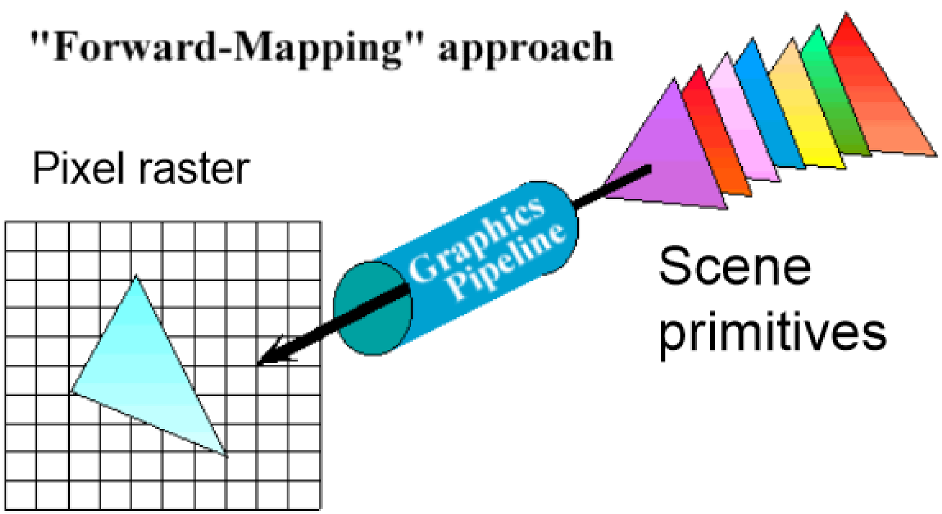
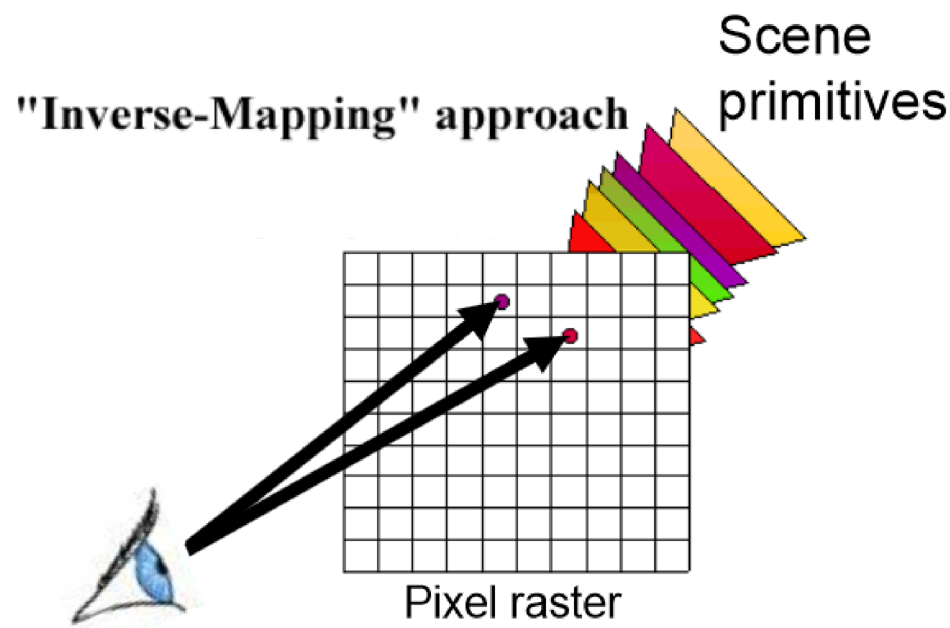
Rasterization VS ray-casting

- For each triangle
  - Project triangle to image plane
  - For each pixel
    - Check pixel in triangle
    - Resolve visibility with z-buffer

- For each pixel
  - Compute pixel ray
  - For each triangle
    - Check ray-triangle intersection
    - Get closest intersection



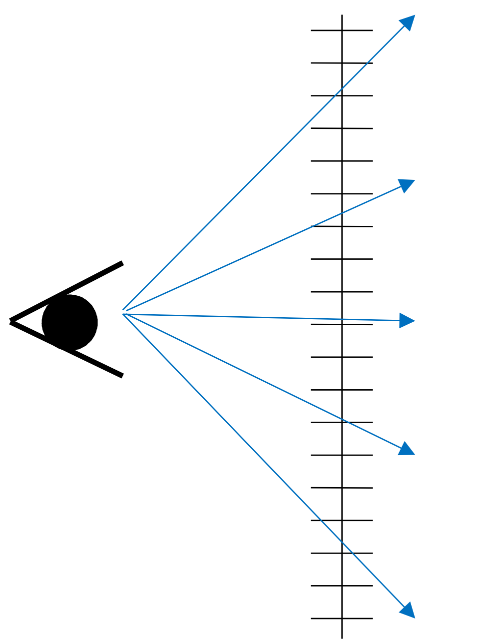


Eye ray and camera

- Persective

r = (x*u,aspect*y*v,D*w), normalized

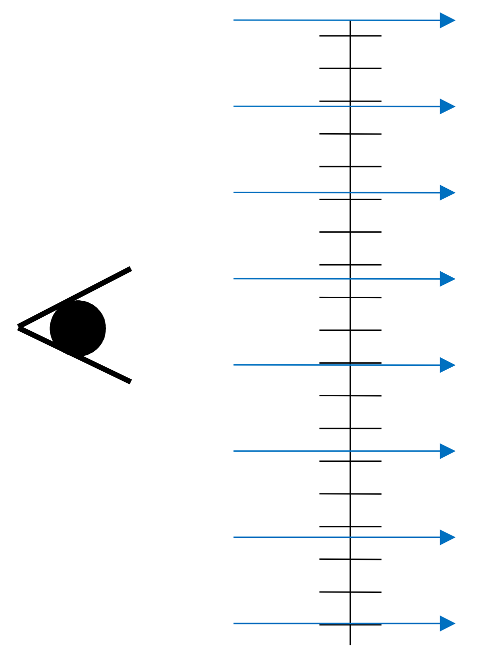
P(t) = e + t*r



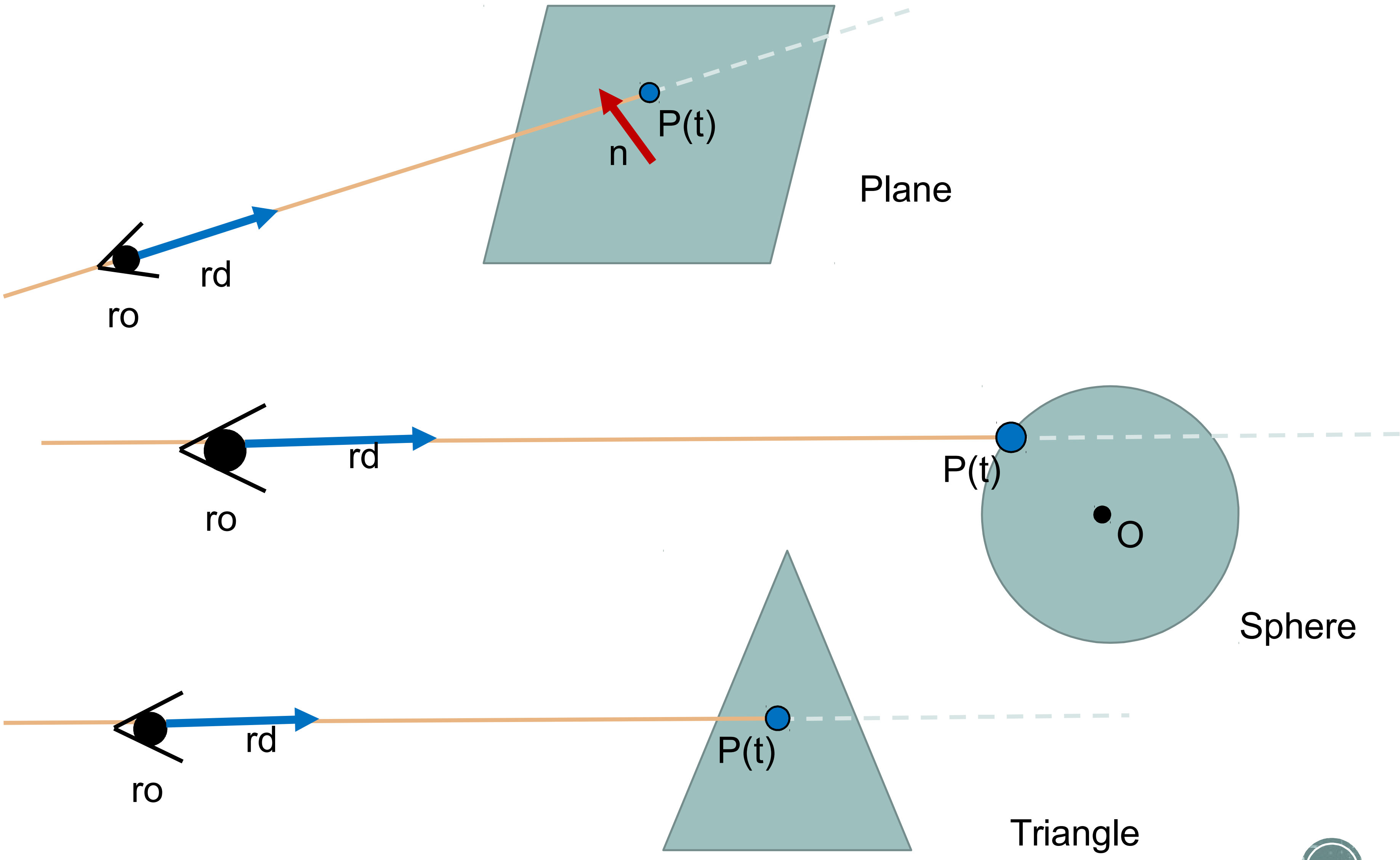
- Orthographic

P(t) = o + t*w

o = e + x*size*u + y*size*v



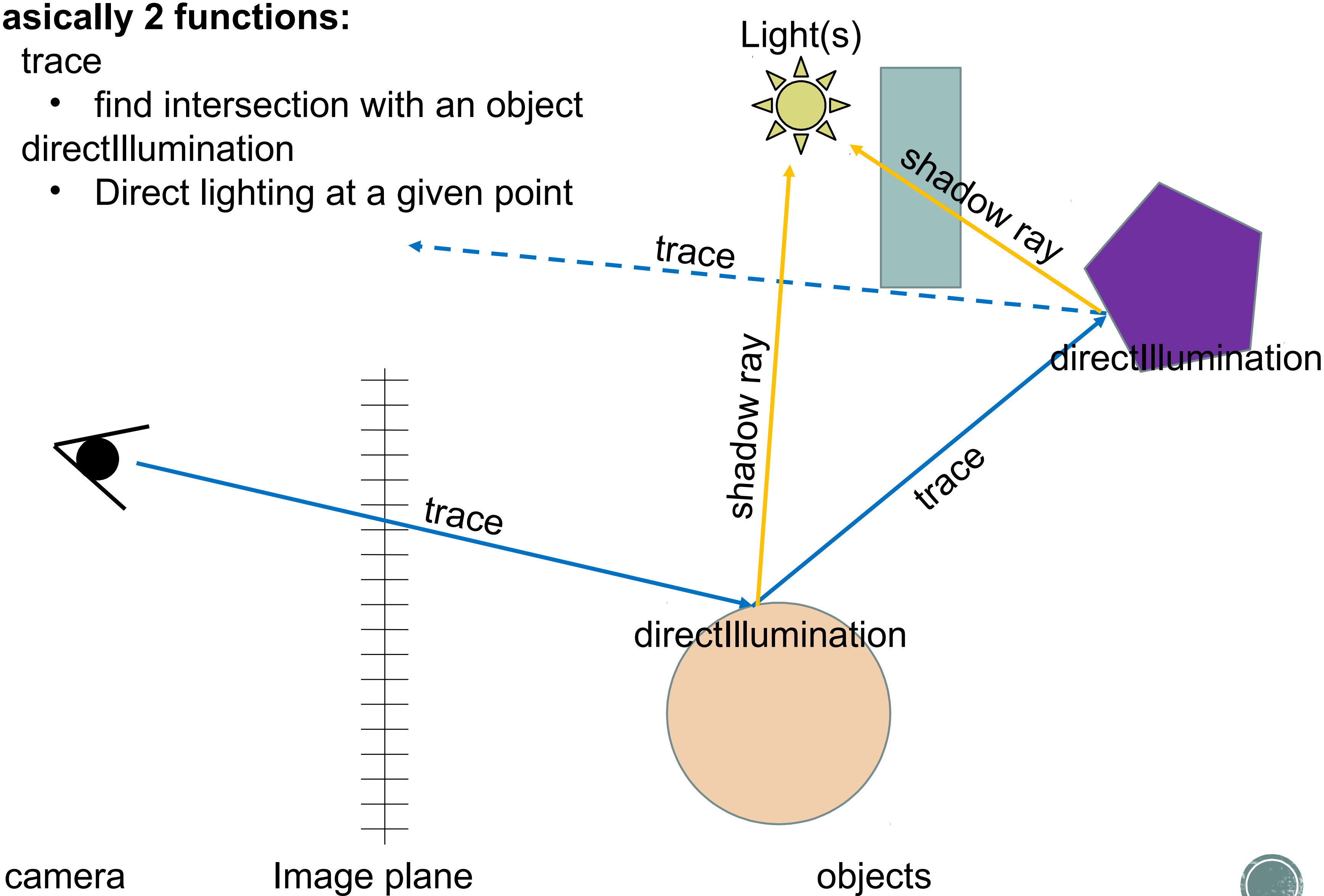
Ray-plane intersection



Ray tracing

Basically 2 functions:

- trace
  - find intersection with an object
- directIllumination
  - Direct lighting at a given point



Ray tracing

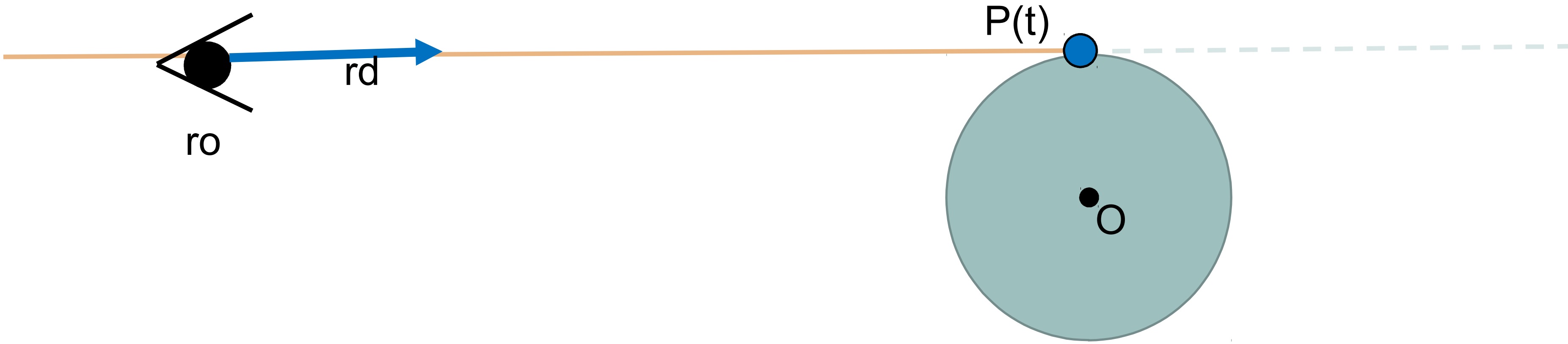
- color trace(ray) {
  - hit = intersectScene(ray)
  - if(hit) {
    - color = directIllumination(hit)
    - if hit is reflective
      - color += c_refl * trace(reflected ray)
    - if hit is transmissive
      - color += c_trans * trace(refracted ray)
  - } else
    - color = background_color
  - return color
- }

Ray tracing

- color directIllumination(hit) {
  - color = (0,0,0)
  - for each light L {
    - T = cast shadow ray to L
    - if hit is not shadowed by L
      - color += Ambient+diffuse+specular terms(L,hit)
  - }
  - return color
- }

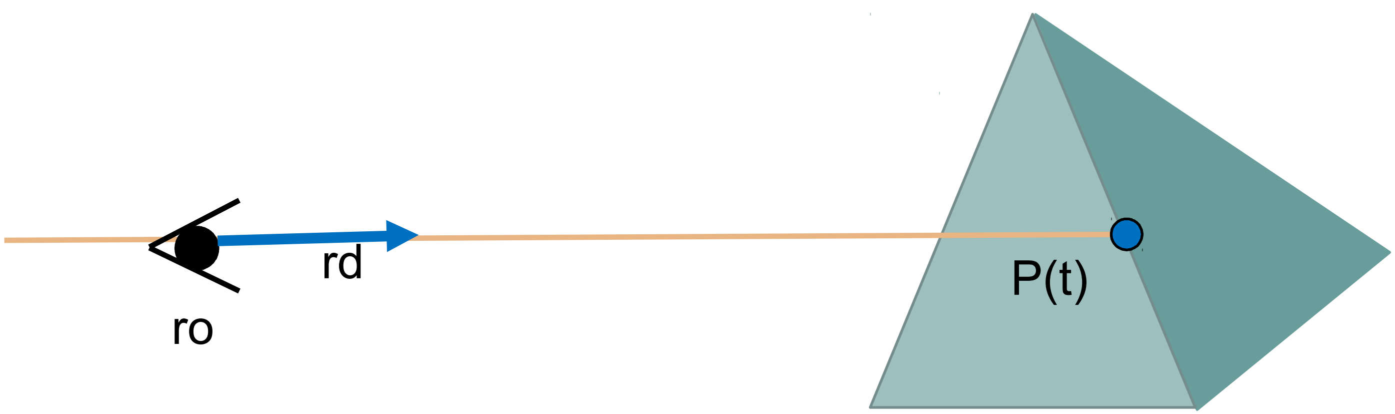
Precision

- Issues
  - Ray origin on an object surface
  - Grazing rays
- Floating point approximation



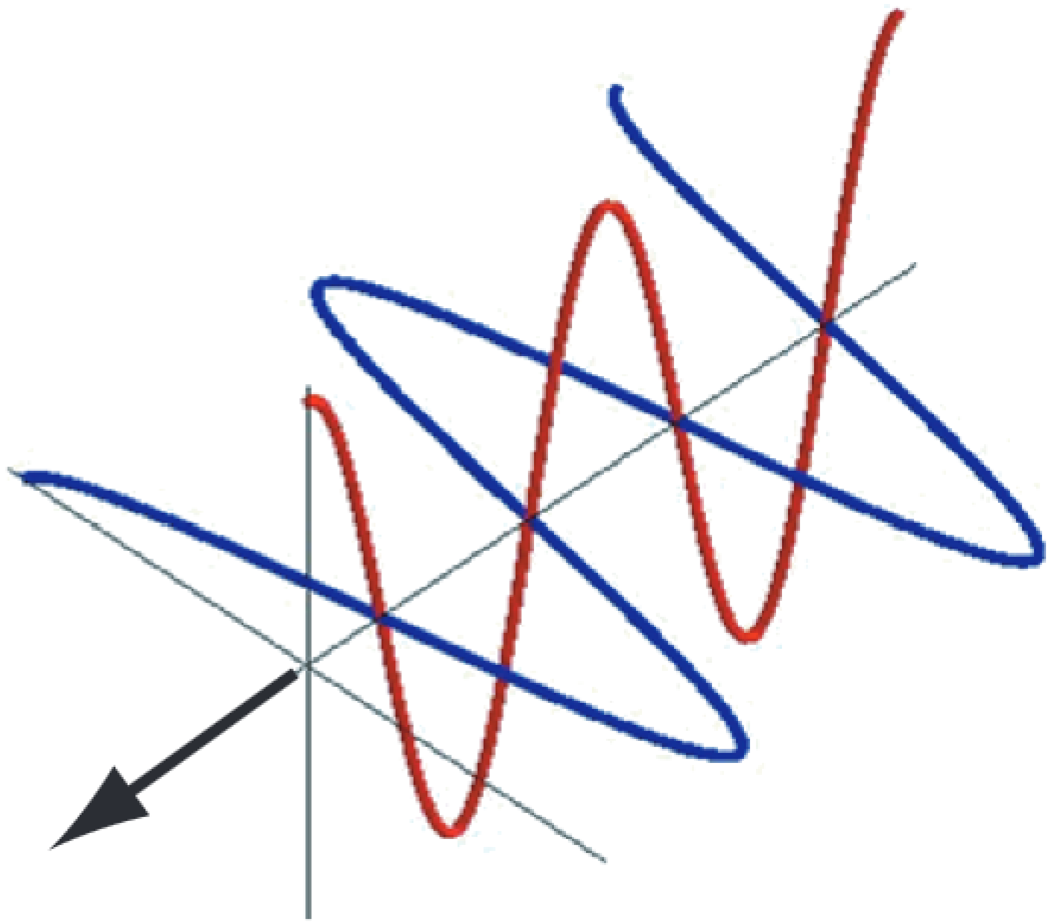
Precision

- Issues
  - Ray origin on an object surface
  - Grazing rays
- Floating point approximation
  - Must report intersection on triangles



Physics of shading

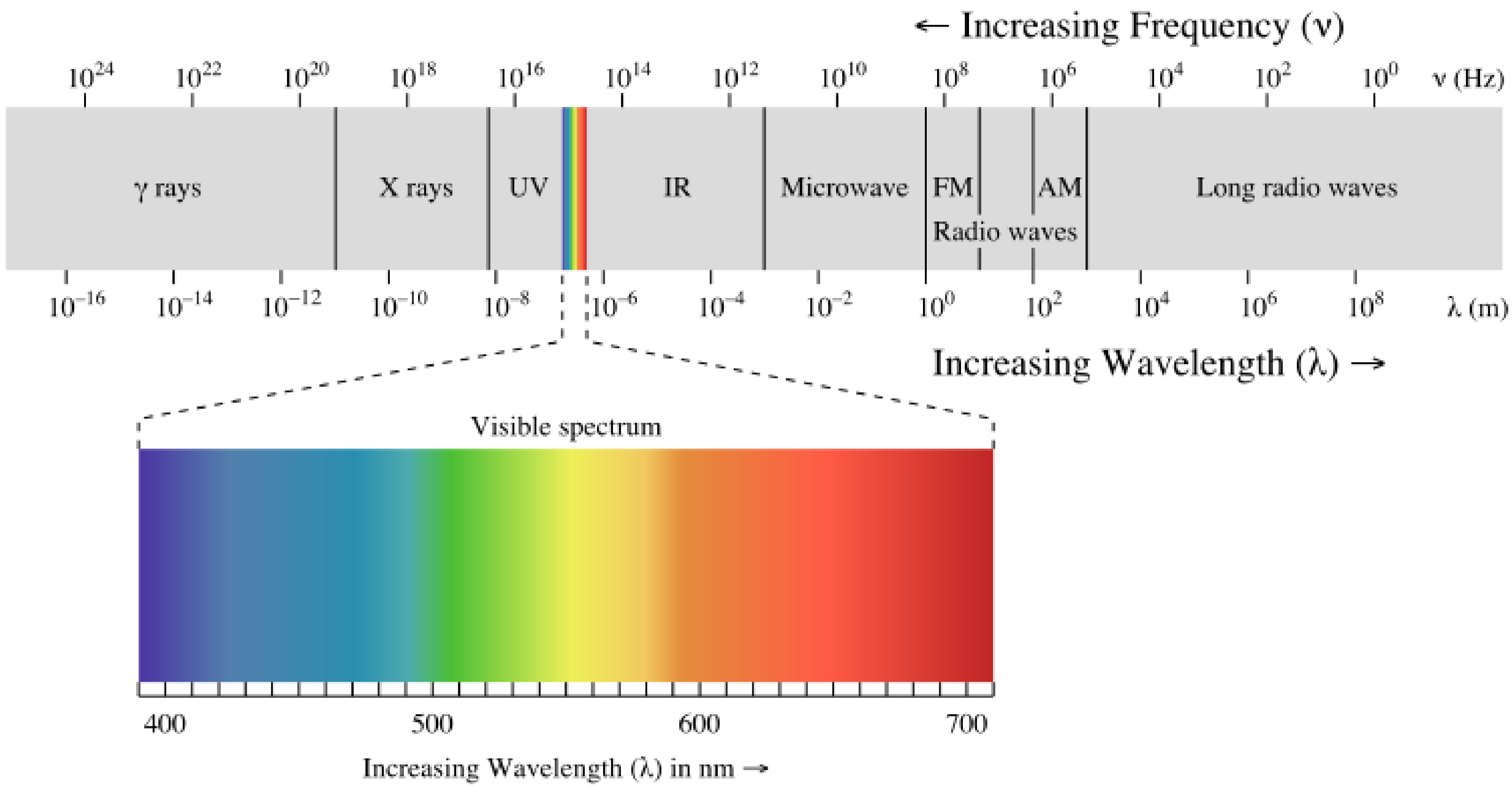
- Light: electromagnetic transverse wave



See Siggraph 2014 course by Naty Hoffman – almost everything from there!!
And images from « Real-time rendering » 3rd edition (A K Peters - 2008)

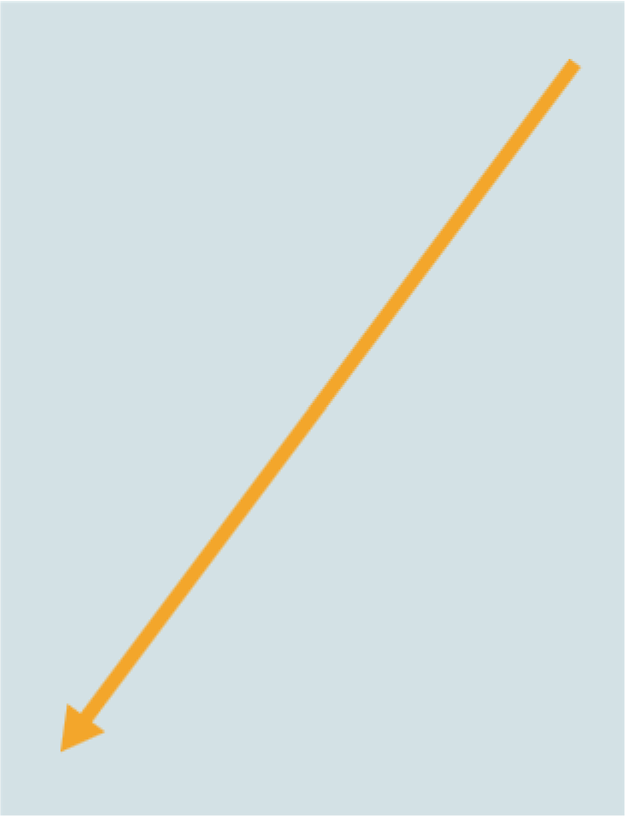
Physics of shading

- Visible light: between 400 and 700 nm



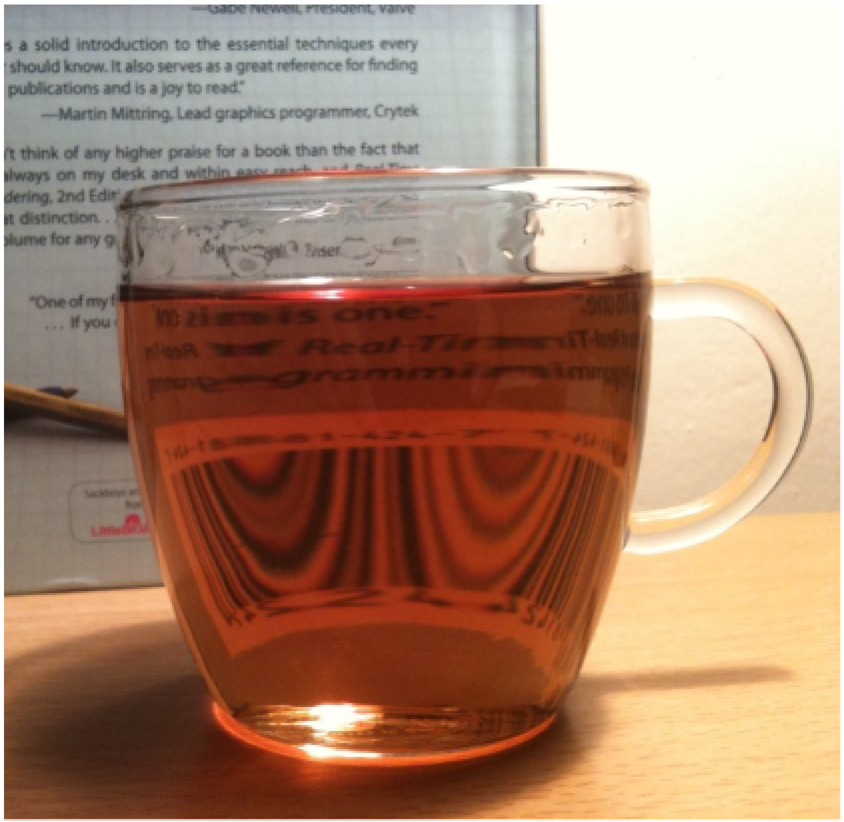
Physics of shading

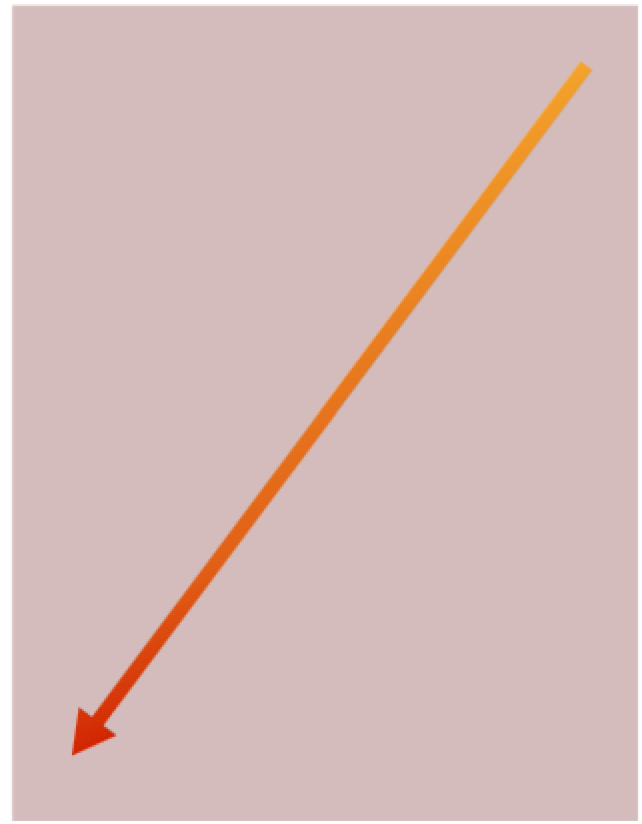
- Light travels in straight line (homogeneous medium)



Physics of shading

- Absorption of parts of visible light





Physics of shading

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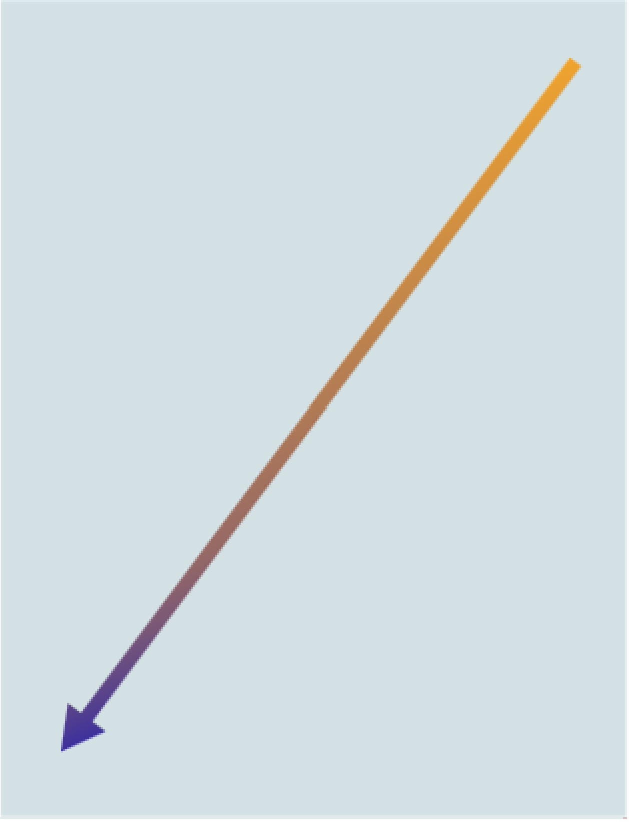
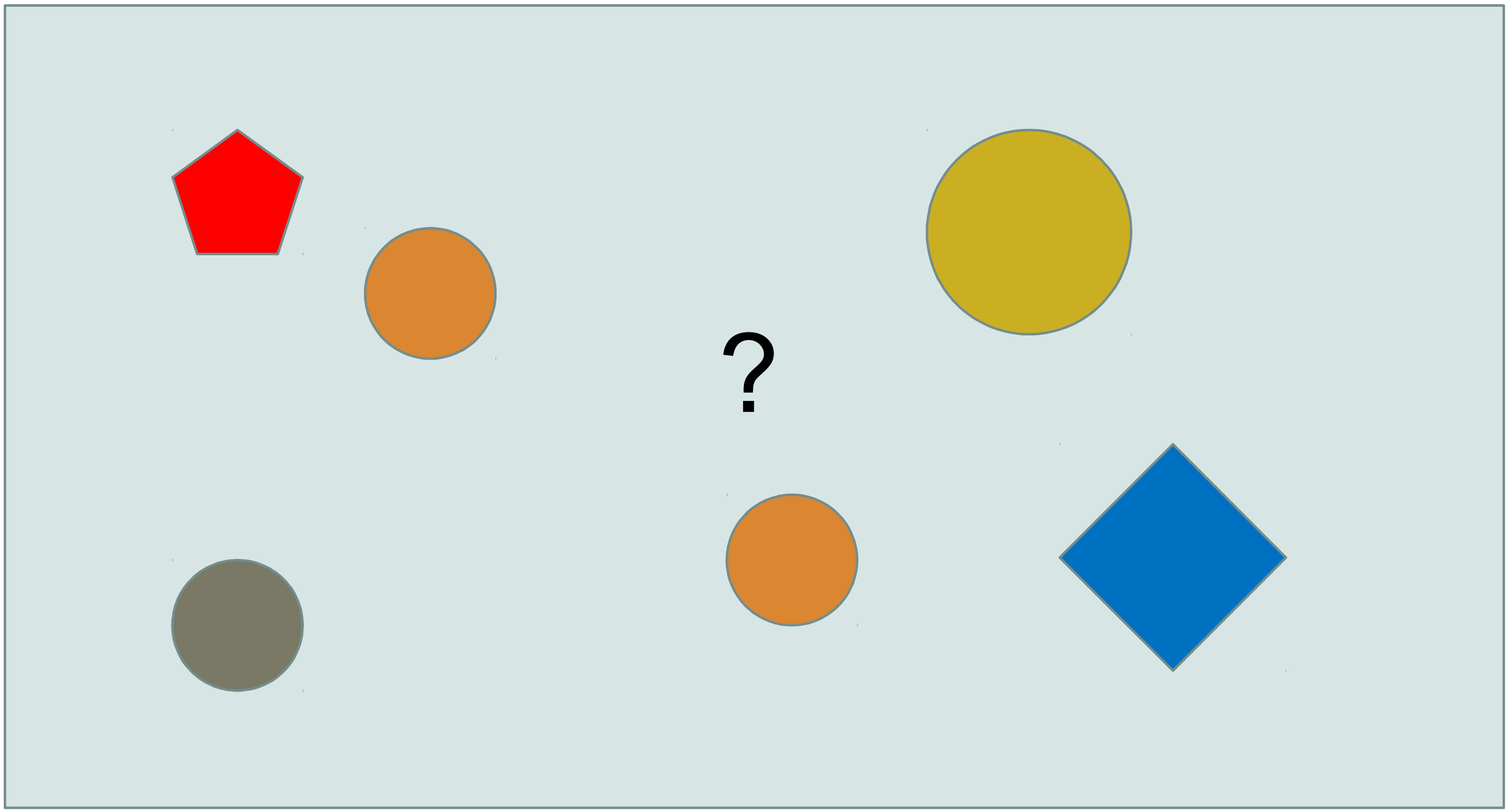


