Computer Graphics 2 - Exam

M2R-MoSIG
duration: 3 hours

2013

The suggested durations are just hints. All documents allowed, no computer. **Sections 1 and 2 must be treated on separate sheets.** The grading scale is approximative.

1 Geometric modeling (90 minutes), 10 points

Consider the curve subdivision scheme given by the following equations:

$$\begin{cases} x_{2i}^{n+1} & = x_i^n \\ x_{2i+1}^{n+1} & = (\frac{9}{16} + 2\theta)(x_i^n + x_{i+1}^n) - (\frac{1}{16} + 3\theta)(x_{i-1}^n + x_{i+2}^n) + \theta(x_{i-2}^n + x_{i+3}^n) \end{cases},$$

where θ is a positive scalar.

Q1 What can you say about this subdivision scheme from the first equation?

Q2 What is the polynomial $\alpha(z)$ related to this subdivision scheme?

Q3 Prove that there exists a real value θ_{max} such that if $\theta < \theta_{max}$ then the subdivision scheme converges towards a continuous limit curve.

2 Animation (90 minutes, 10 points)

2.1 Inverse Kinematics (3 points)

Figure (1) represents a 3-link arm $OABC$ with rotational joints. Point O is fixed. The goal is to drive the end-effector C to point D fixed in world space.

Q1 Write the corresponding linear equation system

Q2 Propose a straighforward solution to this equation system involving only one joint

Q3 Qualitatively compare the straighforward solution with the solution given by the pseudo-inverse (you do not have to compute the pseudo-inverse and solve the system).

Q4 Does the solution of the linear system drive C exactly to D? Why ?

2.2 Motion capture and interpolation (1 point)

Q1 Why is it difficult to re-use captured motion on various characters ? What are the techniques used to fix the problems ?

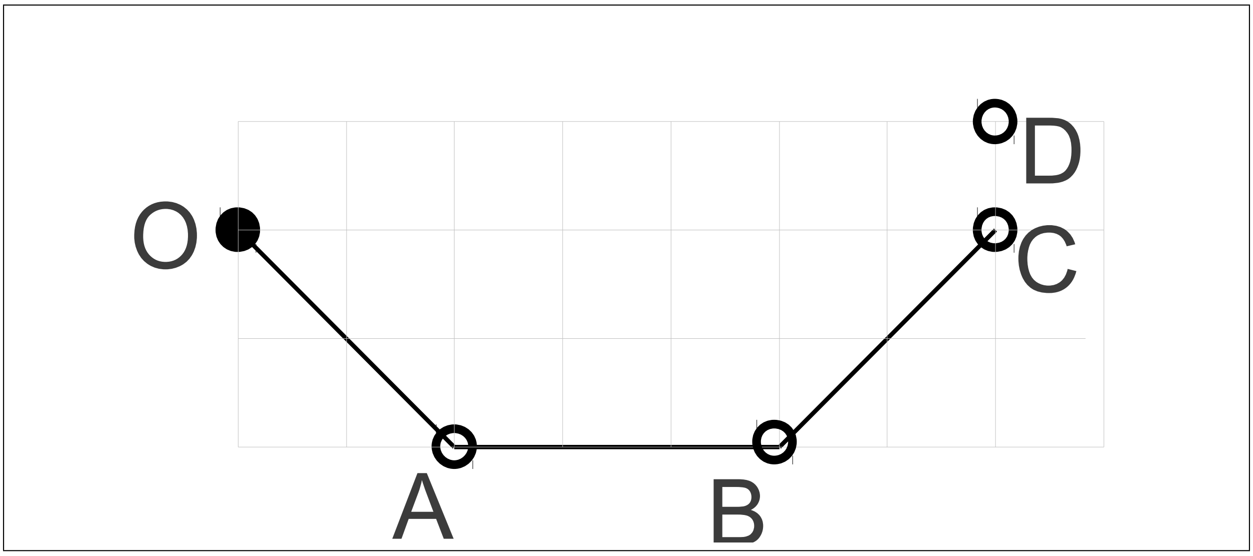

Figure 1: Inverse kinematics of an arm with 3 links: $0A$, AB, BC. The grid size in 1×1.

2.3 Dynamics Integration (2 point)

We consider a particle with unit mass and one-dimensional motion along axis x. It is attached to a spring which applies force $k = -x$ to the particle. No other force is applied. At time $t = 0$, we release the particle from position $x = 1$, with null velocity. We simulate its motion with time step $dt = 1$.

Compute the position and velocity of the particle until $t = 6$ using the following time integration methods:

Q1 explicit Euler

Q2 symplectic Euler

Q3 implicit Euler

Why is implicit Euler often preferred ? Why is it not obvious in these simulations ?

2.4 Finite Elements (1 point)

What are the strengths and weaknesses of the Finite Element method compared to other deformable models ?

2.5 Constrained dynamics (2 points)

Q1 What is the difference between constrained dynamics and penalty-based dynamics ?

Q2 What are Lagrange multipliers ? How are they used in an equation system ?

Q3 What is constraint drift ? How to control it ?

2.6 Collision detection (1 point)

Q1 Summarize the two main approaches to continuous collision detection