High–Order Non–Conforming Finite Element Methods for Acoustic–Elastic Problems

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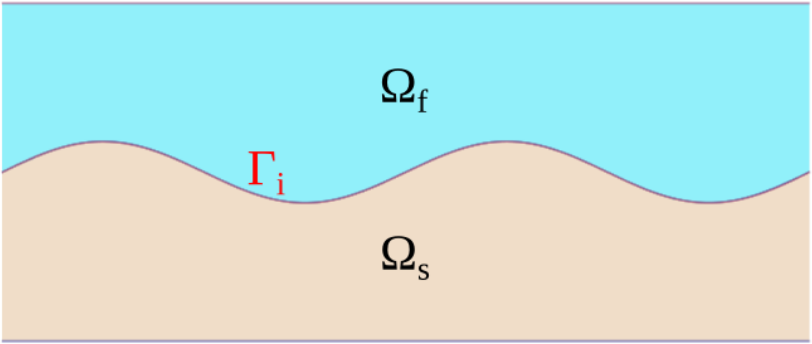
Elastic–Acoustic problem: Domain



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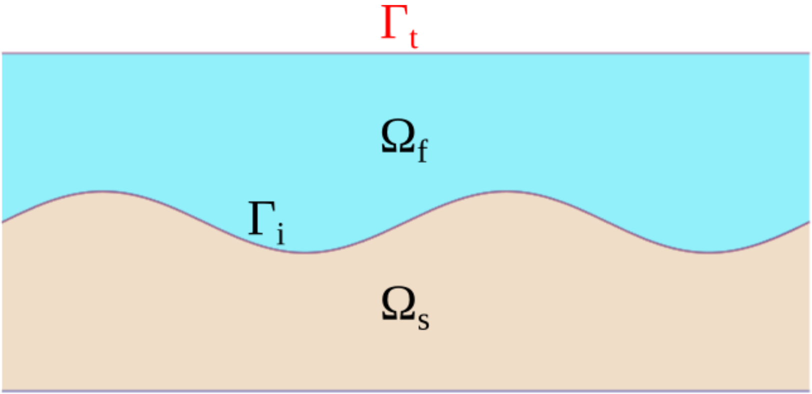

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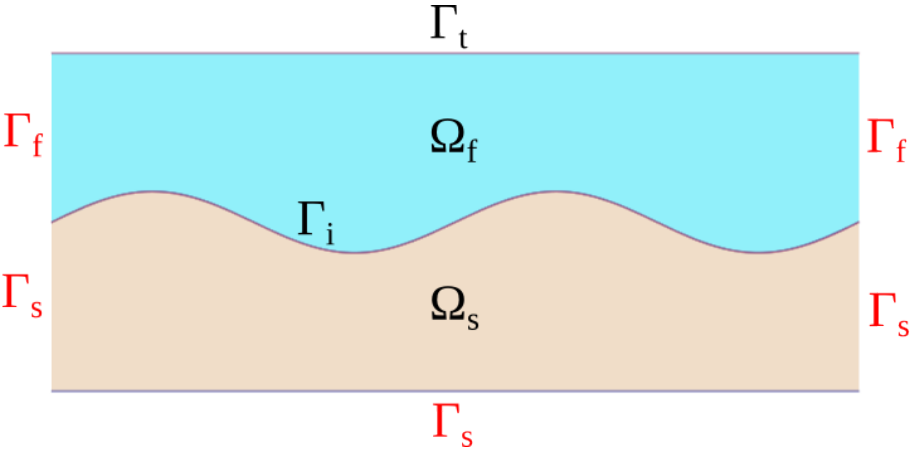

On Γ_i we impose continuation of the normal components of displacement and traction

$$\mathbf{u_s} \cdot \mathbf{n_s} + \mathbf{u_f} \cdot \mathbf{n_f} = 0, \tag{1}$$

$$\underline{\underline{\sigma}}(\mathbf{u_s})\mathbf{n_s} + \underline{\underline{\sigma}}(\mathbf{u_f})\mathbf{n_f} = 0, \tag{2}$$

where, in our case,

$$\sigma_{ij}(\mathbf{u}) = \lambda \, \nabla \cdot \mathbf{u} \, \delta_{ij} + 2 \, \mu \, \epsilon_{ij}(\mathbf{u}), \tag{3}$$

$$\epsilon_{ij}(\mathbf{u}) = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right). \tag{4}$$

For the fluid part $\mu_f = 0$ and so $\sigma_{ij}(\mathbf{u_f}) = -p_f I$.

