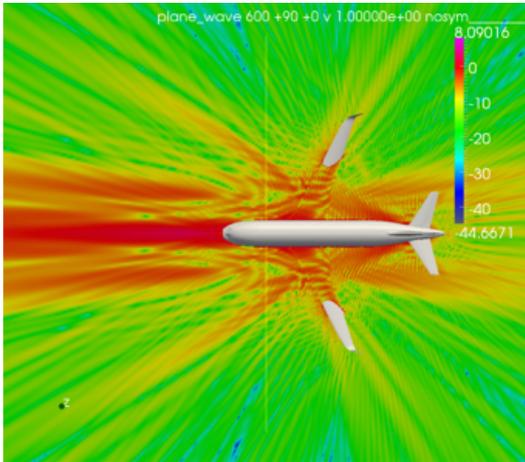


# Fast Solvers for High Frequency Aeroacoustics

Inria / Airbus Central R&T

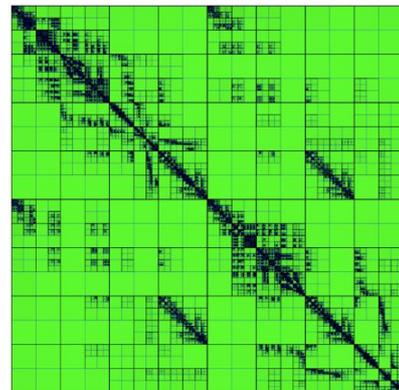


**Industrial Context:** Wave propagation phenomena intervene in many different aspects of systems design at Airbus. They drive the level of acoustic vibrations that mechanical components have to sustain, a level that one may want to diminish for comfort reason (in the case of aircraft passengers, for instance) or for safety reason (to avoid damage in the case of a payload in a rocket fairing at take-off). Numerical simulations of these phenomena plays a central part in the upstream design phase of any such project. Airbus Central R&T has developed over the last decades an in-depth

knowledge in the field of Boundary Element Method (BEM) for the simulation of wave propagation in homogeneous media and in frequency domain. To tackle heterogeneous media (such as the jet engine flows, in the case of acoustic simulation), these BEM approaches are coupled [3] with volumic finite elements (FEM). We end up with the need to solve large (several millions unknowns) linear systems of equations composed of a dense part (coming for the BEM domain) and a sparse part (coming from the FEM domain). Various parallel solution techniques are available today, mixing tools created by the academic world (such as the Mumps [4, 5] and Pastix [6] sparse solvers) as well as parallel software tools developed in-house at Airbus (dense solver SPIDO, multipole solver [1],  $\mathcal{H}$ -matrix solver [2] with an open sequential version available online [9]). In the current state of knowledge and technologies, these methods do not permit to tackle the simulation of aeroacoustics problems at the highest acoustic frequencies (between 5 and 20 kHz, upper limits of human audition) while considering the whole complexity of geometries and phenomena involved (higher acoustic frequency implies smaller mesh sizes that lead to larger unknowns number, a number that grows like  $f^2$  for BEM and  $f^3$  for FEM, where  $f$  is the studied frequency).

**Research Context:** In this research project, we are interested in many aspects that include the design of new advanced techniques to solve these large mixed dense/sparse linear systems, the extensive comparison of these new approaches to the existing ones, and the application of these innovative ideas on realistic industrial test cases.

- The use of  $\mathcal{H}$ -matrix solvers on these problems will be investigated (in the continuation of a PhD currently in progress). Airbus CR&T, in



collaboration with Inria Bordeaux Sud-Ouest, has developed a task-based  $\mathcal{H}$ -matrix solver on top of the runtime engine StarPU [7]. The question of parallel scalability of task-based tools is an active subject of research, using new communication engine such as NewMadeleine [8], that will be investigated during this project, in conjunction with new algorithmic ideas on the task-based writing of  $\mathcal{H}$ -matrix algorithms.

- Ideas coming from the field of sparse direct solvers (such as nested dissection or symbolic factorization) have been tested within H-matrices. This activity will be continued and extended.
- Comparison with existing tools will be performed on large realistic test cases. Coupling schemes between these tools and the hierarchical methods used in  $\mathcal{H}$ -matrix will be developed and benched as well.

