Outline

- Principles of animation
- Keyframe interpolation
- Rigging, skinning and walking
PRINCIPLES OF CHARACTER ANIMATION

~1930, Studios Disney

The Illusion of Life
PRINCIPLES OF TRADITIONAL ANIMATION
APPLIED TO 3D COMPUTER ANIMATION
John Lasseter, Pixar, San Rafael, California
SIGGRAPH 1987
1 Squash & Stretch

The most important principle is called squash and stretch. When an object is moved, the movement emphasizes any rigidity in the object. In real life, only the most rigid shapes (such as chairs, dishes and pans) remain so during motion. Anything composed of living flesh, no matter how bony, will show considerable movement in its shape during an action. For example, when a bent arm with swelling biceps straightens out, only the long sinews are apparent. A face, whether chewing, smiling, talking, or just showing a change of expression, is alive with changing shapes in the checks, the lips, and the eyes. [26]

The most important rule to squash and stretch is that, no matter how squashed or stretched out a particular object gets, its volume remains constant. If an object squashed down without its sides stretching, it would appear to shrink; if it stretched up without its sides squeezing in it would appear to grow. Consider the shape and volume of a half filled flour sack: when dropped on the floor, it squashed out to its fullest shape. If picked up by the top corners, it stretched out to its longest shape. It never changes volume. [26]

The standard animation test for all beginners is drawing a bouncing ball. The assignment is to represent the ball by a simple circle, and then have it drop, hit the ground, and bounce back into the air. A simple test, but it teaches the basic mechanics of animating a scene, introducing timing as well as squash and stretch. If the bottom drawing is flattened, it gives the appearance of bouncing. Elongating the drawings before and after the bounce increases the sense of speed, makes it easier to follow and gives more snap to the action. [26,3] (figure 2)

FIGURE 2. Squash & stretch in bouncing ball.
1 Squash & stretch

An object need not deform in order to squash and stretch. For instance, a hinged object like Luxo Jr. (from the film, Luxo Jr. [21]), squashes by folding over on itself, and stretches by extending out fully. (figure 3)

FIGURE 3. Squash & stretch in Luxo Jr.’s hop.

FIGURE 4a. In slow action, an object’s position overlaps from frame to frame which gives the action a smooth appearance to the eye.

FIGURE 4b. Strobing occurs in a faster action when the object’s positions do not overlap and the eye perceives separate images.

FIGURE 4c. Stretching the object so that it’s positions overlap again will relieve the strobing effect.
Timing, or the speed of an action, is an important principle because it gives meaning to movement—the speed of an action defines how well the idea behind the action will read to an audience. It reflects the weight and size of an object, and can even carry emotional meaning.
2 Timing

Just two drawings of a head, the first showing it leaning toward the right shoulder and the second with it over on the left and its chin slightly raised, can be made to communicate a multitude of ideas, depending entirely on the Timing used. Each inbetween drawing added between these two “extremes” gives a new meaning to the action.

NO inbetweens........... The Character has been hit by a tremendous force. His head is nearly snapped off.

ONE inbetweens........... The Character has been hit by a brick, rolling pin, frying pan.

TWO inbetweens........... The Character has a nervous tic, a muscle spasm, an uncontrollable twitch.

THREE inbetweens.... The Character is dodging a brick, rolling pin, frying pan.

FOUR inbetweens.......... The Character is giving a crisp order, “Get going!” “Move it!”

FIVE inbetweens.......... The Character is more friendly, “Over here.” “Come on-hurry!”

SIX inbetweens.......... The Character sees a good looking girl, or the sports car he has always wanted.

SEVEN inbetweens........... The Character tries to get a better look at something.

EIGHT inbetweens.......... The Character searches for the peanut butter on the kitchen shelf.

NINE inbetweens.......... The Character appraises, considering thoughtfully.

TEN inbetweens.......... The Character stretches a sore muscle.
3 Anticipation

An action occurs in three parts: the preparation for the action, the action proper, and the termination of the action. *Anticipation* is the preparation for the action; the latter two are discussed in the next sections.

