} + \partial_z \sigma_{zz} \\
\partial_t \sigma_{xx} = -2C_{66} \partial_z v_z \\
\partial_t \sigma_{zz} = -2C_{66} \partial_x v_x \\
\partial_t \sigma_{xz} = C_{66} (\partial_x v_z + \partial_z v_x)
\end{cases}$$
(2)

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(1)

P-waves and S-waves splitting

Elastic sub-systems ABC

VTI P-waves:

$$\begin{cases} \sigma_{xx} \approx -\rho V_{\rho} \kappa v_{x} \\ \sigma_{xz} \approx 0 \end{cases}$$
(1)

VTI S-waves:

$$\begin{cases} \sigma_{xx} \approx 0 \\ \sigma_{xz} \approx -\rho V_s v_z \end{cases}$$

(2)

(3)

Elastic P-S system ABC

$$\begin{cases} \sigma_{xx} \approx -\rho V_p \kappa v_x \\ \sigma_{xz} \approx -\rho V_s v_z \end{cases}$$

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Geometrical point of view



Figure: P-waves slowness curves of isotropic (left) and TTI (right) cases

Circle equation:

$$\rho V_p^2 \kappa^2 k_x^{\star 2} + \rho V_p^2 k_z^{\star 2} = 1$$
(4)

Rotated ellipsoid equation:

$$\rho V_p^2 \kappa^2 (k_x \cos \theta + k_z \sin \theta)^2 + \rho V_p^2 (-k_x \sin \theta + k_z \cos \theta)^2 = 1 \qquad (5)$$

Change of coordinates

Propagation modes k from the geometry:

$$k_x = \mu_1 k_x^{\star} + \mu_2 k_z^{\star}$$
 and $k_z = \mu_3 k_x^{\star} + \mu_4 k_z^{\star}$ (6)

Velocity *v* from the second-order equation:

$$v_x = \alpha_1 v_x^{\star} + \alpha_2 v_z^{\star}$$
 and $v_z = \alpha_3 v_x^{\star} + \alpha_4 v_z^{\star}$ (7)

<u>Stress tensor σ from the first-order equation:</u>

$$\sigma_{xx} = \widetilde{\beta}_1 p^*$$
 and $\sigma_{zz} = \widetilde{\beta}_2 p^*$ and $\sigma_{xz} = \widetilde{\beta}_3 p^*$ (8)

 \rightarrow apply the opposite change of coordinate on the isotropic ABCs

Change of coordinates 1/3

Propagation modes

$$k_x = \mu_1 k_x^* + \mu_2 k_z^*$$
 and $k_z = \mu_3 k_x^* + \mu_4 k_z^*$ (9)

For a vertical boundary: $\mu_3 = 0$ (otherwise $\mu_2 = 0$)

Circle equation:

$$\rho V_p^2 \kappa^2 k_x^{\star 2} + \rho V_p^2 k_z^{\star 2} = 1$$
(10)

Rotated ellipsoid equation:

$$\rho V_p^2 \kappa^2 (k_x \cos \theta + k_z \sin \theta)^2 + \rho V_p^2 (-k_x \sin \theta + k_z \cos \theta)^2 = 1 \qquad (11)$$

$$\begin{cases} \mu_1 = \sqrt{1/(\kappa^2 \cos^2 \theta + \sin^2 \theta)} \\ \mu_2 = -(\kappa^2 - 1) \cos \theta \sin \theta / \kappa \sqrt{\kappa^2 \cos^2 \theta + \sin^2 \theta} \\ \mu_4 = \frac{1}{\kappa} \sqrt{\kappa^2 \cos^2 \theta + \sin^2 \theta} \end{cases}$$
(12)

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Change of coordinates 2/3

Velocity v

$$v_x = \alpha_1 v_x^* + \alpha_2 v_z^*$$
 and $v_z = \alpha_3 v_x^* + \alpha_4 v_z^*$ (13)

Propagation modes change of coordinates:

$$k_x = \mu_1 k_x^{\star} + \mu_2 k_z^{\star}$$
 and $k_z = \mu_3 k_x^{\star} + \mu_4 k_z^{\star}$ (14)

Second-order TTI elastic system, with $V = (v_x, v_z)$:

$$\rho \partial_t^2 V = [C_{xx} k_x^2 + C_{xz} k_x k_z + C_{zz} k_z^2] V$$
(15)