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\end{cases}
$$

Elastic–Acoustic problem

Apply $\frac{\partial}{\partial t}(.)$ to the elastic equations and define $v = \partial u/\partial t$ to obtain a more convenient formulation:

$$
(P2) = \begin{cases}
\dfrac{1}{\rho_f \; c^2}\dfrac{\partial^2 p_f}{\partial t^2} - \nabla \cdot (\dfrac{1}{\rho_f}\nabla p_f) = f, \text{ in } \Omega_f \\
\rho_f \dfrac{\partial \mathbf{v_s}}{\partial t} \cdot \mathbf{n}_s - \dfrac{\partial p_f}{\partial \mathbf{n}_f} = 0, \text{ on } \Gamma_i \\
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p = 0, \text{on } \Gamma_t \\
\dfrac{1}{c_f}\dfrac{\partial p}{\partial t} + \nabla p \cdot n = 0, \text{on } \Gamma_f \\
\rho_s B \dfrac{\partial \mathbf{v_s}}{\partial t} + \underline{\underline{\sigma}}(\mathbf{v_s}) \cdot \mathbf{n_s} = 0, \text{on } \Gamma_s
\end{cases}
$$

Weak formulation

Multiply the elastic equation by $w_s = (w_1, w_2)$ and the pressure equation by w_f to proceed as

$$\int_{\Omega_s} \rho_s \frac{\partial^2 v}{\partial t^2} \cdot w_s + \lambda \nabla \cdot v \nabla \cdot w_s \, d\Omega + \int_{\Omega_s} 2\mu \sum_{i,j=1}^{2} \epsilon_{i,j}(v)\epsilon_{i,j}(w_s) \, d\Omega \tag{5}$$

$$+ \int_{\Gamma_s} \rho_s \left(B \frac{\partial v}{\partial t} \right) \cdot w_s \, d\Gamma - \int_{\Gamma_i} \frac{\partial p}{\partial t} n_f \cdot w_s \, d\Gamma = 0,$$

$$\int_{\Omega_f} \rho_f^{-1} c_f^{-2} \frac{\partial^2 p}{\partial t^2} w_f \, d\Omega + \int_{\Omega_f} \rho_f^{-1} \nabla p \cdot w_f \, d\Omega \tag{6}$$

$$- \int_{\Gamma_f} \rho_f^{-1} c_f^{-1} \frac{\partial p}{\partial t} w_f \, d\Gamma - \int_{\Gamma_i} \frac{\partial v}{\partial t} \cdot n_s w_f d\Gamma = \int_{\Omega_f} f w_f \, d\Omega.$$

Let $u = (p, v_1, v_2)^T$, where $p = \sum_{j=1}^{D_f} p_j N_{f,j}$, $v_1 = \sum_{j=1}^{D_s} v_{1,j} N_{s,j}$, $v_2 = \sum_{j=1}^{D_s} v_{2,j} N_{s,j}$ and $F = (f, 0, 0)^T$. The matrix system associated to the problem (P2) is defined as

$$M \frac{\partial^2 u}{\partial t^2} + (S + C) \frac{\partial u}{\partial t} + K u = F, \tag{7}$$

whose matrices are defined as

$$M = \begin{pmatrix} M_f & 0 & 0 \\ 0 & M_{s1} & 0 \\ 0 & 0 & M_{s2} \end{pmatrix}, \qquad S = \begin{pmatrix} S_f & 0 & 0 \\ 0 & S_{s11} & S_{s12} \\ 0 & S_{s21} & S_{s22} \end{pmatrix}, \tag{8}$$

$$C = \begin{pmatrix} 0 & C_{f,s1} & C_{f,s2} \\ C_{s1,f} & 0 & 0 \\ C_{s2,f} & 0 & 0 \end{pmatrix}, \qquad K = \begin{pmatrix} K_f & 0 & 0 \\ 0 & K_{s11} & K_{s12} \\ 0 & K_{s21} & K_{s22} \end{pmatrix}.$$

