Fast inversion of 3D Borehole Resistivity Measurements using Model Reduction Techniques based on 1D Semi-Analytical Solutions.

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INVENTEURS DU MONDE NUMÉRIQUE

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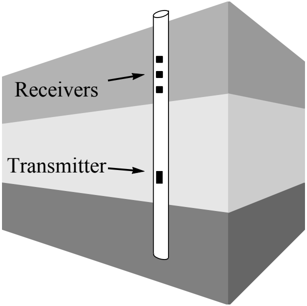
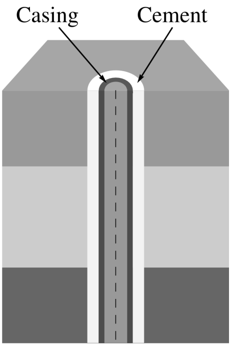
MOTIVATION

- **Main goal:** To obtain a better characterization of the Earth's subsurface
- **How:** Recording borehole resistivity measurements
- **Procedure:**
  - Well
  - Logging Instrument
  - Transmitters
  - Receivers



MOTIVATION

- **Practical Difficulties:**
  - It is not easy to drill a borehole
  - It may collapse
- **Practical Solutions:**
  - Use a metallic casing
  - Surround with a cement layer
- **Problem solved, but...**



MOTIVATION

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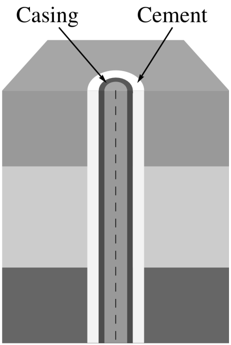
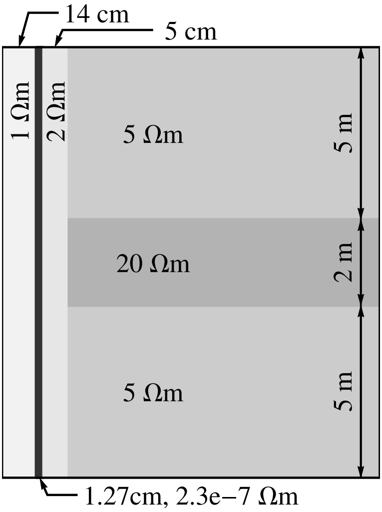


- **Problem solved, but... Numerical problems due to the high conductivity and thinness of the casing**

REALISTIC SCENARIO



- **Scenarios:** As we are considering axisymmetric scenarios, we can work with two dimensional scenarios

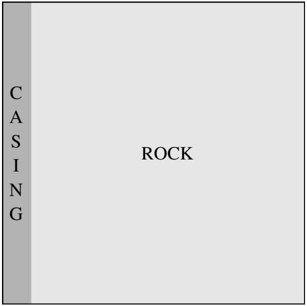
- **Conductivity and casing width:**

$$\begin{cases} \delta & = 1.27\text{e} - 2 \text{ m} \\ \sigma_c & = 4.34\text{e}6 \ \ \Omega^{-1}\text{m}^{-1} \end{cases}$$

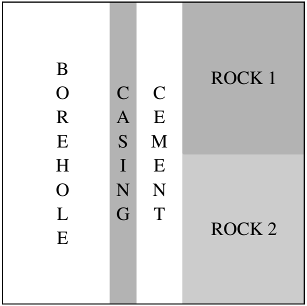
CONFIGURATIONS OF INTEREST

- **Develop:** Asymptotic method for avoiding the conflictive part of the domain (casing)
- **Main idea:** Equivalent conditions for substituting the casing

Configuration A



Configuration B



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- A.A. Kaufman. The electrical field in a borehole with a casing. *Geophysics,* Vol.55, Issue 1, pp. 29-38, 1990.
- D.Pardo, C.Torres-Verdín and Z.Zhang. Sensitivity study of borehole-to-surface and crosswell electromagnetic measurements acquired with energized steel casing to water displacement in hydrocarbon-bearing layers. *Geophysics,* 73 No.6, F261-F268, 2008.
- M. Duruflé, V. Péron and C. Poignard. Thin Layer Models for Electromagnetism.*Communications in Computational Physics* 16(1):213-238, 2014.