Second-order isotropic elastic system:

$$\rho \partial_t^2 V^* = \rho V_\rho^2 [I_{xx} k_x^{2^*} + I_{xz} k_x^* k_z^* + I_{zz} k_z^{2^*}] V^*$$
(16)

Change of coordinates 2/3

Velocity v

$$v_x = \alpha_1 v_x^* + \alpha_2 v_z^*$$
 and $v_z = \alpha_3 v_x^* + \alpha_4 v_z^*$ (13)

Propagation modes change of coordinates Second-order TTI elastic system Second-order isotropic elastic system

$$\begin{cases} v_x = \frac{\sqrt{\rho}V_p}{\sqrt{\kappa^2\cos^2\theta + \sin^2\theta}} [-(\kappa\cos^2\theta + \sin^2\theta)v_x^* - (\kappa - 1)\cos\theta\sin\theta v_z^*] \\ v_z = \frac{\sqrt{\rho}V_p}{\sqrt{\kappa^2\cos^2\theta + \sin^2\theta}} [(\kappa - 1)\cos\theta\sin\theta v_x^* - (\kappa\cos^2\theta + \sin^2\theta)v_z^*] \end{cases}$$

Change of coordinates 3/3

Stress tensor σ

$$\sigma_{xx} = \widetilde{\beta_1} p^*$$
 and $\sigma_{zz} = \widetilde{\beta_2} p^*$ and $\sigma_{xz} = \widetilde{\beta_3} p^*$ (14)

Propagation modes change of coordinates:

$$k_x = \mu_1 k_x^{\star} + \mu_2 k_z^{\star}$$
 and $k_z = \mu_3 k_x^{\star} + \mu_4 k_z^{\star}$ (15)

First-order TTI elastic system:

$$\begin{cases}
\rho \partial_t \mathbf{v}_{\mathsf{x}} = k_{\mathsf{x}} \sigma_{\mathsf{x}\mathsf{x}} + k_{\mathsf{z}} \sigma_{\mathsf{x}\mathsf{z}} \\
\rho \partial_t \mathbf{v}_{\mathsf{z}} = k_{\mathsf{x}} \sigma_{\mathsf{x}\mathsf{z}} + k_{\mathsf{z}} \sigma_{\mathsf{z}\mathsf{z}} \\
\partial_t \sigma_{\mathsf{x}\mathsf{x}} = \widetilde{C_{11}} k_{\mathsf{x}} \mathbf{v}_{\mathsf{x}} + \widetilde{C_{13}} k_{\mathsf{z}} \mathbf{v}_{\mathsf{z}} + \widetilde{C_{16}} (k_{\mathsf{x}} \mathbf{v}_{\mathsf{z}} + k_{\mathsf{z}} \mathbf{v}_{\mathsf{x}}) \\
\partial_t \sigma_{\mathsf{z}\mathsf{z}} = \widetilde{C_{13}} k_{\mathsf{x}} \mathbf{v}_{\mathsf{x}} + \widetilde{C_{33}} k_{\mathsf{z}} \mathbf{v}_{\mathsf{z}} + \widetilde{C_{26}} (k_{\mathsf{x}} \mathbf{v}_{\mathsf{z}} + k_{\mathsf{z}} \mathbf{v}_{\mathsf{x}}) \\
\partial_t \sigma_{\mathsf{x}\mathsf{z}} = \widetilde{C_{16}} k_{\mathsf{x}} \mathbf{v}_{\mathsf{x}} + \widetilde{C_{26}} k_{\mathsf{z}} \mathbf{v}_{\mathsf{z}} + \widetilde{C_{66}} (k_{\mathsf{x}} \mathbf{v}_{\mathsf{z}} + k_{\mathsf{z}} \mathbf{v}_{\mathsf{x}})
\end{cases}$$
(16)

Change of coordinates 3/3

Stress tensor σ

$$\sigma_{xx} = \widetilde{\beta}_1 p^*$$
 and $\sigma_{zz} = \widetilde{\beta}_2 p^*$ and $\sigma_{xz} = \widetilde{\beta}_3 p^*$ (14)

First-order isotropic elastic system:

$$\begin{cases} \rho \partial_t \mathbf{v}_x^{\star} = k_x^{\star} p^{\star} \\ \rho \partial_t \mathbf{v}_z^{\star} = k_z^{\star} p^{\star} \\ \partial_t p^{\star} = \rho V_p^2 k_x^{\star} \mathbf{v}_x^{\star} + \rho V_p^2 k_z^{\star} \mathbf{v}_z^{\star} \end{cases}$$
(15)