There are several facets to Anticipation. In one sense, it is the anatomical provision for an action. Since muscles in the body function through contraction, each must be first be extended before it can contract. A foot must be pulled back before it can be swung forward to kick a ball. [12] Without anticipation many actions are abrupt, stiff and unnatural.
Anticipation example
4 Staging

Staging is the presentation of an idea so it is completely and unmistakably clear; this principle translates directly from 2-D hand drawn animation. An action is staged so that it is understood; a personality is staged so that it is recognizable; an expression so that it can be seen; a mood so that it will affect the audience. [26]

To stage an idea clearly, the audience's eye must be led to exactly where it needs to be at the right moment, so that they will not miss the idea. Staging, anticipation and timing are all integral to directing the eye. A well-timed anticipation will be wasted if it is not staged clearly.

“Present the idea so that it is unmistakably clear”
5 Straight-ahead & pose to pose action

There are two main approaches to hand drawn animation. The first is known as *straight ahead action* because the animator literally works straight ahead from his first drawing in the scene. He knows where the scene fits in the story and the business it has to include. He does one drawing after another, getting new ideas as he goes along, until he reaches the end of the scene. This process usually produces drawings and action that have a fresh and slightly zany look, because the whole process was kept very creative. Straight ahead action is used for wild, scrambling actions where spontaneity is important.

The second approach is called *pose-to-pose*. Here the animator plans his actions, figures out just what drawings will be needed to animate the business, makes the drawings concentrating on the poses, relates them to each other in size and action, and then draws the inbetweens. Pose-to-pose is used for animation that requires good acting, where the poses and timing are all important.
Just as the anticipation is the preparation of an action, follow through is the termination of an action. Actions very rarely come to a sudden and complete stop, but are generally carried past their termination point. For example, a hand, after releasing a thrown ball, continues past the actual point of release.
Slow in and slow out deals with the spacing of the inbetween drawings between the extreme poses. Mathematically, the term refers to second- and third-order continuity of motion.

**FIGURE 9.** Timing chart for ball bounce.

**FIGURE 10a.** This spline controls the Z (up) translation of Luxo Jr. Dips in the spline cause him to intersect the floor.

**FIGURE 10b.** Two extra extremes are added to the spline which removes the dips and prevents Jr. from going into the basement.
8 Arcs

The visual path of action from one extreme to another is always described by an arc. Arcs in nature are the most economical routes by which a form can move from one position to another. In animation, such arcs are used extensively, for they make animation much smoother and less stiff than a straight line for the path of action. In certain cases, an arc may resolve itself into a straight path, as for a falling object, but usually, even in a straight line action, the object rotates. [12]
The meaning of exaggeration is, in general, obvious. However, the principle of exaggeration in animation does not mean arbitrarily distorting shapes or objects or making an action more violent or unrealistic. The animator must go to the heart of anything or any idea and develop its essence, understanding the reason for it, so that the audience will also understand it. If a character is sad, make him sadder; if he is bright, make him shine; worried, make him fret; wild, make him frantic.
10 Secondary actions

A secondary action is an action that results directly from another action. Secondary actions are important in heightening interest and adding a realistic complexity to the animation. A secondary action is always kept subordinate to the primary action. If it conflicts, becomes more interesting, or dominates in any way, it is either the wrong choice or is staged improperly. [26]
The word *appeal* is often misrepresented to suggest cuddly bunnies and soft kittens. It doesn’t; it means anything that a person likes to see: a quality of charm, pleasing design, simplicity, communication, or magnetism. Your eye is drawn to the figure or object that has appeal, and, once there, it is held while you appreciate the object. A weak drawing or design lacks appeal. A design that is complicated or hard to read lacks appeal. Clumsy shapes and awkward moves all have low appeal. Where the live action actor has charisma, the animated character has appeal. [26]
In character animation, all actions and movements of a character are the result of its thought processes. "The thinking animation character becomes a character." [12] Without a thought process, the actions of a character are just a series of unrelated motions. With a thought process to connect them, the actions bring a character to life.
References

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2. Alias Research Inc., 110 Richmond St. East, Suite 500, Toronto, Ontario, Canada m5c- lpl
References