These coupling submatrices' entries are given component-wise as

$$(C_{fs1})_{ij} = -\int_{\Gamma_i} n_{s1} N_{f,i} N_{s,j} d\Gamma, \qquad (C_{fs2})_{ij} = -\int_{\Gamma_i} n_{s2} N_{f,i} N_{s,j} d\Gamma, \tag{9}$$
$$(C_{s1f})_{ij} = -\int_{\Gamma_i} n_{f1} N_{s,i} N_{f,j} d\Gamma, \qquad (C_{s2f})_{ij} = -\int_{\Gamma_i} n_{f1} N_{s,i} N_{f,j} d\Gamma,$$

Notice that, along Γ_i it holds that $n_s = -n_f$ and thus $C_{fs1} = -C_{s1f}^T$ and $C_{fs2} = -C_{s2f}^T$.

Discretization in time and energy analysis

Regarding the stability analysis we will consider the absence of the source term for convenience, i.e., $F = 0$. For the discretization of the second derivative in time we apply the leap-frog scheme, whereas for the first derivative we use central differences to obtain the following second-order scheme

$$M_s \frac{v^{n+1} - 2v^n + v^{n-1}}{\Delta t^2} + S_s \frac{v^{n+1} - v^{n-1}}{2\Delta t} + C_{sf} \frac{p^{n+1} - p^{n-1}}{2\Delta t} + K_s v^n = 0, \tag{10}$$

$$M_f \frac{p^{n+1} - 2p^n + p^{n-1}}{\Delta t^2} + S_f \frac{p^{n+1} - p^{n-1}}{2\Delta t} + C_{fs} \frac{v^{n+1} - v^{n-1}}{2\Delta t} + K_f p^n = 0. \tag{11}$$

By using

$$v^{n+1} - 2v^n + v^{n-1} = (v^{n+1} - v^n) - (v^n - v^{n-1})$$

and

$$v^{n+1} - v^{n-1} = (v^{n+1} - v^n) + (v^n - v^{n-1}),$$

and multiplying equation (10) by $\frac{v^{n+1}-v^{n-1}}{2\Delta t}$ and (11) by $\frac{p^{n+1}-p^{n-1}}{2\Delta t}$ and summing up these two equations we obtain

$$\begin{aligned}
&\frac{1}{2\Delta t} M_s \left(\left(\frac{v^{n+1} - v^n}{\Delta t} \right)^2 - \left(\frac{v^n - v^{n-1}}{\Delta t} \right)^2 \right) \\
&+ S_s \left(\frac{v^{n+1} - v^{n-1}}{2\Delta t} \right)^2 + K_s \frac{< v^{n+1}, v^n > - < v^n, v^{n-1} >}{2\Delta t} \\
&\frac{1}{2\Delta t} M_f \left(\left(\frac{p^{n+1} - p^n}{\Delta t} \right)^2 - \left(\frac{p^n - pv^{n-1}}{\Delta t} \right)^2 \right) \\
&+ S_f \left(\frac{p^{n+1} - p^{n-1}}{2\Delta t} \right)^2 + K_f \frac{< p^{n+1}, p^n > - < p^n, p^{n-1} >}{2\Delta t} \\
&= 0. \qquad (12)
\end{aligned}$$

Discretization in time and energy analysis (III)

A discrete energy can be introduced now for v (and for p) as

$$E_v^{i+1} = \frac{1}{2} \left|\left| \frac{v^{n+1} - v^n}{\Delta t} \right|\right|^2 + \frac{1}{2} \left\langle M_s^{-1} K_s v^{n+1}, v^n \right\rangle , \tag{13}$$

where the following condition holds (in absence of the source F, which vanishes after a given time)

$$\frac{1}{\Delta t}(E_v^{i+1} - E_v^i) = -M_s^{-1} S_s \left|\left| \frac{v^{n+1} - v^{n-1}}{\Delta t} \right|\right|^2 , \tag{14}$$

which shows that the discrete energy decreases in time.

Discretization in time and energy analysis (IV)

Unfortunately, the term $\left\langle M_s^{-1} K_s v^{n+1}, v^n \right\rangle$ does not ensure the positivity of the discrete energy and it must be worked out.