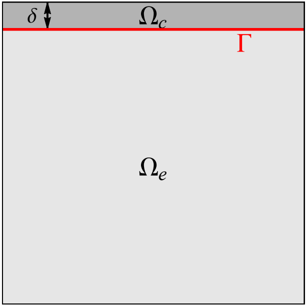
OUTLINE

OUTLINE

MODEL PROBLEM

EQUATIONS FOR THE ELECTRIC POTENTIAL

$$\text{div}\left[(\sigma - i\delta\omega)\nabla u\right] = -\text{div} j \qquad (\omega = 0 \text{ first approach})$$



$$\Omega = \Omega_e \cup \Omega_c \cup \Gamma$$

$$\begin{cases} \sigma_e \Delta u_e = f & \text{in } \Omega_e \\ \sigma_c \Delta u_c = 0 & \text{in } \Omega_c \\ u_e = u_c & \text{on } \Gamma \\ \sigma_c \partial_n u_c = \sigma_e \partial_n u_e & \text{on } \Gamma \\ u_c = 0 & \text{on } \partial\Omega \end{cases}$$

Where the solution is expressed as

$$u = \begin{cases} u_e & \text{in } \Omega_e \\ u_c & \text{in } \Omega_c \end{cases}$$

and σ_e, σ_c, f are known data

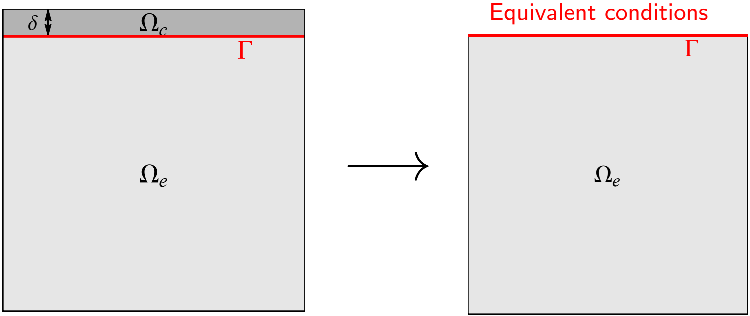
MAIN IDEA

REFERENCE MODEL

Solution: u

ASYMPTOTIC MODEL

Solution: $u^{[n]}$



Definition: Let u be the reference solution. We say an asymptotic model is of **Order n+1**, if its solution $u^{[n]}$ satisfies

$$||u - u^{[n]}||_{L^2} \leq C\delta^{n+1}$$

METHODOLOGY

- **Step1:** Derive an Asymptotic Expansion for u when $\delta \longrightarrow 0$
  - In the casing: $$u_c(t,s) = \sum_{n\in\mathbb{N}} \delta^n U_c^n\left(t, \frac{s}{\delta}\right)$$
  - Outside the casing: $$u_e(x,y) = \sum_{n\in\mathbb{N}} \delta^n u_e^n(x,y)$$
- **Step2:** Obtain Equivalent Conditions of order $k+1$ by identifying a simpler problem satisfied by the truncated expansion
  - $u_{k,\delta} := u_e^0 + \delta u_e^1 + \delta^2 u_e^2 + \ldots + \delta^k u_e^k$

MULTISCALE EXPANSION

- **In the Casing:**

$$\left\{ \begin{array}{rll} \sigma_c \partial_t^2 U_c^{n-2} + \sigma_c \partial_s^2 U_c^n & = 0 & s \in (0,1) \\ \sigma_c \partial_s U_c^n & = \sigma_e \partial_n u_e^{n-1} & s = 0 \\ U_c^n & = 0 & s = 1 \end{array} \right.$$

- **Outside the Casing:**

$$\left\{ \begin{array}{rlll} \sigma_e \Delta u_e^n & = f \delta_0^n & \text{in} & \Omega_e \\ u_e^n & = U_c^n & \text{on} & \Gamma_e \end{array} \right.$$

We collect the equations for $n = 0, 1, 2$ to derive the equivalent conditions

EQUIVALENT MODELS

We identify simpler problems satisfied by truncated expansions outside the casing (up to residual terms)

- **Order 1:** $$\longrightarrow \begin{cases} \sigma_e \Delta u &= f \quad \text{in} \quad \Omega_e \\ u &= 0 \quad \text{on} \quad \Gamma \end{cases}$$