10. Walt Disney Productions, Three Little Pigs, (film), 1933.


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28. Vertigo Systems International Inc., 119 W. Pender St., Suite 221, Vancouver, BC, Canada v6b 1s5
29. Wavefront Technologies, 530 East Montecito, Santa Barbara, CA 93101
Interpolation of translation
Hermite polynomials

\[
\begin{align*}
    x_0 &= a_0 \\
    \dot{x}_0 &= a_1 \\
    x_1 &= a_0 + a_1 + a_2 + a_3 \\
    \dot{x}_1 &= a_1 + 2a_2 + 3a_3
\end{align*}
\]

\[
\begin{align*}
    a_0 &= x_0 \\
    a_1 &= \dot{x}_0 \\
    a_2 &= 3(x_1 - x_0) - 2\dot{x}_0 - \dot{x}_1 \\
    a_3 &= 2(x_0 - x_1) + \dot{x}_0 + \dot{x}_1
\end{align*}
\]

\[
x(t) = \begin{pmatrix}
    \left(1 - 3t^2 + 2t^3\right) x_0 \\
    \left(3t^2 - 2t^3\right) x_1 \\
    \left(t - 2t^2 + t^3\right) \dot{x}_0 \\
    \left(-t^2 + t^3\right) \dot{x}_1
\end{pmatrix}
= \begin{pmatrix}
    h_0(t) & x_0 \\
    h_1(t) & x_1 \\
    h'_0(t) & \dot{x}_0 \\
    h'_1(t) & \dot{x}_1
\end{pmatrix}
\]
Catmull-Rom splines

- Derived from Hermite splines
- Approximate tangents using control points
  \[ D_i = \frac{1}{2}(P_{i+1} - P_{i-1}) \]
- Arbitrary first and last points
Timing curves

\[ x(T) = x(d(t(T))) \]
Bezier curves, B-splines and NURBS

- Bezier: Piece-wise polynomials with tangent continuity
- B-splines: control points, arcs and curves
- NURBS: piece-wise rational curves, i.e. projective splines in projective coordinates
Three-dimensional interpolation

\[ \frac{1}{2} (P_{i+1} - P_{i-1}) \]
Interpolation of rotations

- Cannot interpolate matrices
- Compute $R = R_1^T R_2$
- Compute axis and angle
- Interpolate angles

\[
\begin{align*}
\theta &= \arccos\left(\frac{R_{11} + R_{22} + R_{33} - 1}{2}\right) \\
\sin(\theta) &= 2s \theta \\
n_x &= \frac{R_{32} - R_{23}}{s} \\
n_y &= \frac{R_{13} - R_{31}}{s} \\
n_z &= \frac{R_{21} - R_{12}}{s}
\end{align*}
\]
Quaternion interpolation

\[ q = [w, v], v = (x, y, z), w = scalar \]

Arbitrary axis  Angle of rotation

\( q \) form a sphere of unit length in the 4D space

\[ q_1 \ast q_2 = (w_1.w_2 - v_1.v_2, w_1.v_2 + w_2.v_1 + v_1 \times v_2) \]
Quaternion interpolation

1. Use a quaternion to represent the rotation.

2. Generate a temporary quaternion for the change from the current orientation to the new orientation.

3. PostMultiply the temp quaternion with the original quaternion. This results in a new orientation that combines both rotations.

4. Convert the quaternion to a matrix and use matrix multiplication as normal.
Spherical interpolation

\[ \text{Slerp}(p_0, p_1; t) = \frac{\sin[(1 - t)\Omega]}{\sin\Omega} p_0 + \frac{\sin[t\Omega]}{\sin\Omega} p_1. \]

- **SLERP**: Interpolation on the sphere of unit quaternions
- **LERP**: Linear interpolation then normalization
- **US patent by Budge (2007)**: Fast approximation to the spherical linear interpolation function
Equivalence between Euler angles and quaternions

if you have three Euler angles \((a, b, c)\),

*then* you can form three independent quaternions:

\[
\begin{align*}
Q_x &= [\cos(a/2), \sin(a/2), 0, 0] \\
Q_y &= [\cos(b/2), 0, \sin(b/2), 0] \\
Q_z &= [\cos(c/2), 0, 0, \sin(c/2)]
\end{align*}
\]

And the final quaternion is obtained by \(Q_x \ast Q_y \ast Q_z\).
Rigid motion interpolation

- **Screw Theory**: we can represent any movement of a solid body by a single operation which combines both the rotation and the translation.
  - As [Plucker coordinates](#).
  - As [Dual Quaternions](#).
  - Using [Motor Theory](#) based on [Clifford Algebra](#).