Detailed subjects: Different possibilities exist as M2 internship subjects (to be discussed with the candidates) :

- *Design of a coupled Mumps -  $\mathcal{H}$ -Matrix solver for FEM-BEM applications.* In that approach, the FEM matrix is eliminated by computing a Schur complement using MUMPS. Given the size of the BEM matrix, this can not be done in one operation, so it is done block by block, and added in the  $\mathcal{H}$ -matrix, which is then factorized to complete the process.
- *Implementation of a new approach to handle larger blocks in a task-based  $\mathcal{H}$ -matrix solver.* One of the challenge in solving this kind of problems is the ability to scale efficiently when both the matrix size and machine size increase. The current way of dealing with large  $\mathcal{H}$ -matrix blocks is a limitation in this regard, so a new way will be implemented, and all the  $\mathcal{H}$ -matrix task based algorithms will be adapted to this novelty.
- *Solve a coupled FEM-BEM (sparse/dense) linear system using one unique  $\mathcal{H}$ -matrix.* The current approach in Airbus software is to solve a 2x2 linear system, storing 4 separate matrices (FEM-FEM, FEM-BEM, BEM-FEM and BEM-BEM). If we choose to use the same  $\mathcal{H}$ -matrix solver for both sparse and dense matrices, one can consider storing all the system in one unique matrix with an adapted shape and ordering.
- *Evaluate different scheme of partitioning and ordering for solving sparse linear systems with  $\mathcal{H}$ -matrices.* The nested dissection and the  $\mathcal{H}$ -matrix have both introduced ways of partitioning the unknowns adapted to their respective fields. Solving sparse systems with  $\mathcal{H}$ -matrix allows to test both approaches and all the intermediate steps between the two.
- *Exhaustive performance measure and modelisation of existing solvers for FEM/BEM solver.* Several different solvers exist to tackle the linear system considered, not to mention the one we propose to develop in this study. We want to measure, analyze and understand the advantages and drawbacks of each methods regarding memory footprint, accuracy, performance, scalability, etc.

In all the possible subjects, all the steps for a study will be taken care of : theoretical and bibliographic research, prototyping, implementation, tests, validation, results analysis, report. All these points will be dealt with during the PhD, if time permits. Moreover, this list is not static, other ideas may appear or replace existing topics.

Date: the Master 2 internship will start in Spring'19 with a duration of 5 to 6 months, with a possible PhD starting in its continuity in Fall'19 (the grant for the PhD has already been secured, see below).

Keywords: Linear Algebra, H-matrix, task-based programming, Parallelism, HPC.

Prerequisite:

- Knowledge in the fields of linear algebra and finite element methods;
- Programming skills in C/C++ and script language;
- Knowledge in distributed and shared memory parallelism (MPI, threads);
- Ability to design and conduct a numerical experiment plan.

Localisation: The internship and PhD will take place at Inria Bordeaux Sud-Ouest, in Talence (<https://www.inria.fr/en/centre/bordeaux>).

Funding: The internship will be funded by Inria / Airbus CRT. The PhD takes place in the context of the Regional Project "HPC Scalable Ecosystem". It will be co-funded by the region Nouvelle Aquitaine and Airbus Central R&T.

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[1] Guillaume Sylvand. *La méthode multipôle rapide en électromagnétisme. Performances, parallélisation, applications*. PhD thesis, Ecole des Ponts ParisTech, 2002.

[2] Benoît Lizé. *Résolution Directe Rapide pour les Éléments Finis de Frontière en Électromagnétisme et Acoustique : H-Matrices. Parallélisme et Applications Industrielles*. PhD Thesis. Ecole Doctorale Galilée, 2014.

[3] Fabien Casenave. *Méthodes de réduction de modèles appliquées à des problèmes d'aéroacoustique résolus par équations intégrales*. PhD thesis, Université Paris-Est, 2013

[4] P. R. Amestoy, I. S. Duff, J. Koster and J.-Y. L'Excellent, *A fully asynchronous multifrontal solver using distributed dynamic scheduling*, SIAM Journal of Matrix Analysis and Applications, Vol 23, No 1, pp 15-41 (2001).

[5] P. R. Amestoy, A. Guermouche, J.-Y. L'Excellent and S. Pralet, *Hybrid scheduling for the parallel solution of linear systems*. Parallel Computing Vol 32 (2), pp 136-156 (2006).

[6] P. Hénon, P. Ramet, and J. Roman. *PaStiX: A High-Performance Parallel Direct Solver for Sparse Symmetric Definite Systems*. Parallel Computing, 28(2):301-321, January 2002.

[7] Cédric Augonnet, Samuel Thibault, Raymond Namyst, and Pierre-André Wacrenier. *StarPU: A Unified Platform for Task Scheduling on Heterogeneous Multicore Architectures*. Concurrency and Computation: Practice and Experience, Special Issue: Euro-Par 2009, 23:187-198, February 2011.

[8] Olivier Aumage, Elisabeth Brunet, Nathalie Furmento, Raymond Namyst. *NewMadeleine: a Fast Communication Scheduling Engine for High Performance Networks*. Workshop on Communication Architecture for Clusters (CAC 2007), workshop held in conjunction with IPDPS 2007, Mar 2007, Long Beach, California, United States. 2007

[9] hmat-oss, an open-source sequential h-matrix solver. <https://github.com/jeromerobert/hmat-oss/>