- **Order 3:** $$\longrightarrow \begin{cases} \sigma_e \Delta u &= f \quad \text{in} \quad \Omega_e \\ u + \delta \frac{\sigma_e}{\sigma_c} \partial_n u &= 0 \quad \text{on} \quad \Gamma \end{cases}$$

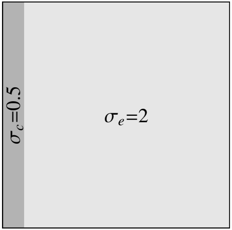
Remark: Second model has already order 3 of convergence due to the flat configuration of the layer

OUTLINE

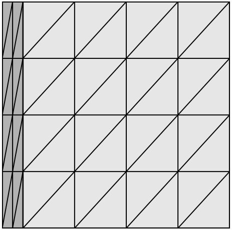
NUMERICAL DISCRETIZATION

- **FINITE ELEMENT METHOD** (Matlab Code)
  - Straight triangular elements
  - Lagrange shape functions of any degree

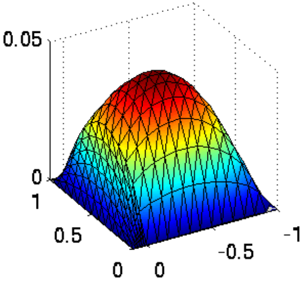
Domain



Mesh

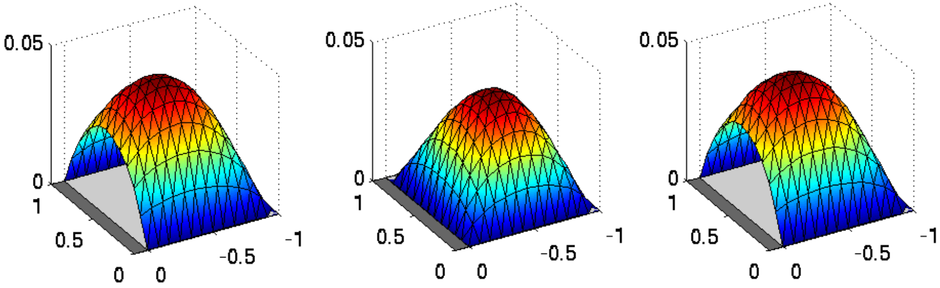


Reference Solution



NUMERICAL SOLUTIONS

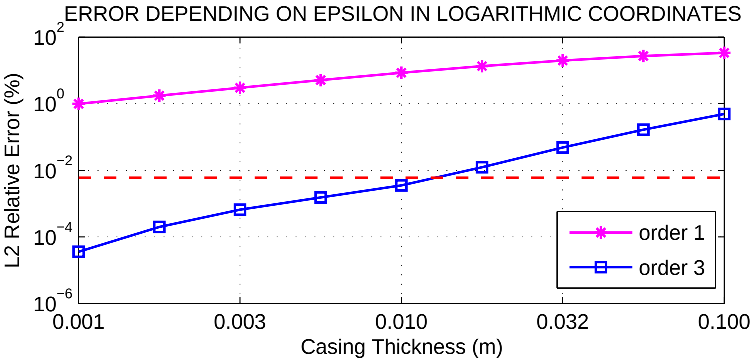
Reference Model (in Ω_e) **Order 1 Model** **Order 3 Model**



Definition: We define the relative error between the reference solution u and the asymptotic solution $u^{[n]}$, as

$$\frac{||u - u^{[n]}||_{L^2}}{||u||_{L^2}}$$

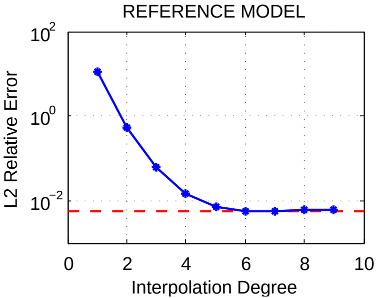
CONVERGENCE RATES

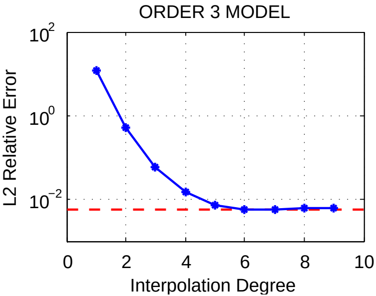


Casing Thickness	0.001	0.002	0.004	0.007	0.013	0.024	0.043	0.078
Order 1 Slopes	0.975	0.956	0.925	0.873	0.792	0.677	0.534	0.382
Order 3 Slopes	2.990	2.074	1.468	1.440	2.188	2.362	2.148	1.872