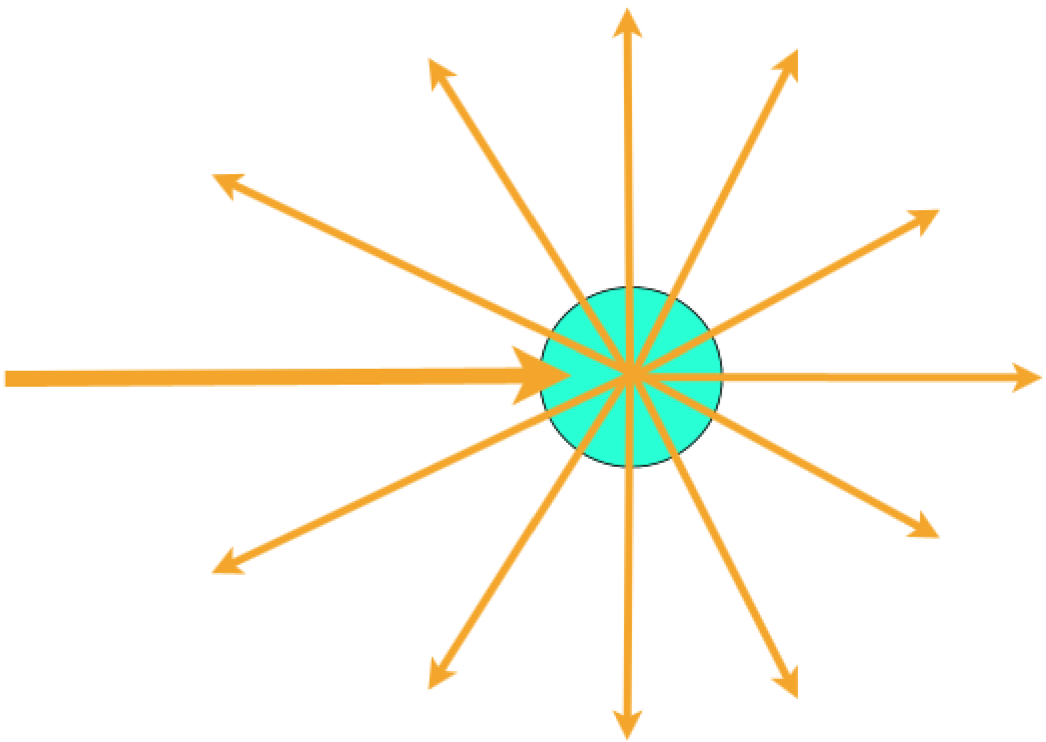
Physics of shading

- Non homogeneous media?



Physics of shading

- Non homogeneous media?
  - Scattering due changes in the index of refraction



Physics of shading

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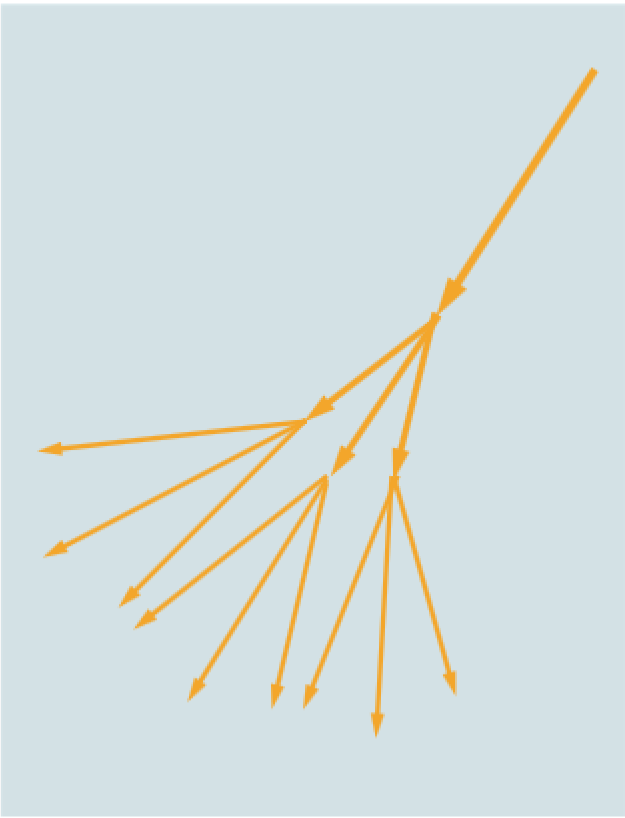

Physics of shading

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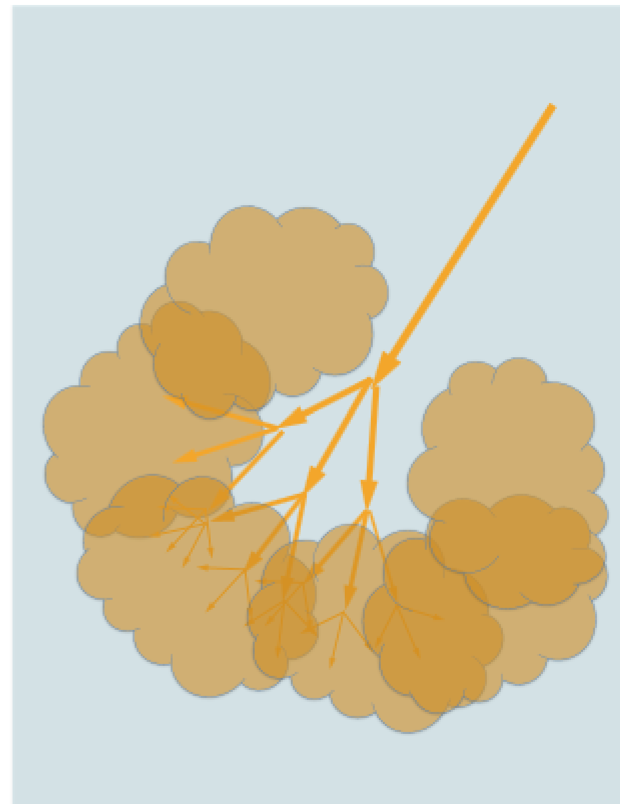
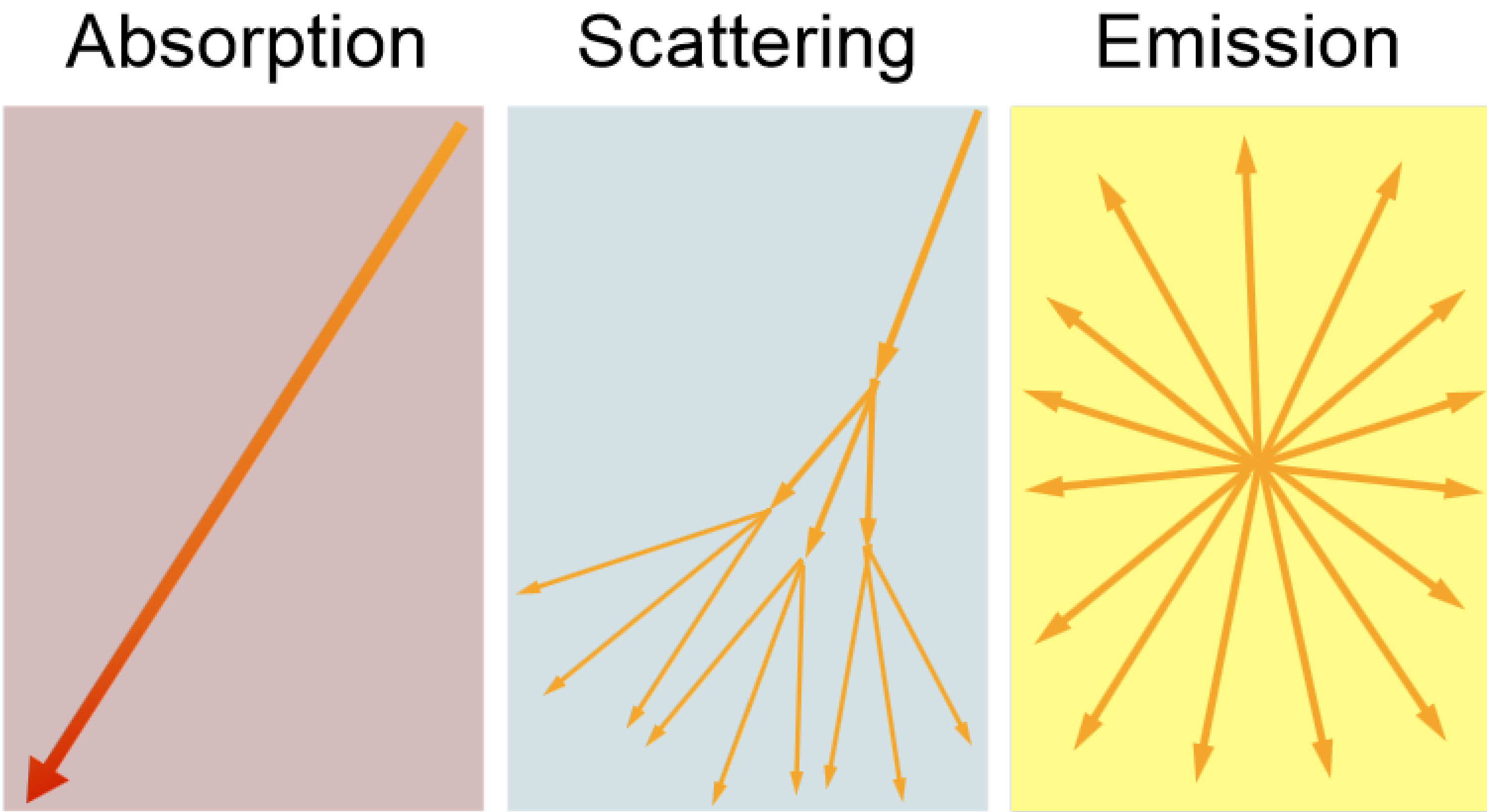


Physics of shading

- Non homogeneous media?
  - Scattering due changes in the index of refraction

Physics of shading

- 3 Modes of light / matter interaction



Physics of shading

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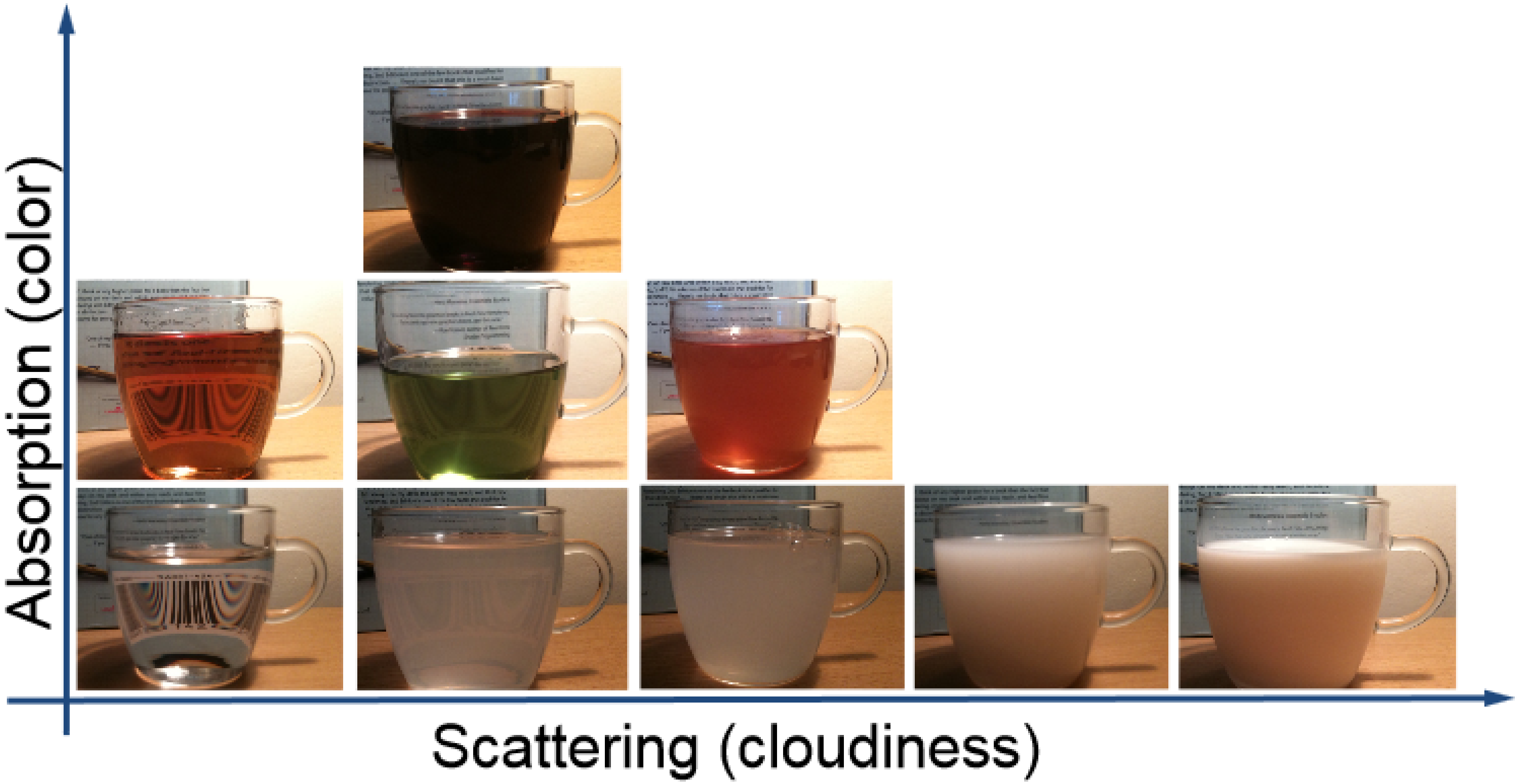
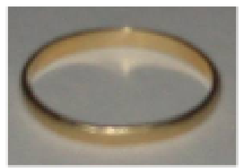
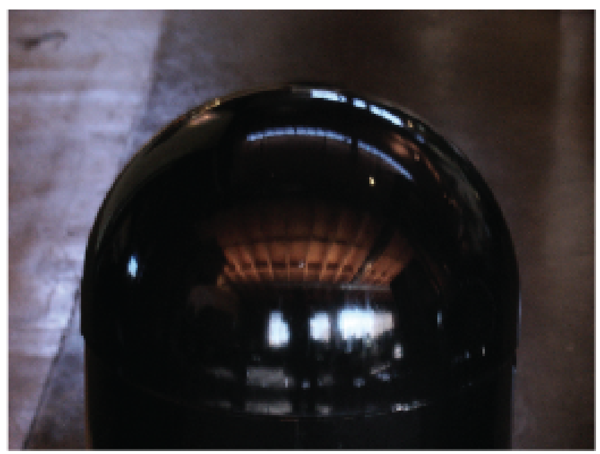


Physics of shading

- What about surfaces?




SIGGRAPH2014

Physics of shading

- Case of (optically) flat surface: Snell Descartes laws

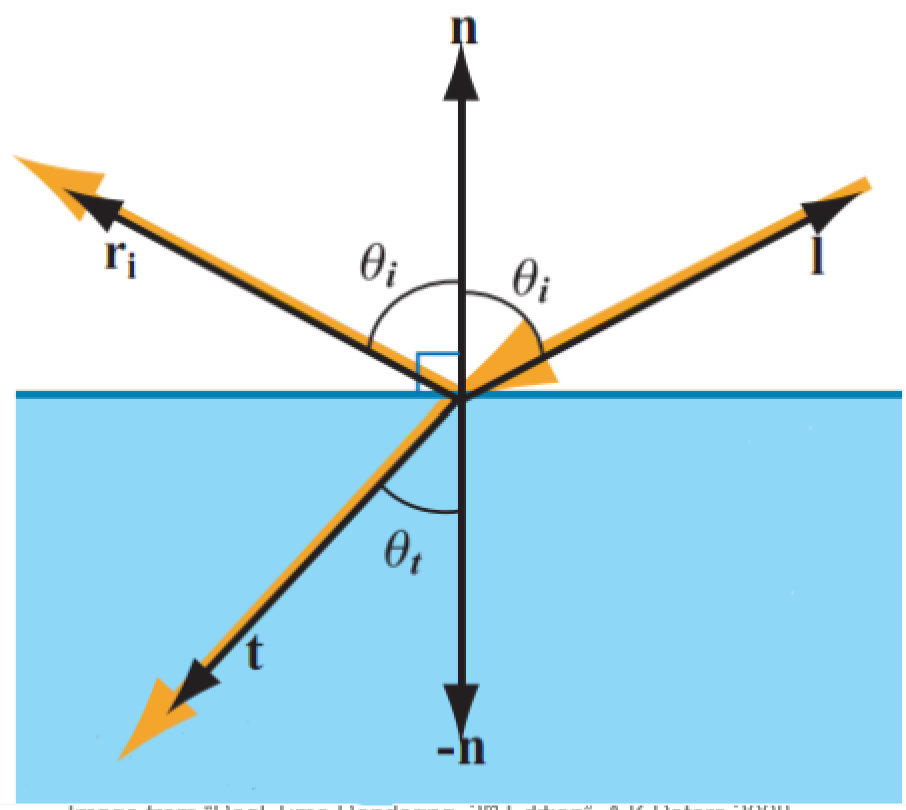
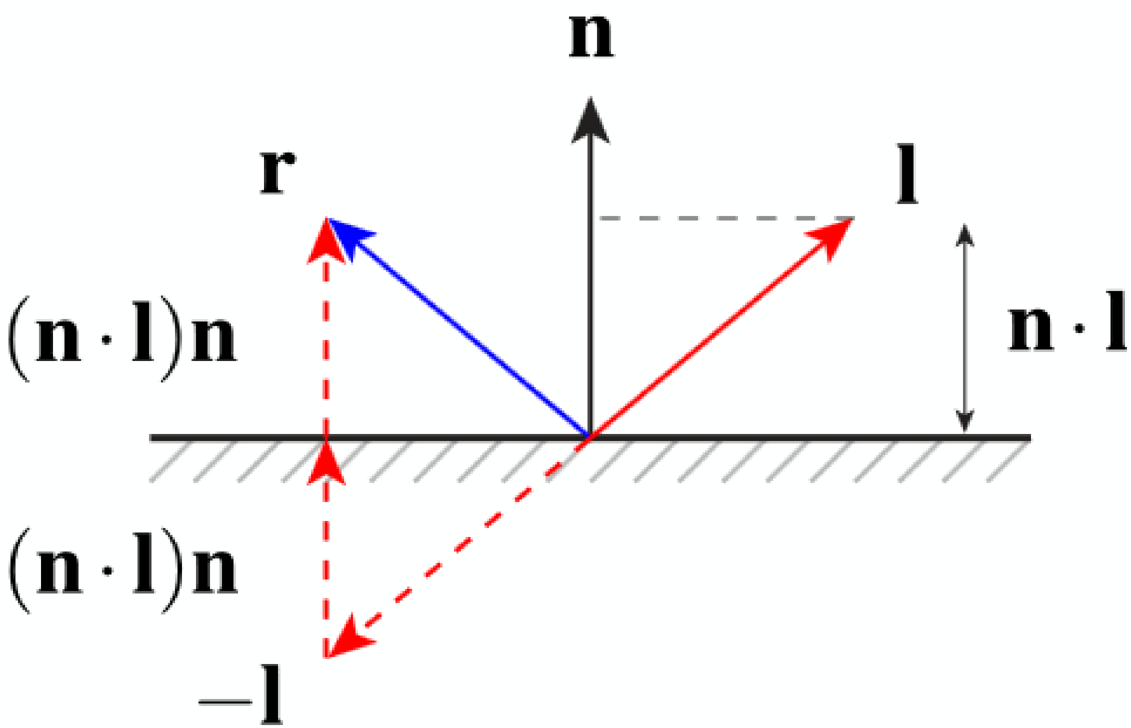


Image from "Real-Time Rendering, 3rd Edition", A K Peters 2008

Physics of shading

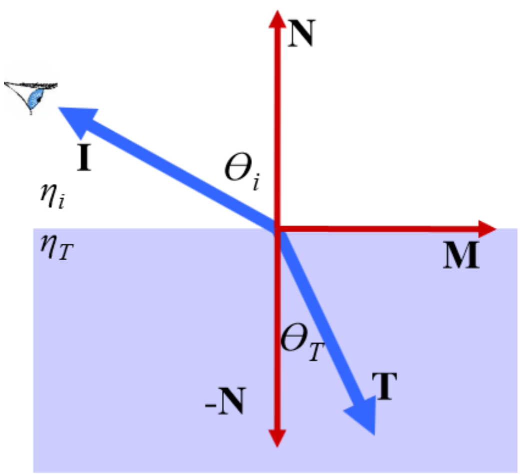
- Case of (optically) flat surface: Snell Descartes laws
  - Incident ray is reflected…



$$\mathbf{r} = 2(\mathbf{n}\cdot\mathbf{l})\mathbf{n} - \mathbf{l}$$

Physics of shading

- Case of (optically) flat surface: Snell Descartes laws
  - Incident ray is reflected…
  - … and refracted



T

$$n_i \sin\theta_i = n_T \sin\theta_T$$

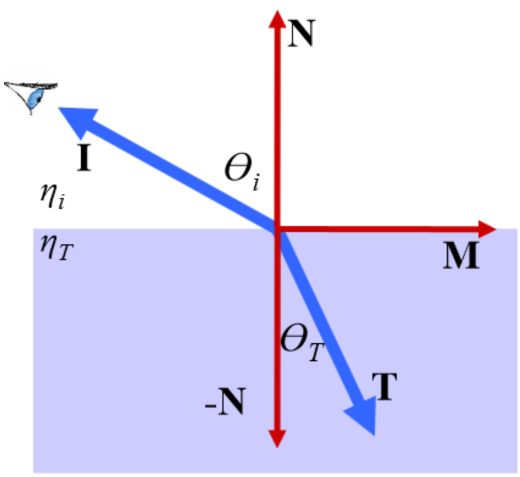
$$\frac{\sin\theta_T}{\sin\theta_i} = \frac{n_i}{n_T} = n_r$$

Physics of shading

- Case of (optically) flat surface: Snell Descartes laws
  - Incident ray is reflected…
  - … and refracted

$$\mathbf{I} = \mathbf{N}\cos\Theta_i - \mathbf{M}\sin\Theta_i$$

$$\mathbf{M} = (\mathbf{N}\cos\Theta_i - \mathbf{I}) / \sin\Theta_i$$



$$n_i \sin\theta_i = n_T \sin\theta_T$$

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Physics of shading

- Case of (optically) flat surface: Snell Descartes laws
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  - … and refracted

$$\mathbf{I} = \mathbf{N}\cos\Theta_i - \mathbf{M}\sin\Theta_i$$

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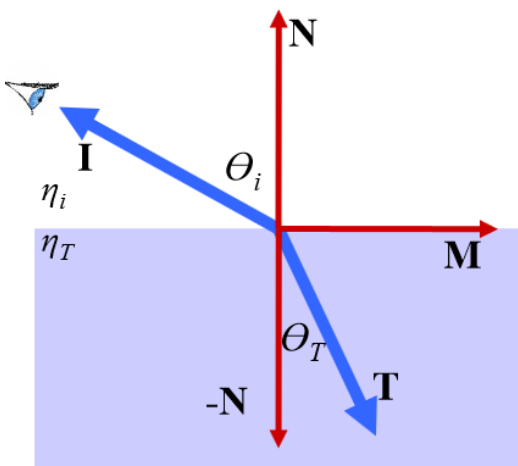

$$\begin{aligned}
\mathbf{T} &= -\mathbf{N}\cos\Theta_T + \mathbf{M}\sin\Theta_T \\
&= -\mathbf{N}\cos\Theta_T + (\mathbf{N}\cos\Theta_i - \mathbf{I})\sin\Theta_T / \sin\Theta_i \quad \textit{Plug M} \\
&= -\mathbf{N}\cos\Theta_T + (\mathbf{N}\cos\Theta_i - \mathbf{I})\,\eta_r \quad \textit{let's get rid of the cos \& sin} \\
&= [\,\eta_r\cos\Theta_i - \cos\Theta_T\,]\,\mathbf{N} - \eta_r\,\mathbf{I} \\
&= [\,\eta_r\cos\Theta_i - \sqrt{1 - \sin^2\Theta_T}\,]\,\mathbf{N} - \eta_r\,\mathbf{I} \\
&= [\,\eta_r\cos\Theta_i - \sqrt{1 - \eta_r^2\sin^2\Theta_i}\,]\,\mathbf{N} - \eta_r\,\mathbf{I} \\
&= [\,\eta_r\cos\Theta_i - \sqrt{1 - \eta_r^2(1 - \cos^2\Theta_i)}\,]\,\mathbf{N} - \eta_r\,\mathbf{I} \\
&= [\,\eta_r(\mathbf{N}\cdot\mathbf{I}) - \sqrt{1 - \eta_r^2(1 - (\mathbf{N}\cdot\mathbf{I})^2)}\,]\,\mathbf{N} - \eta_r\,\mathbf{I}
\end{aligned}$$

$$n_i \sin\theta_i = n_T \sin\theta_T$$

$$\frac{\sin\theta_T}{\sin\theta_i} = \frac{n_i}{n_T} = n_r$$

Physics of shading

- Case of (optically) flat surface: Snell Descartes laws
  - Incident ray is reflected…
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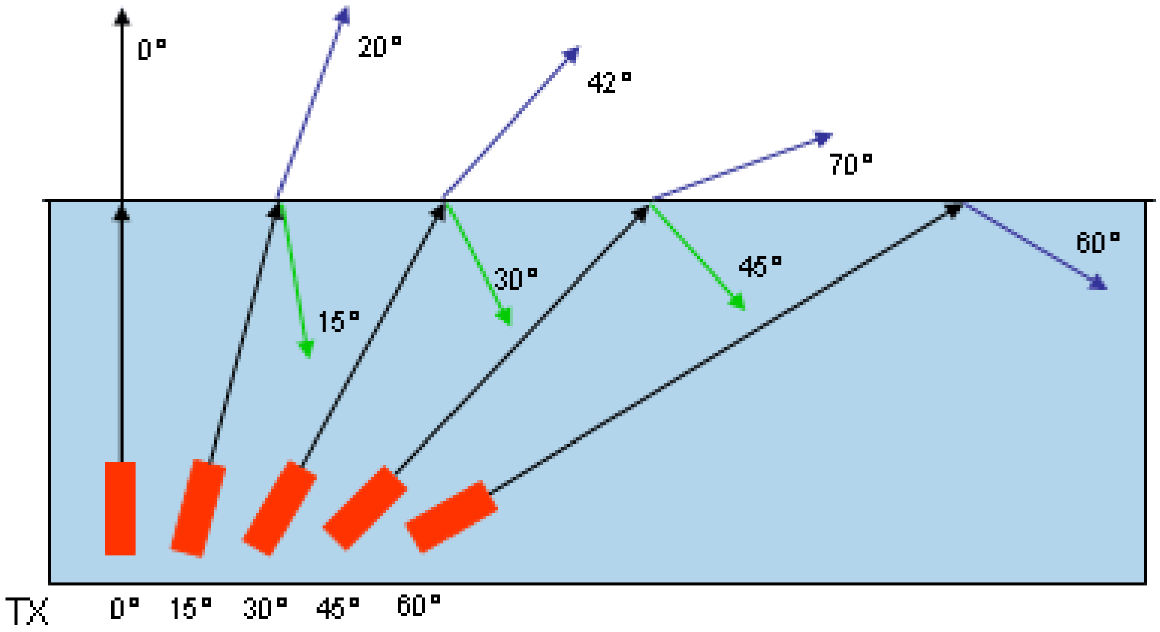
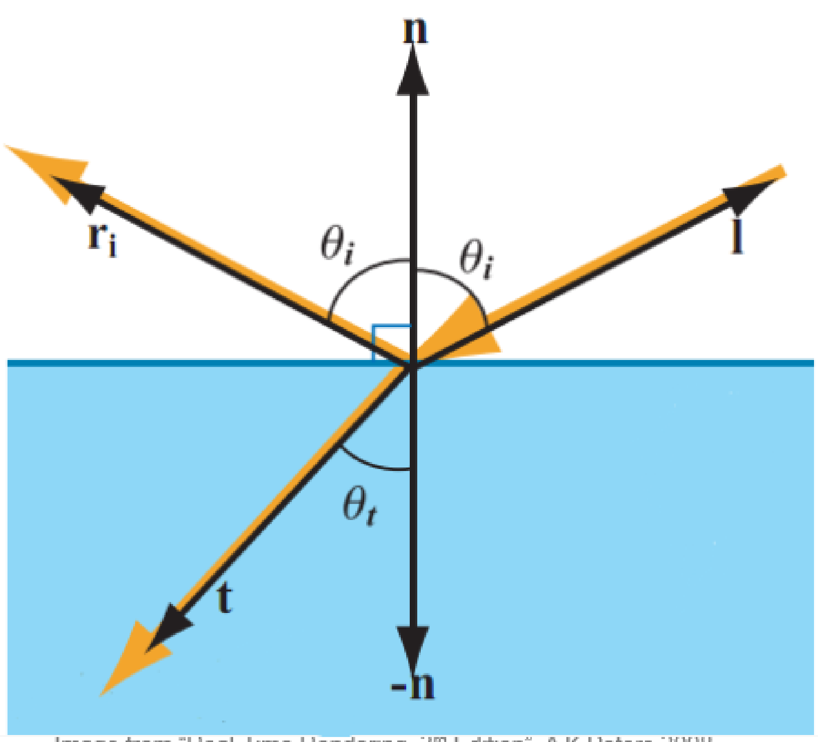


Total internal reflection

Physics of shading

- Case of (optically) flat surface: Snell Descartes laws
  - Incident ray is reflected…
  - … and refracted
- The amount of reflection vs refraction
  - Controlled with Fresnel law (electromagnetic wave)



