Computer Animation

Lesson 9 - Mass Spring Systems

Remi Ronfard, Nov 2019

9 Secondary action

A *secondary action* is an action that results directly from another action. Secondary actions are important in heightening interest and adding a realistic complexity to the animation. A secondary actions is always kept subordinate to the primary action. If it conflicts, becomes more interesting, or dominates in any way, it is either the wrong choice or is staged improperly. [26]

8. Secondary Action

Cartoon physics vs. Real world physics

- Any body suspended in space will remain in space until made aware of its situation.
- Any body in motion will tend to remain in motion until solid matter intervenes suddenly.
- Any body passing through solid matter will leave a perforation conforming to its perimeter.
- The time required for an object to fall twenty stories is greater than or equal to the time it takes for whoever knocked it off the ledge to spiral down twenty flights to attempt to capture it unbroken.
- Certain bodies can pass through solid walls painted to resemble tunnel entrances; others cannot.

Mass spring models (1)

Properties of a spring:

- k_s : stiffness of the spring.
- l_0 : initial spring length.
- L : current spring length.

Spring follow a linear force-deformation: Hooke's law

$$F = k_s(L - l_0) \tag{1}$$

Mass-spring models (2)

A mass-spring model (MSM) is composed by a set of particles:

- Mass,
- Position,
- Speed.

And a set of springs that link them together in pairs.
The springs exerts two elastic forces on each particle of the pair i and j:

$$\mathbf{F}_i = -\mathbf{F}_j = k_s \frac{\mathbf{x}_j - \mathbf{x}_i}{|\mathbf{x}_j - \mathbf{x}_i|} \left(|\mathbf{x}_j - \mathbf{x}_i| - l_0 \right) \tag{2}$$

The forces are proportional to the elongation of the spring.
The value k_s, the rigidity of the spring, determines the behaviour of the model:

- High values $\rightarrow$ Rigid body (and also numerical instabilities!).
- Low values $\rightarrow$ Elastic body.

Mass spring models (3)

In addition to elastic forces, damping can also be added to the springs:

$$\mathbf{F}_i = -\mathbf{F}_j = k_d \left(\mathbf{v}_j - \mathbf{v}_i\right) \frac{\mathbf{x}_j - \mathbf{x}_i}{|\mathbf{x}_j - \mathbf{x}_i|} \tag{3}$$

This force is proportional to the velocity difference of the particles projected along the line of the spring.
Damping is used to:

- Simulate the internal energy loss that happens in deformable bodies.
- Avoid continuous oscillation of the springs.
- Increase the stability of the system.

Mass-spring models (4)

In a generalized spring we define a constraint C:

- This constraint depends on mass positions $C(\mathbf{x}_1, \mathbf{x}_2, \ldots \mathbf{x}_n)$.
- Iff the constraint is met then $C(\mathbf{x}_1, \mathbf{x}_2, \ldots \mathbf{x}_n) = 0$.
- We define a potential energy such as:

$$E(\mathbf{x}_1, \mathbf{x}_2, \ldots \mathbf{x}_n) = \frac{1}{2} k C(\mathbf{x}_1, \mathbf{x}_2, \ldots \mathbf{x}_n)^2 \tag{4}$$

  - $E = 0$ iff the constraint is met.
  - $E > 0$ otherwise.

Mass spring models (5)

Force at point j is based on the energy E:

$$\mathbf{F}_j = -\frac{\partial}{\partial \mathbf{x}_j} E\left(\mathbf{x}_1, \mathbf{x}_2, \ldots \mathbf{x}_n\right) = kC\left(\mathbf{x}_1, \mathbf{x}_2, \ldots \mathbf{x}_n\right) \frac{\partial}{\partial \mathbf{x}_j} C\left(\mathbf{x}_1, \mathbf{x}_2, \ldots \mathbf{x}_n\right) \quad (5)$$

The sum of all this forces for a constraint C is 0:

- Linear and angular momentum are preserved,
- Constraint forces are internal forces.

