Introduction to Parametric interpolation for computer animation

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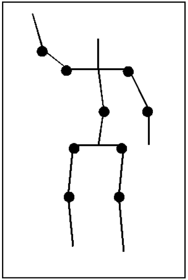
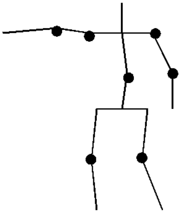
1. Parametric curves
2. Cubic bases
3. Arclength parameterization
4. Velocity control

Main idea

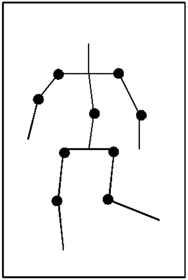
- Series of pairs (time, parameter values)
- interpolate inbetween



key T0,q0



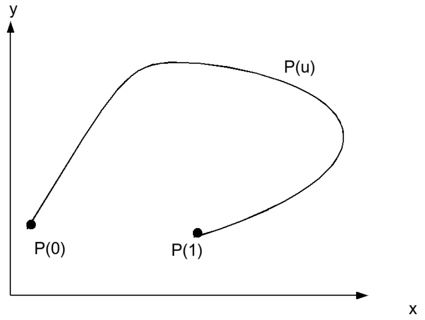
t, interpolated q



key T1,q1

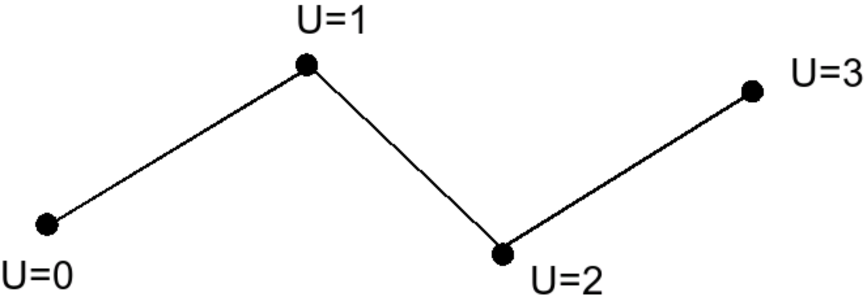
Parametric curves

- General form : $\mathbf{P}(u) = (x(u), y(u), z(u)) \quad 0 \leq u \leq 1$



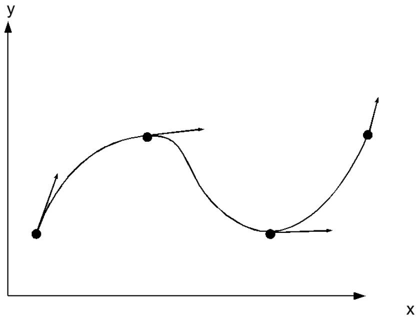
Linear curves

- General form : $\mathbf{P}(u) = (1-u)\mathbf{P}_0 + u\mathbf{P}_1$
- Use fractional value of global parameter U for piecewise curves
- Sharp discontinuities



Cubic curves

- Patches are defined using endpoints and tangents (*Hermite splines*)
- $x(u)$ and $y(u)$ are cubic functions
- Smooth continuity

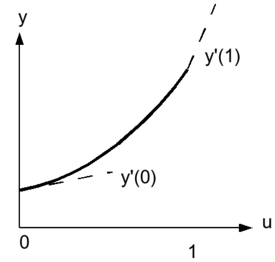


Hermite spline equation

$$
\begin{aligned}
y &= b_0 + b_1 u + b_2 u^2 + b_3 u^3 \\
y' &= b_1 + 2 b_2 u + 3 b_3 u^2
\end{aligned}
$$

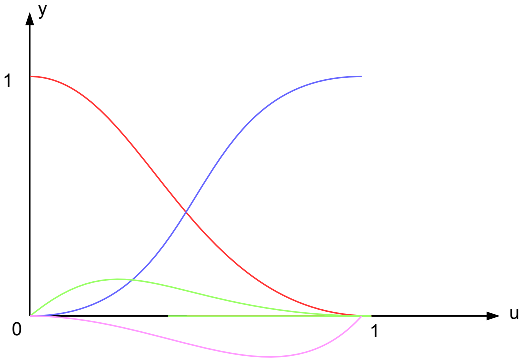
- Match values and slopes

$$
\begin{aligned}
b_0 &= y_0 \\
b_1 &= y'0 \\
b_2 &= 3(y_1 - y_0) - 2y'_0 - y'_1 \\
b_3 &= 2(y_0 - y_1) + y'_0 + y'_1
\end{aligned}
$$