We first notice that

$$\frac{1}{2}\left\|\frac{v^{n+1}-v^n}{\Delta t}\right\|^2 + \frac{1}{2}\left(\left\langle M_s^{-1} K \frac{v^{n+1}+v^n}{2}, \frac{v^{n+1}+v^n}{2}\right\rangle - \frac{\Delta t^2}{4}\left\langle M_s^{-1} K_s \frac{v^{n+1}-v^n}{\Delta t}, \frac{v^{n+1}-v^n}{\Delta t}\right\rangle\right) \tag{15}$$

Discretization in time and energy analysis (V)

Defining

$$||A_s|| = \sup_v \frac{\langle I_s - M_s^{-1} K_s v, v\rangle}{||v||^2}, \tag{16}$$

it holds that

$$\left\langle I_s M_s^{-1} K_s \frac{v^{n+1} - v^n}{\Delta t}, \frac{v^{n+1} - v^n}{\Delta t} \right\rangle \leq ||A_s|| \left|\left| \frac{v^{n+1} - v^n}{\Delta t} \right|\right|^2, \tag{17}$$

and then, the discrete energy satisfies

$$E_v^{i+1} \geq \left(I_s - \frac{\Delta t^2}{4} ||A_s|| \right) \left|\left| \frac{v^{n+1} - v^n}{\Delta t} \right|\right|^2 + \left\langle M_s^{-1} K_s \frac{v^{n+1} + v^n}{2}, \frac{v^{n+1} + v^n}{2} \right\rangle. \tag{18}$$

Discretization in time and energy analysis (VI)

A suficient condition for the positivity of E_v^{i+1} is therefore

$$\Delta t^2 < \frac{4}{||A_s||}. \tag{19}$$

The largest eigenvalue of $||A_s||$ behaves as c_s/h_s^2, c_s depending only on the space discretization method, and so

$$\Delta t < \frac{2h_s}{c_s\sqrt{d}}. \tag{20}$$

Similarly for p, we obtain

$$\Delta t < \frac{2h_p}{c_p\sqrt{d}}, \tag{21}$$

and thus the final CFL condition reads

$$\Delta t < min\left(\frac{2h_p}{c_p\sqrt{d}}, \frac{2h}{c_s\sqrt{d}}\right). \tag{22}$$

Straight–forward solution: Conforming meshes and linear elements

- Advantages:
  - Easy to develop
- Disadvantages:
  - Degree of freedom increases
    - Higher memory requirement
    - Higher computational time
  - Limited accuracy for modeling the geometry of the interface
    - Numerical errors
    - Numerical unstabilities
- Other methods with nonconforming: overlapping issues and/or rather difficult to implement

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Coupling

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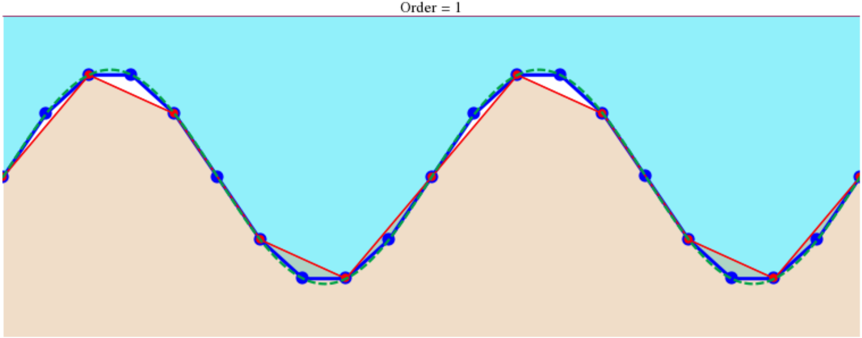
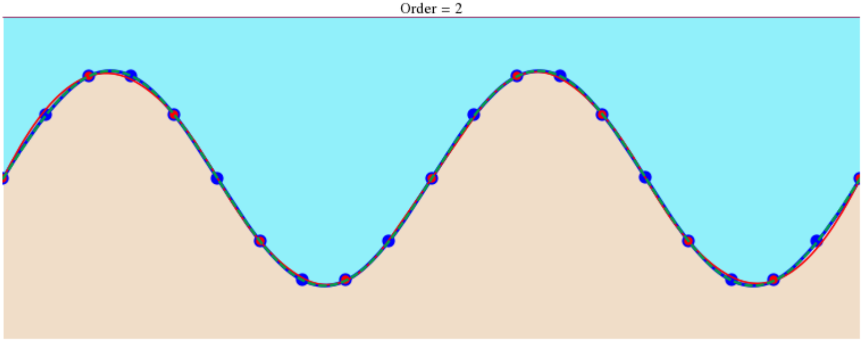
Drawbacks