INTERPOLATION DEGREE

Relative error between a solution of degree 10 and solutions of lower degrees





CONCLUSION: Error analysis is not relevant once we reach a relative error of 10^{-2}

OUTLINE

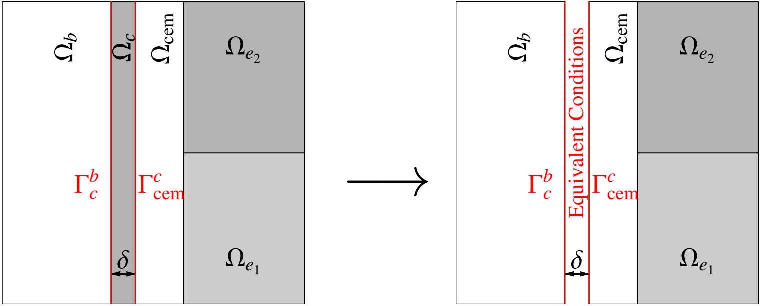
MAIN IDEA

REFERENCE MODEL

Solution: u

ASYMPTOTIC MODEL

Solution: $u^{[n]}$

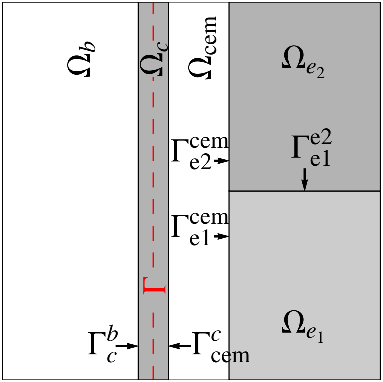


Definition: We define the jump and mean value of the solution u across the casing as

$$[u] = u|_{\Gamma_c^b} - u|_{\Gamma_{\text{cem}}^c}$$

$$\{u\} = \frac{1}{2}\left(u|_{\Gamma_c^b} + u|_{\Gamma_{\text{cem}}^c}\right)$$

MODEL PROBLEM



$$\begin{cases} \sigma_i \Delta u_i &= f_i & \text{in} & \Omega_i \\ \sigma_c \Delta u_c &= 0 & \text{in} & \Omega_c \\ u_i &= u_j & \text{on} & \Gamma_i^j \\ \sigma_i \partial_n u_i &= \sigma_j \partial_n u_j & \text{on} & \Gamma_i^j \\ u &= 0 & \text{on} & \partial\Omega \end{cases}$$

$$i, j = b, c, \text{cem}, e1, e2$$

Where σ_i and f_i are known data

EQUIVALENT CONDITIONS

Using a similar procedure than for the previous configuration we obtain equivalent conditions

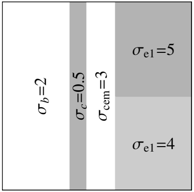
- **Order 1:** $$\longrightarrow \begin{cases} [u] &= 0 \\ [\sigma\partial_n u] &= 0 \end{cases}$$

- **Order 3:** $$\longrightarrow \begin{cases} [u] &= \frac{\epsilon}{\sigma_c}\left\{\sigma\partial_n u\right\} \\ [\sigma\partial_n u] &= -\epsilon\sigma_c\partial_x^2\left\{u\right\} \end{cases}$$

Remark: Second model has already order 3 of convergence due to the flat configuration of the layer

NUMERICAL SOLUTIONS

Domain



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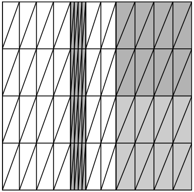
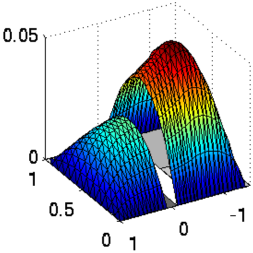
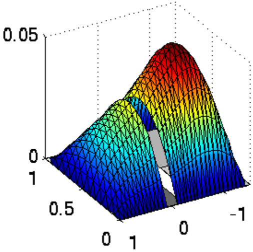