Mass spring models (6)

Preserve distance between two points:

$$C(\mathbf{x}_1, \mathbf{x}_2) = |\mathbf{x}_1, \mathbf{x}_2| - L = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} - L$$

$$\frac{\partial C_d}{\partial \mathbf{x}_1} = \begin{pmatrix} \frac{\partial C_d}{\partial x_1} \\ \frac{\partial C_d}{\partial y_1} \\ \frac{\partial C_d}{\partial z_1} \end{pmatrix} = \frac{1}{|\mathbf{x}_1 - \mathbf{x}_2|} \begin{pmatrix} x_1 - x_2 \\ y_1 - y_2 \\ z_1 - z_2 \end{pmatrix} = \frac{\mathbf{x}_1 - \mathbf{x}_2}{|\mathbf{x}_1 - \mathbf{x}_2|} \tag{6}$$

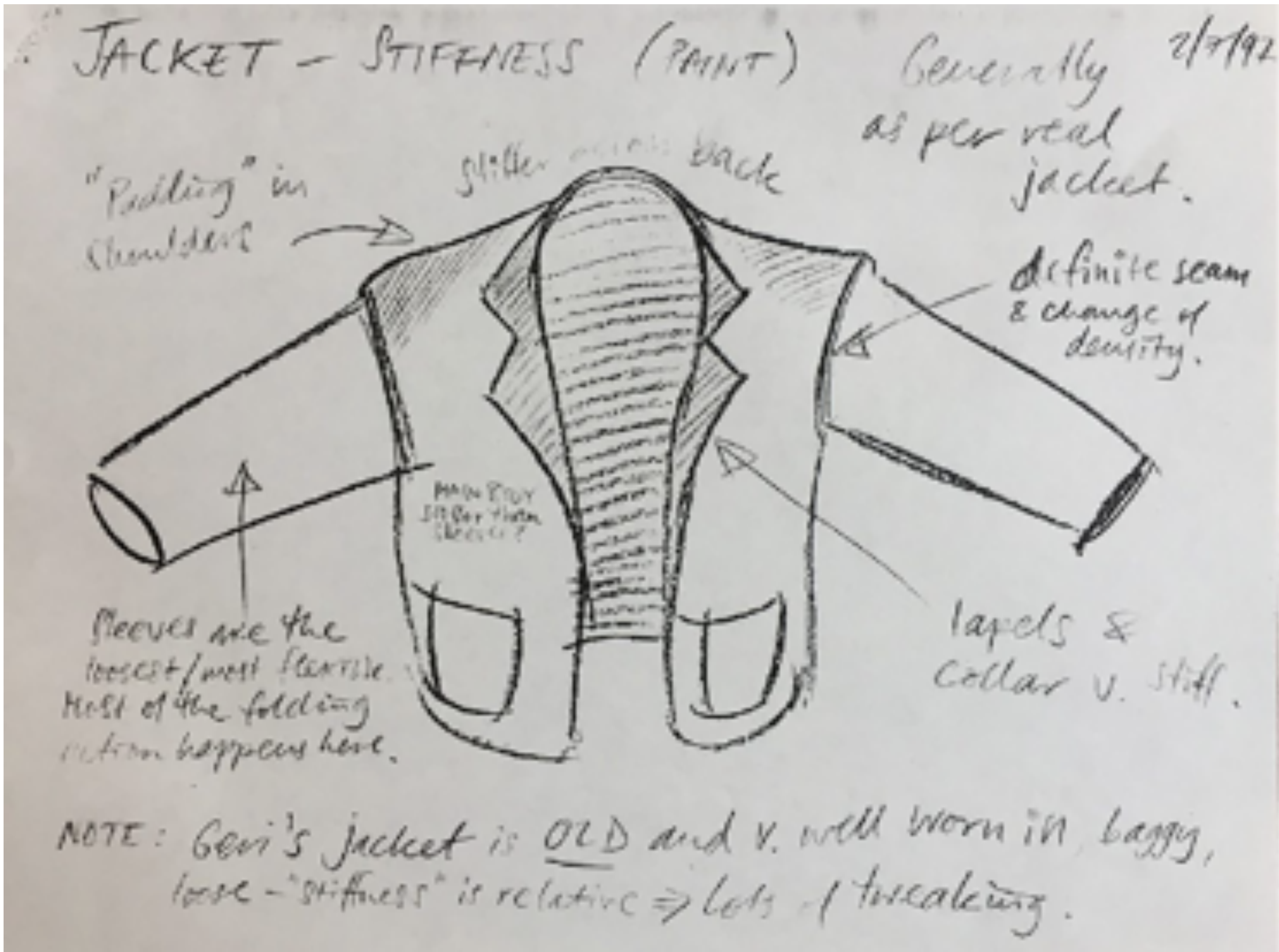
And the force:

$$\mathbf{F}_d = k_d \left(|\mathbf{x}_1 - \mathbf{x}_2| - L \right) \frac{\mathbf{x}_1 - \mathbf{x}_2}{|\mathbf{x}_1 - \mathbf{x}_2|} \tag{7}$$

This is a spring of stiffness k_d !