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Physics of shading

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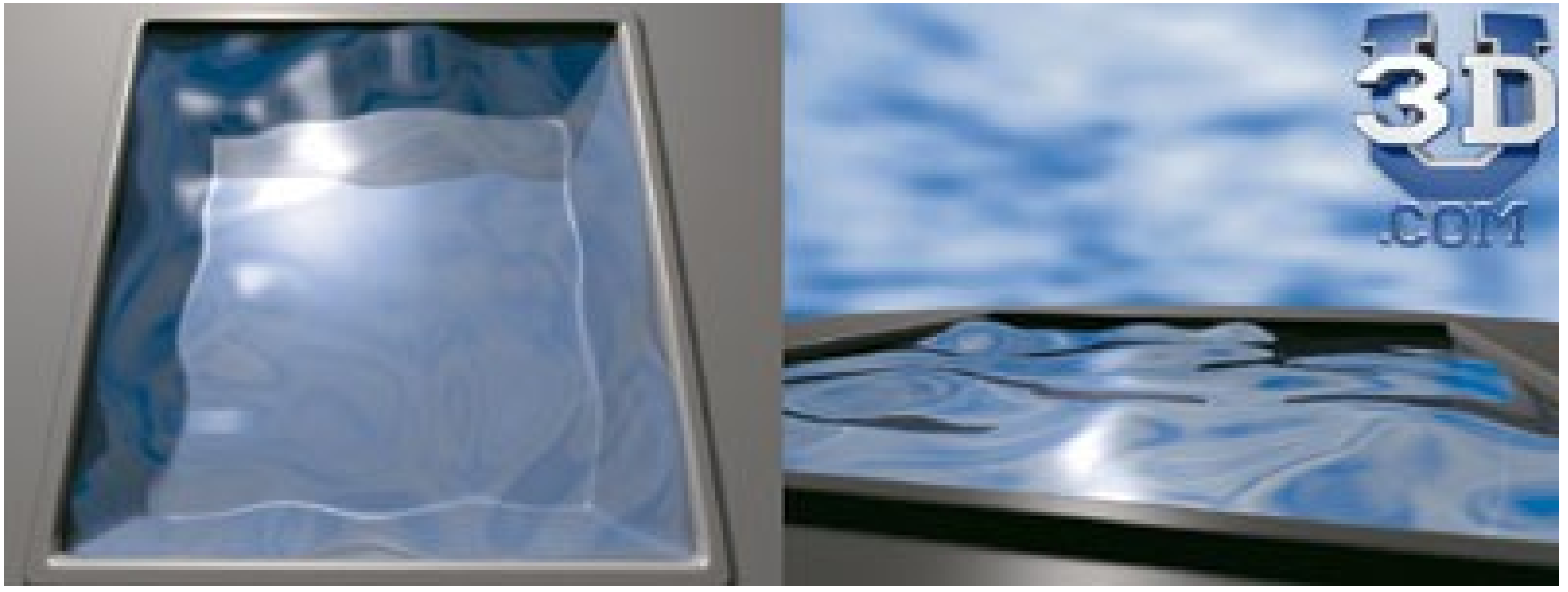

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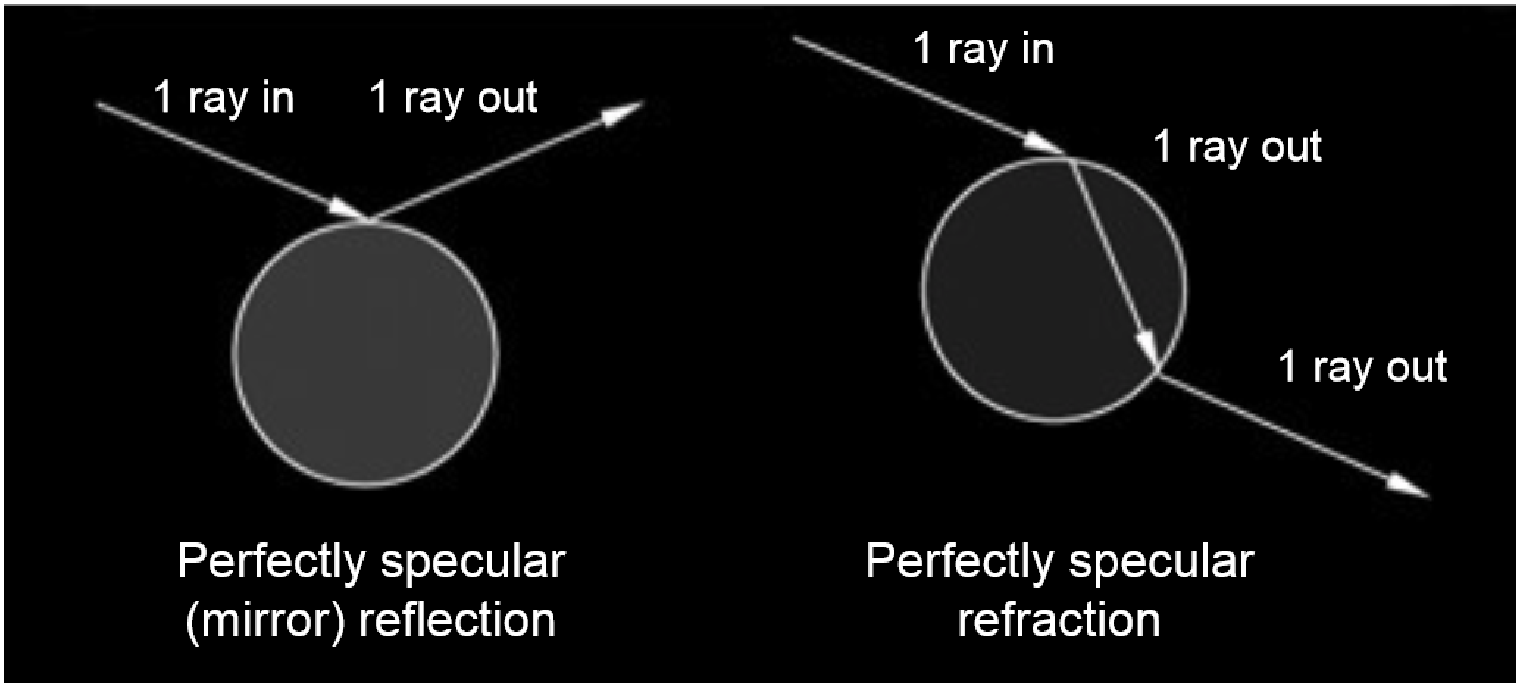

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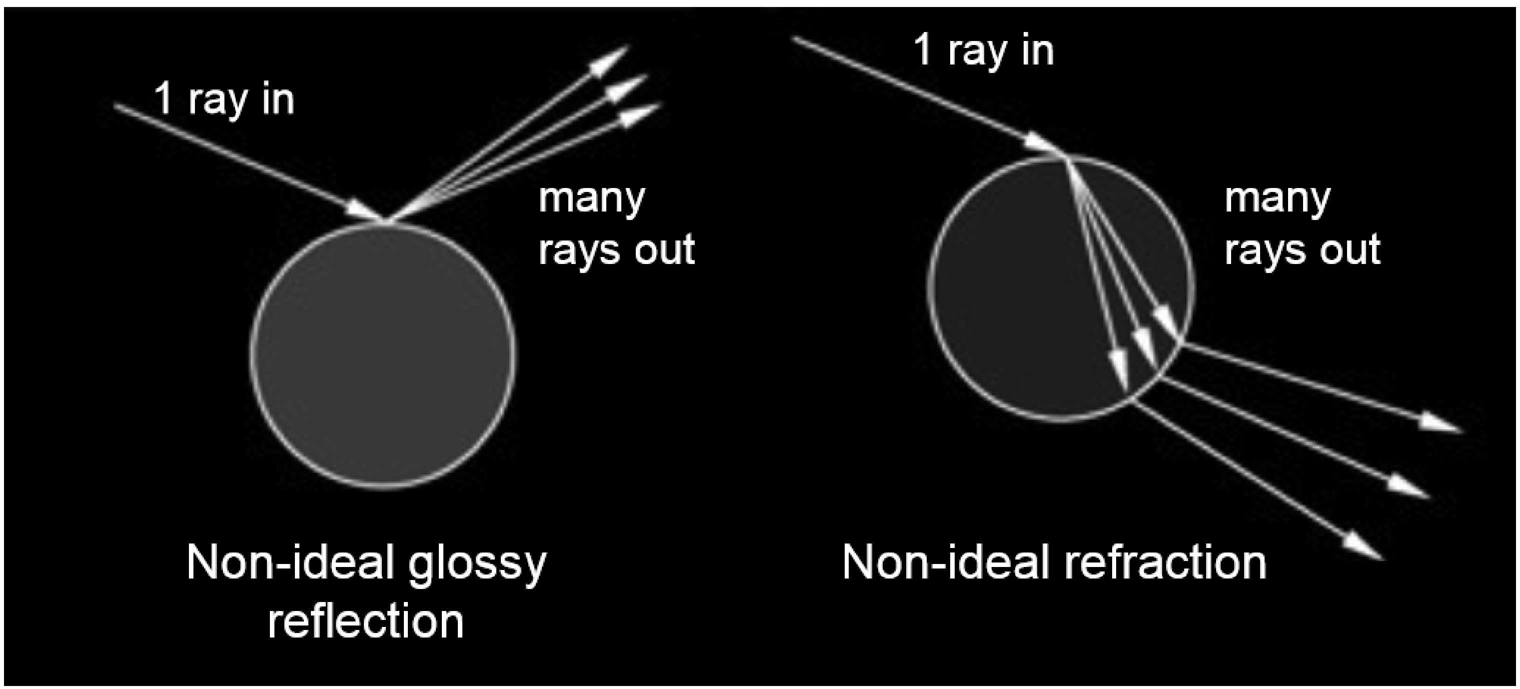
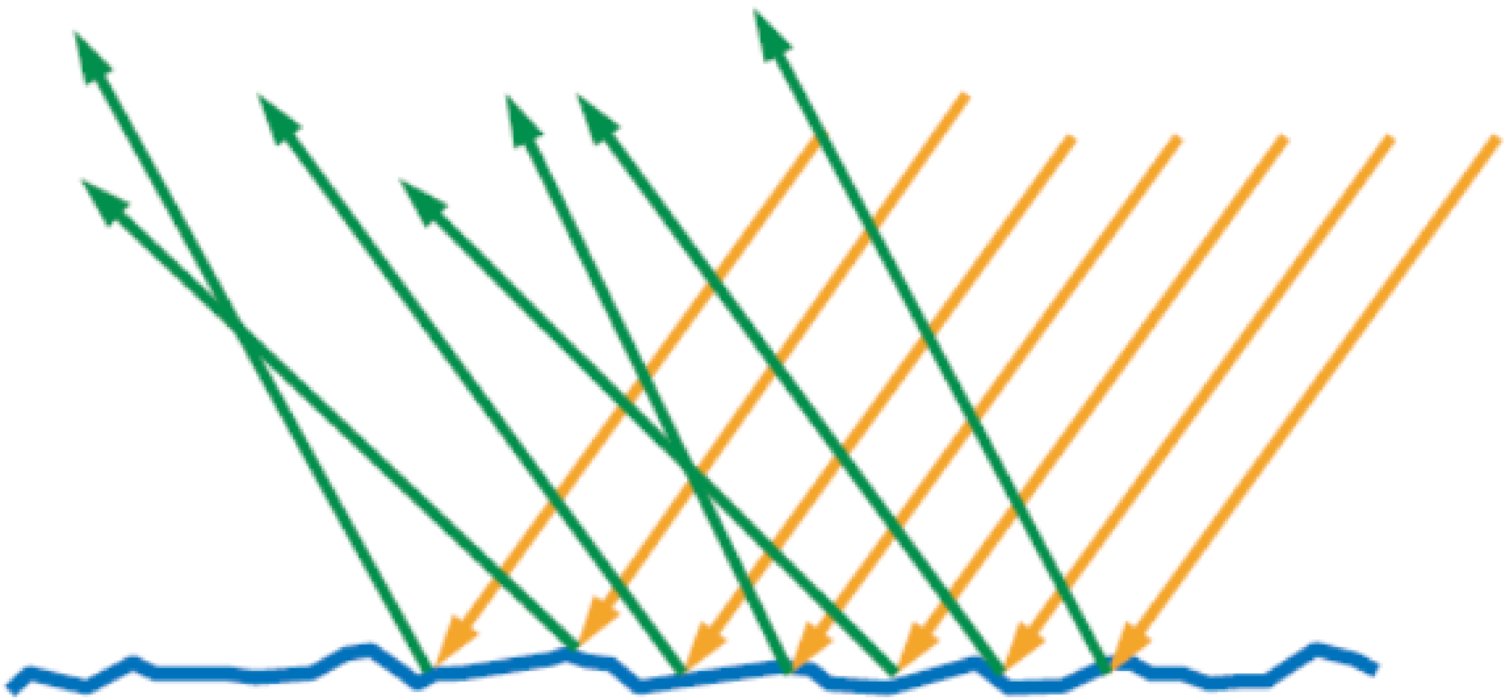


Micro geometry

- Microgeometry bumps
  - Bigger than light wavelength
  - But too small to be visible!
  - Agregate of all response



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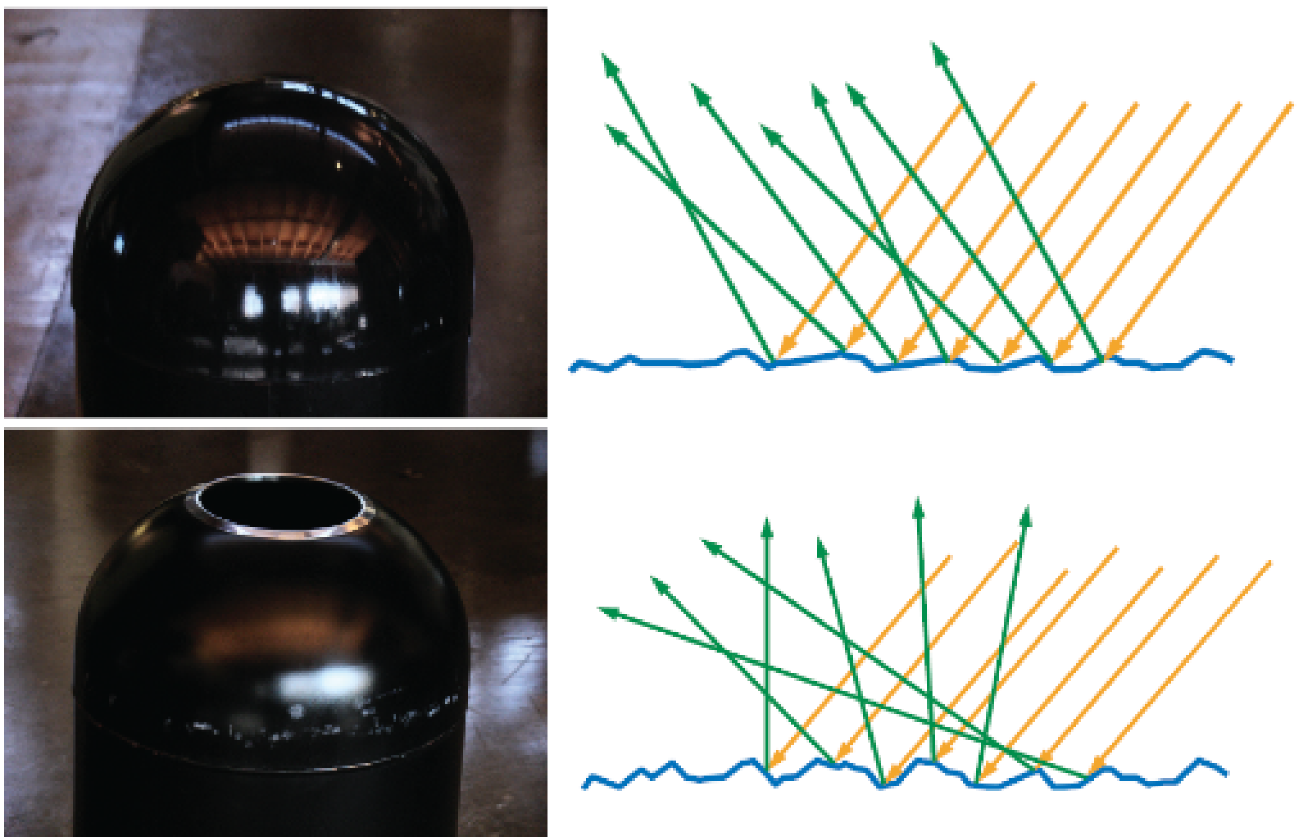

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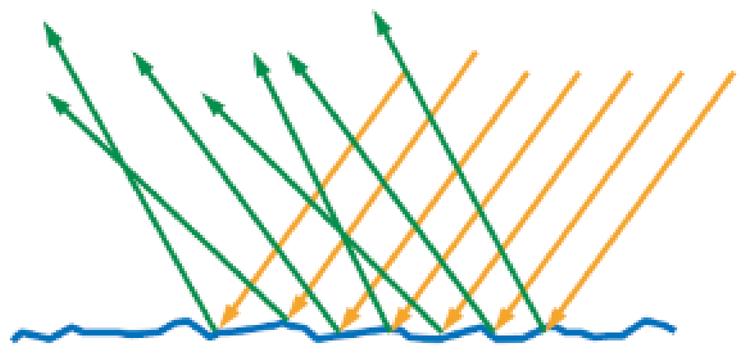
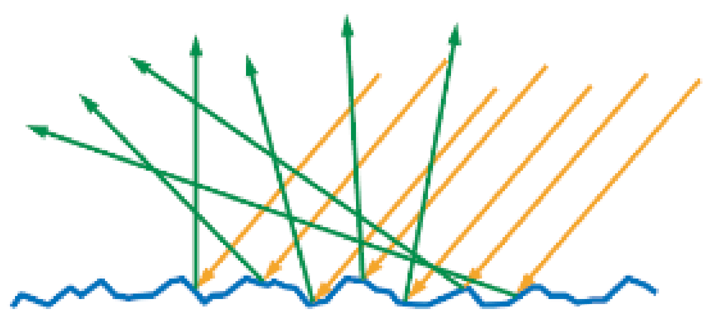
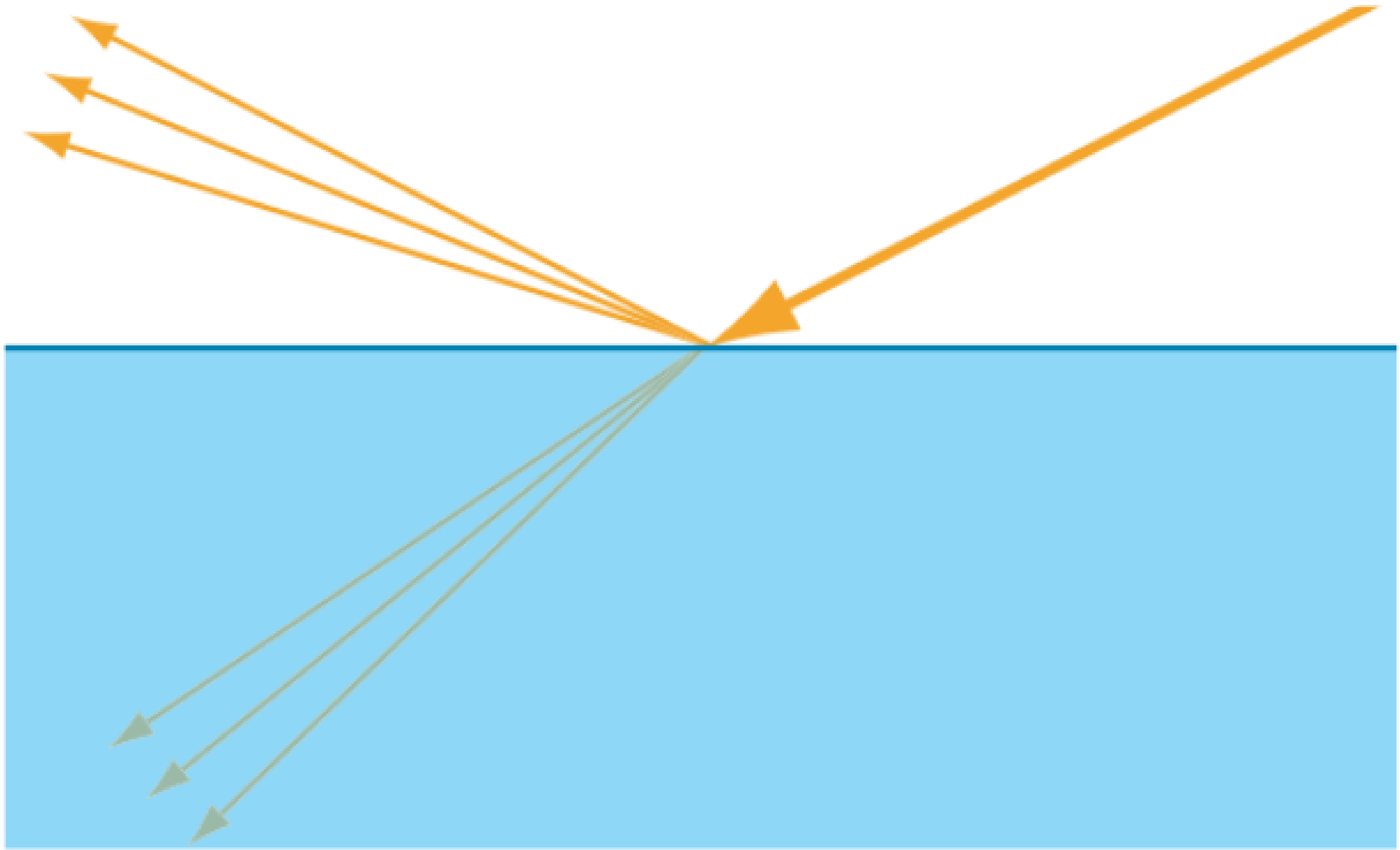


~Mirror



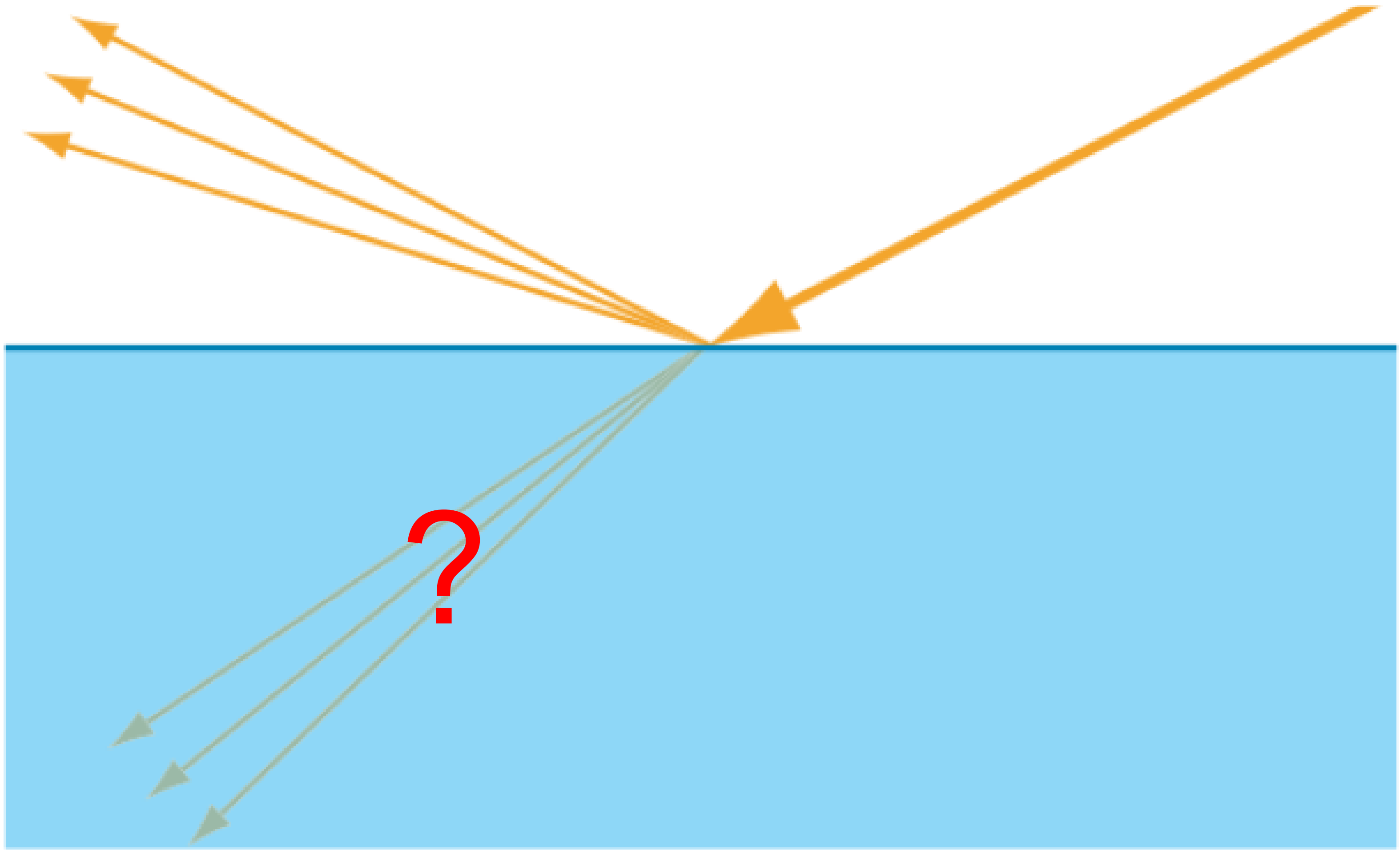
~Rough

Macroscopic view



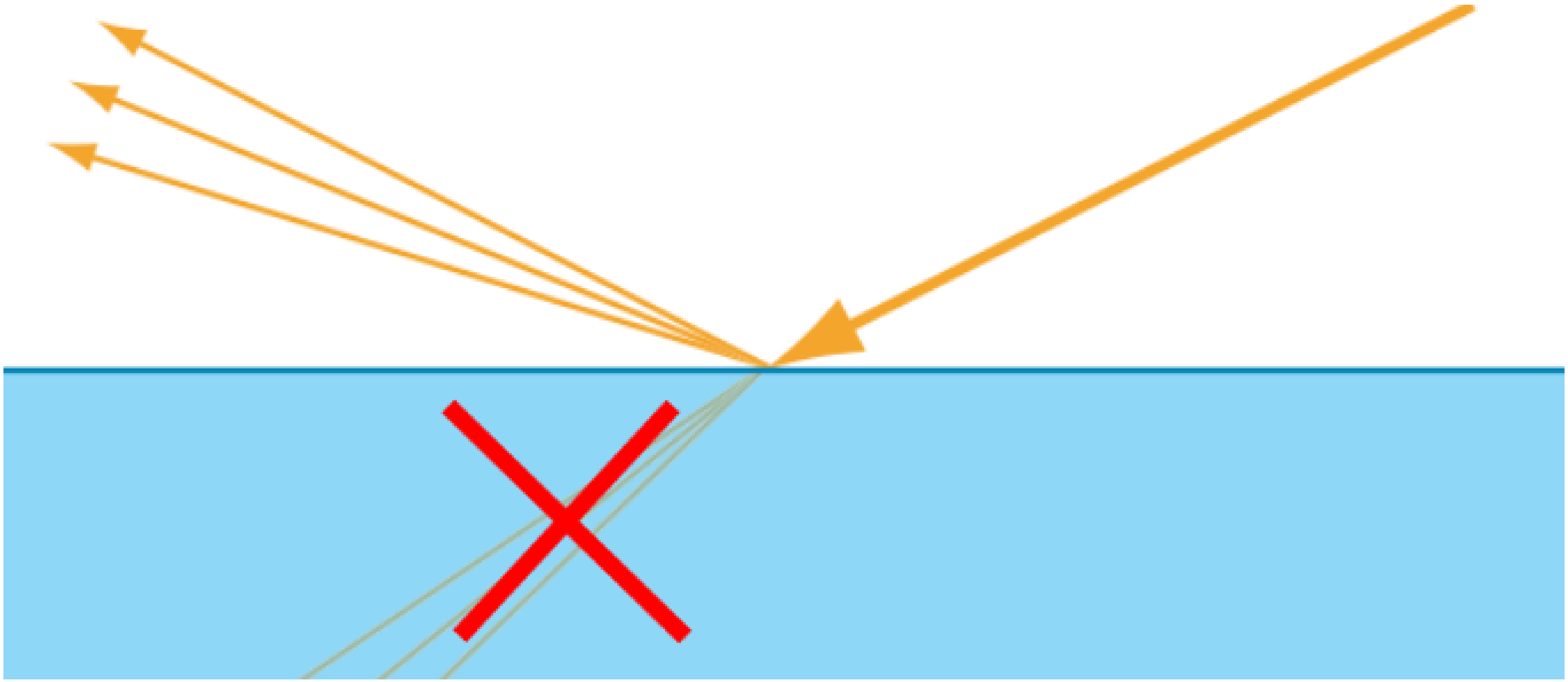
Macroscopic view

- Refractions?



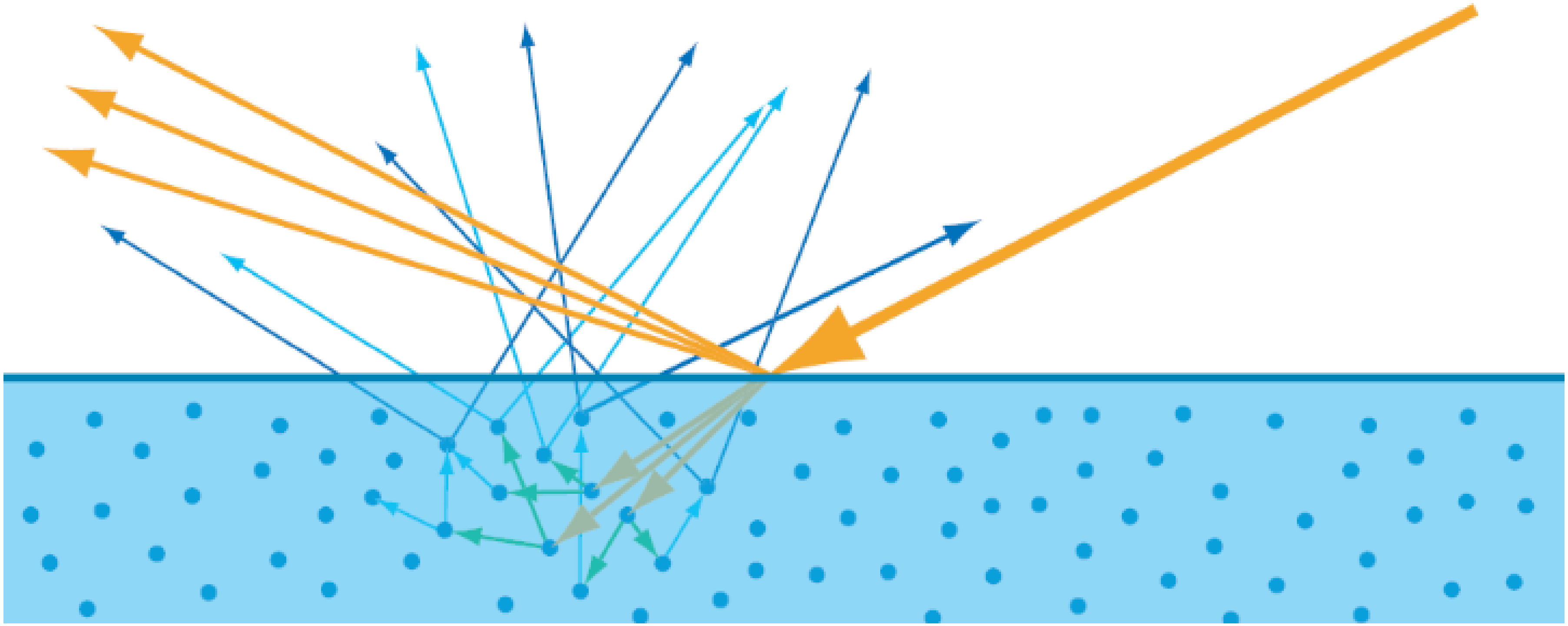
Metals

- Refracted light immediatly absorbed by free electrons



Non-metals

- Behave like regular participating media
  - Light is scattered (enough) are re-emited