Velocity v change of coordinates:

$$v_x = \alpha_1 v_x^{\star} + \alpha_2 v_z^{\star}$$
 and $v_z = \alpha_3 v_x^{\star} + \alpha_4 v_z^{\star}$ (16)

Change of coordinates 3/3

Stress tensor σ

$$\sigma_{xx} = \widetilde{\beta_1} p^*$$
 and $\sigma_{zz} = \widetilde{\beta_2} p^*$ and $\sigma_{xz} = \widetilde{\beta_3} p^*$ (14)

Propagation modes change of coordinates First-order TTI elastic system First-order isotropic elastic system Velocity v change of coordinates

$$\begin{cases} \sigma_{xx} = -\sqrt{\rho} V_{\rho}(\kappa \cos^2 \theta + \sin^2 \theta) p^* \\ \sigma_{zz} = -\sqrt{\rho} V_{\rho}(\kappa \sin^2 \theta + \cos^2 \theta) p^* \\ \sigma_{xz} = \sqrt{\rho} V_{\rho}(\kappa - 1) \cos \theta \sin \theta p^* \end{cases}$$
(15)

TTI ABC formulation

P-wave ABC

Isotropic:

$$\begin{cases} \sigma_{xx}^* = -\rho V_p v_x^* \\ \sigma_{xz}^* = 0 \end{cases}$$

TTI (change of coordinates):

$$\begin{cases} \sigma_{xx} = -\rho V_p \frac{\kappa \cos^2 \theta + \sin^2 \theta}{\sqrt{\kappa^2 \cos^2 \theta + \sin^2 \theta}} \left[(\kappa \cos^2 \theta + \sin^2 \theta) v_x + (\kappa - 1) \cos \theta \sin \theta v_z \right] \\ \sigma_{xz} = -\rho V_p \frac{(\kappa - 1) \cos \theta \sin \theta}{\sqrt{\kappa^2 \cos^2 \theta + \sin^2 \theta}} \left[(\kappa \cos^2 \theta + \sin^2 \theta) v_x + (\kappa - 1) \cos \theta \sin \theta v_z \right] \end{cases}$$

S-wave ABC

$$\begin{cases} \sigma_{xx} = 0 \\ \sigma_{xz} = -\rho V_s v_z \end{cases}$$

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Same methodology in 3D

$$\begin{cases} \sigma_{xx} = -\sqrt{\rho} V_{\rho} & \frac{\kappa \sin^{2} \phi + \cos^{2} \phi (\kappa \cos^{2} \theta + \sin^{2} \theta)}{\sqrt{\kappa^{2} \cos^{2} \theta \cos^{2} \phi + \kappa^{2} \sin^{2} \phi + \sin^{2} \theta \cos^{2} \phi}} [\\ +(\kappa \sin^{2} \phi + \cos^{2} \phi (\kappa \cos^{2} \theta + \sin^{2} \theta))v_{x} \\ -(\kappa - 1) \cos \phi \sin \phi \sin^{2} \theta v_{y} - (\kappa - 1) \cos \theta \sin \theta \cos \phi v_{z}}] \end{cases}$$

$$\begin{cases} \sigma_{xz} = -\sqrt{\rho} V_{\rho} & \frac{(\kappa - 1) \cos \theta \sin \theta \cos \phi}{\sqrt{\kappa^{2} \cos^{2} \theta \cos^{2} \phi + \kappa^{2} \sin^{2} \phi + \sin^{2} \theta \cos^{2} \phi}} [\\ -(\kappa \sin^{2} \phi + \cos^{2} \phi (\kappa \cos^{2} \theta + \sin^{2} \theta))v_{x} \\ +(\kappa - 1) \cos \phi \sin \phi \sin^{2} \theta v_{y} + (\kappa - 1) \cos \theta \sin \theta \cos \phi v_{z}}] \end{cases}$$

$$(16)$$

$$\sigma_{xy} = -\sqrt{\rho} V_{\rho} & \frac{(\kappa - 1) \cos \phi \sin \phi \sin^{2} \theta}{\sqrt{\kappa^{2} \cos^{2} \theta \cos^{2} \phi + \kappa^{2} \sin^{2} \phi + \sin^{2} \theta \cos^{2} \phi}} [\\ -(\kappa \sin^{2} \phi + \cos^{2} \phi (\kappa \cos^{2} \theta + \sin^{2} \theta))v_{x} \\ +(\kappa - 1) \cos \phi \sin \phi \sin^{2} \theta v_{y} + (\kappa - 1) \cos \theta \sin \theta \cos \phi v_{z}}] \end{cases}$$