Paper 9 - Subdivision surfaces in character animation

Paper 9 - Subdivision surfaces in character animation



Paper 9 - Subdivision surfaces in character animation

Paper 9 - Subdivision surfaces in character animation

- Catmull-Clarck subdivision

(A) New face points – the average of all of the old points defining the face.
(B) New edge points – the average of the midpoints of the old edge with the average of the two new face points of the faces sharing the edge.
(C) New vertex points – the average

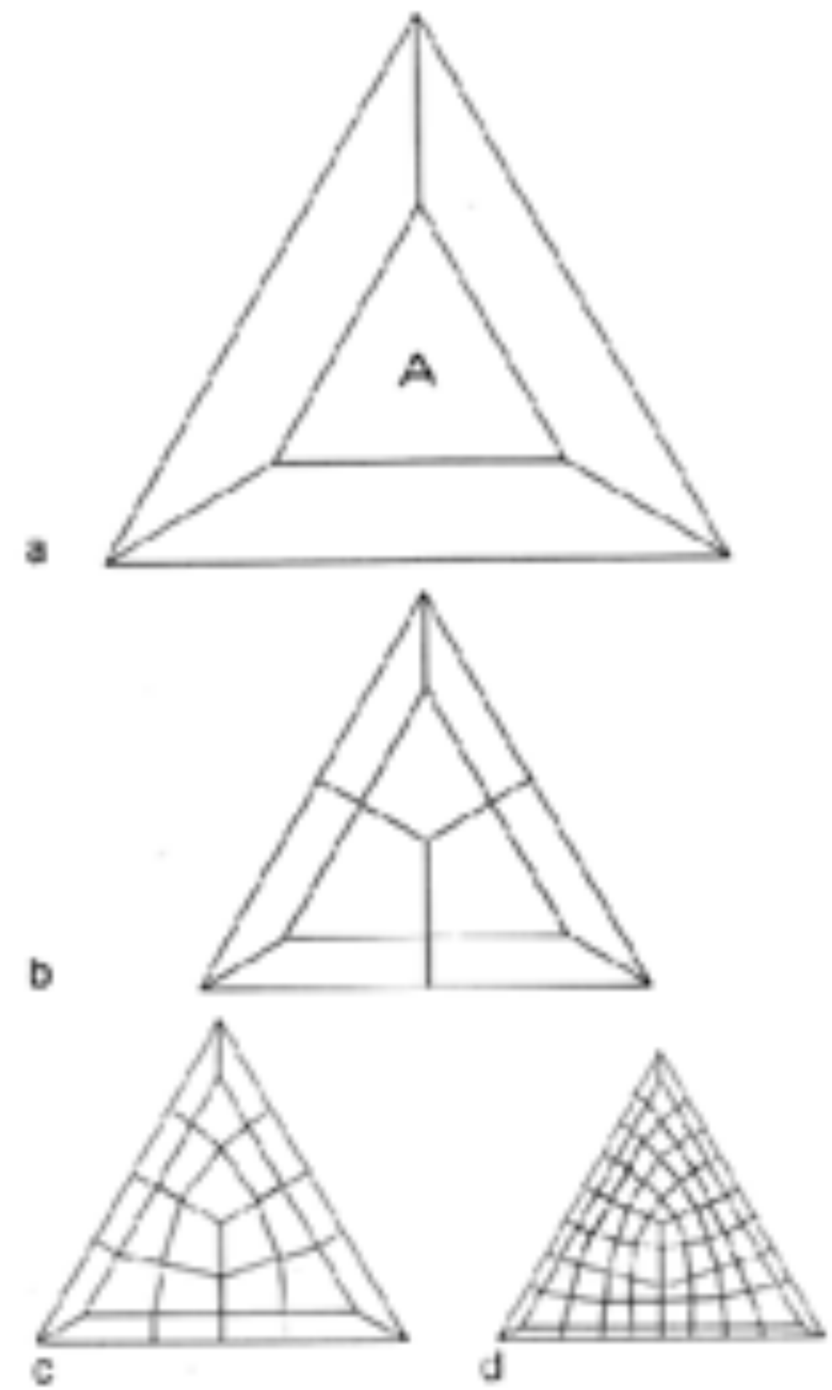
$$\frac{Q}{n} + \frac{2R}{n} + \frac{S(n-3)}{n}$$

where

Q = the average of the new face points of all faces adjacent to the old vertex point.
R = the average of the midpoints of all old edges incident on the old vertex point.
S = old vertex point.