Non-metals

- Behave like regular participating media
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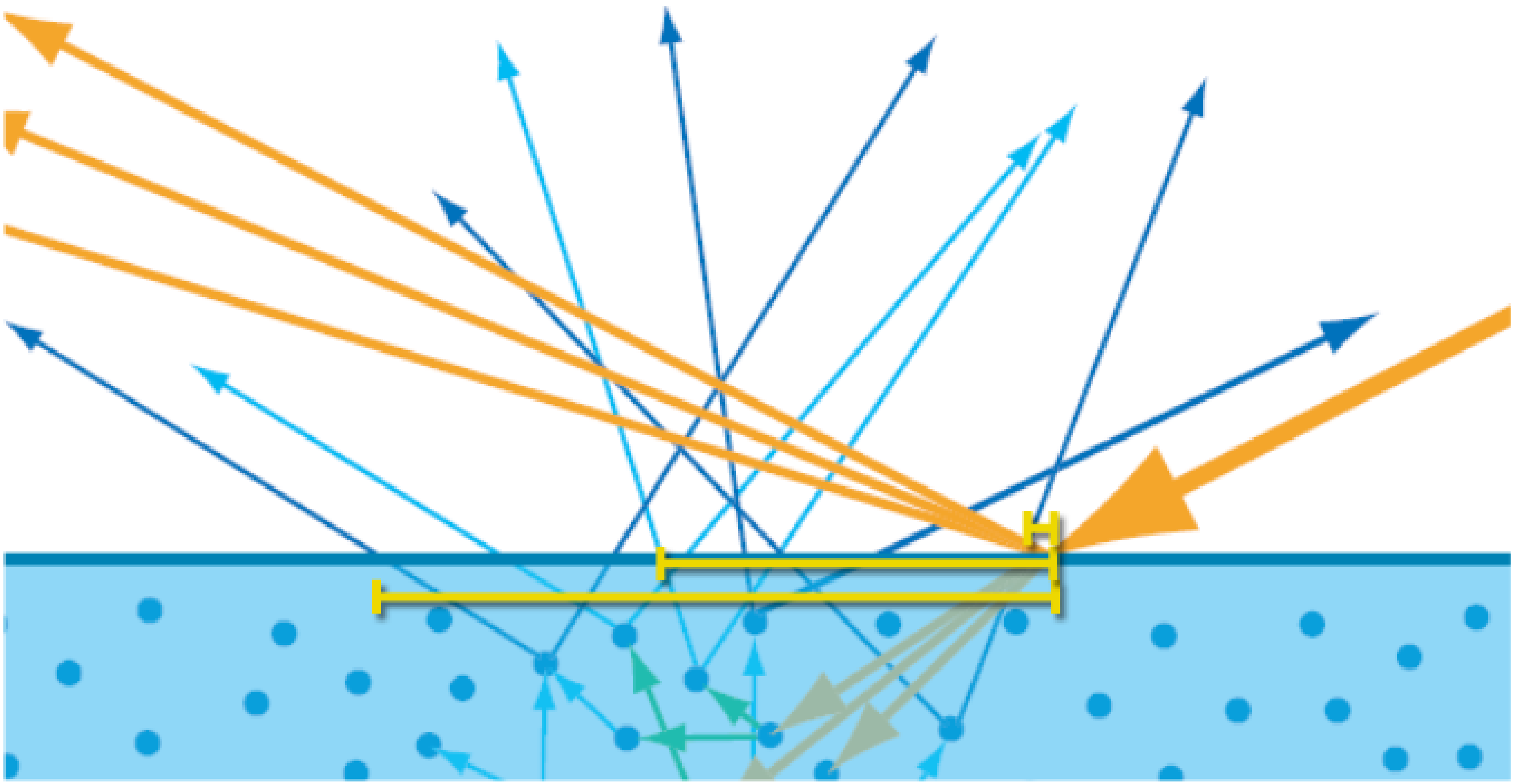
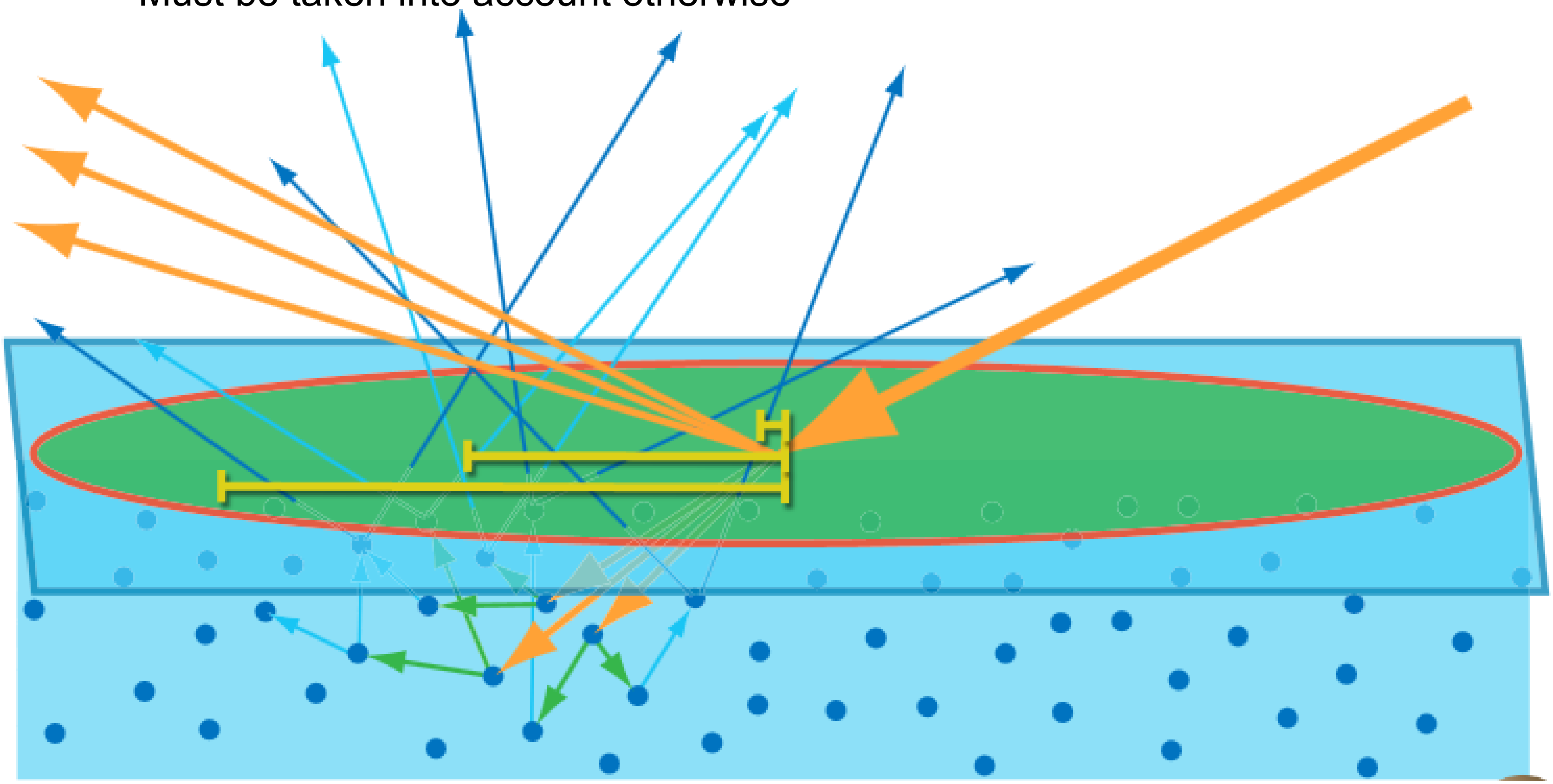
  - Distance depends on particle densities



Non-metals

- Behave like regular participating media
  - Light is scattered (enough) are re-emited
  - Distance depends on particle densities
    - Assume = 0 if shading area (surface point) large enough
    - Must be taken into account otherwise



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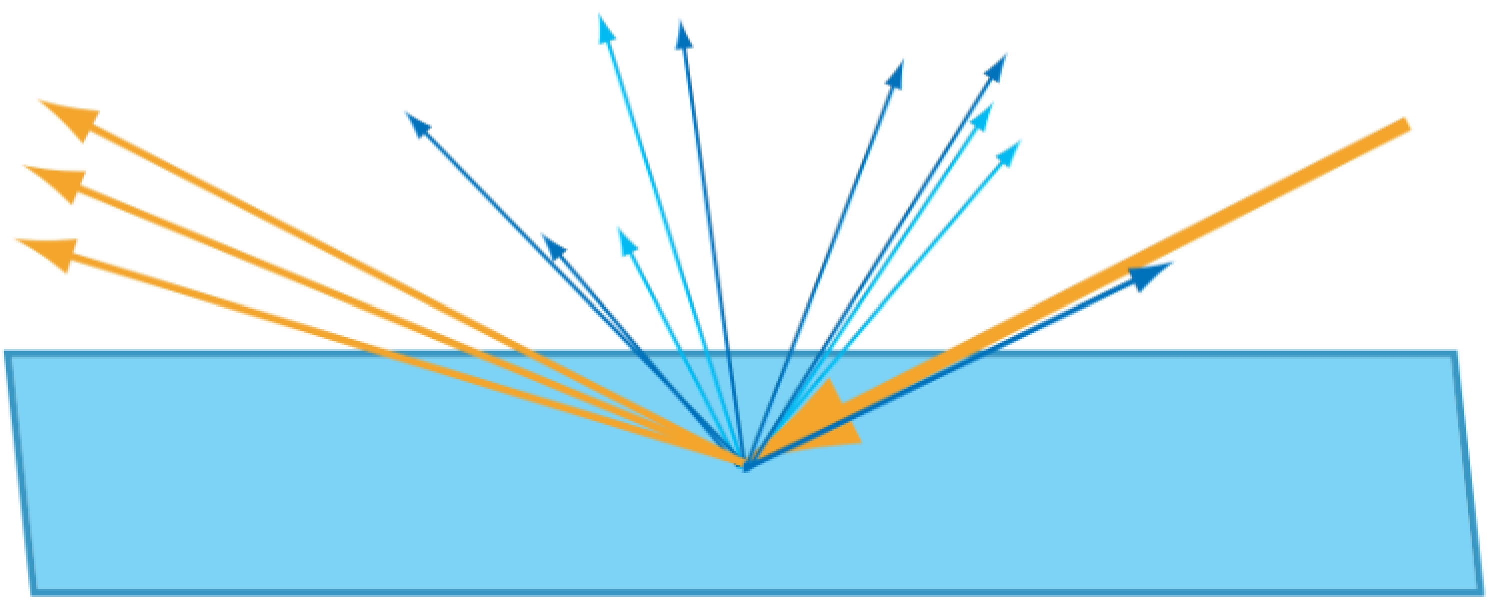

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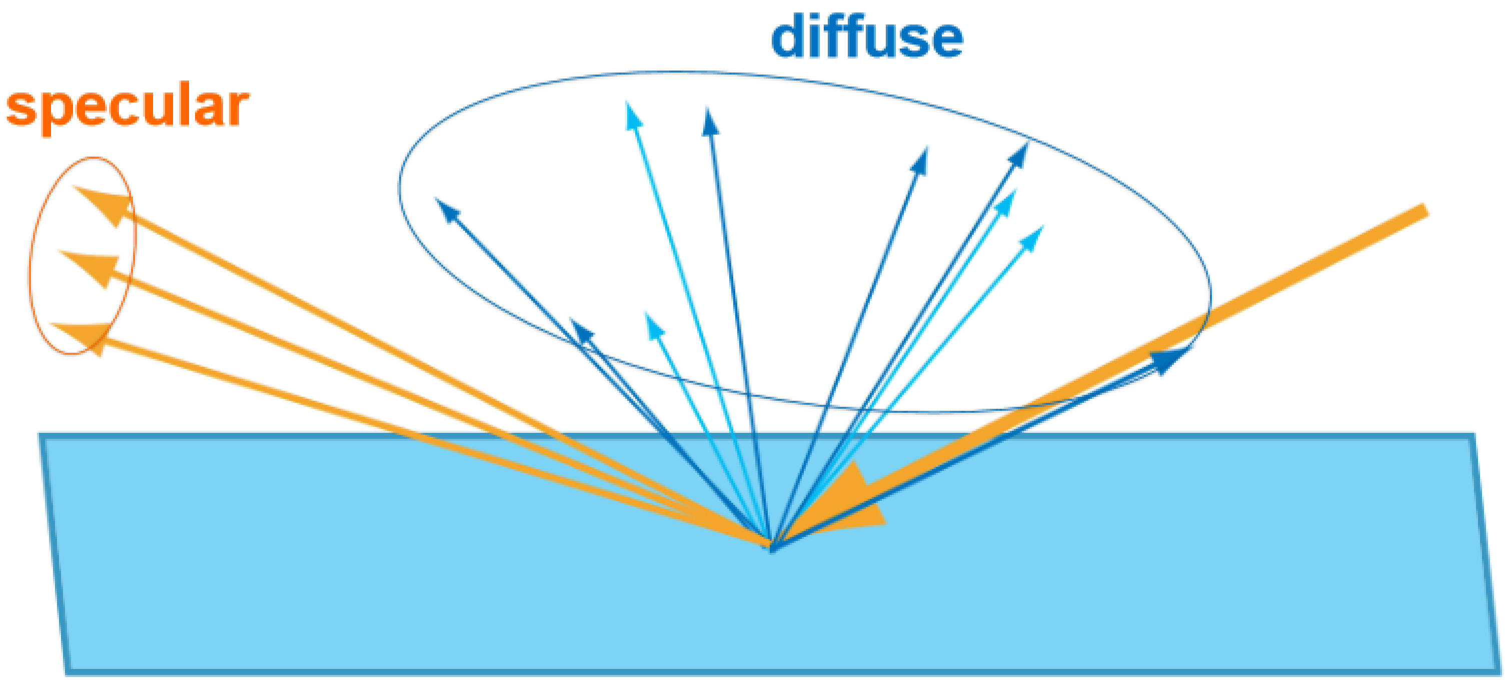
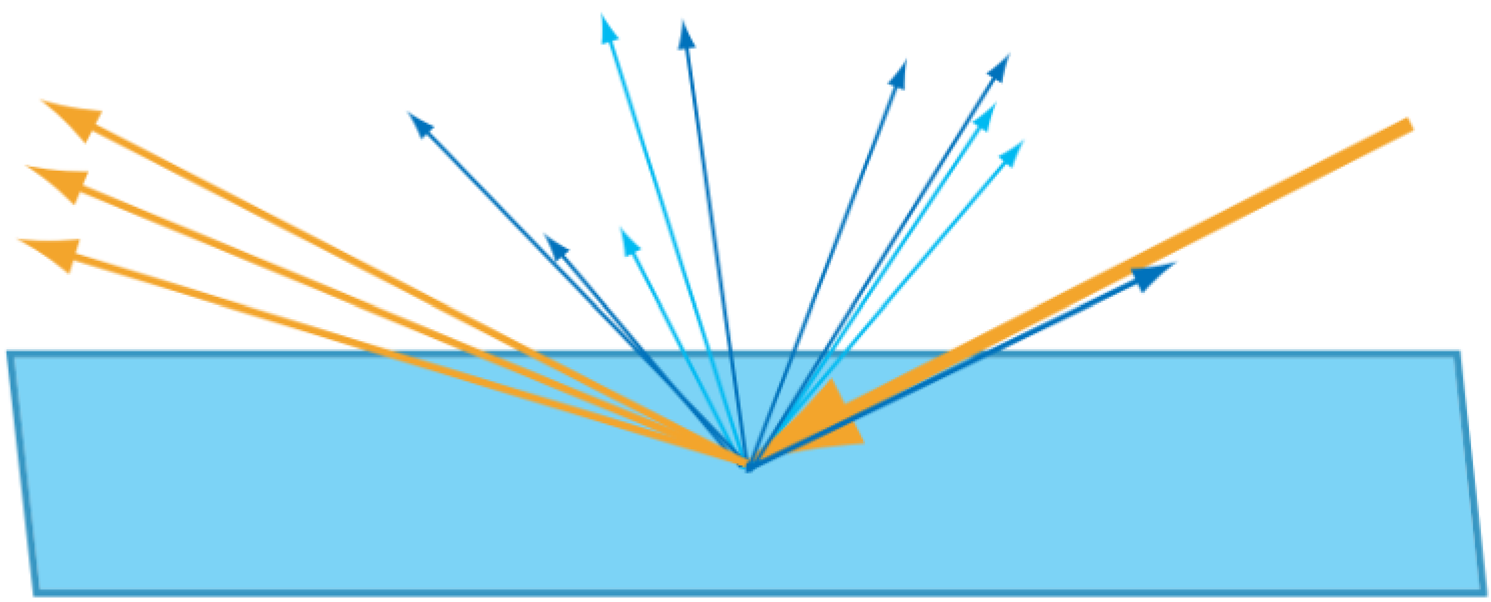


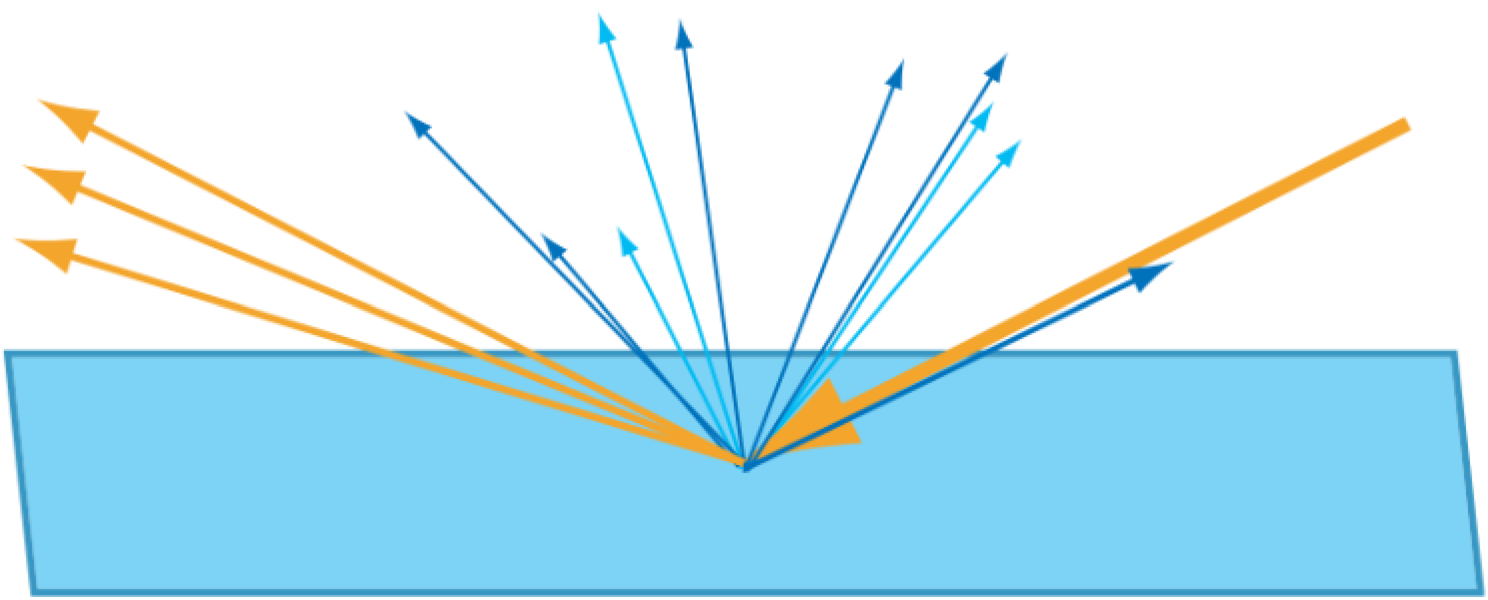
Physics & Maths

- Radiance
  - radiometric quantity used mesure the amount of light along a single ray
  - Spectral quantity (RGB in practice), Watt per steradian per square meter



Physics ⊟ Maths

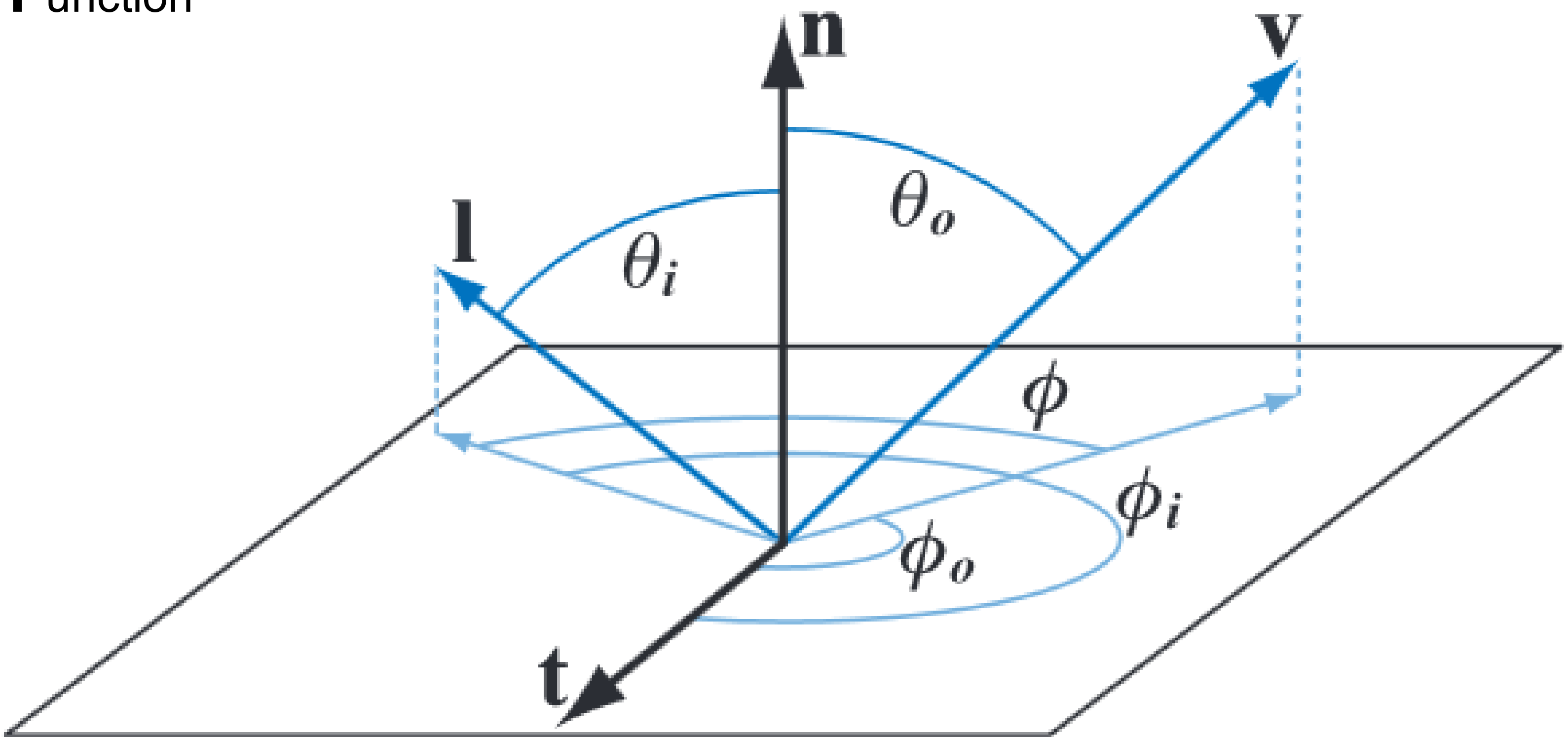
- Radiance
  - radiometric quantity used mesure the amount of light along a single ray
  - Spectral quantity (RGB in practice), Watt per steradian per square meter
- If shading can be handled locally, light response depends on
  - Light direction
  - View direction



Physics ㅂ Maths

- **B**idirectionnal
- **R**eflectance
- **D**istribution
- **F**unction

$$f(\mathbf{l}, \mathbf{v})$$



Physics ㅂ Maths

- **B**idirectionnal
- **R**eflectance
- **D**istribution
- **F**unction

$$f(\mathbf{l}, \mathbf{v})$$

$$L_o(\mathbf{v}) = \int_\Omega f(\mathbf{l}, \mathbf{v}) \otimes L_i(\mathbf{l})(\mathbf{n} \cdot \mathbf{l}) d\omega_i$$

Reflectance equation

Physics ㅂ Maths

- **B**idirectionnal
- **R**eflectance
- **D**istribution
- **F**unction

$$f(\mathbf{l}, \mathbf{v})$$

$$L_o(\mathbf{v}) = \int_\Omega f(\mathbf{l}, \mathbf{v}) \otimes L_i(\mathbf{l})(\mathbf{n} \cdot \mathbf{l}) d\omega_i$$

Outgoing radiance

Reflectance equation

Physics ⊟ Maths

- **B**idirectionnal
- **R**eflectance
- **D**istribution
- **F**unction

$$f(\mathbf{l}, \mathbf{v})$$

$$L_o(\mathbf{v}) = \int_\Omega f(\mathbf{l}, \mathbf{v}) \otimes L_i(\mathbf{l})(\mathbf{n} \cdot \mathbf{l}) d\omega_i$$

Outgoing radiance

Ingoing radiance

Reflectance equation

Physics ⊟ Maths

- **B**idirectionnal
- **R**eflectance
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Outgoing radiance

Ingoing radiance

Surface orientation

Reflectance equation

Physics Maths

- **B**idirectionnal
- **R**eflectance
- **D**istribution
- **F**unction

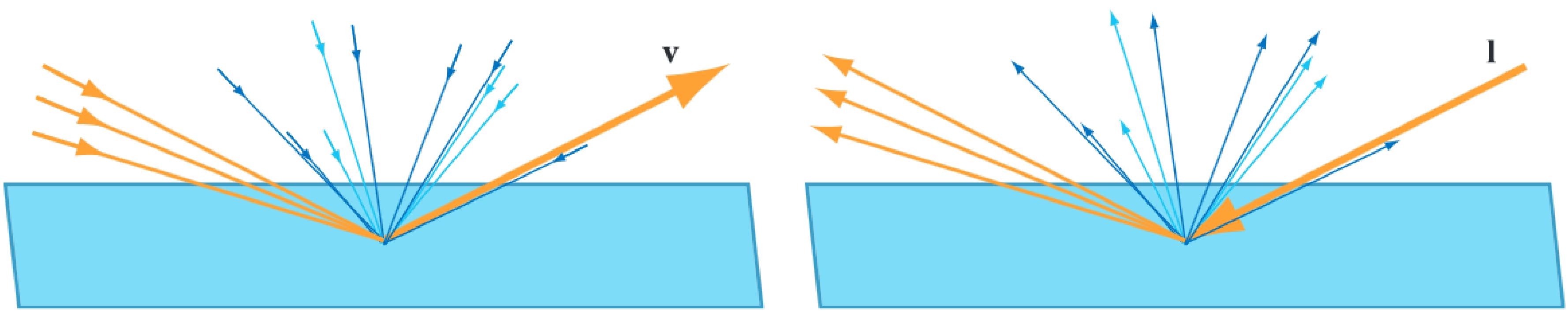
$$f(\mathbf{l}, \mathbf{v})$$

$$\underbrace{L_o(\mathbf{v})}_{\text{Outgoing radiance}} = \int_\Omega \underbrace{f(\mathbf{l}, \mathbf{v})}_{\text{BRDF}} \otimes \underbrace{L_i(\mathbf{l})}_{\text{Ingoing radiance}} \underbrace{(\mathbf{n} \cdot \mathbf{l})}_{\text{Surface orientation}} d\omega_i$$

Reflectance equation

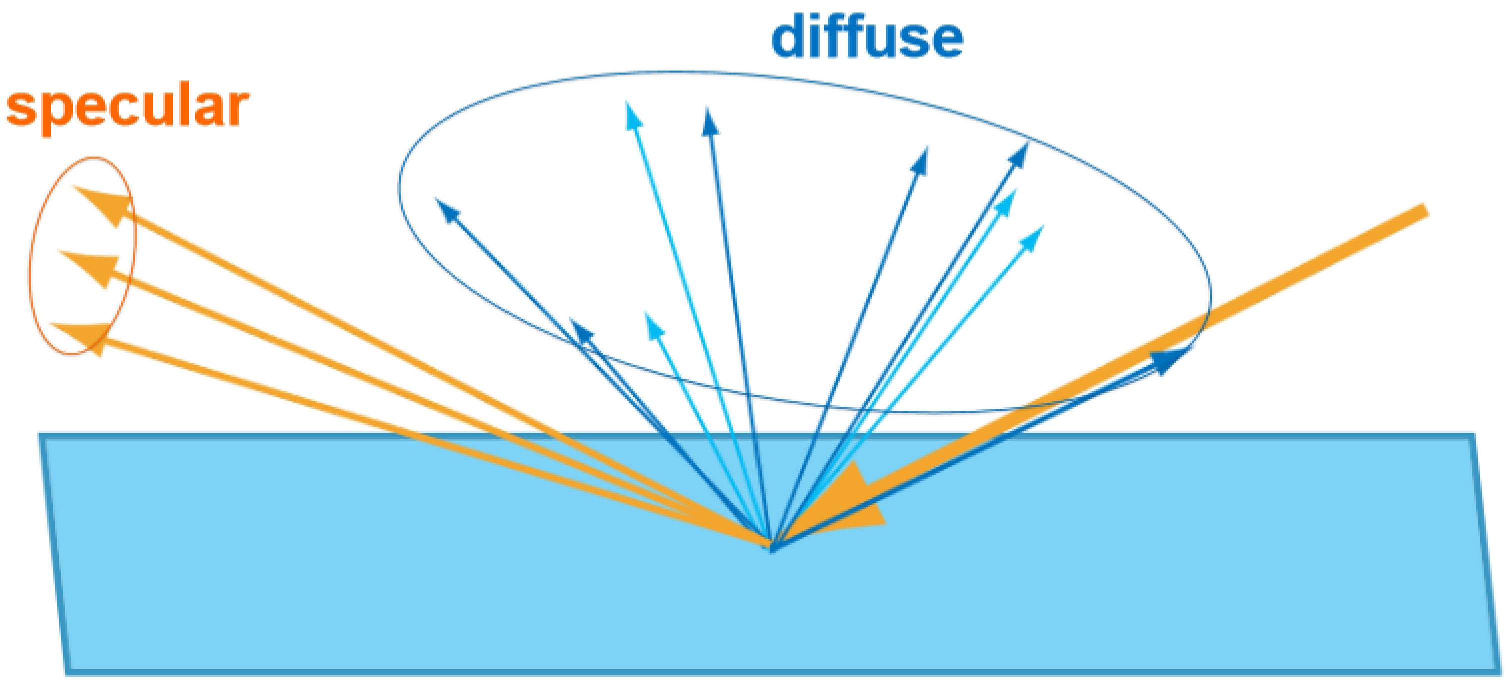
BRDF: intuition

- 2 possible interpretations
  - Given outgoing view → relative contributions of incoming light
  - Given incoming light direction → distribution of outgoing light



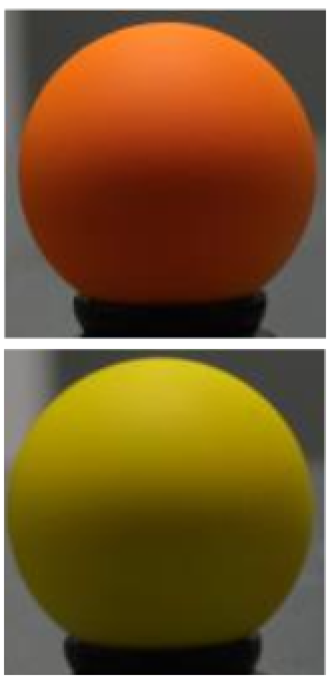
BRDF: intuition

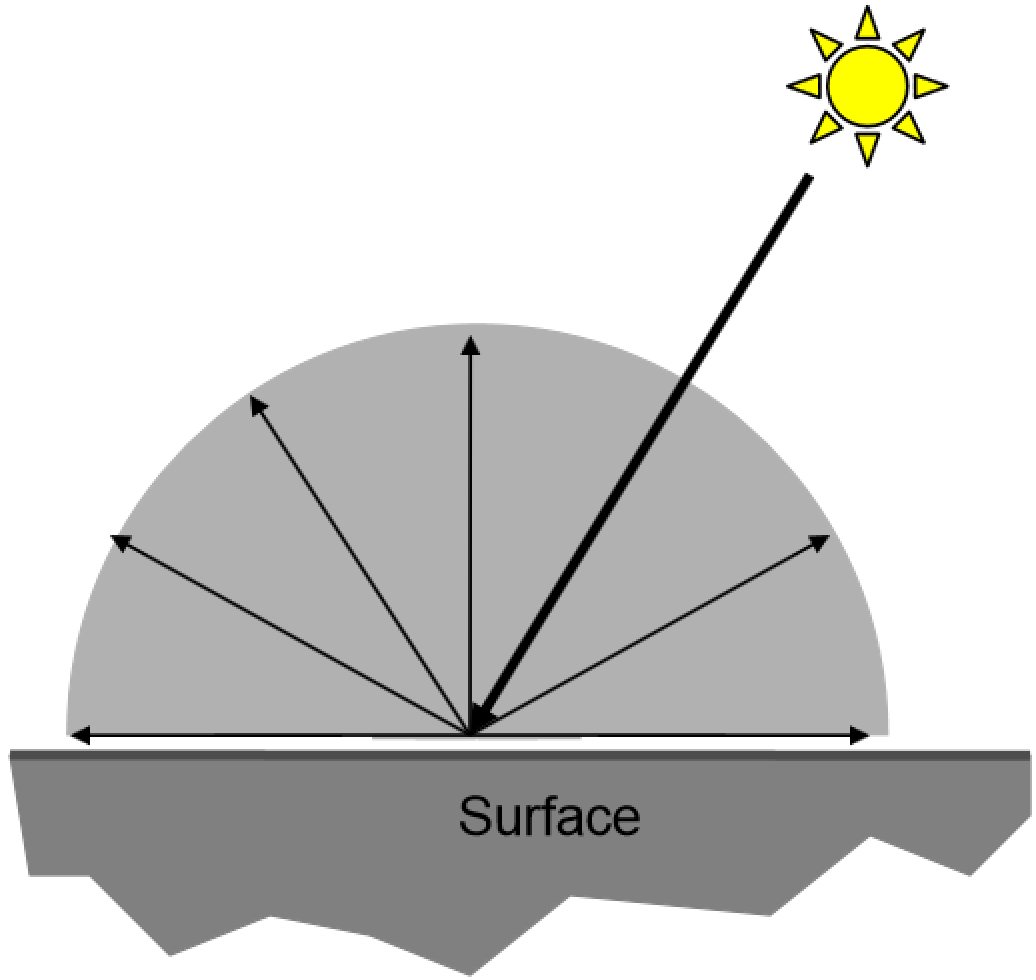
- Phenomena handled separatly:
  - Diffuse term
  - Specular term