- More about this in Class 2.
Slow-in and slow-out

- We want to control the velocity of a moving object along a given path (spline)
- Use arclength parameterization
- Apply velocity control as $s(t)$ with $s = 0$ at starting point and $s = 1$ at end point

![Graphs showing constant speed, accelerated, and decelerated motion](image_url)
Camera interpolation

- 3 translations (dollies)
- 3 rotations: pan left/right, up/do
- Field of view is controlled by zoom = focal length
Camera interpolation

- Screen motion is the composition of camera and actor motion
- Coordinate camera and actor’s movements
Rigging and skinning

- How do we apply kinematics and dynamics to character animation?
  - Skeleton/armature and kinematics
  - Skin and flesh: skinning, smooth skinning, muscles, fat, wrinkles
  - Clothing: particle systems, finite elements
Animation and interpolation

- Series of pairs (time, parameter values)
- Interpolate inbetween

- Keyframe animation
- How to interpolate motion between keyframes
Articulated motion
Articulated motion
Articulated and rigid motion

- Motion of body part is rigid
  - In the parent’s frame
  - In the word frame
- Rigid motion can be represented as a 3 x 4 matrix

\[ M_{b,f} = (S_{b,f} R_{b,f} T_{b,f}) M_{p,f} \]

\[
\begin{pmatrix}
S_x R_{11} & S_x R_{12} & S_x R_{13} \\
S_y R_{21} & S_y R_{22} & S_y R_{23} \\
S_z R_{31} & S_z R_{32} & S_z R_{33}
\end{pmatrix}
\begin{pmatrix}
T_x \\
T_y \\
T_z
\end{pmatrix}
Skinning

- We have a structure of bones, organized as a kinematic tree
- Problem: how do we animation the « skin » of characters given the motion of their bones?
- Rigid skinning: each body part is modeled as a rigid body
  \[ P(v_i) = T_f P_0(v_i) \] where T is the bone transformation
Smooth skinning

- Also known as « Skeleton Subspace Deformation »
- Skin vertices move as a result of several body part motions
  - \( P(v_i) = (\sum_f w_{if} T_f)P_0(v_i) \)
  - Normalized weights: \( \sum_f w_{if} = 1 \)
  - Vertex weights can be computed automatically
  - For example \( w_{if} = 1/d_{if}^2 \)
- Or weights can be drawn by « painting » the skin
Interpolation of matrices

- Transformation matrix $T_f = [SR\mid t]$ with 12 parameters
- Non independent
- 3 translations, 3 rotations, 3 re-scalings
- Better to control them separately
- Automatic weight computation
- Wang et Philips, Multi-weight enveloping: least-squares approximation techniques for skin animation, SCA 2002
Bone Heat Weighting

- Automatic Rigging and Animation of 3D Characters
Examples
Quaternion interpolation

- **Skinning with Dual Quaternions**
Cage deformations

- Build a cage around bones (armatures)
Case study: animation of walking

- Alternance of support and swing phases
- The legs are coordinated
- The motion is near periodic
- The need for realism is high
Walking cycle

- Support and transfer

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<th>rTO</th>
<th>rFS</th>
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<tr>
<td>Right stance</td>
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IFS = left footstrike, rFS = right footstrike, rTO = right takeoff, ITO = left takeoff
Finite State Machine
Contact

- The walk usually starts with the feet at the extended position – where the feet are furthest apart. This is the point where the character’s weight shifts to the forward foot.
Recoil

- As the weight of the body is transferred to the forward foot, the knee bends to absorb the shock. This is called the recoil position, and is the lowest point in the walk.
Passing

- This is halfway through the first step. As the character moves forward, the knee straightens out and lifts the body it’s highest point. This is called the passing position because this is where the free foot passes the supporting leg.
High point

• As the character moves forward, the weight-bearing foot lifts off the ground at the heel, transmitting the force at the ball of the foot. This is where the body starts to fall forward. The free foot swings forward like a pendulum to catch the ground.
• The free leg makes contact. This is exactly half the cycle. The second half is an exact mirror of the first. If it differs, the character may appear to limp.
Foot placement and synchronisation

Basic idea:
drive the body and the feet using kinematic constraints

We have to coordinate:
– each leg with each other
– the legs with the body

Splines are used to drive the constrained points

The footsteps are placed according to the trajectory of the character

Velocity curves can be applied to the body of the character

\[ v = \text{stepLength} \times \text{frequency} \]
From footprints to animation

Figure 3: Footprints with timing information and manipulation handles.

