Logic-Based Argumentation with N-ary Graphs
Présentation Journée GraphIK 2018

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What is argumentation? (Part 1)

- Argumentation is a way of reasoning that is based on arguments and attacks between them.
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It is usually modelled using the Dung’s framework and represented as a directed graph.

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Example

- Other generalisations exist and implement supports relation, weights on attacks, preferences on arguments, different kind of attacks and arguments, etc.
We use *argumentation semantics* (preferred, stable semantics) to extract meaningful consistent subsets of the set of arguments.

Example

The set \( \{a, c\} \) is a preferred extension (maximal conflict-free and defend itself).
We consider inconsistent $Datalog^{\pm}$ knowledge bases.

Example (Knowledge Base)

- $\mathcal{F} =$
  \[
  \{ \text{contains}(m, \text{saltC}), \text{contains}(m, \text{sugar}), \text{contains}(m, \text{yogurt}),
  \text{notSoup}(m), \text{edible}(m) \} 
  \]

- $\mathcal{R} = \{ \forall x (\text{contains}(x, \text{saltC}) \land \text{contains}(x, \text{yogurt}) \rightarrow 
  \text{tzaziki}(x)) \}$

- $\mathcal{N} = \{ \forall x (\text{contains}(x, \text{saltC}) \land \text{contains}(x, \text{sugar}) \land 
  \text{contains}(x, \text{yogurt}) \rightarrow \bot), \forall x (\text{tzaziki}(x) \land \text{notSoup}(x) \rightarrow \bot) \}$
Why n-ary argumentation?

Let $\mathcal{K}$ be a KB, $\mathcal{AS}_\mathcal{K} = (\mathcal{A}, \mathcal{C})$ where $\mathcal{A}$ is a set of arguments and $\mathcal{C}$ a set of attacks defined as follows.

**Definition (Old arguments)**

An argument is a tuple $(H, C)$ with $H$ a non-empty $\mathcal{R}$-consistent subset of $\mathcal{F}$ and $C$ a set of facts such that:

- $H \subseteq \mathcal{F}$ and $H$ is $\mathcal{R}$-consistent (consistency);
- $C \subseteq \mathcal{C}_\mathcal{R}(H)$ (entailment);
- $\nexists H' \subset H$ s.t. $C \subseteq \mathcal{C}_\mathcal{R}(H')$ (minimality).

**Example**

An argument is:

$$a_1 = (\{\text{contains}(m, \text{saltC}), \text{contains}(m, \text{yogurt})\}, \{\text{tzaziki}(m)\})$$
Let $\mathcal{K}$ be a KB, $\mathcal{AS}_\mathcal{K} = (\mathcal{A}, \mathcal{C})$ where $\mathcal{A}$ is a set of arguments and $\mathcal{C}$ a set of attacks defined as follows.

**Definition (Old attacks)**

We say that $a = (H, C)$ attacks $b = (H', C')$ denoted by $(a, b) \in \mathcal{C}$ iff there exists $\phi \in H'$ such that $C \cup \{\phi\}$ is $\mathcal{R}$-inconsistent.

**Example**

$a_1 = (\{contains(m, saltC), contains(m, yogurt)\}, \{tzaziki(m)\})$ attacks $a_2 = (\{notSoup(m)\}, \{notSoup(m)\})$. 

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**Problem:** we have too many arguments (and attacks). Here, we have 33 arguments and 360 attacks for a knowledge base with 5 facts, 1 rule and 2 negative constraints. We need a way for arguments to jointly attack other arguments.
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- Problem: we have too many arguments (and attacks). Here, we have 33 arguments and 360 attacks for a knowledge base with 5 facts, 1 rule and 2 negative constraints.

- We need a way for arguments to jointly attack other arguments.

Example

a: “Martin is on the tandem bicycle”
b: “Madalina is on the tandem bicycle”
c: “Pierre is on the tandem bicycle”

We need attacks of the form (\{a, b\}, c)
Let us consider the KB $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$. $\mathcal{AS} = (\mathcal{A}, \mathcal{C})$ with $\mathcal{C} \subseteq 2^\mathcal{A} \times \mathcal{A}$ is such that:

**Definition (Argument)**

An argument $a \in \mathcal{A}$ is:

- $f$, where $f \in \mathcal{F}$. $\text{Conc}(a) = f$ and $\text{Prem}(a) = \{f\}$
- $a_1, \ldots, a_n \rightarrow f'$ if $a_1, \ldots, a_n$ are arguments such that there exists a tuple $(r, \pi)$ where $r \in \mathcal{R}$, $\pi$ is a homomorphism from the body of $r$ to $\{\text{Conc}(a_1), \ldots, \text{Conc}(a_n)\}$ and $f'$ is the resulting atom from the rule application. $\text{Conc}(a) = f'$ and $\text{Prem}(a) = \text{Prem}(a_1) \cup \cdots \cup \text{Prem}(a_n)$

where $\text{Prem}(a)$ is $\mathcal{R}$-consistent.
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**Definition (Attack)**