BRDF: intuition

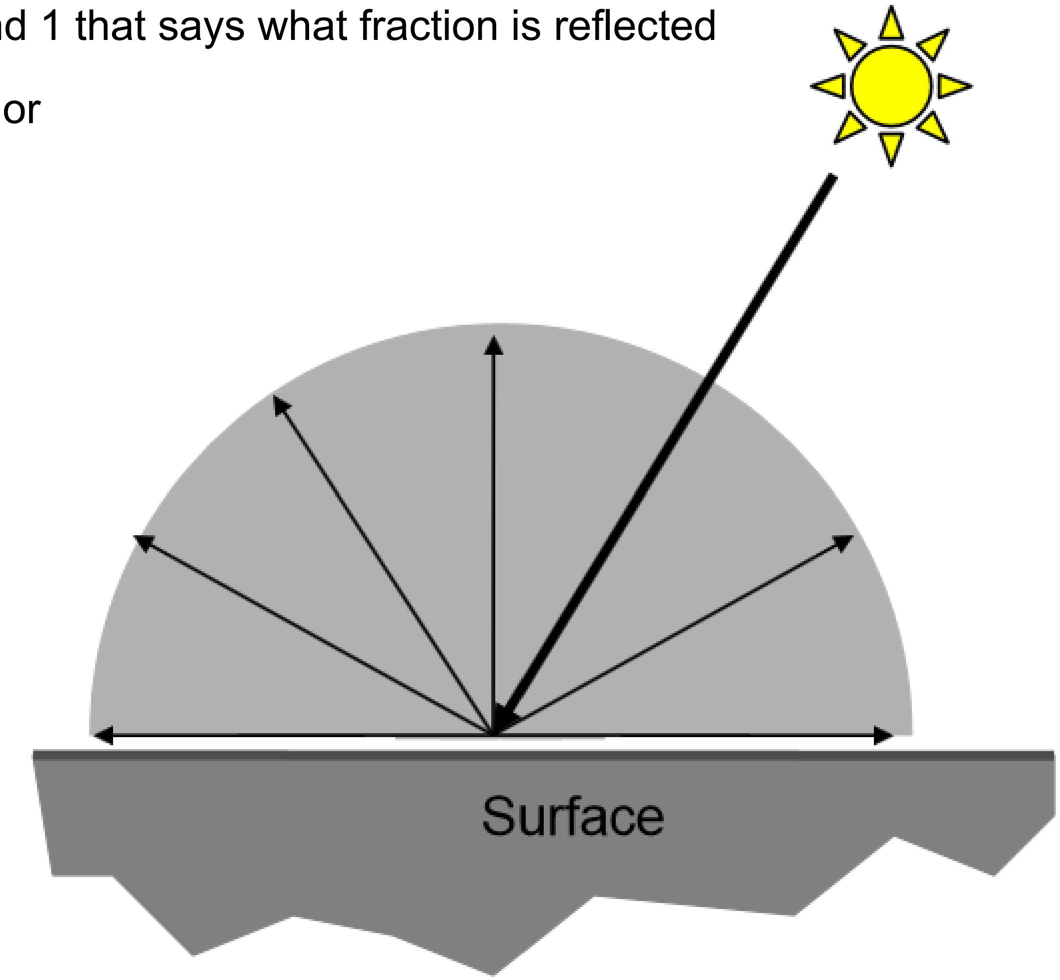
- Ideal diffuse reflectance (matte materials)
  - Assume surface reflects equally in all directions





BRDF: intuition

- Ideal diffuse reflectance (matte materials)
  - Assume surface reflects equally in all directions
  - Coefficient between 0 and 1 that says what fraction is reflected
  - Usually called diffuse color



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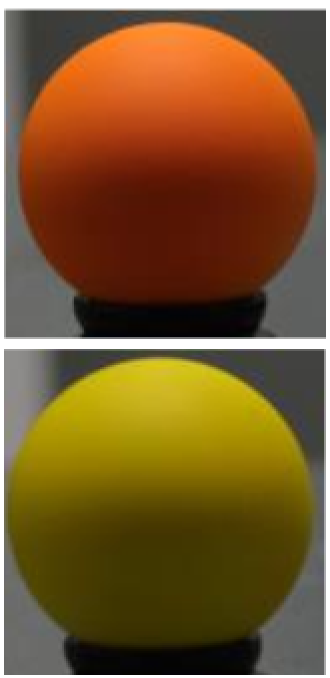
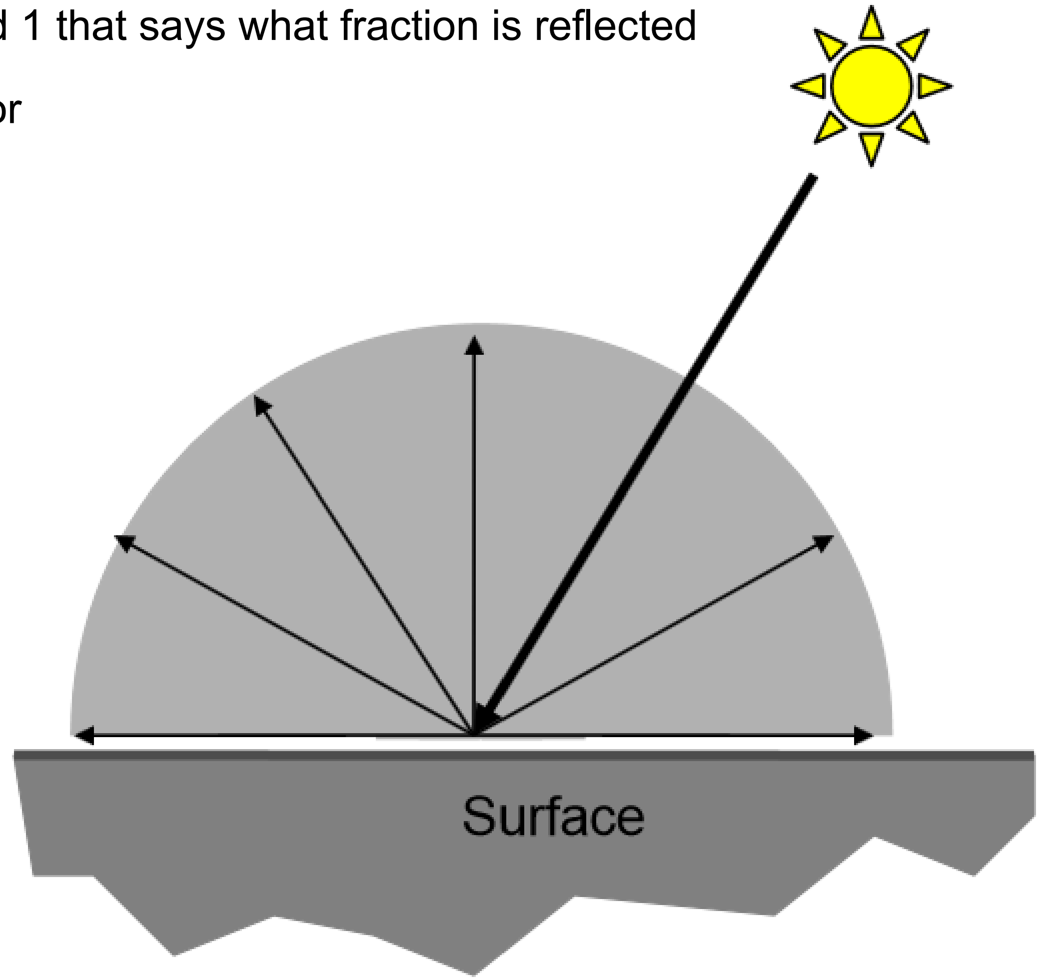
$$f(\mathbf{l}, \mathbf{v}) = const$$





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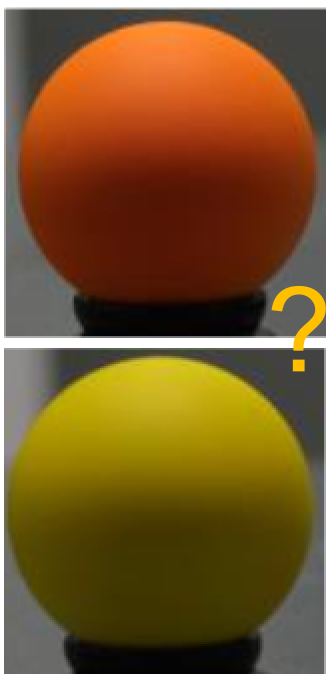

Why does color change?

BRDF: intuition

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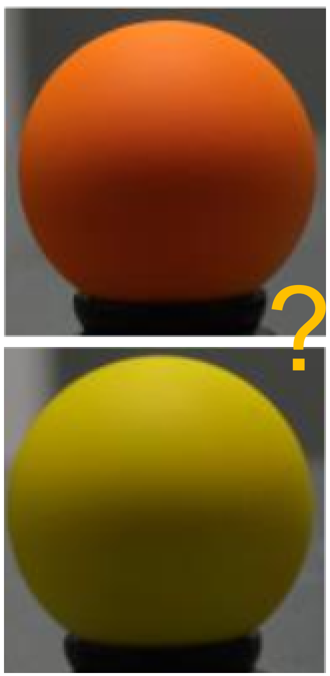
$$f(\mathbf{l}, \mathbf{v}) = const$$



Why does color change?

$$L_o(\mathbf{v}) = \int_{\Omega} f(\mathbf{l}, \mathbf{v}) \otimes L_i(\mathbf{l}) \boxed{(\mathbf{n} \cdot \mathbf{l})} d\omega_i$$

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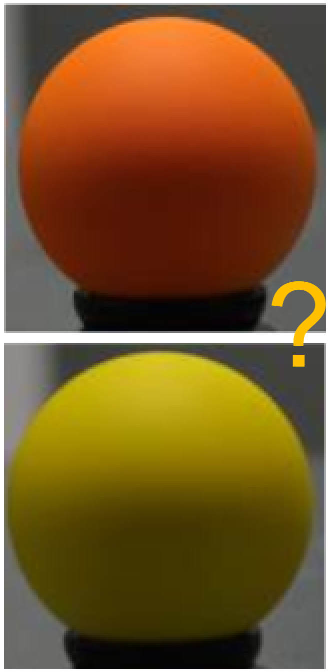

?

Why does color change?

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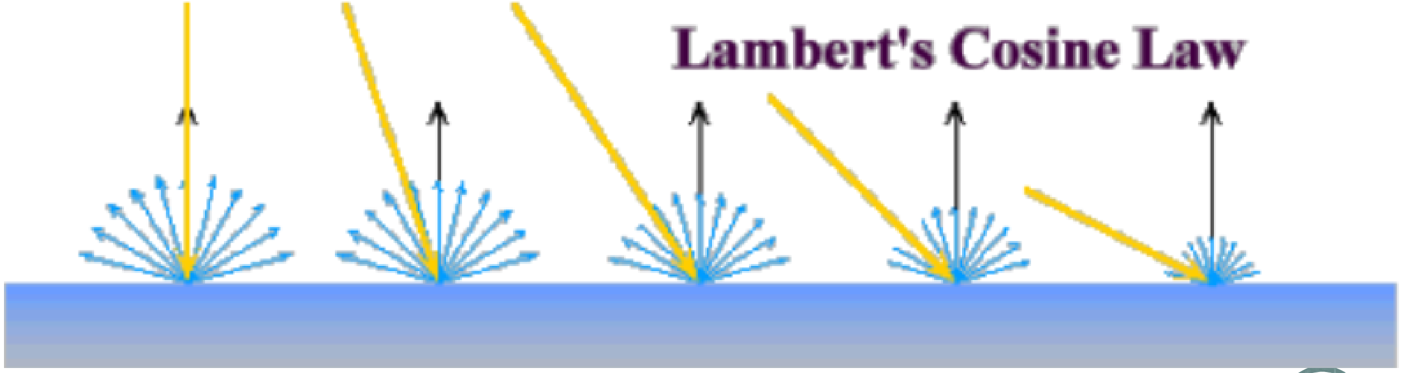
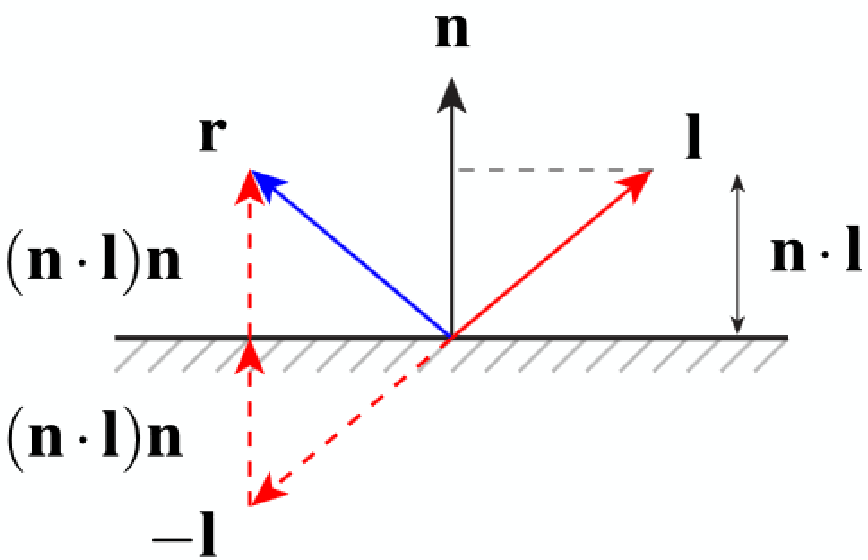


BRDF: intuition

- Ideal specular reflectance (mirror materials)
  - Delta dirac in the reflected direction
  - Not usefull for point lights… better for reflections of other surfaces

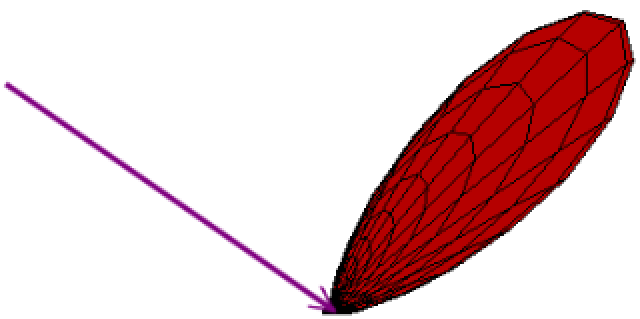
$$f(\mathbf{l}, \mathbf{v}) = dirac$$

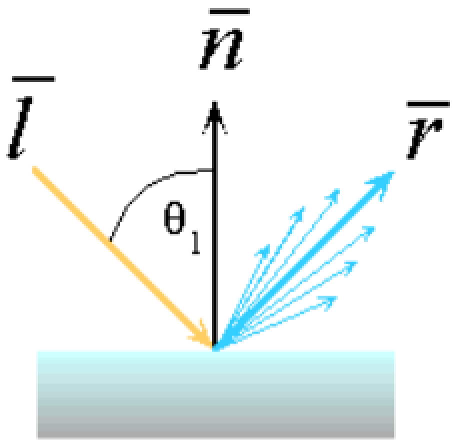


$$\mathbf{r} = 2(\mathbf{n} \cdot \mathbf{l})\mathbf{n} - \mathbf{l}$$

BRDF: intuition

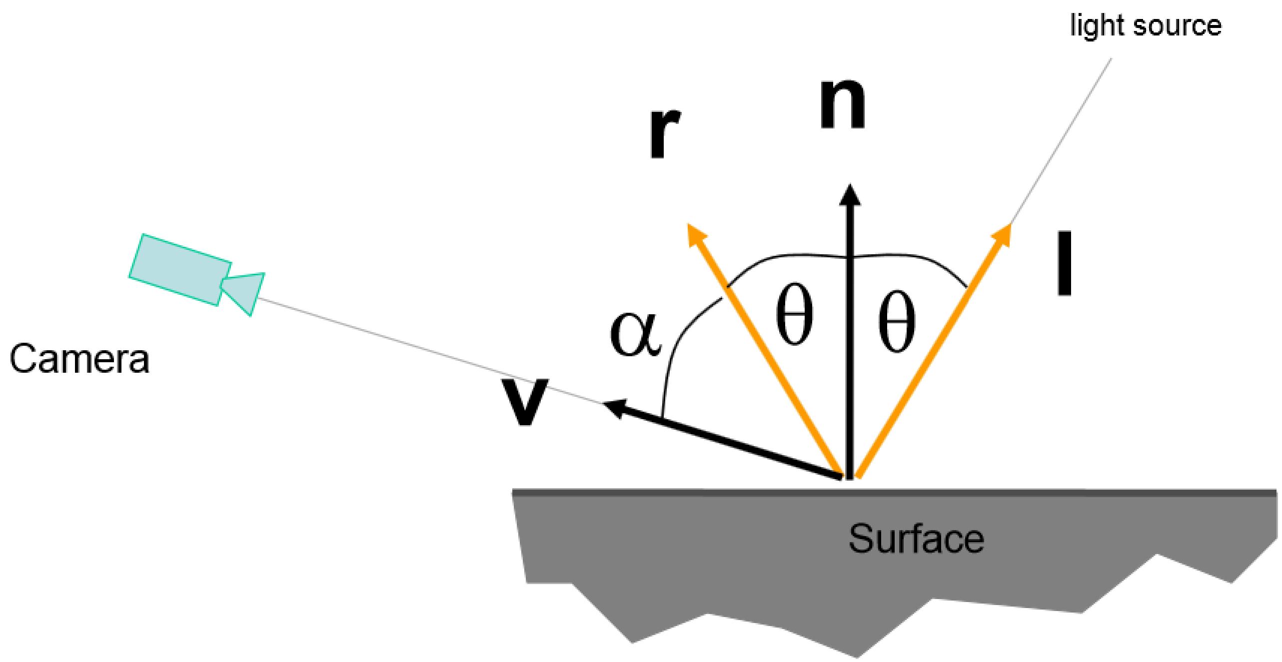
- Non ideal reflectors (glossy material)
  - Expect most of reflected light to travel in the direction of the ideal mirror ray
  - Some of the light should also be reflected slightly offsetted from the mirror ray
  - As we move farther and farther from the mirror ray, we expect to see less light reflected





BRDF: Phong model

- Reflection depends on the angle between the ideal reflection and the view vectors



$$L_o = \quad k_s \, (\cos \alpha)^q \, \frac{L_i}{r^2} \quad = k_s \, (\boldsymbol{v} \cdot \boldsymbol{r})^q \, \frac{L_i}{r^2}$$

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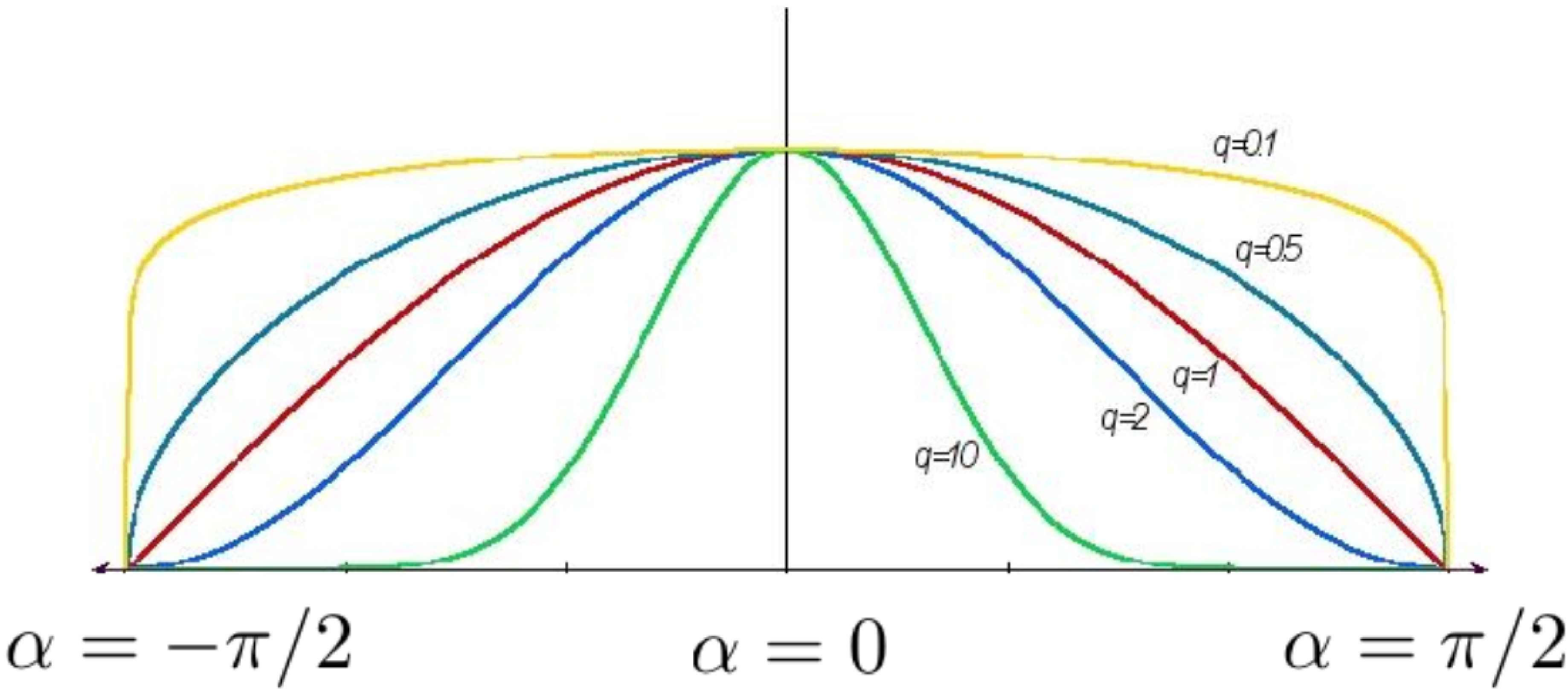

$$L_o = k_s (\cos\alpha)^q \frac{L_i}{r^2} = k_s (\boldsymbol{v} \cdot \boldsymbol{r})^q \frac{L_i}{r^2}$$

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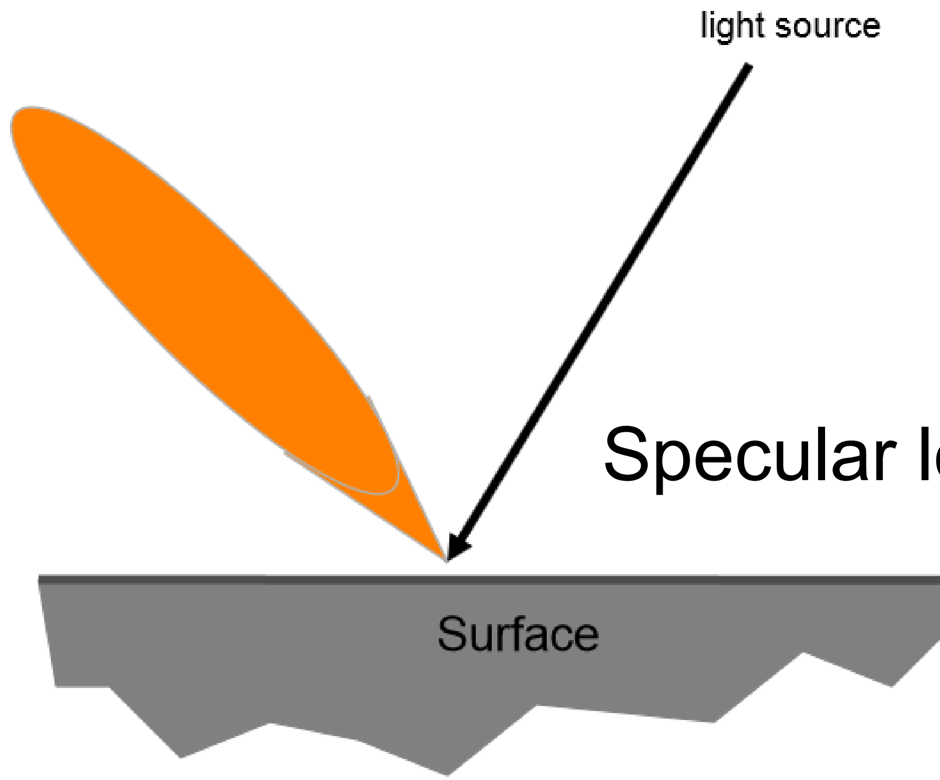
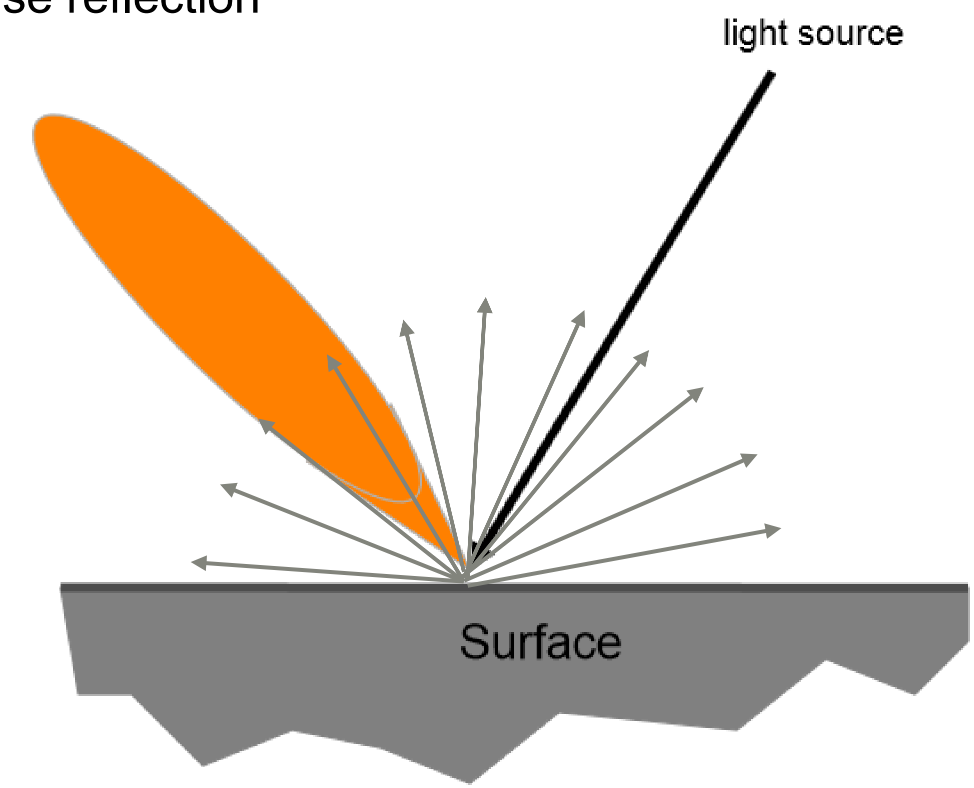


Specular lobe $(\boldsymbol{r} \cdot \boldsymbol{v})^q$

$$L_o = \; k_s \, (\cos\alpha)^q \, \frac{L_i}{r^2} \; = k_s \, (\boldsymbol{v} \cdot \boldsymbol{r})^q \, \frac{L_i}{r^2}$$

BRDF: Phong model

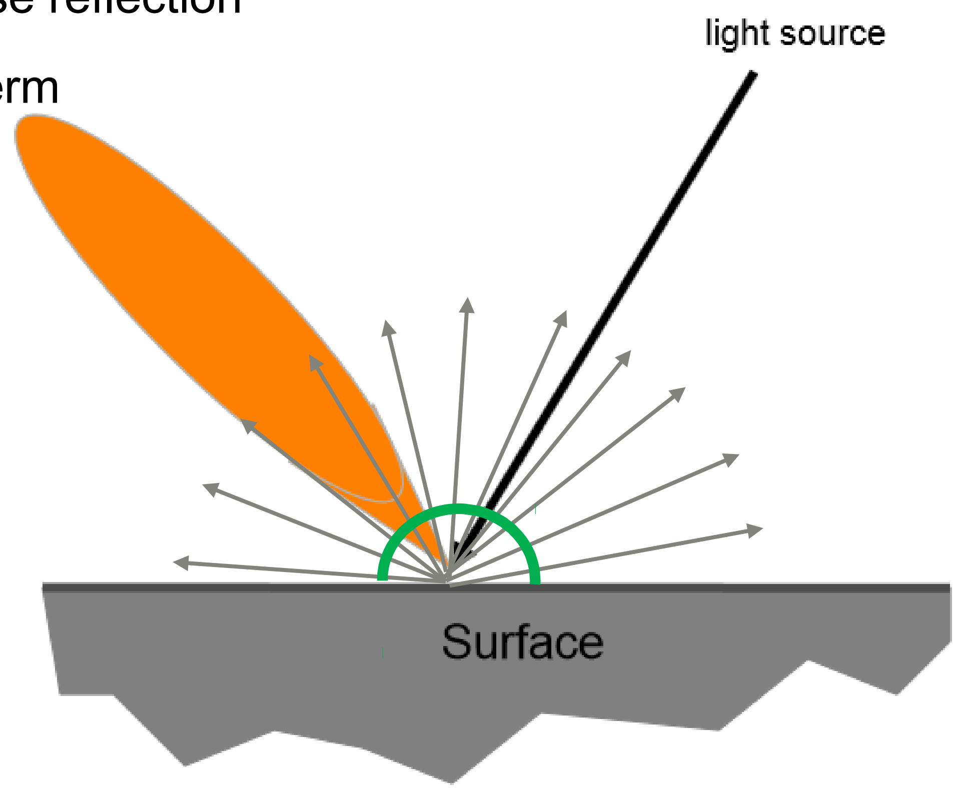
- Reflection depends on the angle between the ideal reflection and the view vectors
- + ideal diffuse reflection



$$L_o = \left[k_d \left(\boldsymbol{n} \cdot \boldsymbol{l} \right) + k_s \left(\boldsymbol{v} \cdot \boldsymbol{r} \right)^q \right] \frac{L_i}{r^2}$$

BRDF: Phong model

- Reflection depends on the angle between the ideal reflection and the view vectors
- + ideal diffuse reflection
- + ambient term



$$L_o = \left[k_a + k_d\,(\boldsymbol{n}\cdot\boldsymbol{l}) + k_s\,(\boldsymbol{v}\cdot\boldsymbol{r})^q\right]\frac{L_i}{r^2}$$

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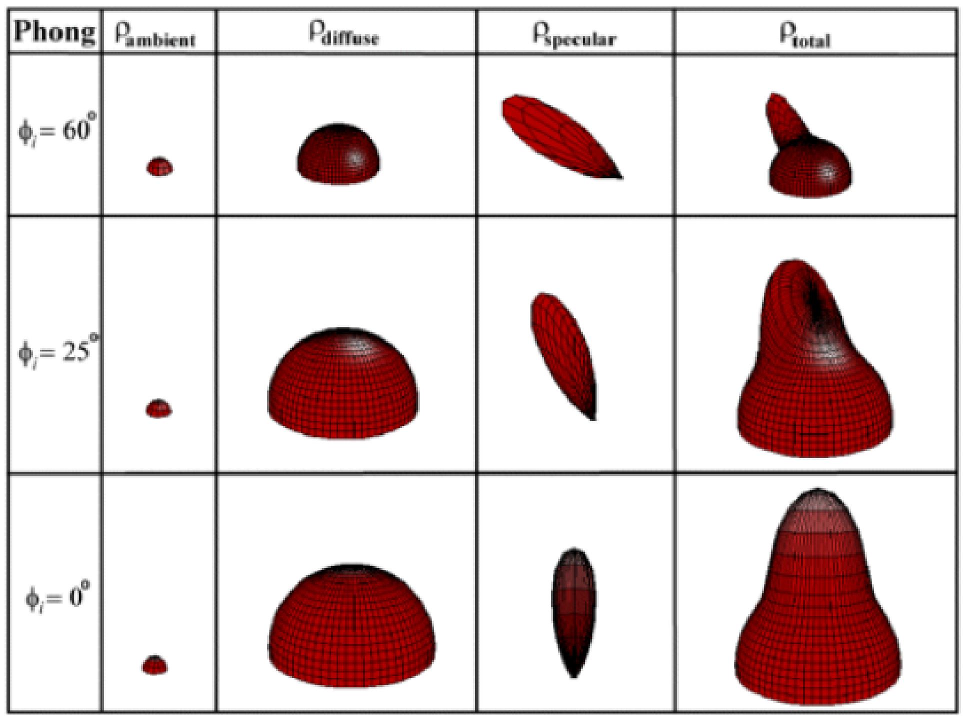

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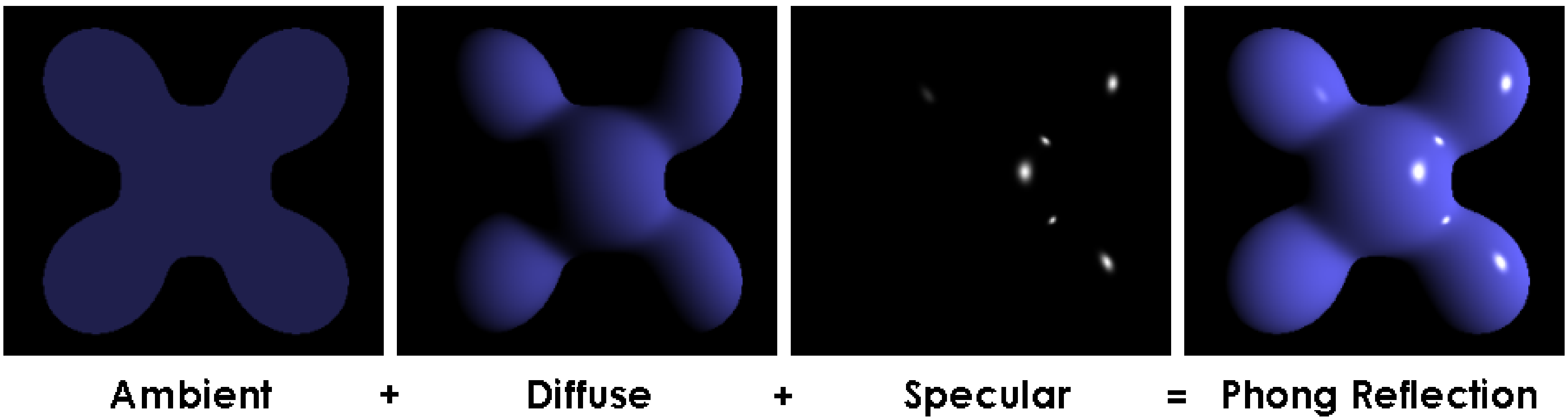