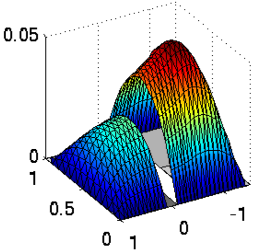
Reference Solution outside Ω_c



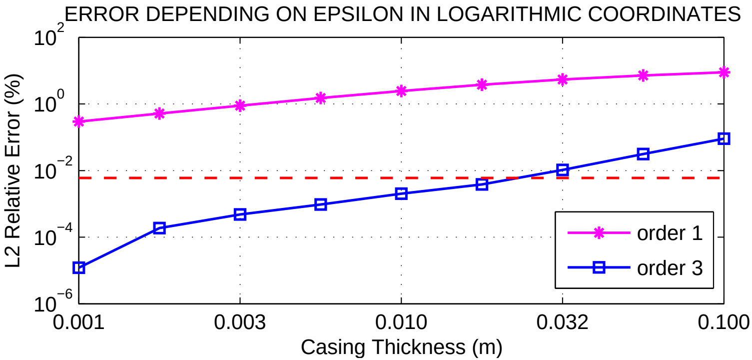
Order 1 Model



Order 3 Model



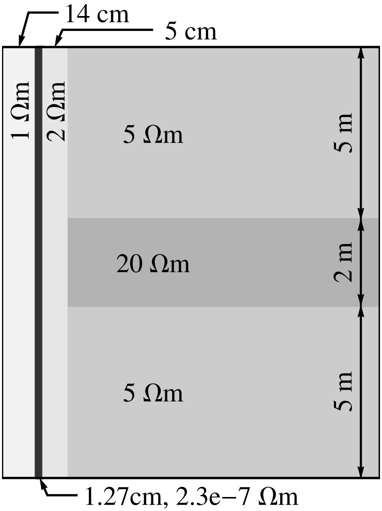
CONVERGENCE RATES



Casing Thickness	0.001	0.002	0.004	0.007	0.013	0.024	0.043	0.078
Order 1 Slopes	0.967	0.944	0.904	0.843	0.751	0.631	0.493	0.365
Order 3 Slopes	4.764	1.637	1.187	1.291	1.116	1.742	1.911	1.846

OUTLINE

REALISTIC SCENARIO



- **Conductivity and casing width:**

$$\begin{cases} \delta & = 1.27\text{e}-2 \ \text{m} \\ \sigma_c & = 4.34\text{e}6 \ \ \Omega^{-1}\text{m}^{-1} \end{cases}$$

$$\Rightarrow \sigma_c \approx \delta^{-3}$$

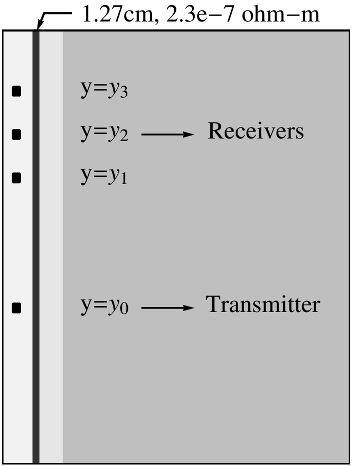
- **First approach:**

$$\sigma_c = \alpha \qquad \alpha \in \mathbb{R}$$

- **Case to be studied:**

$$\sigma_c = \alpha\delta^{-3} \qquad \alpha \in \mathbb{R}$$

SECOND DIFFERENCE OF POTENTIAL



- **Equation:**

$$\text{div}\left[(\sigma - i\delta\omega)\,\nabla u\right] = f$$

- **Right hand side:**

$$f = \begin{cases} 1 & \text{In the transmitter} \\ 0 & \text{Outside the transmitter} \end{cases}$$

- **Objective:** Measure the second difference of potential on the Receivers

$$U_2 = u(y_1) - 2u(y_2) + u(y_3)$$

- **Expected Result:** The second difference of potential proportional to the rock resistivity

$$U_2 = \alpha \cdot \rho_{\text{rock}} \qquad \alpha \in \mathbb{R}$$

Perspectives

- **Short Term:**
  - Asymptotic models with $\sigma_c = \alpha\delta^{-3} \quad \alpha \in \mathbb{R}$
  - Measure the second difference of potential on the receivers
- **Long Term:**
  - Consider physically more realistic scenarios
  - Develop 3D electromagnetic models
  - Study highly deviated boreholes

THANK YOU FOR

YOUR ATTENTION