Non-conforming coupling with independent spline interpolation

Coupling: Non-conforming meshes (II)

Drawbacks



Non-conforming coupling with independent spline interpolation

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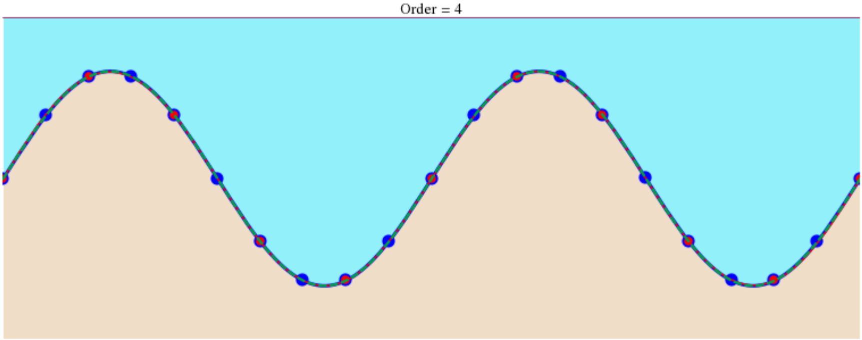
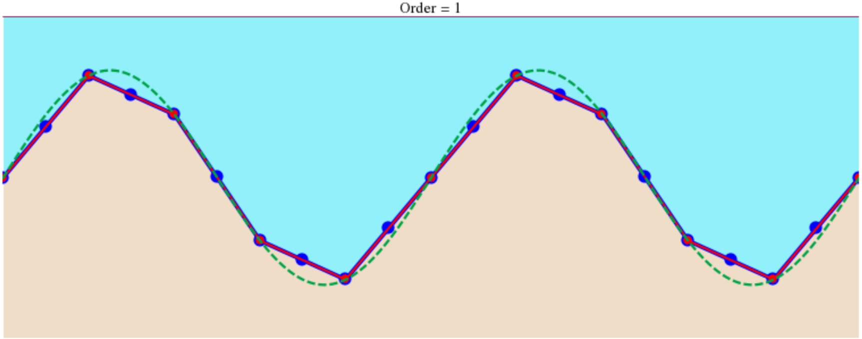


Non-conforming coupling with independent spline interpolation

The solid part holds the dominant spline:



Non-conforming coupling with **joint spline interpolation**

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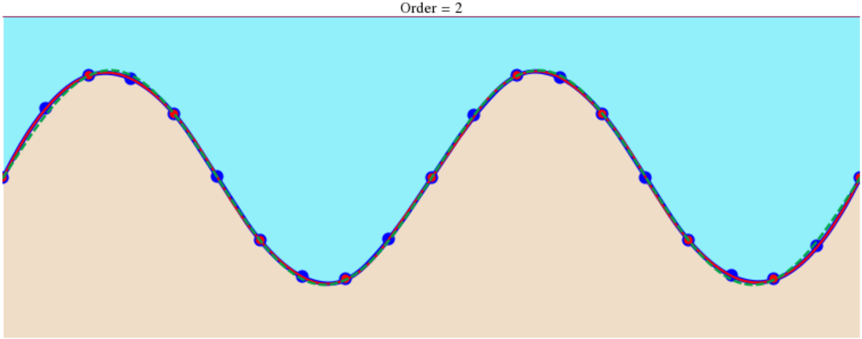

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Coupling: Solution (III)