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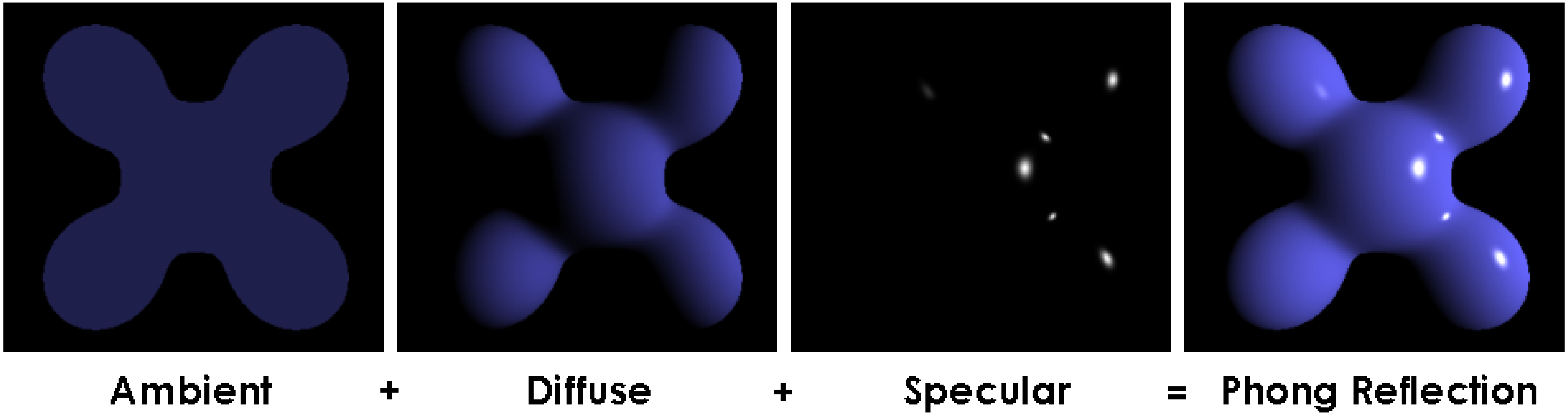
BRDF: Phong model

- Problems:
  - Does not conserve energy (may reflect more than it receives)
  - Not conform to BRDF model (cosine)
  - Ambient is a total hack



$$L_o = \left[k_a + k_d \left(\boldsymbol{n} \cdot \boldsymbol{l} \right) + k_s \left(\boldsymbol{v} \cdot \boldsymbol{r} \right)^q \right] \frac{L_i}{r^2}$$

Physically plausible BRDFs

$$L_o(\mathbf{v}) = \int_\Omega f(\mathbf{l}, \mathbf{v}) \otimes L_i(\mathbf{l})(\mathbf{n} \cdot \mathbf{l}) d\omega_i$$

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Physically plausible BRDFs

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- Positivity $$f(\mathbf{l}, \mathbf{v}) >= 0$$
- Reciprocity $$f(\mathbf{l}, \mathbf{v}) = f(\mathbf{v}, \mathbf{l})$$

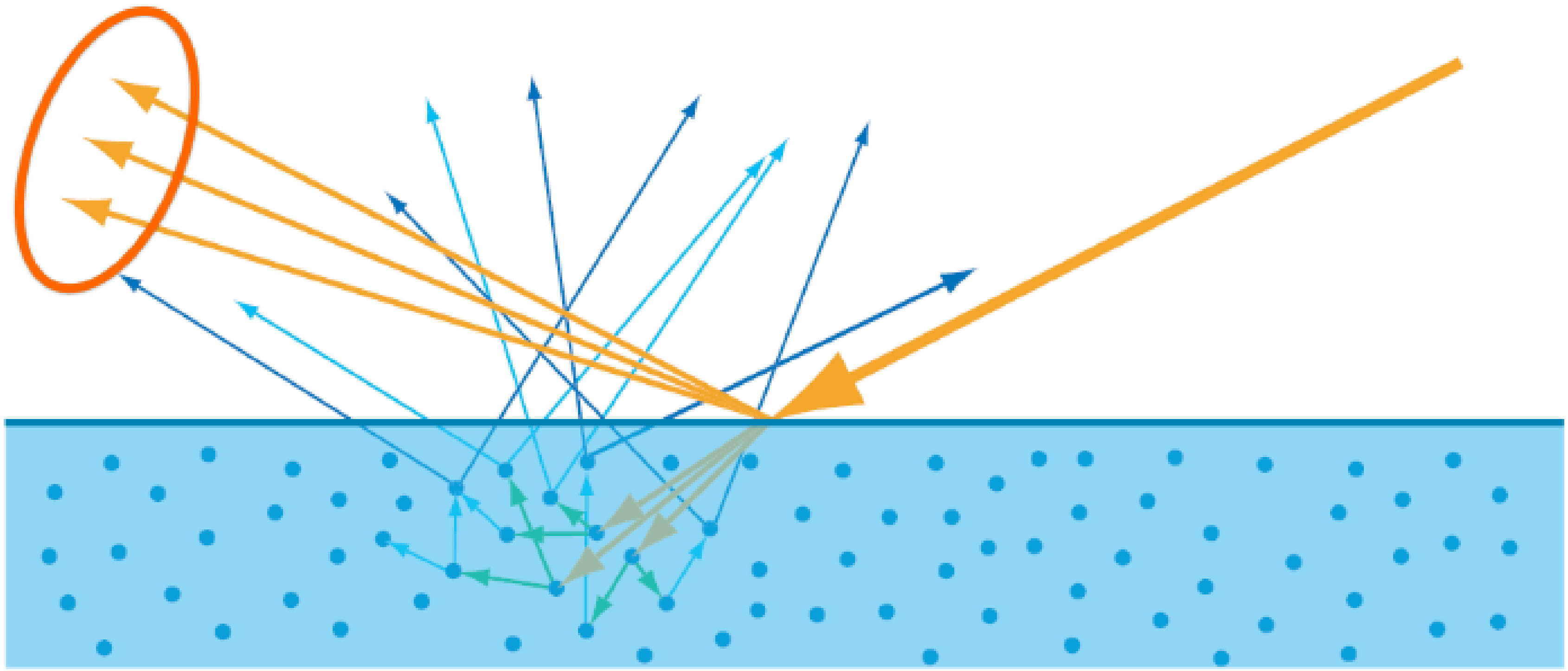
Physically plausible BRDFs

$$L_o(\mathbf{v}) = \int_\Omega f(\mathbf{l}, \mathbf{v}) \otimes L_i(\mathbf{l})(\mathbf{n} \cdot \mathbf{l}) d\omega_i$$

- Positivity $$f(\mathbf{l}, \mathbf{v}) >= 0$$
- Reciprocity $$f(\mathbf{l}, \mathbf{v}) = f(\mathbf{v}, \mathbf{l})$$
- Energy conservation $$\forall \mathbf{l}, \int_\Omega f(\mathbf{l}, \mathbf{v})(\mathbf{n} \cdot \mathbf{v})\, d\omega_o \leq 1$$

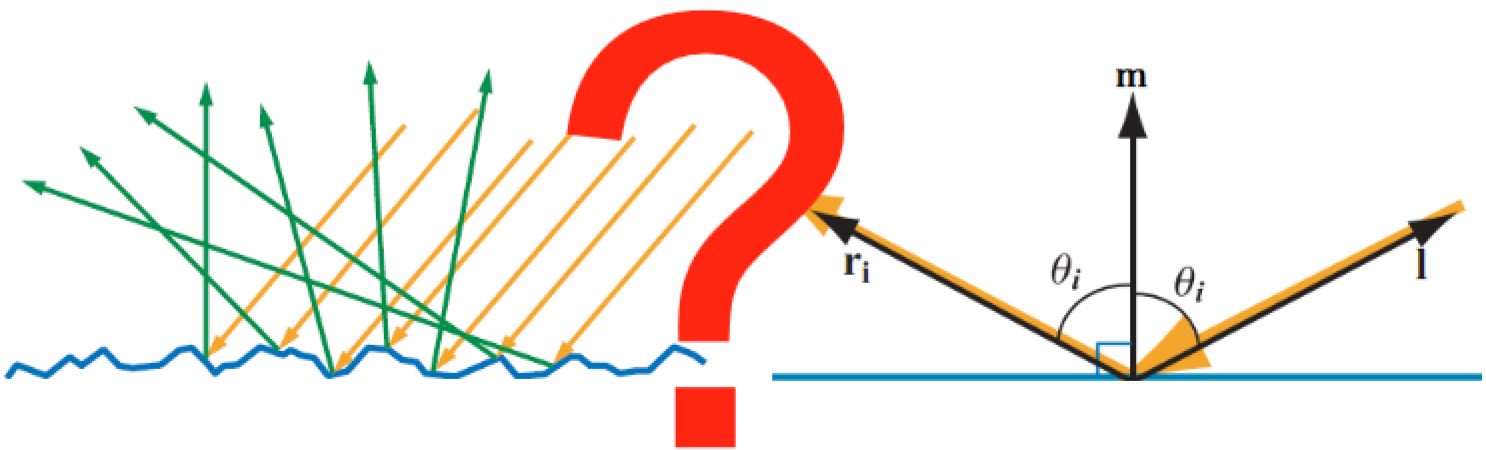
Microfacet theory

- Surface reflection (specular term)



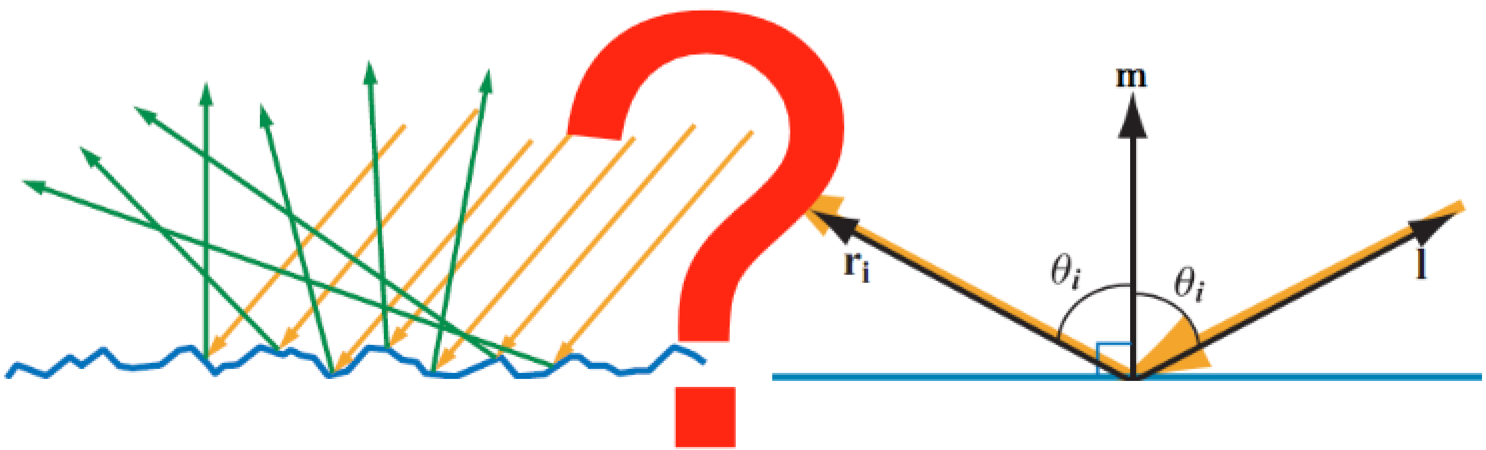
Microfacet theory

- Derive BRDF from non optically flat surfaces
  - Details too small to be visible
  - But large compared to light wavelength



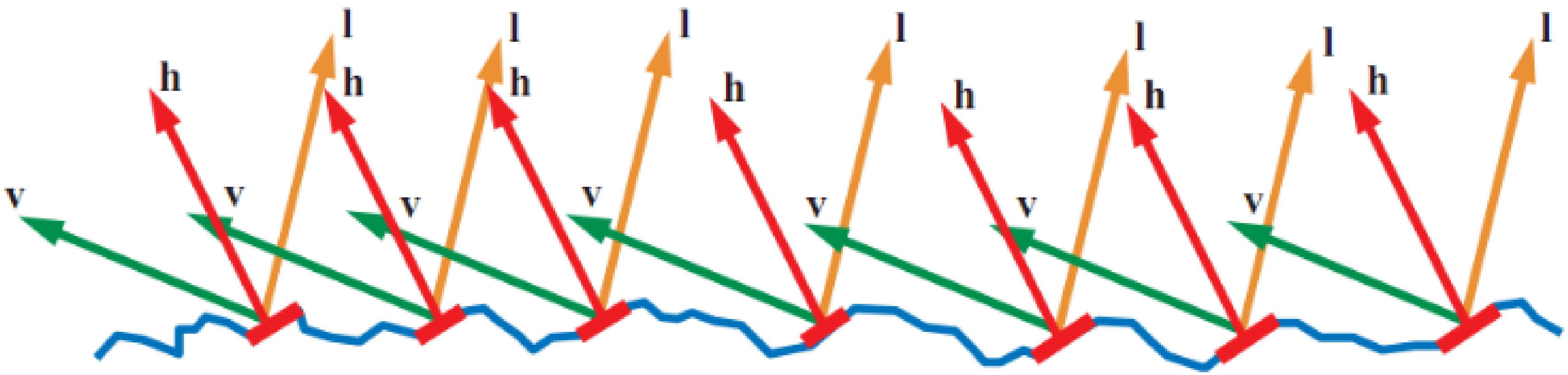
Microfacet theory

- Derive BRDF from non optically flat surfaces
  - Details too small to be visible
  - But large compared to light wavelength
- Each facet considered as a perfect mirror
  - Reflection depends on light direction and microfacet normal



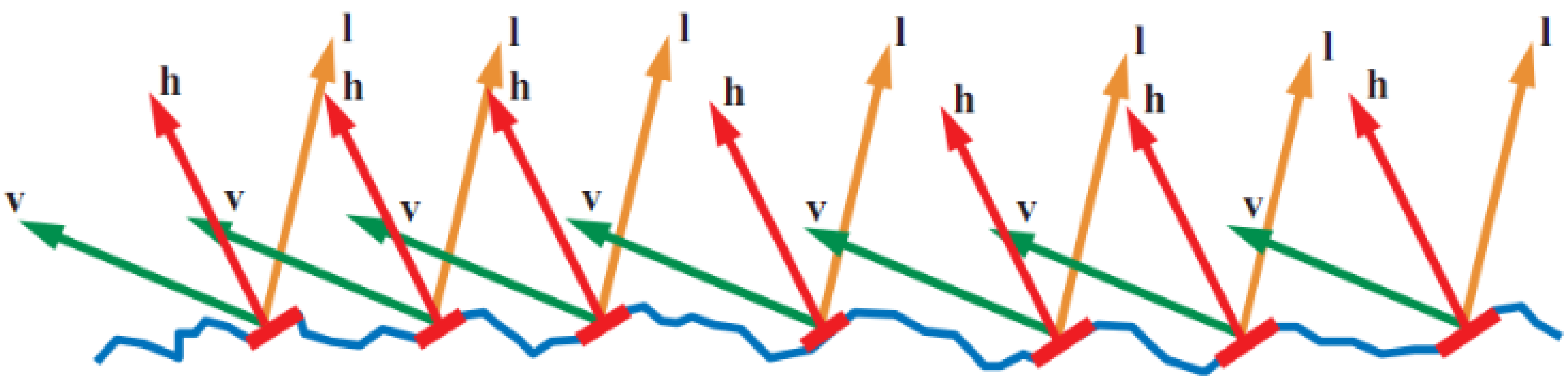
Microfacet theory

- Half vector
  - ▭ microfact normal



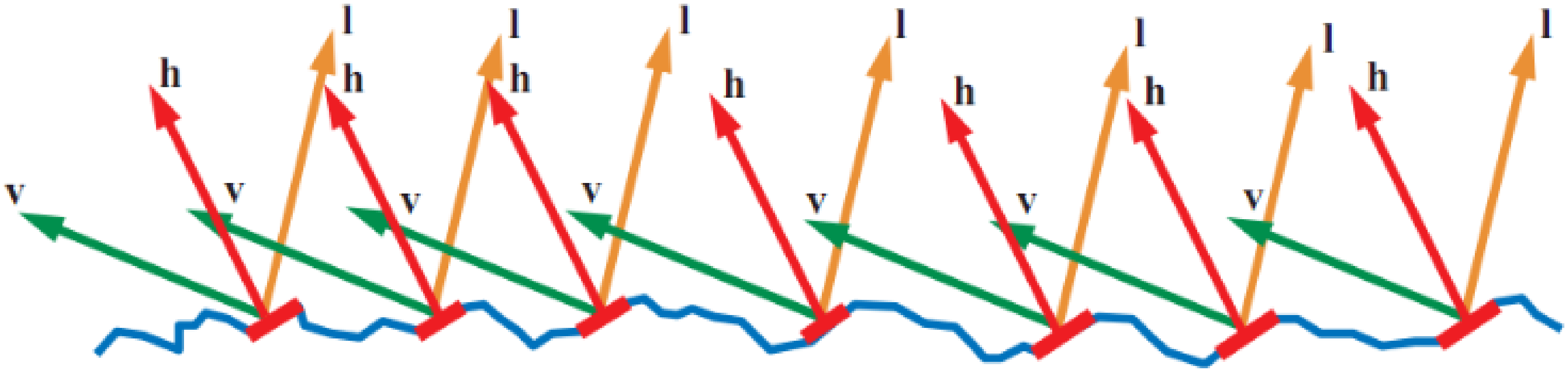
Microfacet theory

- Half vector
  - microfact normal
  - Only microfacets having their normals halfway between the view and light direction will reflect something!



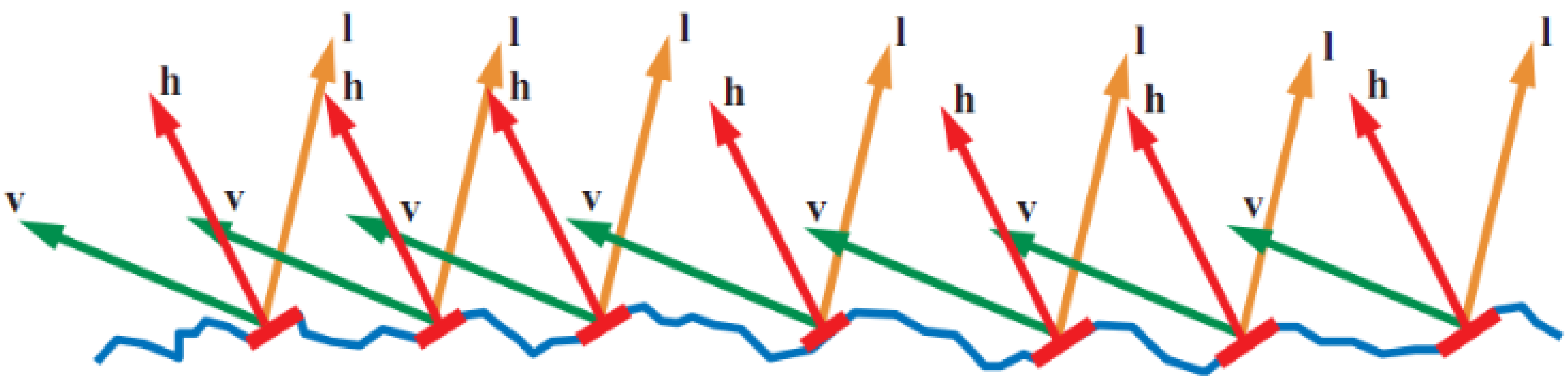
Microfacet theory

- Half vector
  - microfact normal
  - Only microfacets having their normals halfway between the view and light direction will reflect something!
  - Parametrized by h: give me the percent number of facets having this orientation



Microfacet theory

- Shadowing and masking
  - Not all microfacets oriented by a given h will contribute…
  - Some will be blocked by other microfacets from either



Microfacet theory

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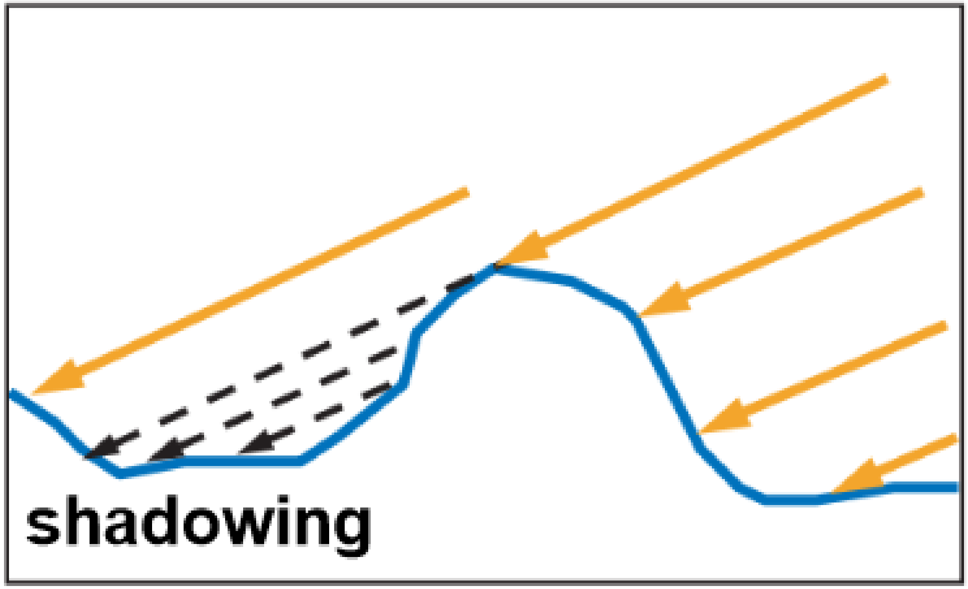
    - The light direction (shadowing)



Microfacet theory

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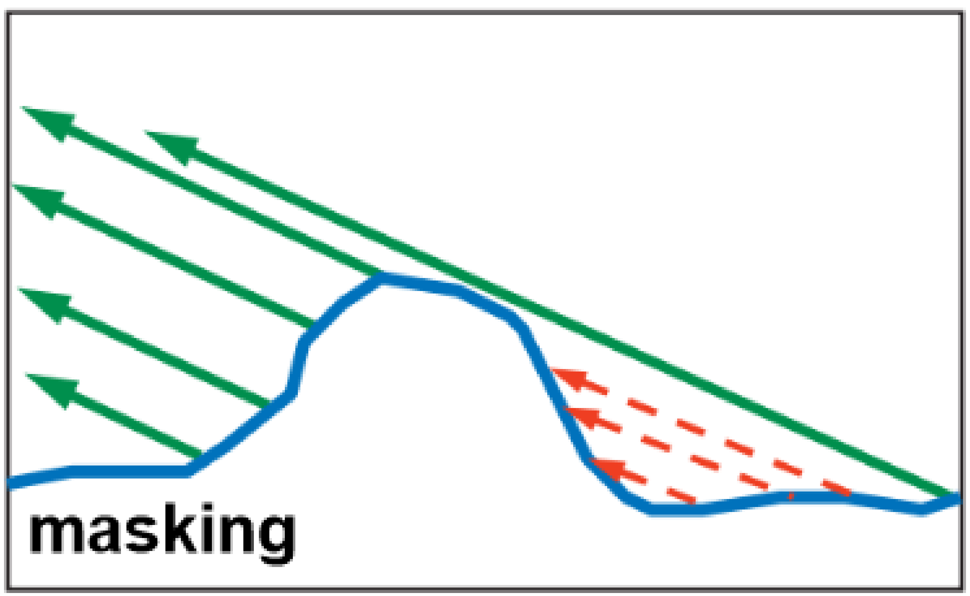
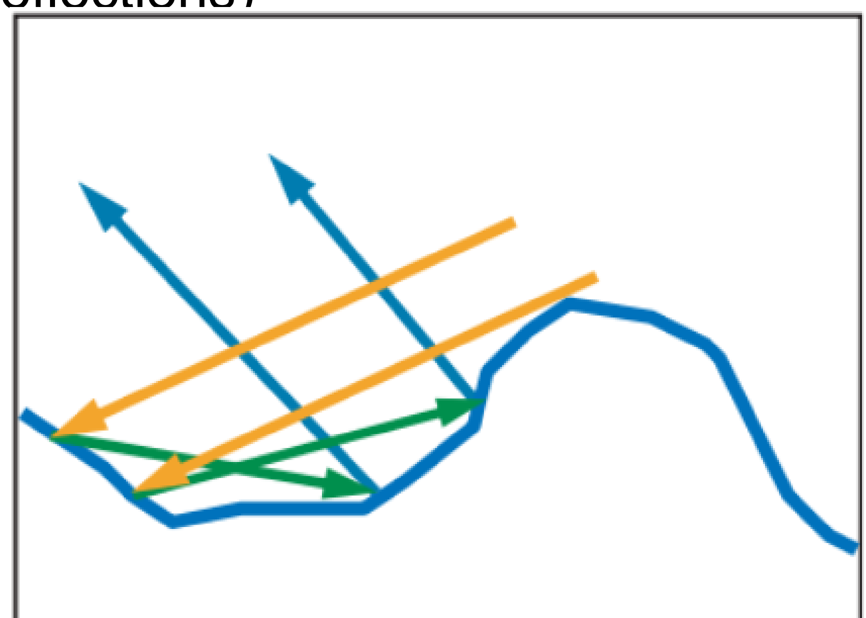


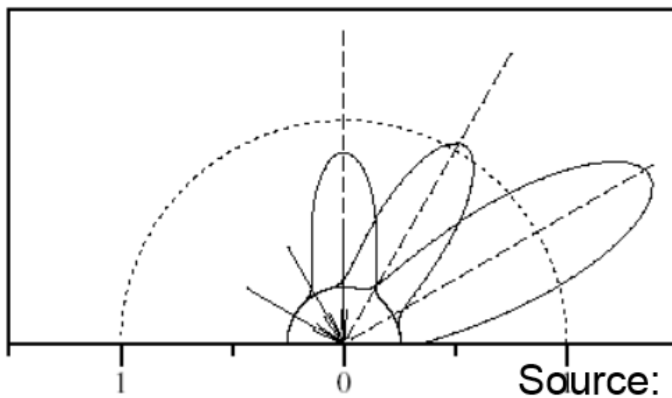
Microfacet theory

- Shadowing and masking
  - Not all microfacets oriented by a given h will contribute…
  - Some will be blocked by other microfacets from either
    - The light direction (shadowing)
    - The view direction (masking)
  - Not completely true (interreflections)
    - Microfacet limitation…



Microfacet theory

- Fresnel effect
  - Increase specularity near grazing angles



Source: Lafortune et al. 97

Microfacet theory

- Summary
  - Fresnel effect

$$f(\mathbf{l}, \mathbf{v}) = \quad F(\mathbf{l}, \mathbf{h})$$

Microfacet theory

- Summary
  - Fresnel effect
  - Masking shadowing

$$f(\mathbf{l}, \mathbf{v}) = \quad F(\mathbf{l}, \mathbf{h}) G(\mathbf{l}, \mathbf{v}, \mathbf{h})$$

Microfacet theory

- Summary
  - Fresnel effect
  - Masking shadowing
  - Amount of microfacets at a particular orientation

$$f(\mathbf{l}, \mathbf{v}) = \frac{F(\mathbf{l}, \mathbf{h})G(\mathbf{l}, \mathbf{v}, \mathbf{h})D(\mathbf{h})}{}$$

Microfacet theory

- Summary
  - Fresnel effect
  - Masking shadowing
  - Amount of microfacets at a particular orientation

$$f(\mathbf{l}, \mathbf{v}) = \frac{F(\mathbf{l}, \mathbf{h})G(\mathbf{l}, \mathbf{v}, \mathbf{h})D(\mathbf{h})}{4(\mathbf{n} \cdot \mathbf{l})(\mathbf{n} \cdot \mathbf{v})}$$

correction factor for quantities being transformed between the microgeometry local space and the overall macrosurface

forshortening

Microfacet theory

- Summary (cook-terrance model)
  - Fresnel effect
  - Masking shadowing
  - Amount of microfacets at a particular orientation

$$f(\mathbf{l}, \mathbf{v}) = \frac{F(\mathbf{l}, \mathbf{h}) G(\mathbf{l}, \mathbf{v}, \mathbf{h}) D(\mathbf{h})}{4(\mathbf{n} \cdot \mathbf{l})(\mathbf{n} \cdot \mathbf{v})}$$

correction factor for quantities being transformed between the microgeometry local space and the overall macrosurface

Fresnel reflectance

- Fraction of incoming light that is reflected
- In this case:
  - How much of the light hitting the relevant microfacets is reflected

$$f(\mathbf{l}, \mathbf{v}) = \frac{\boxed{F(\mathbf{l}, \mathbf{h})} G(\mathbf{l}, \mathbf{v}, \mathbf{h}) D(\mathbf{h})}{4(\mathbf{n} \cdot \mathbf{l})(\mathbf{n} \cdot \mathbf{v})}$$

Fresnel reflectance

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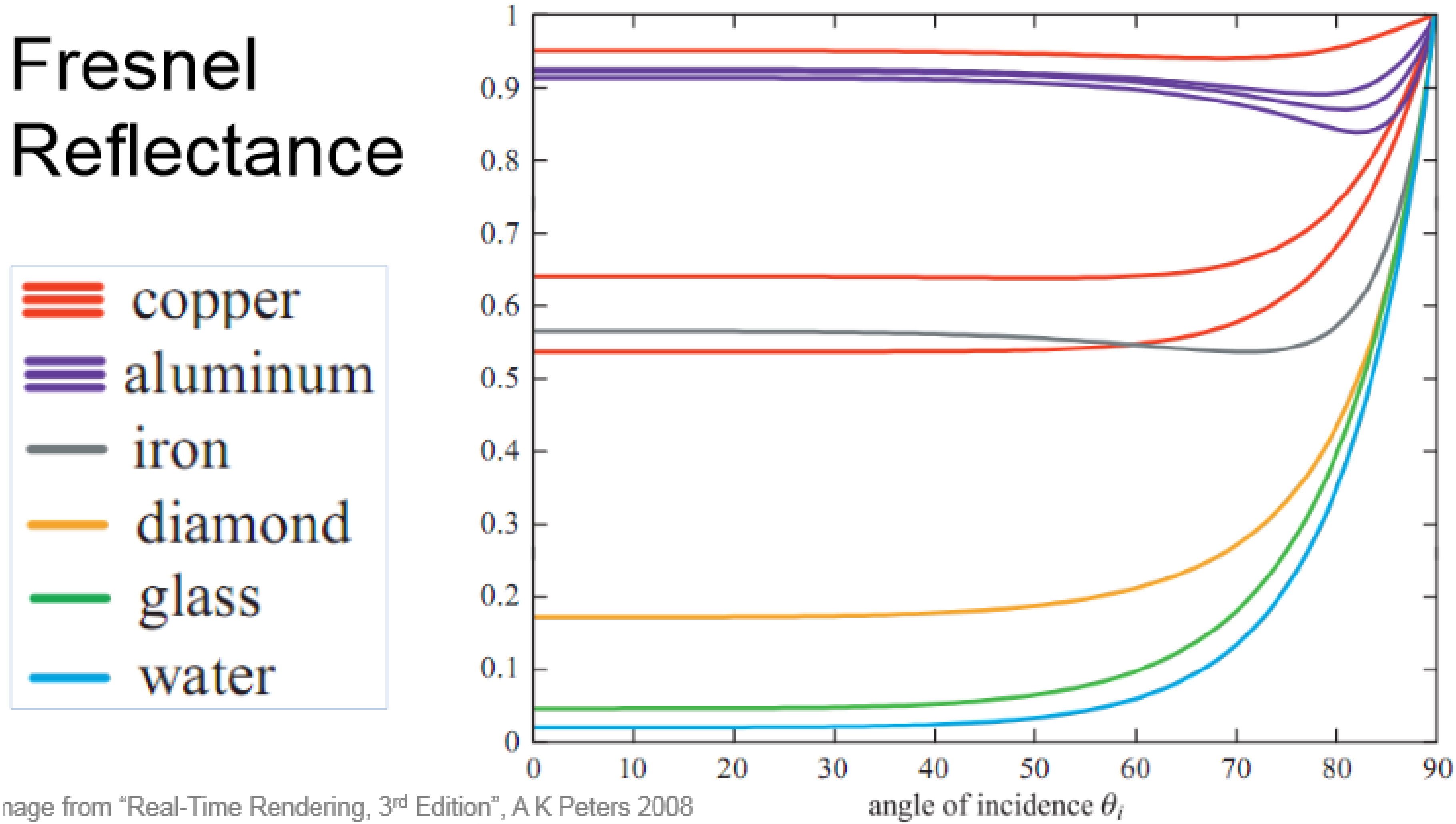

nage from "Real-Time Rendering, 3rd Edition", A K Peters 2008

Fresnel reflectance

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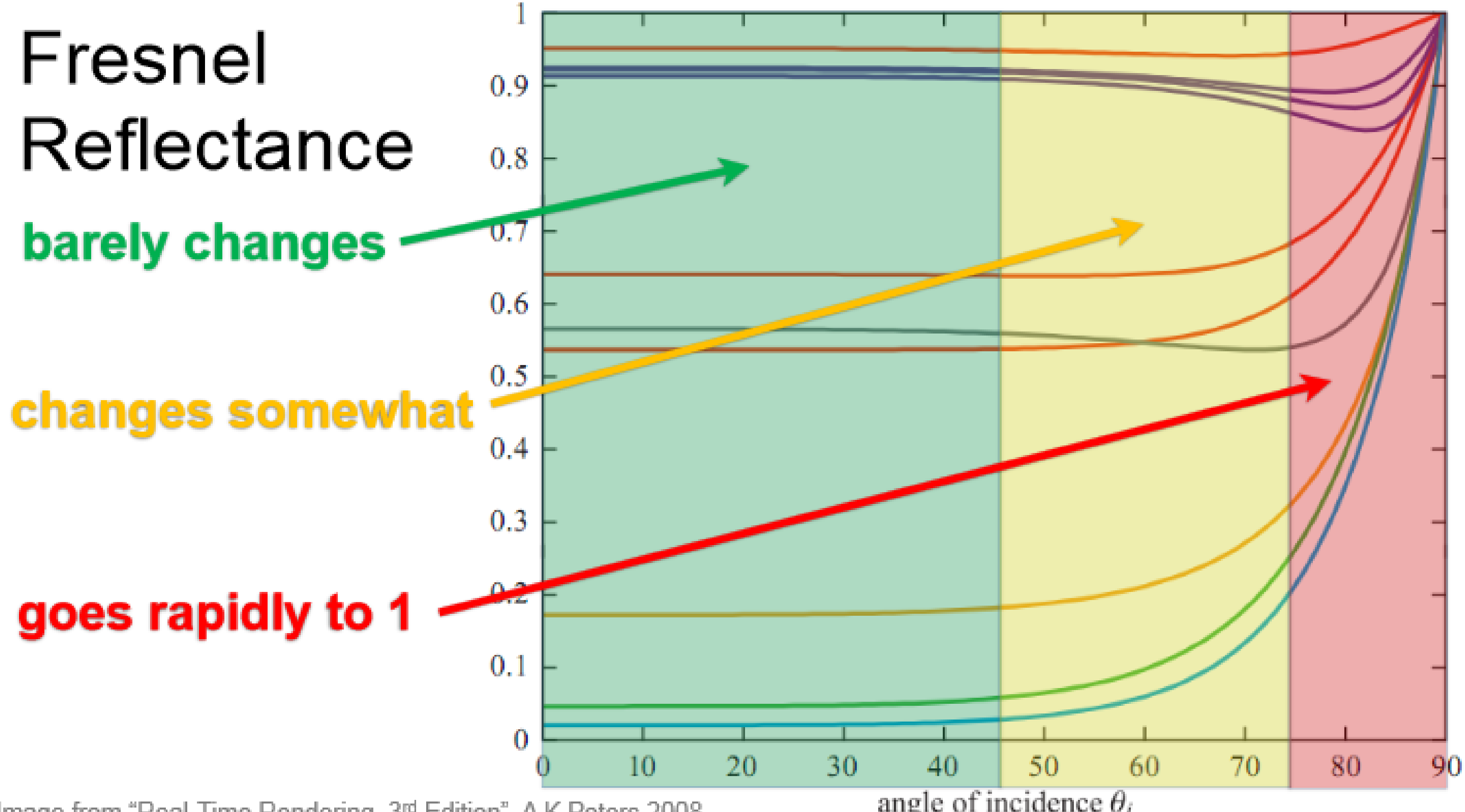