Figure 4: Automatic footprint generation from a path.
From footprints to animation

Figure 1: Specification of a Walking Motion (a) plan view of footprints (b) timing diagram.

Figure 2: Block diagram for motion synthesis from footprints.
From footprints to animation

Figure 5: The generic biped model.

\[ E = \int_{0}^{T} (\varepsilon_{\text{physics}} + \varepsilon_{\text{comfort}}) \, dt \]

\[ \varepsilon_{\text{physics}} = |F + mg - ma| \]

\[ \varepsilon_{\text{comfort}} = k \left( \ell - \ell_{\text{nom}} \right)^2 \]
Control

- Top: body trajectory and velocity
- Footprints and foot trajectories
- Synchronization
- Bottom: inverse kinematics

- The available global degrees of freedom can be used to optimize various criteria
  - position of the mass center
  - mechanical energy
  - beauty
- Finding the good criteria is hard!

Other applications

- Running
- Animal gaits (4 legs, 6 legs,...)
High-level control

- Walking style
- Physics
- Aesthetics
- Expressivity
- Goal-driven
Combined direct and inverse kinematics

• Principle:
  – using available data on human walk
  – correct constraint violations using inverse kinematics

• Advantage: data may model a realistic gait
Controlling and editing the walk cycle

- Coach-trainee metaphor
- Can also be used to learn walking styles
Kinematic corrections

- Constraint violations are corrected using inverse kinematics

![Diagram showing kinematic corrections](image)
The coach-trainee metaphor

no constraint  a constraint is enforced  The constraint has vanished

In Joint space

Reference motion, or Coach
Corrected motion, or Trainee
Both Coach and Trainee coincide when there is no initial cartesian constraint
As the constraint has vanished, only the Coach configuration tracking controls the Trainee
Control

Key-frame System  |  Rotoscopy Process  |  Functional, Behavioural Models  |  Interactive Acquisition  |  Other Generators

joint space  |  reference model

trainee  |  coach

Coach-Trainee Control

joint space  |  corrected motion

Cartesian half-space constraints

Sampling  |  Adaptation

recording
Human walk using forward kinematics

- Use available data (motion capture, biomechanics, ...)

![Graph showing hip, knee, and ankle movement during a step cycle.](image)
Model data

- Control points allow us to:
  - interpolate for any time $t$
  - modify the laws using intuitive parameters
Editing motion

• Starting from given data, we want to
  – edit the motion
  – combine it with other data
  – concatenate the motion to another one
• Data may be represented as:
  – a function of time $dof(t) \rightarrow$ motion warping
  – a signal $\rightarrow$ motion signal processing
Using signal representation

- Frequency filtering: tuning amplitudes
  - low frequencies: basic motion
  - high frequencies: noise, emotion
Using signal representation (continued)

• Mixing two motions

\[ \theta^{(1)}_i(t) = B_{0i} + \sum_{n \geq 1} B_{ni} \sin(nt + \phi^{(1)}_{ni}) \]

\[ \theta^{(2)}_i(t) = (1-s)A_{oi} + sB_{0i} + \sum_{n \geq 1} C_{ni} \sin(nt + \phi^{(2)}_{ni}) \]

\[ C_{ni} = (1-s)A_{ni} + sB_{ni}, \phi^{(2)}_{ni} = (1-s)\phi_{ni} + s\phi^{(1)}_{ni} \]

• Transition between motions: use s
Using signal representation (continued)

- Blending motions
Conclusion

- Principles of animation
- Keyframe interpolation
- Rigging, skinning and walking
- Next: Forward and inverse kinematics