After these points have been computed, new edges are formed by

- connecting each new face point to the new edge points of the edges defining the old face
- connecting each new vertex point to the new edge points of all old edges incident on the old vertex point



Paper 9 - Subdivision surfaces in character animation

- Cloth dynamics : springs, diagonals and penalties

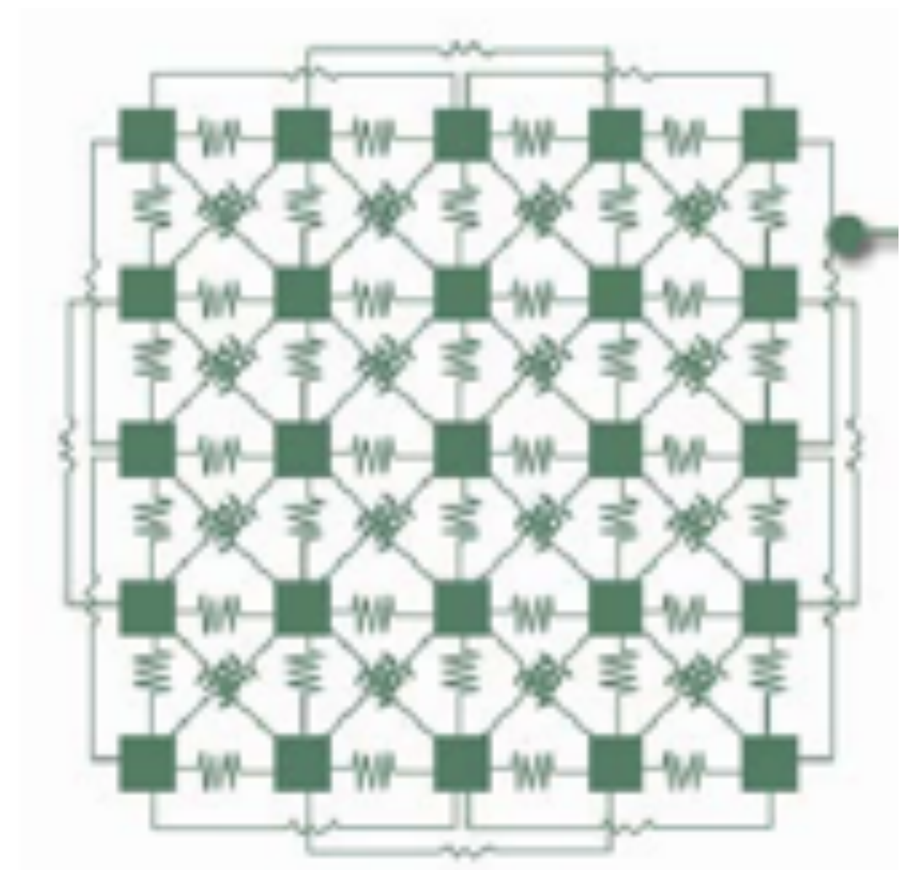
points. We chose to avoid these special cases by adopting a finite-difference approach, approximating the clothing with a mass-spring model [18] in which all the mass is concentrated at the control points.

$$E_s(p_1,p_2) = \frac{1}{2}\left(\frac{|p_1-p_2|}{|p_1^*-p_2^*|}-1\right)^2. \tag{4}$$

$$E_d(p_1,p_2,p_3,p_4) = E_s(p_1,p_2)E_s(p_3,p_4). \tag{5}$$

$$E_p(p_1,p_2,p_3) = \frac{1}{2} * [C(p_1,p_2,p_3) - C(p_1^*,p_2^*,p_3^*)]^2 \tag{6}$$

$$C(p_1,p_2,p_3) = \left|\frac{p_3-p_2}{|p_3^*-p_2^*|} - \frac{p_2-p_1}{|p_2^*-p_1^*|}\right| \tag{7}$$



Paper 9 - Subdivision surfaces in character animation

- Collisions

We build a coarsening hierarchy for each of the cloth meshes, as well as for each of the kinematic obstacles. To determine collisions between a cloth mesh and a kinematic obstacle, we test each vertex of the cloth mesh against the hierarchy for the obstacle. To determine collisions between a cloth mesh and itself, we test each vertex of the mesh against the hierarchy for the same mesh.