Image from "Real-Time Rendering, 3rd Edition", A K Peters 2008

Fresnel reflectance

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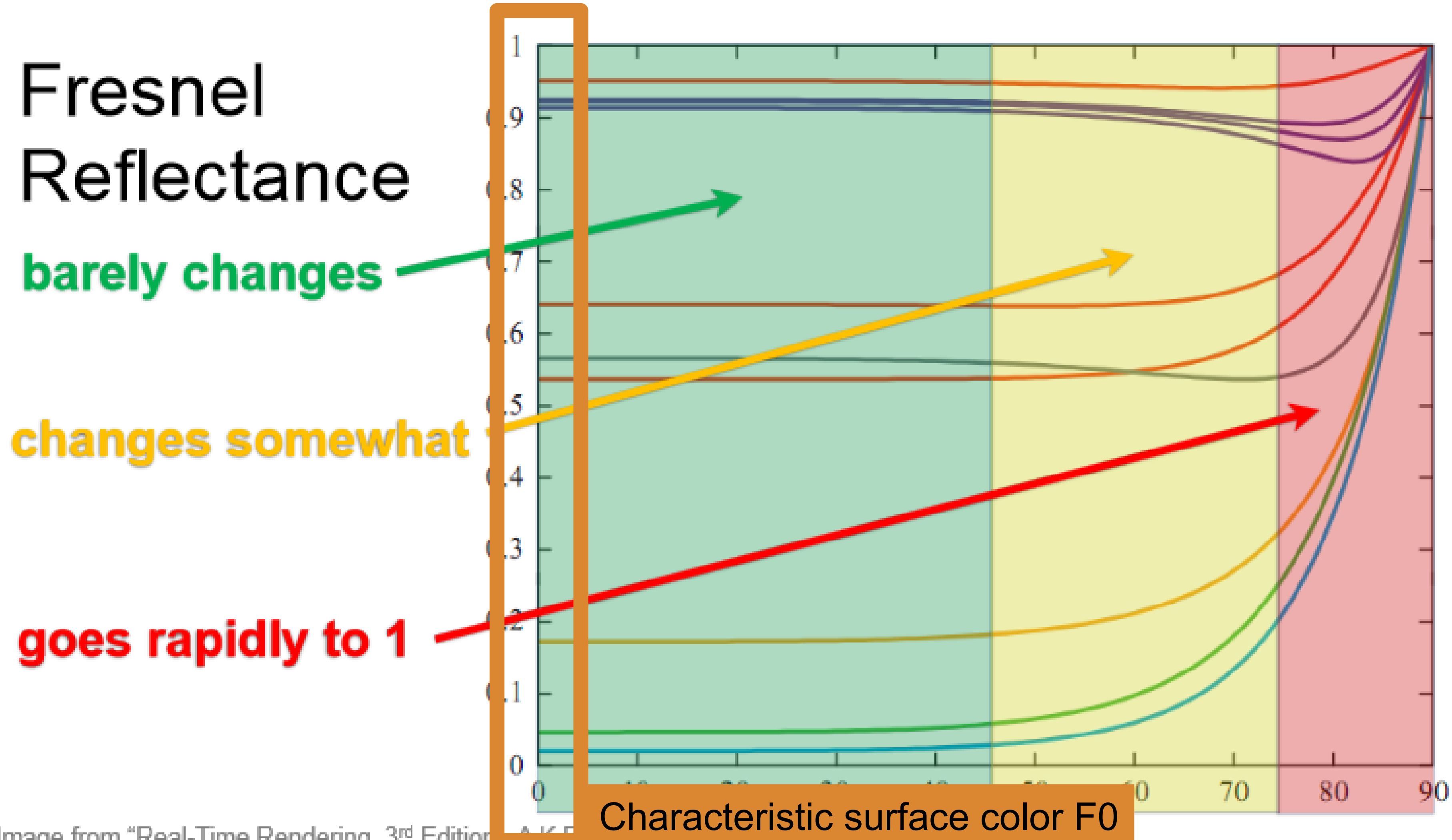
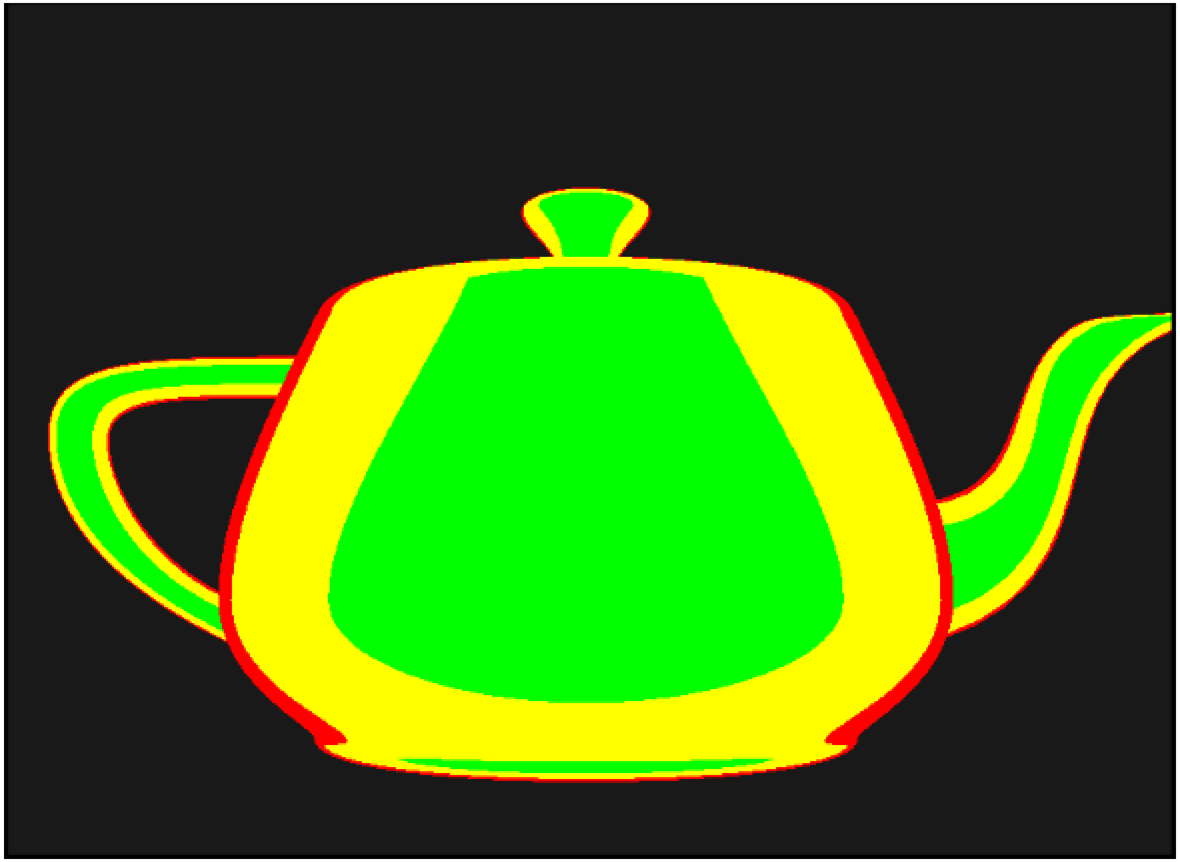


Image from “Real-Time Rendering, 3rd Edition

Fresnel reflectance

- Fraction of incoming light that is reflected
  - Mainly affect edges



Fresnel reflectance

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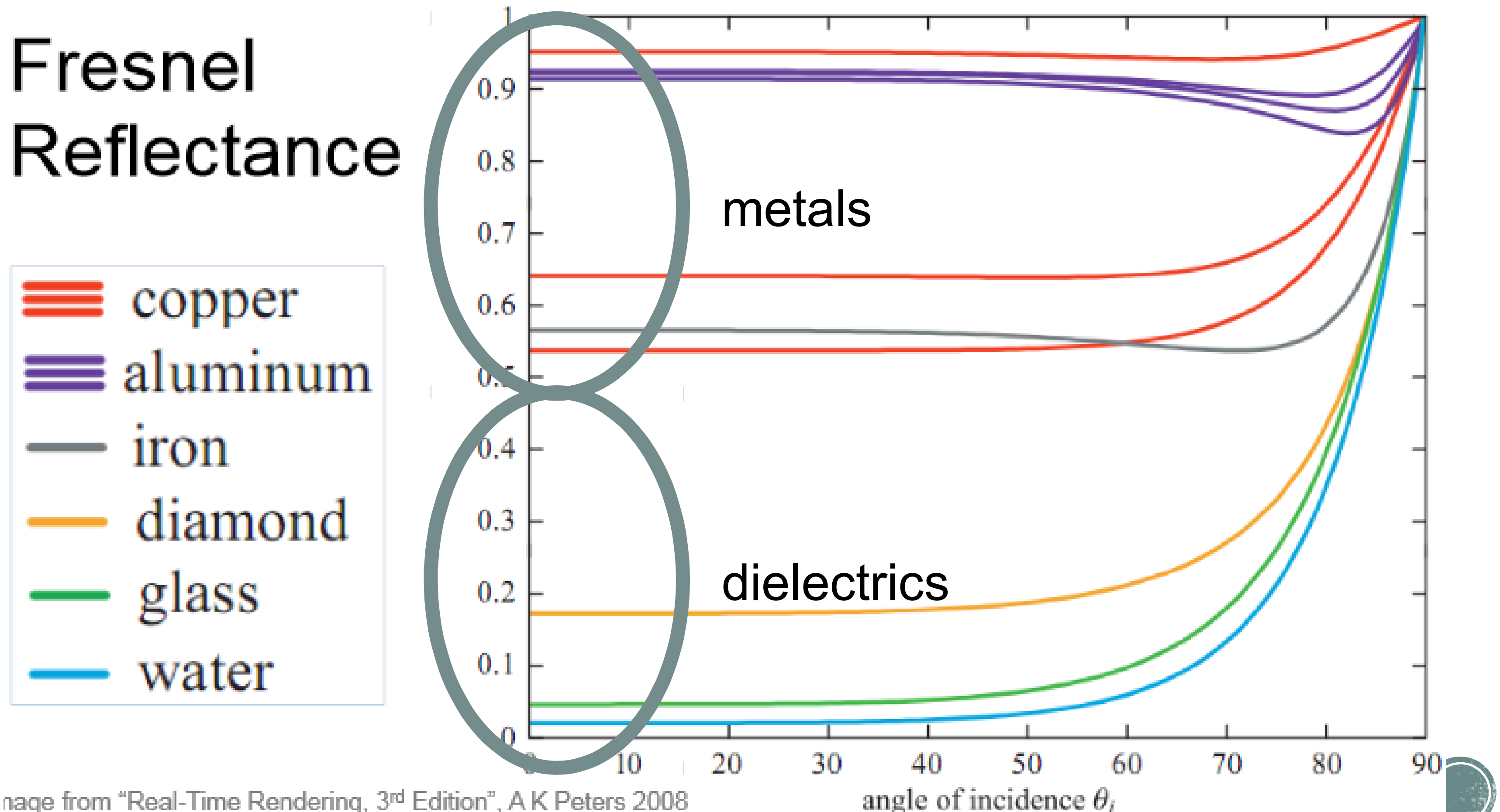


nage from "Real-Time Rendering, 3rd Edition", A K Peters 2008

Fresnel reflectance

- Fraction of incoming light that is reflected
  - Mainly affect edges
- Schlick approximation
  - Accurate, cheap and parametrized by F0

$$F_{\text{Schlick}}(F_0, \mathbf{l}, \mathbf{n}) = F_0 + (1 - F_0)(1 - (\mathbf{l} \cdot \mathbf{n}))^5$$

  - For microfacet models

$$F_{\text{Schlick}}(F_0, \mathbf{l}, \mathbf{h}) = F_0 + (1 - F_0)(1 - (\mathbf{l} \cdot \mathbf{h}))^5$$

Normal Distribution Functio

- Statistical distribution of orientation h
  - Determine size, brightness and shape of specular highlight

$$f(\mathbf{l}, \mathbf{v}) = \frac{F(\mathbf{l}, \mathbf{h})G(\mathbf{l}, \mathbf{v}, \mathbf{h})\boxed{D(\mathbf{h})}}{4(\mathbf{n} \cdot \mathbf{l})(\mathbf{n} \cdot \mathbf{v})}$$

Normal Distribution Functio

- Statistical distribution of orientation h
  - Determine size, brightness and shape of specular highlight

$$D_p(\mathbf{m}) = \frac{\alpha_p + 2}{2\pi}(\mathbf{n}\cdot\mathbf{m})^{\alpha_p}$$

$$D_{uabc}(\mathbf{m}) = \frac{1}{\left(1 + \alpha_{abc1}\left(1 - (\mathbf{n}\cdot\mathbf{m})\right)\right)^{\alpha_{abc2}}}$$

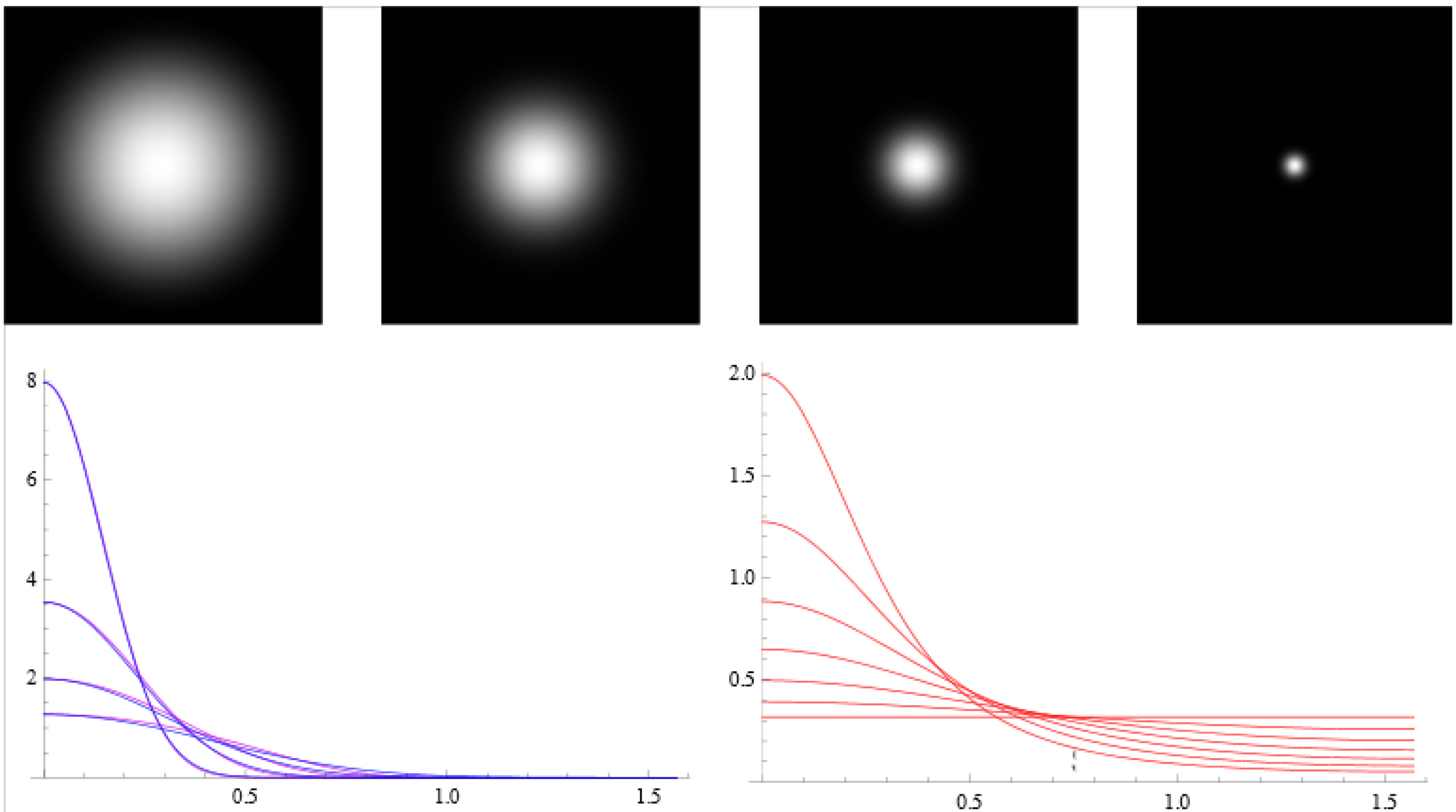
$$D_{tr}(\mathbf{m}) = \frac{\alpha_{tr}^2}{\pi\left((\mathbf{n}\cdot\mathbf{m})^2\left(\alpha_{tr}^2 - 1\right) + 1\right)^2}$$

$$D_b(\mathbf{m}) = \frac{1}{\pi\alpha_b^2(\mathbf{n}\cdot\mathbf{m})^4} e^{-\left(\frac{1-(\mathbf{n}\cdot\mathbf{m})^2}{\alpha_b^2(\mathbf{n}\cdot\mathbf{m})^2}\right)}$$

$$D_{\mathrm{sgd}}(\mathbf{m}) = \frac{\mathrm{p22}\left[\frac{1-(\mathbf{n}\cdot\mathbf{m})^2}{(\mathbf{n}\cdot\mathbf{m})^2}\right]}{\pi(\mathbf{n}\cdot\mathbf{m})^4}$$

Normal Distribution Functio

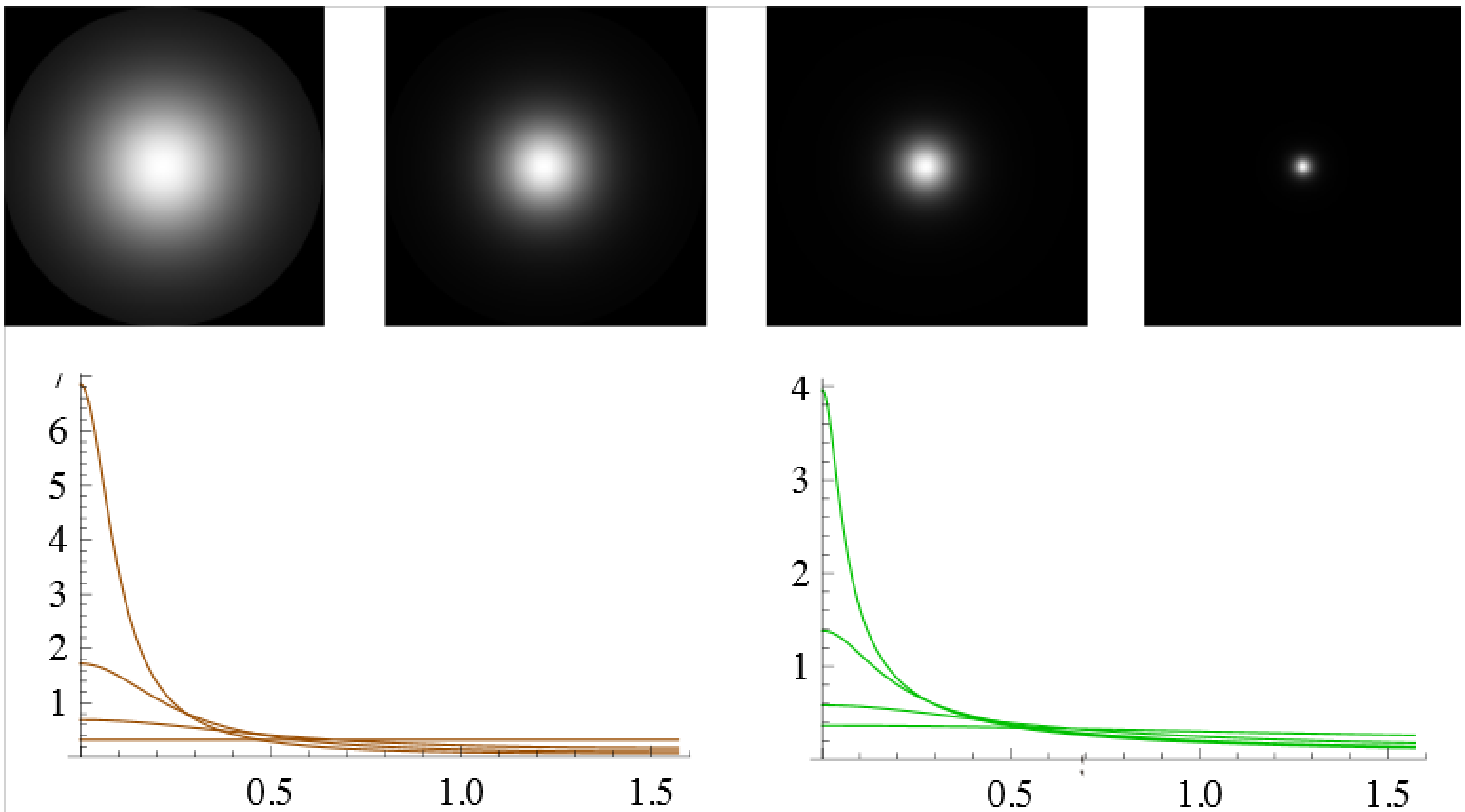
- Statistical distribution of orientation h
  - Determine size, brightness and shape of specular highlight



Gaussian shapes

Normal Distribution Functio

- Statistical distribution of orientation h
  - Determine size, brightness and shape of specular highlight



Spiky shapes

Normal Distribution Functio

- Statistical distribution of orientation h
  - Determine size, brightness and shape of specular highlight
- Commonly used NDFs
  - Phong distribution

$$D_p(\mathbf{m}) = \frac{\alpha_p + 2}{2\pi}(\mathbf{n} \cdot \mathbf{m})^{\alpha_p}$$

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Normalization factor: $(\mathbf{v} \cdot \mathbf{n}) = \int_{\Theta} D(\mathbf{m})(\mathbf{v} \cdot \mathbf{m})\, d\omega_m$

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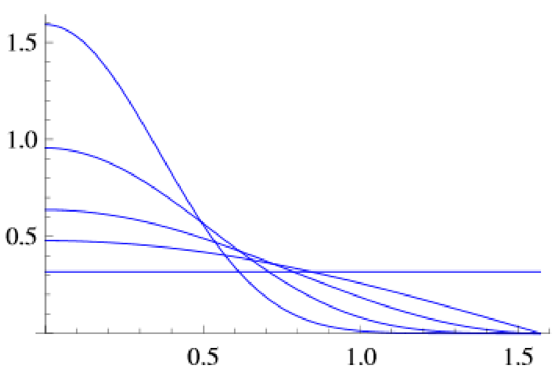
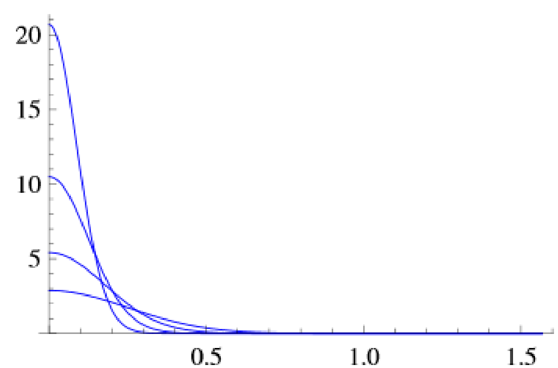
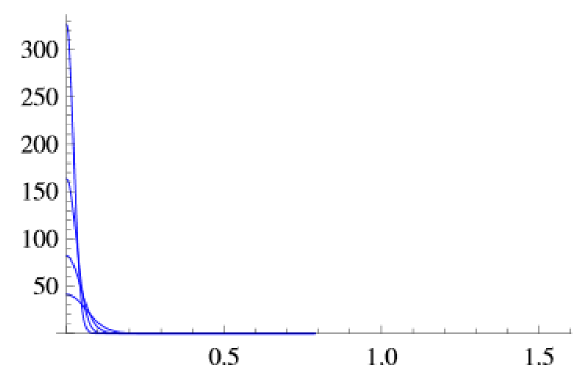


0 ▭ 8



16 ▭ 128



256 ▭ 2048

Normal Distribution Functio

- Statistical distribution of orientation h
  - Determine size, brightness and shape of specular highlight
- Commonly used NDFs
  - Phong distribution
  - Beckmann distribution

$$D_b(\mathbf{m}) = \frac{1}{\pi \alpha_b^2 (\mathbf{n} \cdot \mathbf{m})^4} e^{-\left(\frac{1-(\mathbf{n} \cdot \mathbf{m})^2}{\alpha_b^2 (\mathbf{n} \cdot \mathbf{m})^2}\right)}$$

0 ⎵ 8 16 ⎵ 128 256 ⎵ 2048

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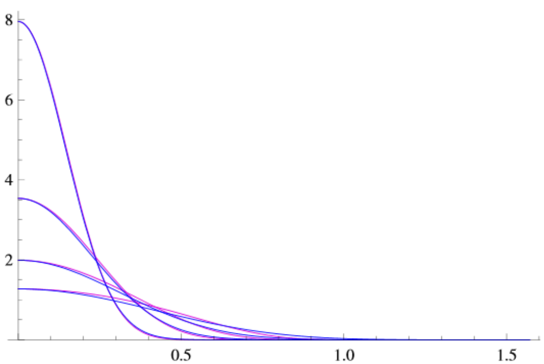
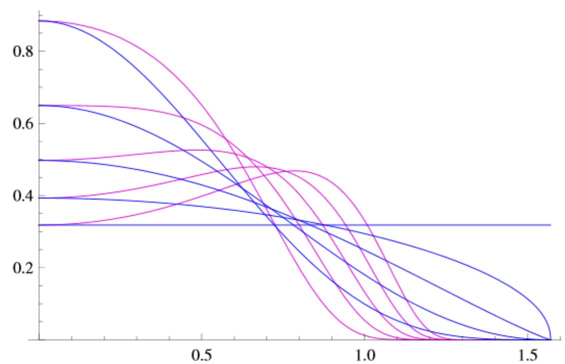
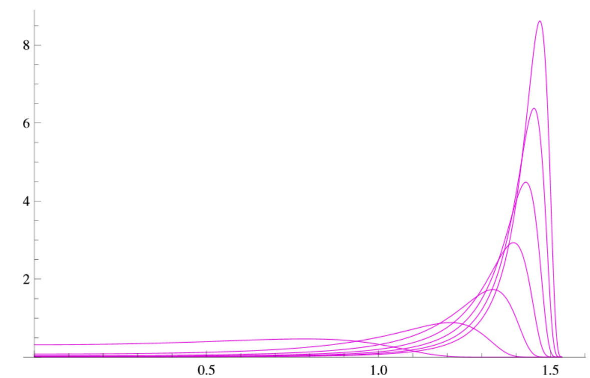






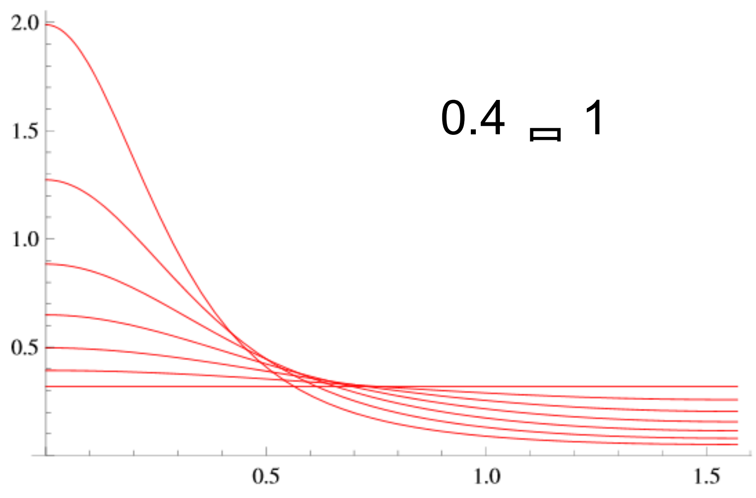
$$\alpha_p = 2\alpha_b^{-2} - 2$$

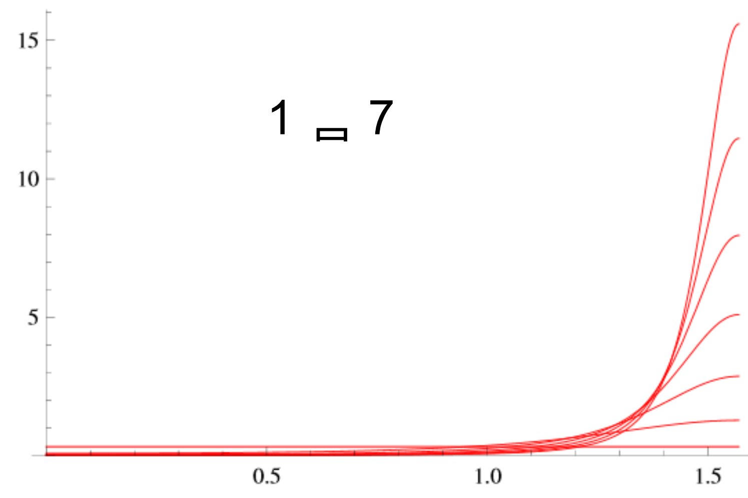
1 ▭ 7
Super rough

Normal Distribution Functio

- Statistical distribution of orientation h
  - Determine size, brightness and shape of specular highlight
- Commonly used NDFs
  - Phong distribution
  - Beckmann distribution
  - GGX distribution

$$D_{\mathrm{tr}}(\mathbf{m}) = \frac{\alpha_{\mathrm{tr}}^2}{\pi\left((\mathbf{n}\cdot\mathbf{m})^2\left(\alpha_{\mathrm{tr}}^2 - 1\right) + 1\right)^2}$$





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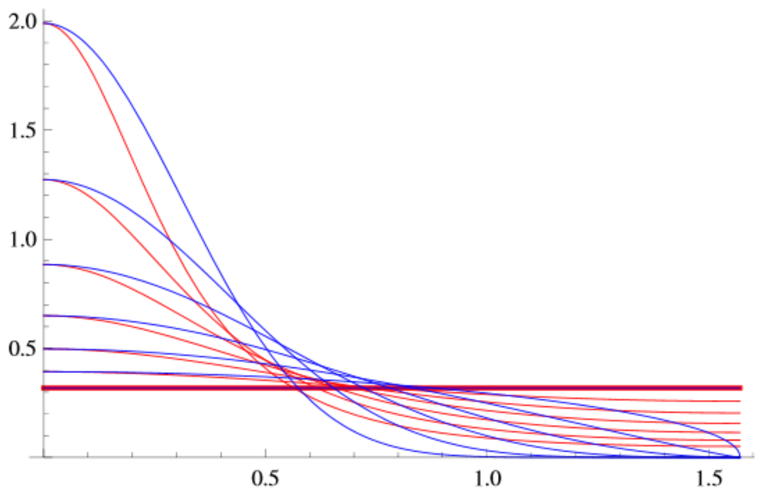