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Image: A mathematical states and a mathem

TTI results



Figure: Velocity magnitude at different time steps of the simulation

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TTI ABC versus simple ABCs

TTI medium, with different kind of ABCs:



Figure: TTI ABC versus simple ABCs

 \Rightarrow TTI ABC is better than isotropic or VTI ABCs!

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HPC evolution



- Monoprocessors: easy to program, automatic updates
- Multicores: start thinking parallel, local and remote memories
- Accelerators: peak vs real performances, special language
- Manycores: at least two levels of parallelism

The problem: heterogeneity



Classical approach:

- MPI over CPUs
- CUDA over GPUs

implies:

- big programming effort
- difficult to maintain
- hardware-dependent

Idea: use a runtime and task-based programming



Runtime system:

- abstraction layer
- hiding heterogeneity

Scheduler decides:

- where to execute
- when to execute

Memory manager:

- does the transfer
- guarantees consistency

Discretization

Elastic wave equation (first order)

$$\begin{cases}
\rho(\mathbf{x})\partial_t \mathbf{v}(\mathbf{x},t) &= \nabla \underline{\underline{\sigma}}(\mathbf{x},t) \\
\partial_t \underline{\underline{\sigma}}(\mathbf{x},t) &= \underline{\underline{\Gamma}}(\mathbf{x}) : \underline{\underline{\epsilon}}(\mathbf{v}(\mathbf{x},t))
\end{cases}$$
(17)

Discontinuous Galerkin with Leap-frog scheme

Iteration on n:

$$\begin{cases} M_{v} \frac{v_{h}^{n+1} - v_{h}^{n}}{\Delta t} + R_{\underline{\underline{\sigma}}} \underline{\underline{\sigma}}_{h}^{n+1/2} = 0 \\ M_{\underline{\underline{\sigma}}} \underline{\underline{\underline{\sigma}}}_{\underline{\underline{m}}}^{n+3/2} - \underline{\underline{\sigma}}_{\underline{\underline{n}}}^{n+1/2} + R_{v} v_{h}^{n+1} = 0 \end{cases}$$
(18)

 M_{v} and M_{σ} block-diagonal matrices \Rightarrow easily invertible!

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Sequential algorithm

Quasi-explicit reformulation

$$\begin{cases} v_h^{n+1} &= v_h^n + M_v^{-1} [\Delta t R_{\underline{\sigma}} \underline{\sigma}_h^{n+1/2}] \\ (\underline{\sigma}_h^{n+3/2} &= \underline{\sigma}_h^{n+1/2} + M_{\underline{\sigma}}^{-1} [\Delta t R_v v_h^{n+1}] \end{cases}$$
(19)

Algorithme 1 : DIP sequential

$$\begin{array}{l} \textbf{Data}: \ \textit{N}_{h}, \Delta_{t}, \textit{N}_{t} \\ \textbf{Result}: \ \textit{v}_{h}, \ \underline{\sigma}_{h} \\ [\textit{v}_{h}^{1}, \underline{\sigma}_{h}^{3/2}] \leftarrow \textbf{Initialization}(\textit{N}_{h}, \Delta_{t}); \\ \textbf{for} \ \underline{n = 1..N_{t}} \ \textbf{do} \\ & \left| \begin{array}{c} \textbf{for} \ \underline{K = 1..N_{h}} \ \textbf{do} \\ | \ v_{h_{K}}^{n+1} \leftarrow \textbf{UpdateVelocity}(v_{h_{K}}^{n}, \underline{\sigma}_{h_{\overline{K}}}^{n+1/2}, \Delta_{t}); \\ \textbf{end} \\ \textbf{for} \ \underline{K = 1..N_{h}} \ \textbf{do} \\ | \ \underline{\sigma}_{h_{K}}^{n+3/2} \leftarrow \textbf{UpdateStress}(\underline{\sigma}_{h_{K}}^{n+1/2}, v_{h_{\overline{K}}}^{n+1}, \Delta_{t}); \\ \textbf{end} \end{array} \right| \\ \textbf{end} \end{array}$$