An attack is a pair $(X, a)$ where $X \subseteq \mathcal{A}$ and $a \in \mathcal{A}$ such that $X$ is minimal for set inclusion such that $\bigcup_{x \in X} \text{Prem}(x)$ is $\mathcal{R}$-consistent and there exists $\varphi \in \text{Prem}(a)$ such that $(\bigcup_{x \in X} \text{Conc}(x)) \cup \{\varphi\}$ is $\mathcal{R}$-inconsistent.
Example

Let $\mathcal{K}$ be the previous knowledge base about the choice of an appetiser, the resulting argumentation graph is composed of six arguments and 11 attacks:

- $a_1 = \text{contains}(m, \text{sugar})$
- $a_2 = \text{contains}(m, \text{saltC})$
- $a_3 = \text{contains}(m, \text{yogurt})$
- $a_4 = \text{notSoup}(m)$
- $a_5 = \text{edible}(m)$
- $a_6 = a_2, a_3 \rightarrow \text{tzaziki}(m)$

An example attack of $\mathcal{C}$ is $\{a_1, a_2\}, a_3$. 
We have the one-to-one correspondence between preferred/stable extensions and maximal consistent subset of facts.

Example

\{ \text{contains}(m, \text{saltC}), \text{contains}(m, \text{yogurt}), \text{edible}(m) \} \uparrow \downarrow \{a_2, a_3, a_5, a_6\}
We have an upper-bound on the number of attacks with respect to the number of arguments and we have an upper-bound on the number of arguments if there are no rules.

**Example (Attack upper-bound)**

Let $\mathcal{K}$ be a knowledge base and $\mathcal{AS}_\mathcal{K} = (\mathcal{A}, \mathcal{C})$ be the corresponding argumentation framework. If $|\mathcal{A}| = n$ then

$$|\mathcal{C}| \leq \sum_{i=1}^{n-1} \binom{n}{i} (n - i).$$
Properties

We satisfy the basic rationality postulates (closure, indirect and direct consistency)

Example (Indirect consistency)

Let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ be a knowledge base, $\mathcal{AS}_\mathcal{K}$ the corresponding argumentation framework and $x \in \{s, p, g\}$. Then:

- for every $E \in Ext_x(\mathcal{AS}_\mathcal{K})$, $Concs(E)$ is a $\mathcal{R}$-consistent.
- $Output_x(\mathcal{AS}_\mathcal{K})$ is $\mathcal{R}$-consistent.
Properties

- Presence of structural properties (cycle, etc.)

**Example (Self-attacking Arguments)**

Let $\mathcal{K}$ be a knowledge base and $\mathcal{AS}_\mathcal{K} = (\mathcal{A}, \mathcal{C})$ be the corresponding argumentation framework. There is no $(S, t) \in \mathcal{C}$ such that $t \in S$.

**Example (Defense)**

Let $\mathcal{K}$ be a knowledge base and $\mathcal{AS}_\mathcal{K} = (\mathcal{A}, \mathcal{C})$ be the corresponding argumentation framework. If there is $(S, t) \in \mathcal{C}$ then there exists $(S', s) \in \mathcal{C}$ such that $s \in S$. 
We generated this n-ary argumentation graph on a set of 134 existing knowledge bases.
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- A set of $A$ composed of 108 knowledge bases. This dataset is further split into three smaller set of knowledge bases:
  - A set of $A_1$ of 31 knowledge bases without rules, two to seven facts, and one to three negative constraints.
  - A set $A_2$ of 51 knowledge bases generated by fixing the size of the set of facts and successively adding negative constraints until saturation.
  - A set $A_3$ of 26 knowledge bases with only ternary negative constraints, three to four facts and one to three rules.
- A set $B$ of 26 knowledge bases with eight facts, six rules and one or two negative constraints. This set contains more free-facts than the knowledge bases in set $A$. 
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We compared the number of argument and attacks with the existing argumentation framework.
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<table>
<thead>
<tr>
<th>$\mathcal{K}$</th>
<th># Arg.</th>
<th># Att.</th>
<th>Gen. Time (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>22</td>
<td>128</td>
<td>160</td>
</tr>
<tr>
<td>$A_2$</td>
<td>25</td>
<td>283</td>
<td>133</td>
</tr>
<tr>
<td>$A_3$</td>
<td>85</td>
<td>1472</td>
<td>399.5</td>
</tr>
<tr>
<td>$B$</td>
<td>5967</td>
<td>11542272</td>
<td>533089</td>
</tr>
</tbody>
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<th># Att.</th>
<th>% Att.</th>
<th>G. Time</th>
<th>% Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>5</td>
<td>77.27</td>
<td>6</td>
<td>93.75</td>
<td>276.00</td>
<td>-81.48</td>
</tr>
<tr>
<td>$A_2$</td>
<td>7</td>
<td>72.00</td>
<td>8</td>
<td>92.93</td>
<td>342.00</td>
<td>-183.57</td>
</tr>
<tr>
<td>$A_3$</td>
<td>7</td>
<td>91.76</td>
<td>9</td>
<td>99.26</td>
<td>369.50</td>
<td>1.66</td>
</tr>
<tr>
<td>$B$</td>
<td>14</td>
<td>99.77</td>
<td>20.5</td>
<td>99.99</td>
<td>7814.5</td>
<td>98.08</td>
</tr>
</tbody>
</table>
We developed the Naked tool for visualising and generating n-ary graphs from Datalog\(^\pm\) knowledge bases.