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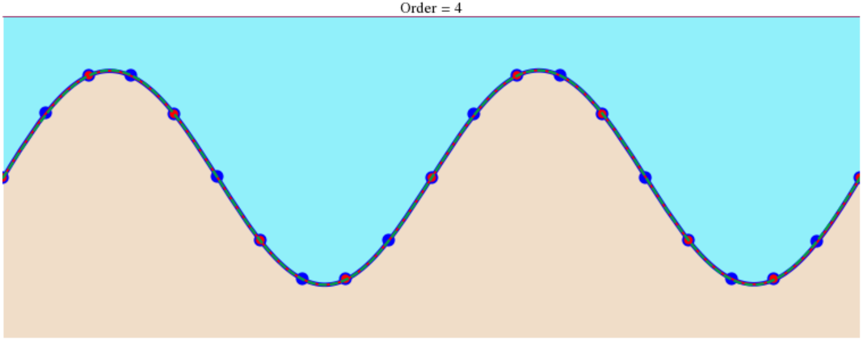
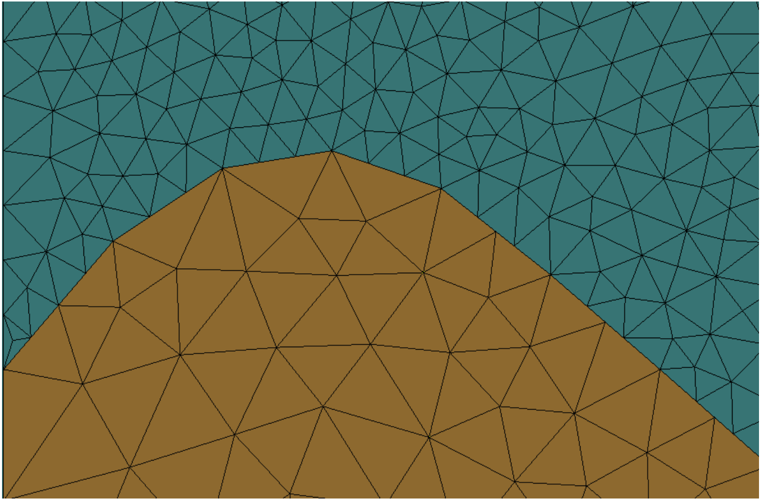
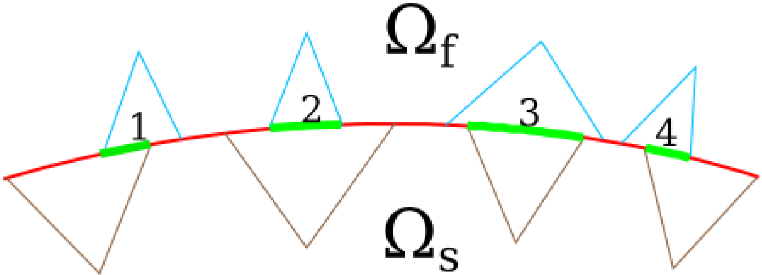


Non-conforming coupling with **joint spline interpolation**

Coupling: General nonconforming mesh





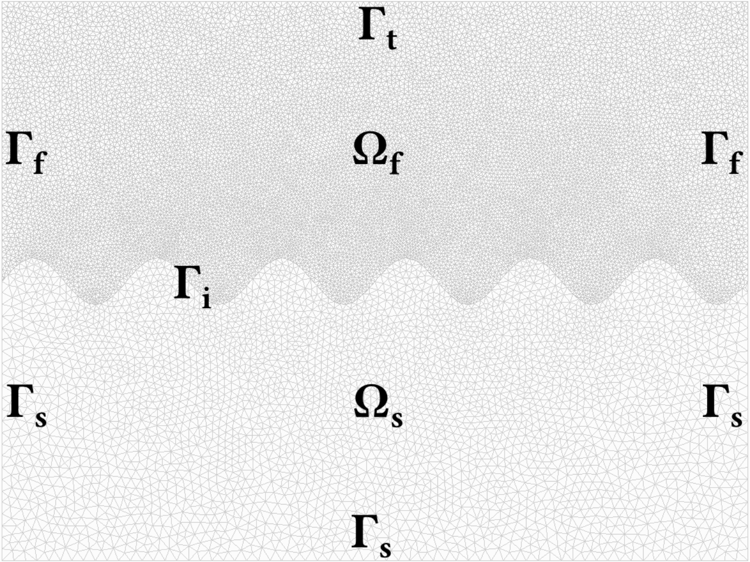
These four cases must be treated in order to determine the right limits of integration.