end

Parallel algorithm

Algorithme 2 : DIP parallel **Data** : N_p , N_h , Δ_t , N_t **Result** : v_h , $\underline{\sigma}_h$ $N_{h_{loc}} \leftarrow \text{DomainDecomposition}(N_p);$ $[v_h^1, \underline{\sigma}_h^{3/2}] \leftarrow \text{Initialization}(N_{h_{loc}}, \Delta_t);$ for $n = 1..N_t$ do $\underline{\sigma}_{h=}^{n+1/2} \leftarrow \text{Communication}(\underline{\sigma}_{h\nu}^{n+1/2});$ for $K = 1..N_{h_{loc}}$ do $v_{h\kappa}^{n+1} \leftarrow \text{UpdateVelocity}(v_{h\kappa}^n, \underline{\sigma}_{h\pi}^{n+1/2}, \Delta_t);$ end $v_{h_{\tau}}^{n+1} \leftarrow \text{Communication}(v_{h_{\nu}}^{n+1});$ for $K = 1..N_{h_{loc}}$ do $\overline{\sigma_{h}^{n+3/2}} \leftarrow U$ pdateStress $(\underline{\sigma_{h}}^{n+1/2}, v_{h-1}^{n+1}, \Delta_t);$ end end

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HPC contribution DIVA issue in MPI

MPI version = one domain per process



Load balancing limitations:

- order of (discretization of) each cell and direct neighbors
- architecture specifications (e.g. SIMD, cache size, ...)

 \Rightarrow not straightforward to estimate the computation time for a domain!

Load imbalance example



Figure: Trace analysis of 10 timesteps on 32 MPI processes

Each line represents a core activity during the execution time:

- ORANGE computation of the velocity field
- RED computation of the stress tensor
- GREY waiting time (synchronization)

HPC contribution Task-based programming

First step: the DAG (Direct Acyclic Graph)

Sequential code

 $A \leftarrow fun1();$ $B \leftarrow fun2(A);$ $C \leftarrow fun2(A);$ $C \leftarrow fun3(B,C);$

- Describe the functions as tasks
- Form the dataflow: specify data dependencies



Second step: task description

Parametrized model

 $\begin{array}{l} \mathsf{Task_fun1()} \\ \mathsf{WRITE} \ \mathsf{A} \rightarrow \mathsf{A} \ \mathsf{Task_fun2(1)} \\ & \rightarrow \mathsf{A} \ \mathsf{Task_fun2(2)} \end{array}$

 $\begin{array}{l} Task_fun2(n) \\ n{=}1..2 \\ READ \quad A \leftarrow A \; Task_fun1() \\ WRITE \; X \rightarrow (n{=}{=}1) \; ? \; B \; Task_fun3() \\ \quad \rightarrow (n{=}{=}2) \; ? \; C \; Task_fun3() \end{array}$



Basic COMPUTE and EXCHANGE model:





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Basic COMPUTE and EXCHANGE model:



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Image: A matrix of the second seco

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Basic COMPUTE and EXCHANGE model:





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Fine granularity



rigure. Subdivision examp

More than one domain per CPU

- exhibit deeper parallelism
- allow dynamic flexibility
- reduce the boundary size

Results

Geophysics test case

Realistic test case:

- 3D elastic
- TTI (anisotropy)
- multi-layers

Hybrid discretization:

- unstructured tetrahedra
- P1-P2-P3 orders
- boundary conditions



Results

One processor behavior



 \Rightarrow both fine granularity and work-stealing are essential!

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Anisotropic elastic wave modeling

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ccNUMA machine

- 8 processors (previous Intel Xeon E7-8837)
- Total of 64 CPU cores in ccNUMA architecture



Figure: cache-coherent Non-Uniform Memory Access (ccNUMA) scheme

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HPC contribution

Results

ccNUMA results - efficiency



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Results

Trace comparison



Figure: PaRSEC version (NUMA-aware, granularity x6) t = 2.060s

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Intel Xeon Phi results - efficiency

HPC contribution

Results



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Conclusion & perspectives

Context

- Geophysics: Seismic Imaging (waves)
- TOTAL collaboration: DIP (TMBM)

Results

- TI without additional cost: ABC (with taper) vs PML
- (portable) perfect efficiency: task-based vs MPI parallelism

Next

- many updates in TMBM
- GPU option with OpenACC