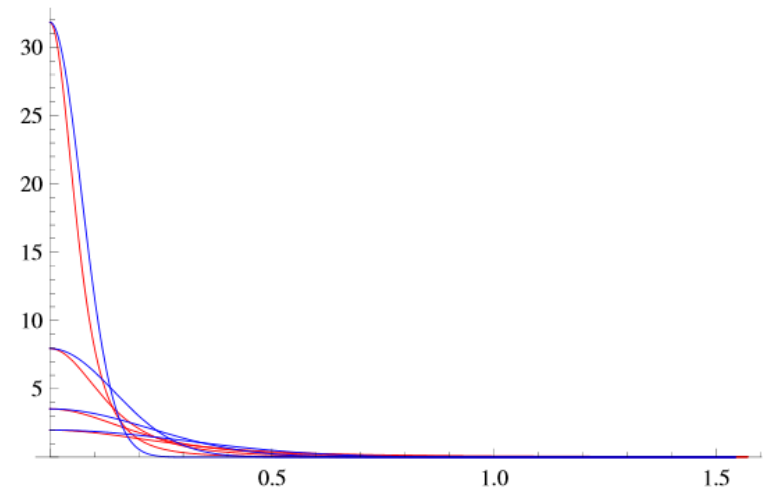

Phong comparison

Normal Distribution Functio

- Statistical distribution of orientation h
  - Determine size, brightness and shape of specular highlight
- Commonly used NDFs
  - Phong distribution
  - Beckmann distribution
  - GGX distribution
  - And many others…

Normal Distribution Functio

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  - Determine size, brightness and shape of specular highlight
- Commonly used NDFs
  - Phong distribution
  - Beckmann distribution
  - GGX distribution
  - And many others…
- Choice of NDF?
  - Depends on evaluation cost (applications)
  - Material properties (rough, isotropic, etc)
  - Artistic controls

Geometry function

- Shadowing and masking
- Probability that points with given microfacet normal
  - is visible from light
  - And from view

$$f(\mathbf{l}, \mathbf{v}) = \frac{F(\mathbf{l}, \mathbf{h}) \boxed{G(\mathbf{l}, \mathbf{v}, \mathbf{h})} D(\mathbf{h})}{4(\mathbf{n} \cdot \mathbf{l})(\mathbf{n} \cdot \mathbf{v})}$$

Geometry function

- Shadowing and masking
- Probability that points with given microfacet normal
  - is visible from light
  - And from view
- Commonly used geometry functions
  - No visibility

$$G_{\text{implicit}}(\mathbf{l}, \mathbf{v}, \mathbf{m}) = (\mathbf{n} \cdot \mathbf{l_c})(\mathbf{n} \cdot \mathbf{v})$$

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  - Cook-Terrance

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$$G_{\text{ct}}(\mathbf{l}, \mathbf{v}, \mathbf{h}) = \min\left(1, \frac{2(\mathbf{n} \cdot \mathbf{h})(\mathbf{n} \cdot \mathbf{v})}{(\mathbf{v} \cdot \mathbf{h})}, \frac{2(\mathbf{n} \cdot \mathbf{h})(\mathbf{n} \cdot \mathbf{l})}{(\mathbf{v} \cdot \mathbf{h})}\right)$$

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  - Smith $G(\mathbf{l}, \mathbf{v}, \mathbf{h}) = G_1(\mathbf{l})G_1(\mathbf{v})$ depends on NDF

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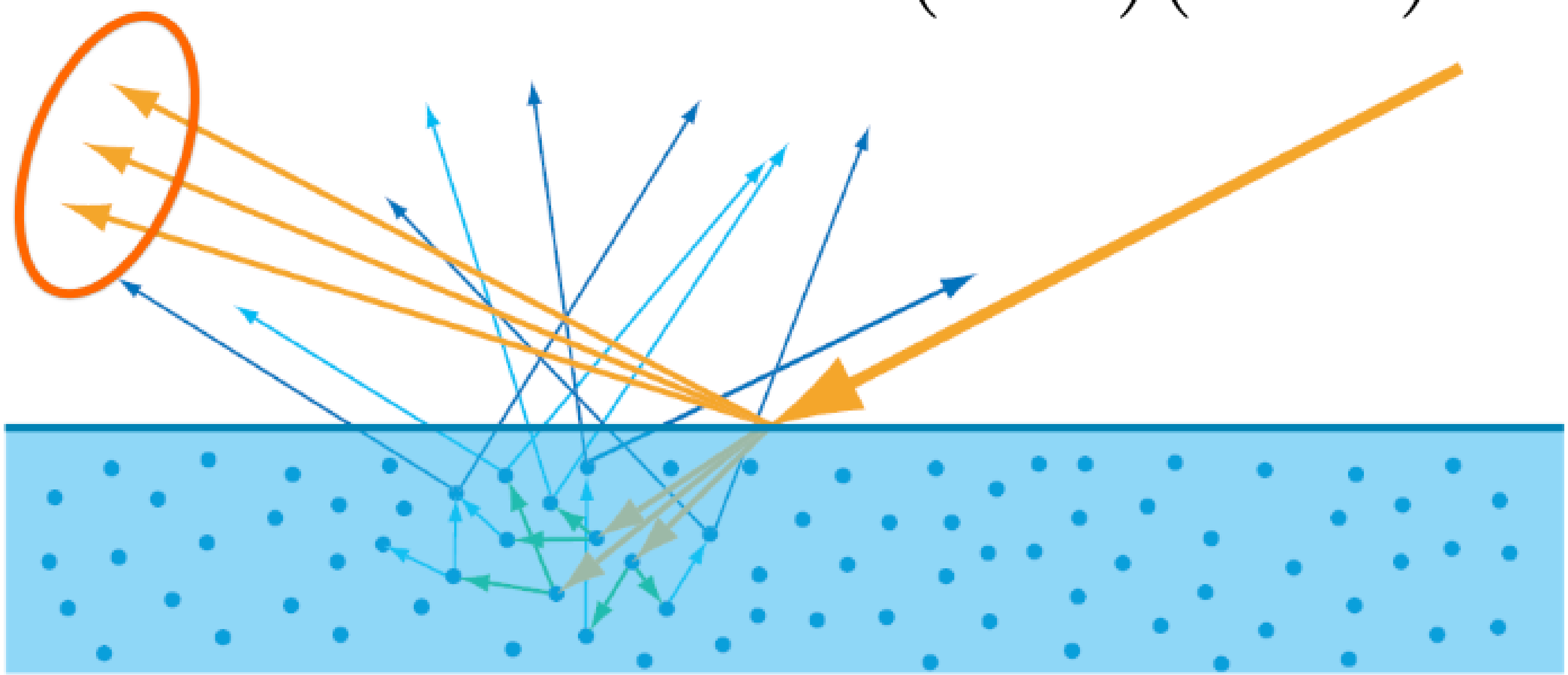
- More about the masking shadowing function:
  - Understanding the Masking-Shadowing Function in Microfacet-Based BRDFs [Heitz - JCGT 2014]

Microfacet theory

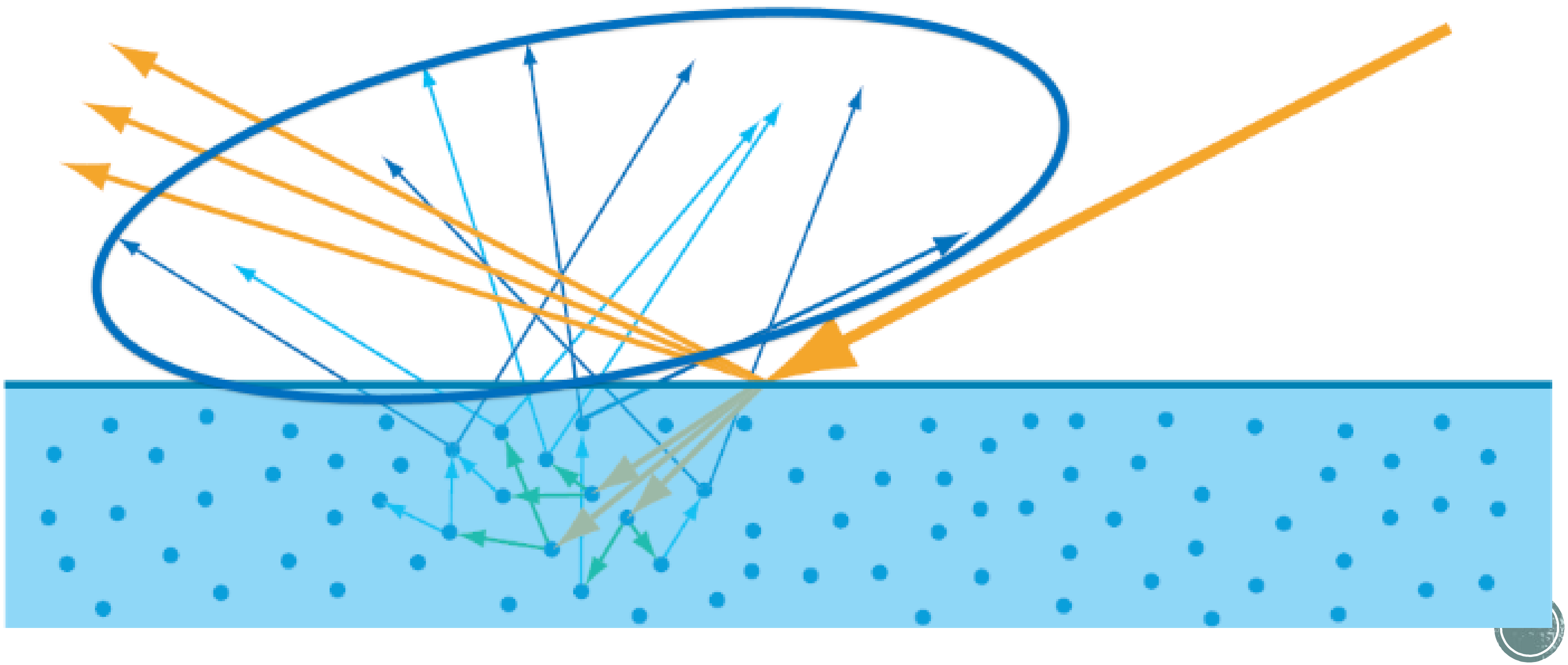
- Surface reflection (specular term)

$$f(\mathbf{l}, \mathbf{v}) = \frac{F(\mathbf{l}, \mathbf{h})G(\mathbf{l}, \mathbf{v}, \mathbf{h})D(\mathbf{h})}{4(\mathbf{n} \cdot \mathbf{l})(\mathbf{n} \cdot \mathbf{v})}$$



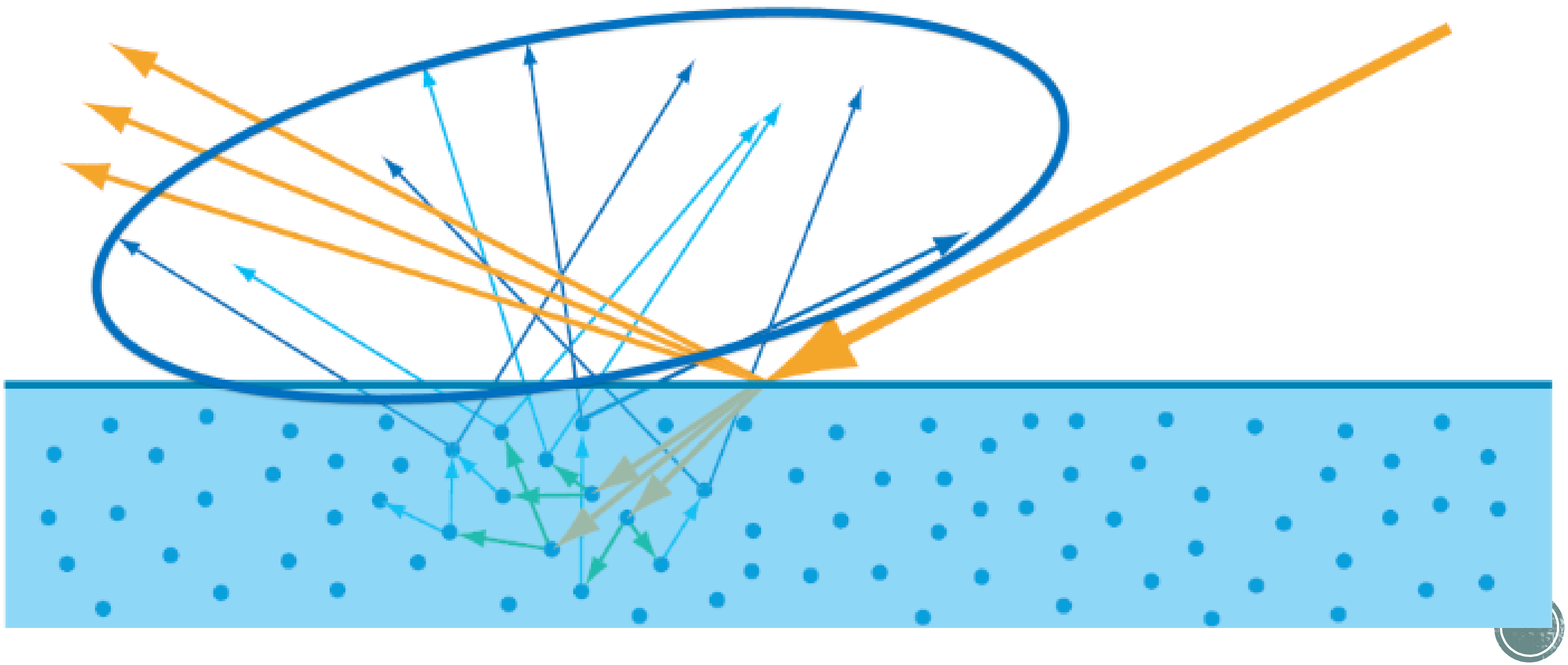
Microfacet theory

- Subsurface reflection (diffuse term)



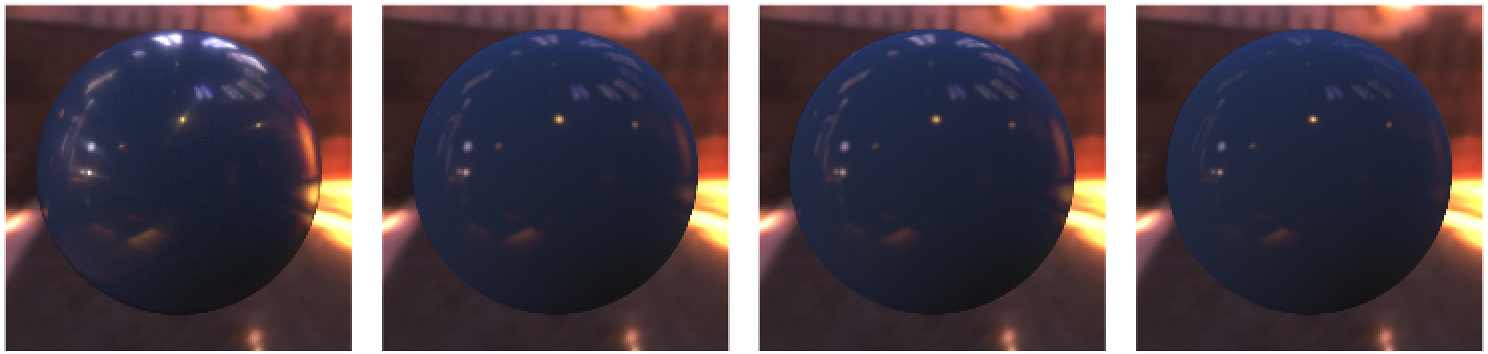
Microfacet theory

- Subsurface reflection (diffuse term)
  - Constant: $f_{\mathrm{Lambert}}(\mathbf{l}, \mathbf{v}) = \dfrac{\mathbf{c}_{\mathrm{diff}}}{\pi}$

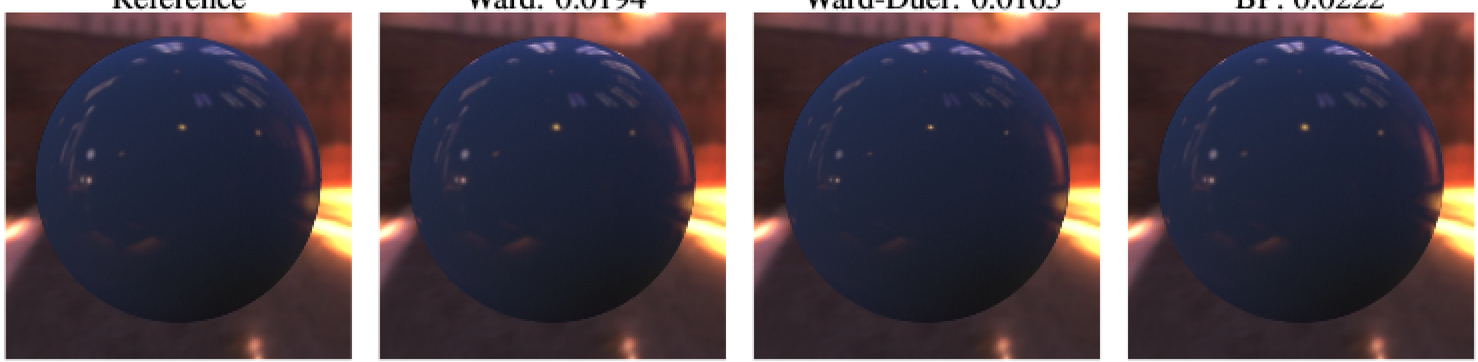


BRDF comparison

- Ngan et al. 2005



Reference | Ward: 0.0194 | Ward-Duer: 0.0165 | BP: 0.0222



Lafortune: 0.0167 | CT: 0.0155 | He: 0.0141 | Ash: 0.0153

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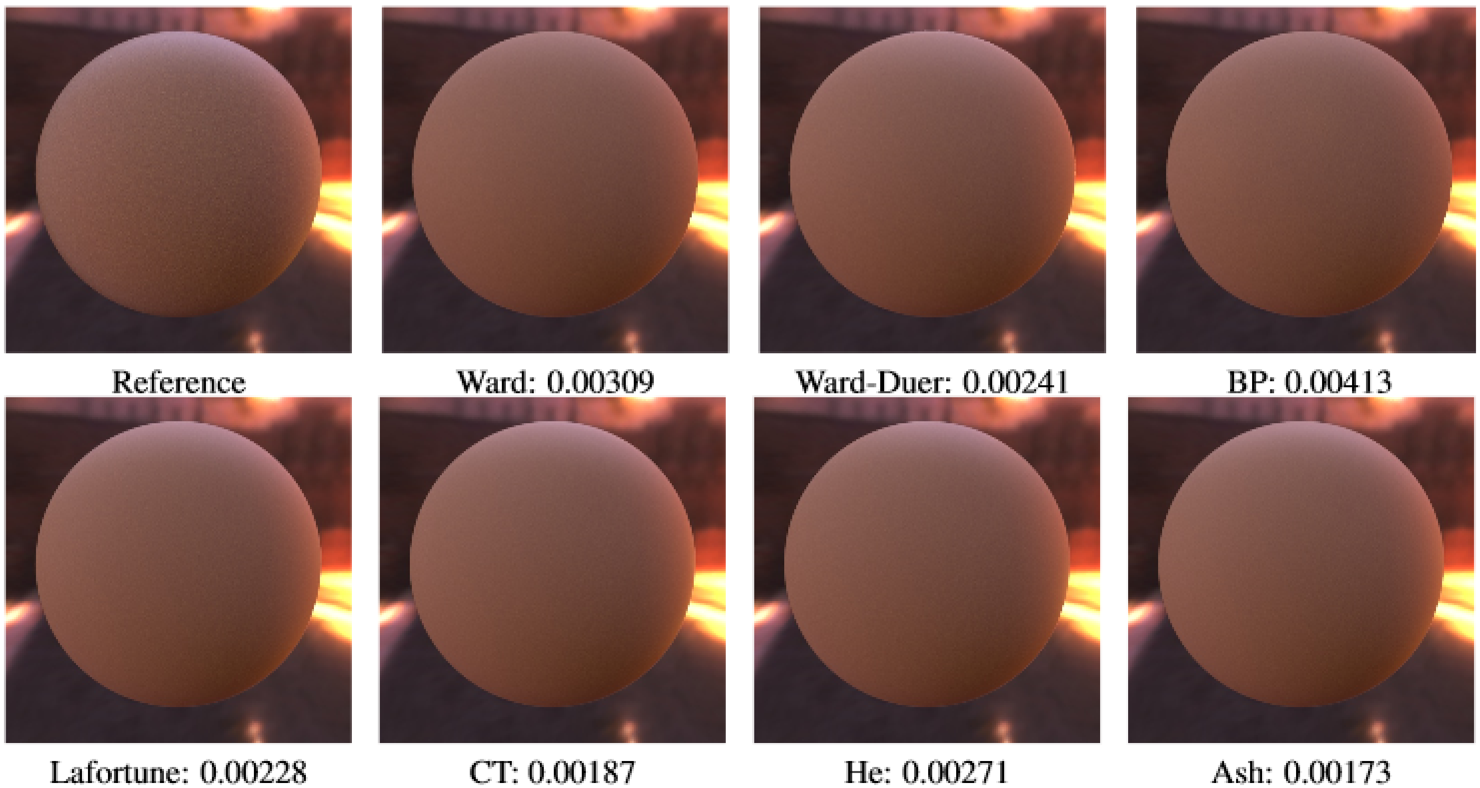

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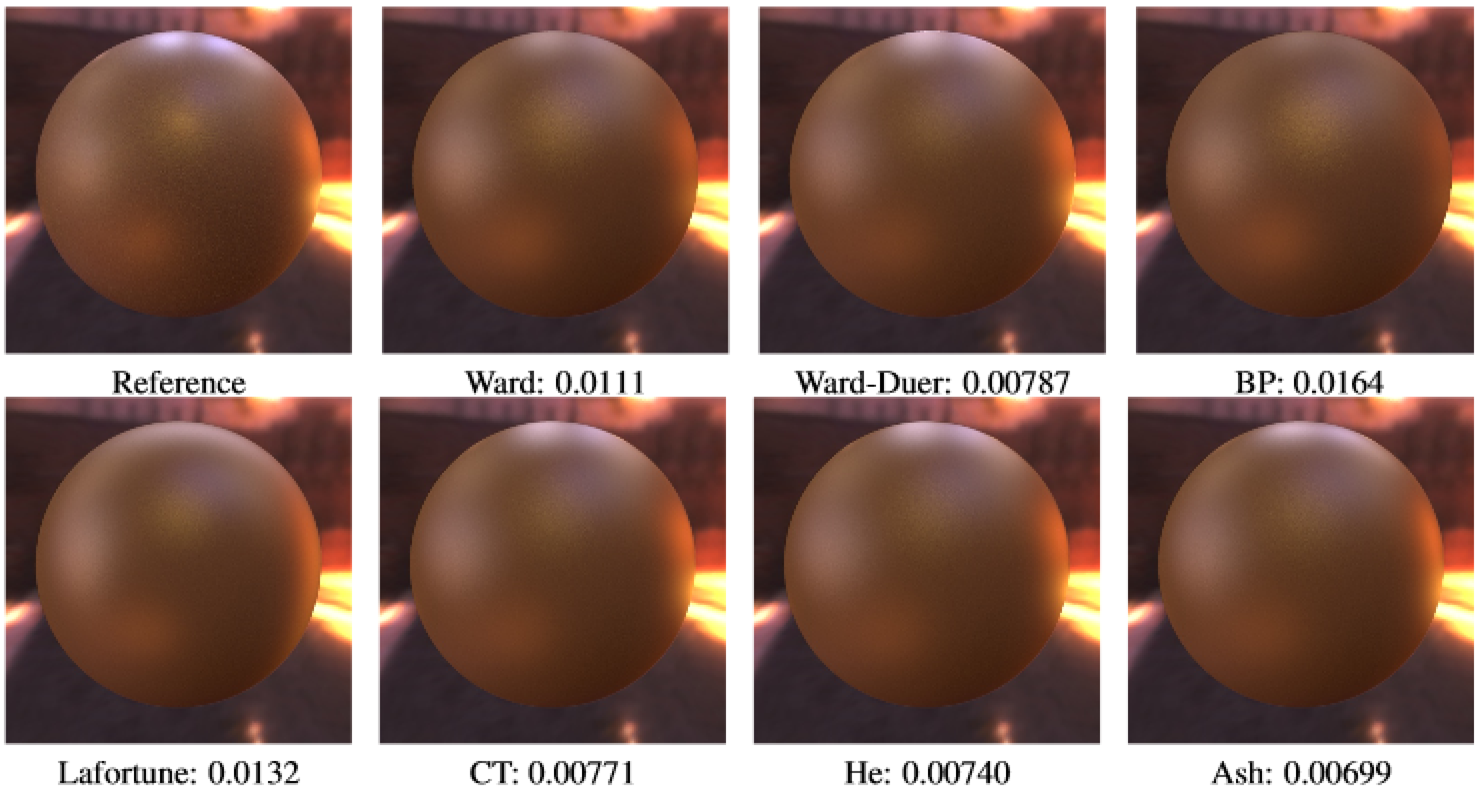

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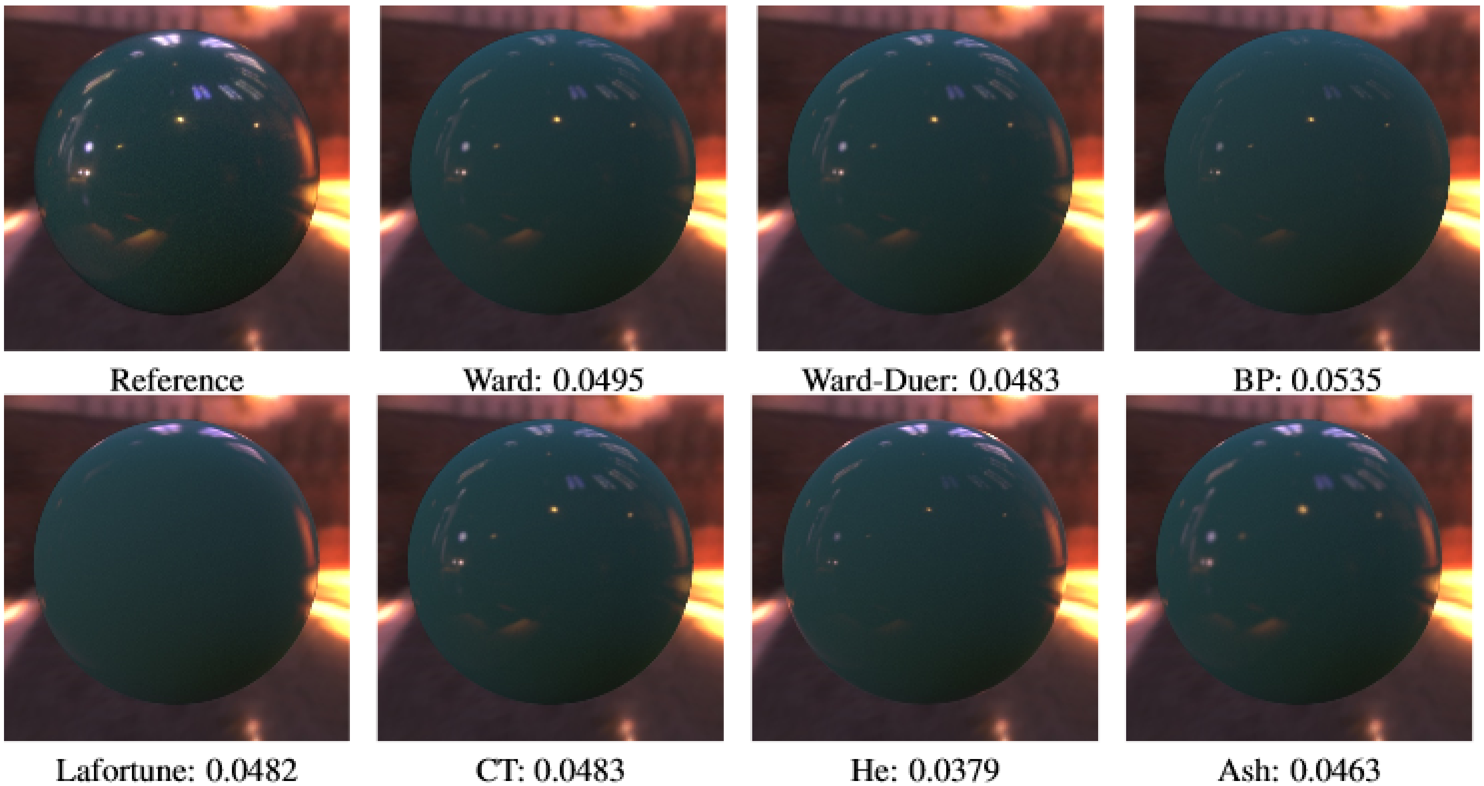

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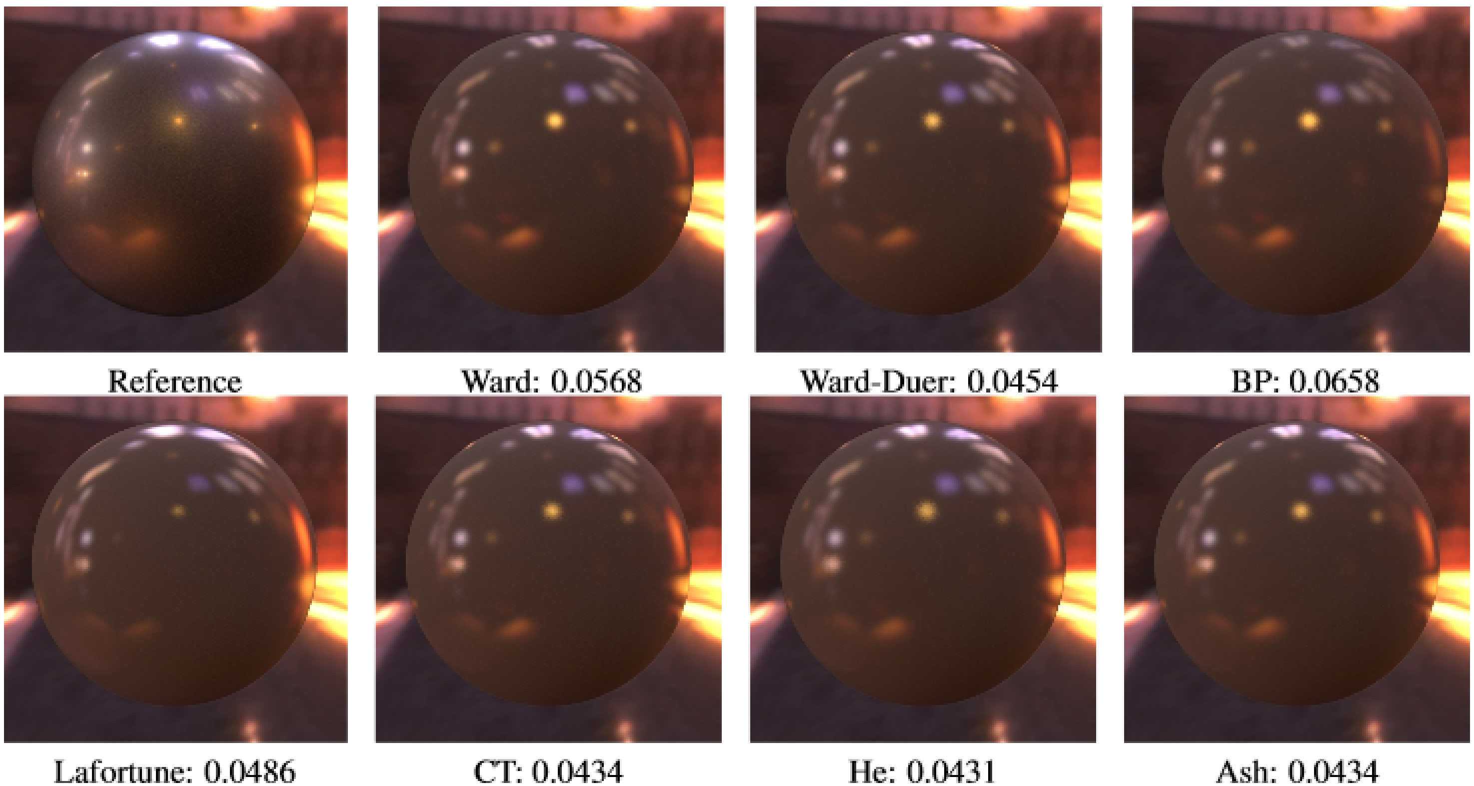

What are we missing?

- BRDF: Bidirectionnal Reflectance Distribution Function

$$f(\mathbf{l}, \mathbf{v})$$

  - 4D or 3D for isotropic materials

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$$f(\mathbf{l}, \mathbf{x}_\mathbf{l}, \mathbf{v}, \mathbf{x}_\mathbf{v})$$

- BSSRDF: Bidirectionnal Surface Scattering DF
  - 8D: light exits at another location

What are we missing?

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- BSSRDF: Bidirectionnal Surface Scattering DF $f(\mathbf{l}, \mathbf{x_l}, \mathbf{v}, \mathbf{x_v})$
  - 8D: light exits at another location
- BSDF: Bidirectionnal Scattering Distribution Function $f(\mathbf{l}, \mathbf{x_l}, \lambda_\mathbf{l}, \mathbf{v}, \mathbf{x_v}, \lambda_\mathbf{v})$
  - XD: General formulation

References

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- http://web.cs.wpi.edu/~emmanuel/courses/cs563/write_ups/zackw/realistic_raytracing.html