Simulations

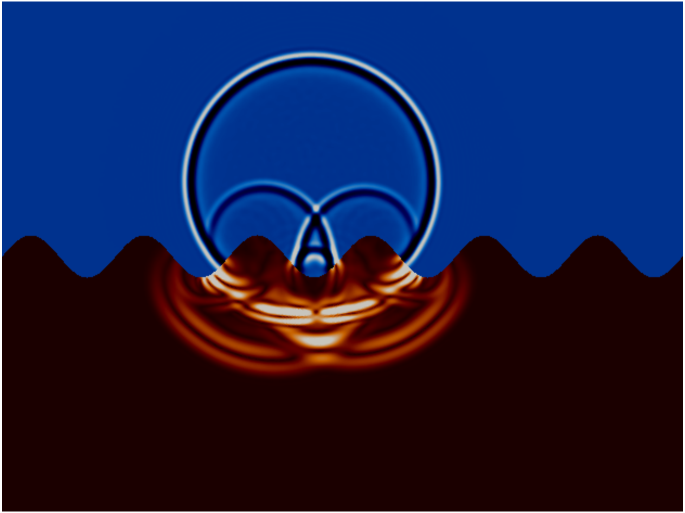
Configuration and parameters:

- domain: [0,64] x [0, 48] (km), time: [0,10] (s)
- coupling interface: sinusiodal
- point source location: (29.0833, 31)
- point source dominating frequence: 10 Hz
- $c_p = 1500$ m/s, $v_p = 3400$ m/s, $v_s = 1963$ m/s
- $\rho_p = 1020$ kg$/m^3$, $\rho_s = 2500$ kg$/m^3$
- boundary conditions: absorbing on the left, bottom and right boundaries, and free on top
- Time discretization: Leap-frog
- Numerical linear algebra solver: MUMPS

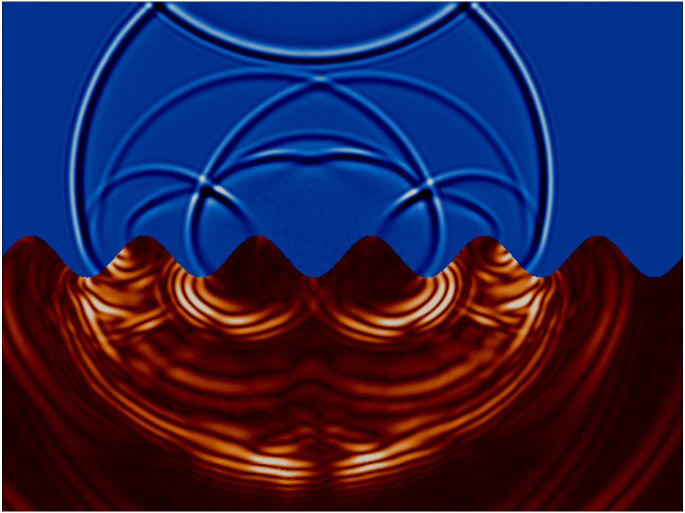
Simulations: Nonconforming mesh



Simulations: Snapshots (I)



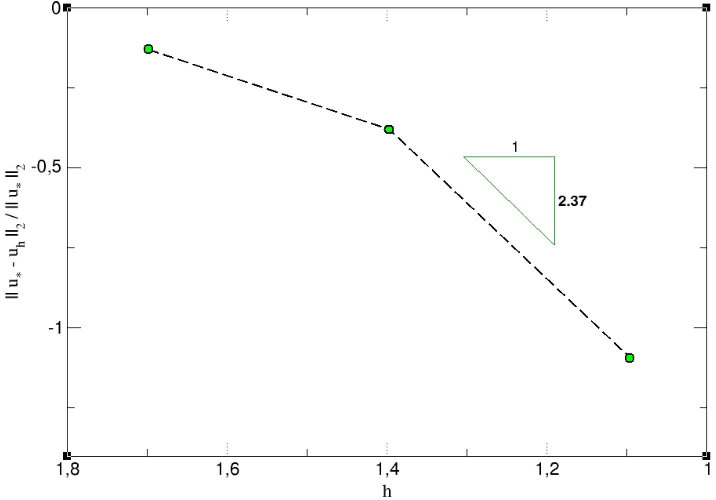
$t = 0.87$



$t = 1.57$

Simulations: Convergence

Convergence in P1 (Log-log scale)



Conclusions

- High-order nonconforming methods for wave propagation problems are introduced
- Overlapping and empty regions problems are avoided
- Stability proved for certain caises
- High efficiency

Future work

- Anisotropic media
- Parallelization in MPI
- Extension to DG
- Multiphysics: coupling with electromagnetic waves