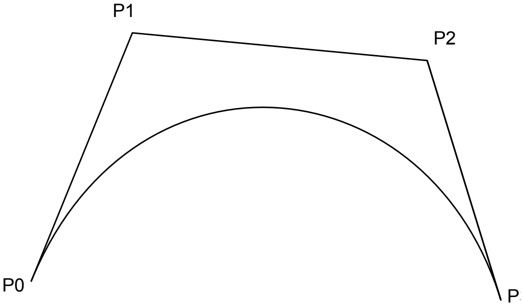
Hermite blending functions

$$y(t) = (1-3u^2+2u^3)y_0+(3u^2-2u^3)y_1+(u-2u^2+u^3)y_0'+(u^3-u^2)y_1'$$



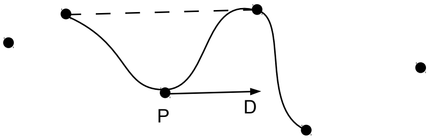
Beziers curves

- 4 points
- tangents along the first and last line segments



Catmull-Rom splines

- Derived from Hermite splines
- Approximate tangents using control points
  $D_i = \frac{1}{2}(P_{i+1} - P_{i-1})$
- Arbitrary first and last points



Kochaneck-Bartels splines

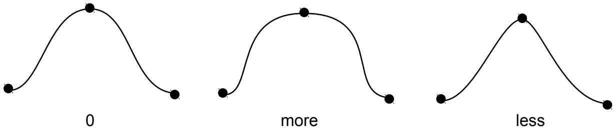
- Add intuitive control parameters to Catmull-Rom splines
  - Tension t
  - Bias b
  - Continuity c

$$
\begin{aligned}
D^{-} &= \frac{(1-t)(1-b)(1+c)}{2}(P_{i+1}-P_i) \\
&\quad + \frac{(1-t)(1+b)(1-c)}{2}(P_i-P_{i-1}) \\
D^{+} &= \frac{(1-t)(1-b)(1-c)}{2}(P_{i+1}-P_i) \\
&\quad + \frac{(1-t)(1+b)(1+c)}{2}(P_i-P_{i-1})
\end{aligned}
$$

Tension

- Tension t is responsible for sharpness

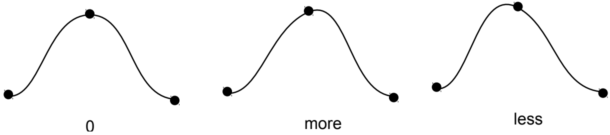
$$D = \frac{1-t}{2}(P_{i+1} - P_{i-1})$$



Bias

- Bias b modifies the slope

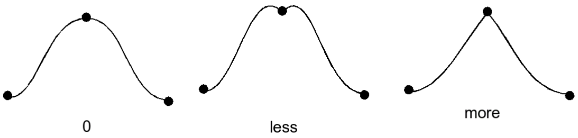
$$D = \frac{1-b}{2}(P_{i+1} - P_i) + \frac{1+b}{2}(P_i - P_{i-1})$$



Discontinuity

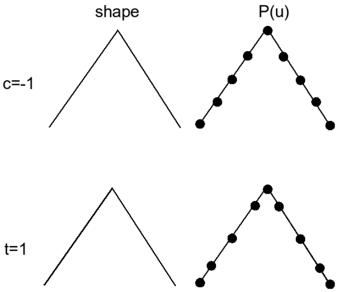
- Discontinuity c splits the tangent in two pieces

$$
\begin{aligned}
D^- &= \frac{1+c}{2}(P_{i+1} - P_i) + \frac{1-c}{2}(P_i - P_{i-1}) \\
D^+ &= \frac{1-c}{2}(P_{i+1} - P_i) + \frac{1+c}{2}(P_i - P_{i-1})
\end{aligned}
$$



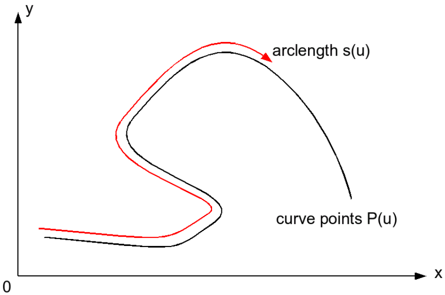
Arclenght parameterization

- It is difficult to control P and $\frac{dP}{du}$ independently
- Example using two Kochaneck-Bartels with same shape



Arclength parameterization

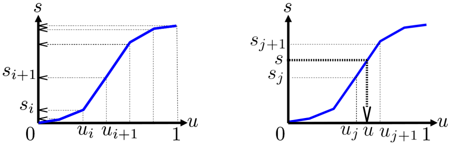
- We want to control the arclength $s(u(t))$



- Problem : $s(u)$ is far from trivial

Approximate arclength parameterization

- Approximate $s(u)$ using distances between regularly sampled points
- Model $s(u)$ as a tabulated function



- For a given s, find the corresponding interval and apply linear approximation
- Complete computation : $P(u(s(t)))$

Velocity

- We want to control the velocity of a moving object along a given path (spline)
- Use arclength parameterization
- Apply velocity control as $s(t)$ with $s = 0$ at starting point and $s = 1$ at end point